Probability models that relate nondestructive test methods to lumber design values of plantation loblolly pine

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Within-grade variability in mechanical properties for visually graded lumber has led to increased deployment of nondestructive testing (NDT) methods, even though the relationships between static bending and NDT-predicted values are often highly variable. Dynamic modulus of elasticity (MOEdyn) was measured using two acoustic velocity instruments and one transverse vibration instrument, along with specific gravity, for 819 pieces of visually graded loblolly pine lumber. Static modulus of elasticity (MOE) and bending strength (Fb) were measured via destructive testing. The probability of meeting design values was compared using (1) normal distribution linear and power regression models and (2) binomial distribution logistic regression models; the parameters of both models were fit using maximum likelihood estimation. For the normal distribution models, the standard error of the estimate, which ranged from 1.28 to 1.82 GPa for MOE and 4.47 to 5.07 MPa for Fb, was incorporated into predictions in order to calculate the probability of meeting design values. At 50 per cent probability, transverse vibration MOEdyn values of 10.9 (normal) and 11.0 (binomial) GPa would meet the No. 2 MOE design value (9.7 GPa). At probabilities of 75 per cent and 95 per cent, the required values were 12.1 and 13.8 (normal) GPa and 12.0 and 13.5 (binomial) GPa, respectively. The normal and binomial approaches required similar NDT values to meet thresholds, although the advantage of the normal approach is that the regression parameters do not need to be recalculated for each threshold value, but at the expense of increased model complexity.

Introduction

Evaluation of wood properties often relies on nondestructive testing (NDT) methods that can determine the suitability of the material for a particular end use without diminishing either its properties or its product performance (Ross, 2015a). For structural lumber produced in North America, visual qualitative inspection of knots and other characteristics (e.g. wane and shakes) have long been used to determine lumber grades (ASTM D245, 2011); however, as variability in modulus of elasticity (MOE; i.e. stiffness) and modulus of rupture (MOR; i.e. strength) is high within these qualitatively determined lumber grades (Dahlen et al., 2013), quantitative NDT methods are increasingly being employed (Briggs, 2010; Baillères et al., 2012). Recent technological improvements have allowed quantitative systems to become more reliable at sorting lumber into similar categories than is possible with qualitative systems (Wang et al., 2008; Pellerin and Ross, 2015). In both types of systems, lumber is sorted into a grade category containing theoretically similar pieces that carry specific design values as determined using mechanical destructive testing (ASTM D4761, 2013; ASTM D198, 2015). For any grading method using NDT, there is a need to understand the relationships between NDT-predicted and statically tested mechanical properties. The design values for MOE are at the mean level of the population, while those for bending strength (Fb) are at the fifth percentile level, with the Fb values being the MOR values reduced for safety, uncertainty and for performance over time (ASTM D1990, 2016).

The common NDT systems for lumber grading include those that evaluate either density or MOE (Carter et al., 2006; ALSC, 2014; Galligan et al., 2015; Ross, 2015b). Density, often used interchangeably with the term specific gravity (SG), is an important physical property for wood that is measured in production environments using X-rays (Galligan et al., 2015). Dynamic MOE (MOEdyn) can be determined using different methods; when determined by transverse vibration (MOEdyn,TV), it is calculated as follows:

\[
\text{MOEdyn,TV} = \frac{f^2 \text{WL}^3}{k\text{MOIg}} = \frac{f^2 \text{ML}^3}{k\text{MOI}}
\]

where the unit for MOEdyn,TV is Pa, \( f \) is the resonant frequency (Hz), \( W \) is the beam weight (N), \( L \) is the span length (m), \( k \) is a constant (2.46) for a beam simply supported at its ends for free

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vibration, MOI is the moment of inertia (m²) and g is the acceleration due to gravity (9.80665 m s⁻²) (Ross, 2015b). The equation is simplified by dividing W (beam weight) by g (acceleration due to gravity) to get mass (M) (kg) in the numerator. The MOE dyn.TV units in Pa are converted to GPa to allow reporting without scientific notation and to match reported values of GPa in the literature. MOE dyn determined by acoustic velocity (MOE dyn.AV) is calculated as follows:

\[
\text{MOE}_{\text{dyn.AV}} = \rho AV^2
\]

(2)

where the unit for MOE dyn.AV is Pa, \(\rho\) is the density (kg m⁻³) and AV is the acoustic velocity (m s⁻¹) (Wang, 2013); again, the MOE dyn.AV units are converted to GPa. The acoustic velocity is determined from the frequency of numerous acoustic pulses as follows:

\[
AV = 2f_0L
\]

(3)

where AV is weighted mean acoustic velocity (m s⁻¹), \(f_0\) is the first harmonic frequency of an acoustic wave signal (Hz) and L is the length (m) of the material (Wang, 2013). Applying both techniques, Yang et al. (2015) found good relationships between static MOE and both MOE dyn.TV (\(R^2 = 0.86\), root mean square error (RMSE) = 0.98 GPa) and MOE dyn.AV (\(R^2 = 0.82\), RMSE = 1.12 GPa) for southern pine No. 2 grade lumber. In addition to evaluating lumber, NDT methods are frequently employed in the field in an attempt to segregate low-quality material before processing (Wessels et al., 2011b; Moore et al., 2013; Murphy and Cown, 2015; Bérubé-Deschenes et al., 2016; Tippner et al., 2016; Butler et al., 2017). Paradis et al. (2013) utilized the time-of-flight acoustic technique on standing trees to identify those trees likely to yield lumber with high stiffness. Wang et al. (2013) combined information on log diameter and position along the stem with acoustic velocity to better predict lumber MOE and visual grade yield in Douglas-fir (Pseudotsuga menziesii (Mirb.) Franco). An acoustic method has also been adopted to assess seedlings to screen families with high wood stiffness (Emms et al., 2012, 2013).

Although NDT techniques have been widely used to evaluate lumber (Grabianowski et al., 2006; Ross, 2015a), the relationships between NDT-predicted values and the corresponding static bending values can be highly variable (Yang et al., 2015). As a result, there is uncertainty associated with the prediction of mechanical properties. Thus, there is a need to establish the degree of confidence in the NDT-predicted values, and this can be done through the determination of confidence intervals that are likely to include the true value (e.g. a 95 per cent confidence interval). When applying an unbiased model, there is a 50 per cent probability that the expected static value will be lower than the predicted mean value and the same probability that it will be higher. The specified confidence level can be adjusted accordingly for applications that tolerate either higher or lower uncertainty. For example, a floor assembly composed of multiple members may tolerate a higher level of uncertainty, whereas the outermost tension layer in an unbalanced glulam beam, which is subjected to the highest stresses, may require a lower level of uncertainty (Moody, 1977).

The Southern Pine Inspection Bureau (SPIB) implemented a monitoring program whereby lumber MOE dyn.TV was used to determine potential changes in the visually graded southern pine lumber resource (Kretschmann et al., 1999). The observed decline in southern pine lumber quality was rationalized to be a consequence of the increasing percentage of juvenile wood in the timber resource (Kretschmann et al., 1999; Larson et al., 2001; Clark et al., 2008). For southern pine visually graded lumber, the impact of the decline in mechanical properties was realized in 2013 after the design values for southern pine were decreased (ALSC, 2013). However, this decline was not detected via the monitoring program (SPIB, 2011), likely due to differences between MOE dyn and static MOE values. Calculated dynamic properties are higher than measured static properties due to the effect of creep in the static test that is not present in dynamic testing (Divós and Tanaka, 2005). Thus, when comparing NDT and static values, these differences should be accounted for.

The primary goals of this study were therefore (1) to quantify the relationship between both the static MOE and the characteristic bending strength (\(f_b\)) and NDT methods (SG, MOE dyn.AV and MOE dyn.TV) in loblolly pine (Pinus taeda L.) dimension lumber (38 mm thickness) and (2) to base these relationships, to find, for a given probability, the NDT values needed to meet specific design value thresholds at 50 per cent, 75 per cent and 95 per cent confidence. The probability of meeting a specific threshold was determined using a normal distribution approach employing linear and power regression models, and a binomial distribution approach using logistic regression models. The parameters for both distributions were fit using maximum likelihood estimation (MLE). The information generated in this study could be incorporated into prediction equations used by industry to determine, with increased confidence, whether lumber properties predicted using NDT methods meet design value requirements.

**Materials and methods**

**Origin of lumber samples**

The lumber data used in this study originate from a sawmill study described by Butler et al. (2016). A total of 93 loblolly pine trees from five stands were harvested in 2013 from the Lower Coastal Plain near Brunswick, Georgia, USA. The productivity of the sites was high; site index at base age 25 years (\(S_{25}\)) ranged from 25.3 to 27.4 m. Sample trees were selected from across the diameter distribution range of each stand. A chainsaw was used to fell, de-limb and then buck each sample tree into lumber length (5.2 m) logs numbered 1 (butt), 2 (middle) and 3 (top). The logs were transported to the participating mill (Hoboken, GA, USA) and processed into dimension lumber 38 mm (thickness) by 89 mm (2 × 4), 140 mm (2 × 6), 184 mm (2 × 8) and 235 mm (2 × 10) (width). The lumber was dried, planed and visually graded into No. 1 and better (No. 1), No. 2 and No. 3 grade categories by certified graders from the cooperating mill. Altogether, the 244 logs yielded a total of 819 pieces of lumber after grading (Table 1); for the 2 × 4, 2 × 6, 2 × 8 and 2 × 10 sizes, the respective counts of lumber pieces were 115, 295, 343 and 66. Separating the lumber by grade gave 157, 593 and 69 pieces for the No. 1, No. 2 and No. 3 visual grades, respectively.

**NDT and destructive testing procedures**

The lumber was transported to the University of Georgia's wood quality laboratory in Athens, GA, USA, for testing. The SG of the lumber was calculated using oven dry weight and the volume determined from the dimensions at ambient conditions. The average moisture content of the lumber was 11.2 per cent with a range from 8.5 to 17.2 per cent. The SG
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Table 1 Total number of pieces by grade for 38-mm thick by 89-mm (2 × 4), 140-mm (2 × 6), 184-mm (2 × 8) and 235-mm (2 × 10) width lumber used in this study.

<table>
<thead>
<tr>
<th>Grade</th>
<th>Size</th>
<th>No. 1</th>
<th>No. 2</th>
<th>No. 3</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>89 mm</td>
<td></td>
<td>25</td>
<td>74</td>
<td>16</td>
<td>115</td>
</tr>
<tr>
<td>140 mm</td>
<td></td>
<td>36</td>
<td>230</td>
<td>29</td>
<td>295</td>
</tr>
<tr>
<td>184 mm</td>
<td></td>
<td>71</td>
<td>251</td>
<td>21</td>
<td>343</td>
</tr>
<tr>
<td>235 mm</td>
<td></td>
<td>25</td>
<td>38</td>
<td>3</td>
<td>66</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>157</td>
<td>593</td>
<td>69</td>
<td>819</td>
</tr>
</tbody>
</table>

Values were then corrected to 15 per cent moisture content (oven dry weight, volume at 15 per cent moisture content) in accordance with a corresponding adjustment for mechanical properties (ASTM D1990, 2016). The longitudinal acoustic velocity was measured using two different instruments – the fibre-gen Hitman HM200 (AV[1010] Fibre-gen, Christchurch, New Zealand) and the Fakopp Portable Lumber Greader (PLG) (AV[1000]) (Fakopp BT, Agfalfa, Hungary). Both systems calculate the acoustic velocity (m s⁻¹) using the first harmonic frequency of a resonant wave (Wang, 2013) and thus were expected to yield similar results. MOE dyn was calculated for each instrument (MOE dyn.HM and MOE dyn.PLG) using the density and the acoustic velocity determined using equation (2). The MOE dyn.TV was measured flatwise (i.e. with the beam depth equal to the thickness dimensions of the lumber) using a Metriguard 340 E-Computer (Metriguard, Pullman, WA, USA) (ASTM D6874, 2012). The E-Computer measures the density of the material and the resonant frequency for each sample, then calculates MOE dyn.TV using equation (1). Because both sample size and length influence the resonant frequency, the resonant frequency was standardized by size and length to allow comparisions between sizes. To do this, the measured frequency for each sample was divided by the calculated mean frequency from each size and length (2 × 4: 0.011 Hz, 2 × 6: 0.023 Hz, 2 × 8: 0.038 Hz, 2 × 10: 0.059 Hz); these frequencies corresponded to the No. 2 design value (9.7 GPa) as the MOE dyn.TV value, the length for each test (2 × 4: 1.81 m, 2 × 6: 2.69 m, 2 × 8: 3.45 m, 2 × 10: 4.30 m) and the mean weight for each size (2 × 4: 1.81 N, 2 × 6: 4.15 N, 2 × 8: 7.02 N, 2 × 10: 11.25 N).

Each piece of lumber was then tested in static edgewise destructive bending (ASTM D4761 2013; ASTM D1998, 2015) using a four-point bending setup in ‘third-point’ loading (load heads positioned one-third of the span distance from the reactions) on a universal testing machine. The span-to-depth ratio was 17:1 (2 × 4: 1511–89 mm, 2 × 6: 2375–140 mm, 2 × 8: 3131–184 mm, 2 × 10: 399–235 mm). Following testing, the mechanical properties were adjusted to 15 per cent moisture content (ASTM D1990, 2016) and the MOR values were also adjusted to account for design uncertainty and differences in strength gain loading time to give the characteristic bending strength at the 2 × 8 size (Fb) (Evans et al., 2001; ASTM D1990, 2016; Butler et al., 2016). Results were compared with the design values for southern pine, with the design values for MOE being the mean values for visually graded southern pine lumber, i.e. 11.0 GPa (No. 1), 9.7 GPa (No. 2) and 9.0 (No. 3), respectively (ALSC, 2013). The design values for Fb are the nonparametric fifth percentile values determined at 75 per cent confidence and are 8.6 MPa (No. 1), 6.4 MPa (No. 2) and 3.6 MPa (No. 3), respectively (ALSC, 2013; ASTM D1990, 2016).

Statistical analyses
All statistical analyses were performed and associated graphics produced in the R statistical programming environment (R Core Team, 2017) with the RStudio interface (RStudio, 2017) and several R packages (Sarkar, 2008; Chang, 2014; Auguie, 2016; Wickham and Francois, 2016). Boxplots of lumber variability by tree nested within stand were constructed for SG, MOE and Fb. Pearson correlation coefficients were calculated between different NDT property values and between the NDT property values and the static values.

Two approaches were used to develop models for predicting static values for MOE and Fb. The first employed linear and power models fitted using the normal distribution and the second approach employed logistic regression models fitted using the binomial distribution. For both model types, MLE was used to find the parameters for each model. The MLE methodology finds the most likely parameters, where possible parameters are compared with other sets of parameters within a recurrent optimization framework for a particular density function, for example, the normal distribution (Millar, 2011). For the linear and power model approach, initial models were developed to predict static MOE or Fb using SG and MOE dyn values (i.e. MOE dyn.HM, MOE dyn.PLG and MOE dyn.TV). The base regression models were linear in form:

$$y = N(p_0 + p_1x, \sigma^2)$$

where $y$ follows a normal distribution (N), $p_0$ is the model intercept, $p_1$ is the slope coefficient, $x$ is a vector of the independent variables (NDT method), $\sigma^2$ is the variance or the mean squared error, and thus $\sigma$ is the standard error of the estimate. Assumptions of normality were checked using a Kalmagorov-Smirnov test and by visual assessment of quantile-quantile normal graphs. Where deviations from normality were detected, a revised model was fit with a power transformation to improve normality:

$$y = N((p_0 + p_1x)^\lambda, \sigma^2)$$

where $\lambda$ is a scaling coefficient. The starting values for the model parameters were estimated using the linear model function in R. The model parameters and $\sigma$ were then explicitly estimated through MLE. The probability of any measurement ($x$) given an expected value ($\mu$) and $\sigma$ was calculated from the normal distribution density function:

$$f(x|\mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-(x-\mu)^2}{2\sigma^2}\right)$$

where $x$ is a specific value of the independent variable, $\mu$ is the expected value and $\sigma$ the standard error of the estimate; the expected value ($\mu$) is twice the standard error of the estimate contained approximately 95 per cent of the target population. Equation (6) identifies the likelihood of one observation given the expected value and the standard error of the estimate. In order to find the likelihood for the entire dataset, the product of the individual likelihoods was maximized, which is the summation of the natural logarithms of every likelihood value (log-likelihood) (Millar, 2011). An updated set of regression parameters ($p_0$, $p_1$ and $\lambda$, if used) and the standard error of the estimate ($\sigma$) of the model were then found using an interactive optimization process using the objective function:

$$\max I(f(.), \sigma|x, y) = \sum_{i=1}^{n} \left[\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-(y_i-\mu_i)^2}{2\sigma^2}\right)\right]$$

where $I$ is the maximum log-likelihood, $f(.)$ is the linear function ($y = p_0 + p_1x$ or $y = (p_0 + p_1x)^\lambda$), $\sigma$ is the standard error of the estimate, $x_i$ is the value for each independent variable, $y_i$ is the value of the dependent variable and $n$ is the natural logarithm. The parameter estimates were obtained using the quasi-Newton method (Broyden-Fletcher-Goldfarb-Shanno (BFGS)) implemented using the general purpose optimization function in R. The confidence intervals for the parameters were calculated from the model's Hessian matrix (second-order partial derivatives of the equation with respect to the parameter), where the square root from the product of the inverse Hessian and the identity matrix were found using the delta method (Ver Hoef, 2012). The models
were compared across NDT methods using the maximized sum of the log-likelihoods (where a higher number indicates a better model) and Akaike’s Information Criteria (AIC) (Akaike, 1974) (where a lower number is better).

After selecting the optimized parameters for the regression model for each NDT variable, the probability of achieving a certain threshold at a given confidence level was calculated by integrating the likelihood over the desired level of certainty. The probability (P) of failing a certain design value threshold (y_{threshold}) is a function described by the normal cumulative function:

\[ P(x ≥ y_{threshold}) = \Phi(x; \mu, \sigma^2) \]

where P is the probability and \( \Phi \) is the normal cumulative function, which is calculated by integrating the normal distribution function from \( x \) to infinity (\( \infty \)):

\[ \Phi(x; \mu, \sigma^2) = \frac{1}{\sigma \sqrt{2\pi}} \int_{x}^{\infty} \exp\left(\frac{-1}{2\sigma^2}(x - \mu)^2\right)dx \]

Combining equation (7) and equation (9) results in:

\[ \Phi(x; \mu, \sigma^2) = \int_{x}^{\infty} \frac{1}{\sqrt{2\pi} \sigma} \exp\left(\frac{-1}{2\sigma^2}(x - \mu)^2\right)dx \]

where the parameters are defined as previously. Plots showing the relationship between the dependent variables (MOE and F_s) and the independent variables (the NDT-derived values) were produced. Regression mean confidence intervals for each model were calculated using the function’s gradient estimated from a Taylor series expansion making use of the previously estimated Hessian matrix.

For the binomial distribution approach, using logistic regression analysis, the probability (P) of meeting (pass = 1) or not meeting (fail = 0) the design values for a given value of a dependent variable was calculated using the inverse-logit function:

\[ P = \frac{\exp(y_0)}{\exp(y_0) + 1} \]

where P is the probability and y_0 is the predicted value calculated from:

\[ y_0 = \beta_0 + \beta_1x \]

where \( \beta_0 \) and \( \beta_1 \) are the regression parameters specific to the binomial regression model (and thus not the same as the regression parameters estimated using the linear and power models). The probability and regression parameters are specific to each design value and thus the model is fit for each value. The starting \( \beta_0 \) and \( \beta_1 \) parameters were calculated using a generalized linear model with the binomial family and a logit link, and then an updated set of parameters was explicitly estimated through MLE. For the whole dataset, the likelihood that the \( \beta_0 \) and \( \beta_1 \) parameters make the observed data most likely to occur is the product of the likelihood for each individual observation:

\[ L(\beta_0, \beta_1) = \prod_{i=1}^{N} \left( \frac{N}{k_i} \right)^{N-k_i} P_i(1 - P_i)^{N-k_i} \]

where L is the likelihood, \( \prod \) is the product, N represents the sample size, \( k_i \) is the response of the ith observation in meeting (1) or not meeting (0) the design value, and \( P \) is the probability calculated using equations (11) and (12) (Bolker, 2008). The log-likelihood expression for equation (13) is then maximized:

\[ \max \{ L(\beta_0, \beta_1) \} = \sum_{i=1}^{N} \left[ \log \left( \frac{N}{k_i} \right) + k_i \log P + (N-k_i) \log (1-P) \right] \]

where max I is the objective function to be maximized by taking the log-likelihood for all observations and the rest of the parameters the same as equation (13). The parameter estimates were obtained using the quasi-Newton method (BFGS) implemented using the general purpose optimization function.

The NDT values needed to meet the mean design values for MOE (No. 1 = 11.0 GPa, No. 2 = 9.7 GPa, No. 3 = 9.0 GPa) and 82 per cent of the of the mean design values for MOE (No. 1 = 9.0 GPa, No. 2 = 8.0 GPa, No. 3 = 7.4 GPa), at the 50 per cent, 75 per cent and 95 per cent confidence levels, were calculated for the normal distribution and binomial distribution models. The mean design values were selected as one threshold value because the design values for MOE are specified at the mean level. Values corresponding to 82 per cent of each mean threshold value were selected because machine-stress-rated (MSR) lumber grading requires that 95 per cent of the pieces must be greater than 82 per cent of the mean design value (ALSC, 2013). The stress wave tools (Hitman and PLG) used in this study are similar to commercial acoustic systems that measure MOE_{dyn} to allocate grades to MSR lumber (ALSC, 2014). The design values for the No. 2 grade material were used as the design values for the overall data because this grade made up the majority of the pieces in the study (72 per cent) and also it represents the grade of the majority of southern pine dimension lumber produced (SPFA, 2009), while the design values for each grade were used for the grade-specific data. For F_s, only eight pieces (1 per cent) failed below the design values, with two pieces from the No. 1 grade (1 per cent), six from the No. 2 grade (1 per cent) and none from the No. 3 grade. The reason for this low number is that for F_s, the design values are at the fifth percentile level. Because so few pieces failed below the design values, models for predicting the necessary NDT value to meet the design values are less useful than the MOE results and are not presented here.

Results

Boxplots of the measured lumber properties showed large variability in lumber SG, MOE and F_s both within and between trees (Figure 1). In the boxplots, trees within each stand were displayed in ascending order of the median SG value for the lumber obtained from each tree. The median MOE values closely followed the ascending order arrangement for SG, but the F_s values were more randomly distributed, thus emphasizing the often variable relationship between wood physical properties and lumber mechanical properties. For MOE, 312 pieces (38 per cent) were below the design values, comprising 58 (37 per cent), 222 (37 per cent) and 32 pieces (46 per cent) from the No. 1, No. 2 and No. 3 grades, respectively.

Of the explanatory variables used, SG had the highest correlation with MOE (R = 0.72) and F_s (R = 0.58) (Table 2). The three nondestructive instruments had slightly lower correlations with MOE (R = 0.63, 0.62, −0.61) and F_s (R = 0.45, 0.43, −0.39) for the Hitman, PLG and transverse vibration systems, respectively. The measurements of acoustic velocity from the Hitman (AV_Hit) and PLG (AV_PLLG) instruments were strongly correlated (R = 0.87), with only a few outlying observations reducing the correlation. The three MOE_{dyn} measurements each had good correlations with static MOE (MOE_{dyn-HIT}, R = 0.80; MOE_{dyn-PLL}, R = 0.78; MOE_{dyn-LTV}, R = 0.83).

The regression parameters for static MOE and F_s using the normal distribution approach are shown in Table 3, and scatterplots of static MOE vs. the four NDT measurements are shown in Figure 2. For static MOE, linear models were fit to the MOE_{dyn-LTV} data, and since normality deviations were detected for SG, MOE_{dyn-HIT} and MOE_{dyn-PLL}, power models were fit for these variables. Because of the large sample size (n = 819), the standard error around the mean regression line was small for MOE predictions. For all four NDT measures, the ranges of the predicted MOE values were quite
variable, with the $\sigma$ ranging from 1.28 to 1.61 GPa. The NDT tools used in this study, i.e. the two acoustic velocity instruments (Hitman and PLG), and the transverse vibration instrument (E-Computer), calculate $\text{MOE}_{\text{dyn}}$ using the density of the material combined with either acoustic velocity or frequency information. The best performing model for predicting static $\text{MOE}$ was the $E$-Computer, with a log-likelihood value of $-1365.7$ (higher is better) and a $\alpha$ value of 1.28 GPa. The Hitman model $\alpha$ (1.38 GPa) was similar to the PLG model (1.43 GPa), and both were an improvement over the SG model (1.61 GPa).

For static $F_b$, a linear model was used for each NDT measure except $\text{MOE}_{\text{dyn,PLG}}$, where a power model was used (Table 3). Prediction of static $F_b$ from the four NDT measures varied considerably, with $\sigma$ ranging from 4.47 to 4.58 MPa. The standard error around the mean was also variable, as indicated by the grey polygon around the mean (Figure 3). The best performing model in terms of the log-likelihood ($-2387.6$) and $\sigma$ (4.47 MPa) values was the $\text{MOE}_{\text{dyn,TV}}$ model. However, the differences between SG (4.58 MPa) and the three NDT instruments were minimal, and thus overall, each of the NDT measures showed a similar level of performance in predicting $F_b$.

The probability parameters for the binomial distribution models are shown in Table 4. The model parameters were used to calculate the probability of meeting the specified design values when used with the inverse-logit function (equation (11)). For meeting the 9.7 GPa mean design value, 82 per cent of the 9.7 GPa design value (8.0 GPa), and the $F_b$ design value (6.4 MPa), the $\text{MOE}_{\text{dyn,TV}}$ model had the lowest AIC values. Note that the AIC values between the normal and binomial distribution models should not be directly compared.

Tables 5 and 6 show the results of the four NDT measures needed to meet the design values (Table 5) vs 82 per cent of

Figure 1 Variation in SG, MOE and bending strength ($F_b$) for each sampled tree from each stand with tree number sorted by median SG within stand. The boxplots show outliers as dots, the minimum value not including outliers, first quartile (25 per cent), median, third quartile (75 per cent) and maximum value sans outliers.

Table 2 Pearson correlation matrix among wood properties measured, all coefficients were statistically significant ($\alpha < 0.05$)

<table>
<thead>
<tr>
<th>Property</th>
<th>$AV_{\text{prompt}}$ (m/s)</th>
<th>$AV_{\text{PLG}}$ (m/s)</th>
<th>Freq</th>
<th>$\text{MOE}_{\text{dyn,Hitman}}$ (GPa)</th>
<th>$\text{MOE}_{\text{dyn,PLG}}$ (GPa)</th>
<th>$\text{MOE}_{\text{dyn,TV}}$ (GPa)</th>
<th>MOE (GPa)</th>
<th>$F_b$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SG</td>
<td>0.36</td>
<td>0.39</td>
<td>-0.30</td>
<td>0.74</td>
<td>0.75</td>
<td>0.74</td>
<td>0.72</td>
<td>0.58</td>
</tr>
<tr>
<td>$AV_{\text{prompt}}$</td>
<td>0.87</td>
<td>-0.88</td>
<td>0.89</td>
<td>0.79</td>
<td>0.82</td>
<td>0.63</td>
<td>0.45</td>
<td></td>
</tr>
<tr>
<td>$AV_{\text{PLG}}$</td>
<td>-0.88</td>
<td>0.80</td>
<td>0.90</td>
<td>0.83</td>
<td>0.62</td>
<td>0.43</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Freq</td>
<td>-0.77</td>
<td>-0.76</td>
<td>0.93</td>
<td>0.94</td>
<td>0.80</td>
<td>0.59</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\text{MOE}_{\text{dyn,Hitman}}$</td>
<td>0.95</td>
<td>0.78</td>
<td>0.83</td>
<td>0.78</td>
<td>0.58</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\text{MOE}_{\text{dyn,PLG}}$</td>
<td>0.75</td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>$\text{MOE}_{\text{dyn,TV}}$</td>
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<td></td>
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<tr>
<td>MOE</td>
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<td></td>
</tr>
</tbody>
</table>

SG at 15 per cent moisture content, $AV_{\text{prompt}}$ = acoustic velocity Hitman HM200 (m/s), $AV_{\text{PLG}}$ = acoustic velocity Portable Lumber Grader (m/s), Freq = standardized frequency from transverse vibration, $\text{MOE}_{\text{dyn,Hitman}}$ = dynamic modulus of elasticity from Hitman (GPa), $\text{MOE}_{\text{dyn,PLG}}$ = dynamic modulus of elasticity from PLG (GPa), $\text{MOE}_{\text{dyn,TV}}$ = dynamic MOE from transverse vibration (GPa), MOE = static modulus of elasticity (GPa), $F_b$ = static bending strength (MPa).
Table 3  Regression parameters for linear and power models using MOE (GPa) and bending strength ($F_b$) (MPa) as the dependent variables vs nondestructive evaluation predictions

<table>
<thead>
<tr>
<th>Model</th>
<th>Regression statistics</th>
<th>Model parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>Equation</td>
<td>Log-likelihood</td>
</tr>
<tr>
<td>MOE</td>
<td>($\beta_0 + \beta_1 \text{ SG}$)\textsuperscript{1}</td>
<td>−1547.3</td>
</tr>
<tr>
<td></td>
<td>($\beta_0 + \beta_1 \text{ MOE}_{\text{dyn.HM}}$)\textsuperscript{1}</td>
<td>−1425.9</td>
</tr>
<tr>
<td></td>
<td>($\beta_0 + \beta_1 \text{ MOE}_{\text{dyn.PLG}}$)\textsuperscript{1}</td>
<td>−1456.1</td>
</tr>
<tr>
<td>$F_b$</td>
<td>$\beta_0 + \beta_1 \text{ SG}$</td>
<td>−1365.7</td>
</tr>
<tr>
<td></td>
<td>$\beta_0 + \beta_1 \text{ MOE}_{\text{dyn.HM}}$</td>
<td>−2405.0</td>
</tr>
<tr>
<td></td>
<td>$\beta_0 + \beta_1 \text{ MOE}_{\text{dyn.PLG}}$</td>
<td>−2397.0</td>
</tr>
<tr>
<td></td>
<td>$\beta_0 + \beta_1 \text{ MOE}_{\text{dyn.TV}}$</td>
<td>−2397.0</td>
</tr>
</tbody>
</table>

SG at 15 per cent moisture content, MOE\textsubscript{dyn.HM} = dynamic modulus of elasticity from Hitman (GPa), MOE\textsubscript{dyn.PLG} = dynamic modulus of elasticity from PLG (GPa), MOE\textsubscript{dyn.TV} = dynamic MOE from transverse vibration (GPa), MOE = static modulus of elasticity (GPa), $F_b$ = static bending strength (MPa), $\sigma$ = standard error of the estimate.

Figure 2  Plots showing the linear or power relationship between MOE (GPa) and SG, and MOE\textsubscript{dyn} measured from the Hitman HM200, PLG and transverse vibration (GPa). The solid line is the mean, the grey polygon around the solid line is the standard error around the mean and the dashed lines are the 95 per cent prediction intervals.

the design values (Table 6) for MOE at 50 per cent, 75 per cent and 95 per cent confidence using both the normal and the binomial distribution approaches. The tables include all lumber pieces, irrespective of grade, grouped together with the applied threshold of the No. 2 grade MOE (9.7 GPa vs 8.0 GPa) and then separated by grade to meet the No. 1 grade (11.0 GPa vs 9.0 GPa), No. 2 grade (9.7 GPa vs 8.0 GPa) and No. 3 grade (9.0 GPa vs 7.4 GPa) design values. For the No. 2 grade material, using the normal and binomial distribution approaches, the MOE\textsubscript{dyn.TV} value needed at 50 per cent confidence using the normal approach was 10.9 GPa, whereas the corresponding value for the binomial approach was 11.0 GPa. At the 95 per cent confidence level, the normal approach required a 26 per cent increase to 13.6 GPa, and the binomial approach needed a 20 per cent increase to 13.0 GPa. A graphical comparison between the normal and binomial approaches at the different confidence levels is shown in Figure 4 for the MOE\textsubscript{dyn.TV} variable.

Discussion

The focus of this study was to quantify the relationships between both static MOE and $F_b$, and different NDT methods for loblolly pine dimension lumber, and based on the resultant relationships, determine the probability that a given NDT value meets a specific design value threshold. The use of NDT on wood and lumber has received considerable attention since the 1960s (Ross, 2015a) and research has continued on the use of NDT in evaluating timber, boards and dimension lumber (Carter et al., 2006; Grabianowski et al., 2006; Baillères et al., 2012; Yang et al., 2015, 2017; Llana et al., 2016). We found that NDT measurements were moderately to strongly correlated with lumber MOE and less so for $F_b$; similar results were found in other lumber studies. Yang et al. (2015) reported strong relationships between static MOE and NDT-derived values in southern pine lumber using three instruments – the Hitman
Figure 3 Plots showing the linear or power relationship between bending strength ($f_b$) (MPa) and SG, and MOE$_{dyn}$ measured from the Hitman HM200, PLG and transverse vibration (GPa). The solid line is the mean, the grey polygon around the solid line is the standard error around the mean and the dashed lines are the 95 per cent prediction intervals.

Table 4 Regression parameters for the logistic binomial regression models for dependent variables MOE (GPa) at the No. 2 mean value (9.7 GPa), 82 per cent of the No. 2 mean value (8.0 GPa) and bending strength ($f_b$) (MPa) at the No. 2 fifth percentile value (6.4 MPa) vs NDT predictions

<table>
<thead>
<tr>
<th>Design value</th>
<th>Model</th>
<th>Regression statistics</th>
<th>Model parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.7 GPa</td>
<td>MOE SG</td>
<td>-381.55</td>
<td>767.11, 16.90, 35.16</td>
</tr>
<tr>
<td></td>
<td>MOE$_{dyn, HM}$</td>
<td>-318.39</td>
<td>640.78, 10.72, 0.99</td>
</tr>
<tr>
<td></td>
<td>MOE$_{dyn, PLG}$</td>
<td>-314.63</td>
<td>633.26, 10.75, 0.99</td>
</tr>
<tr>
<td></td>
<td>MOE$_{dyn, TV}$</td>
<td>-285.62</td>
<td>575.25, 13.02, 1.18</td>
</tr>
<tr>
<td>8.0 GPa</td>
<td>MOE SG</td>
<td>-224.86</td>
<td>453.72, 11.34, 27.77</td>
</tr>
<tr>
<td></td>
<td>MOE$_{dyn, HM}$</td>
<td>-171.12</td>
<td>346.23, 9.42, 1.11</td>
</tr>
<tr>
<td></td>
<td>MOE$_{dyn, PLG}$</td>
<td>-179.26</td>
<td>362.53, 8.25, 0.98</td>
</tr>
<tr>
<td></td>
<td>MOE$_{dyn, TV}$</td>
<td>-161.93</td>
<td>327.86, 10.28, 1.18</td>
</tr>
<tr>
<td>6.4 MPa</td>
<td>$F_b$ SG</td>
<td>-65.50</td>
<td>135.00, 0.53, 9.35</td>
</tr>
<tr>
<td></td>
<td>MOE$_{dyn, HM}$</td>
<td>-60.06</td>
<td>124.11, 1.27, 0.50</td>
</tr>
<tr>
<td></td>
<td>MOE$_{dyn, PLG}$</td>
<td>-62.24</td>
<td>128.48, 0.07, 0.37</td>
</tr>
<tr>
<td></td>
<td>MOE$_{dyn, TV}$</td>
<td>-57.75</td>
<td>119.49, 2.24, 0.59</td>
</tr>
</tbody>
</table>

SG at 15 per cent moisture content, MOE$_{dyn, HM}$ = dynamic modulus of elasticity from Hitman (GPa), MOE$_{dyn, PLG}$ = dynamic modulus of elasticity from PLG (GPa), MOE$_{dyn, TV}$ = dynamic MOE from transverse vibration (GPa), MOE = static modulus of elasticity (GPa), $F_b$ = static bending strength (MPa), $\sigma$ = standard error of the estimate.

($R^2 = 0.82$, RMSE = 1.12 GPa), the A-Grader (Falcon Engineering Ltd.; $R^2 = 0.77$, RMSE = 1.21 GPa) and the E-Computer ($R^2 = 0.86$, RMSE = 0.98 GPa). The first and third instruments were used in the current study, whereas the A-grader is similar to the PLG instrument. We can account for the slightly better results reported by Yang et al. (2015) since that study focused on No. 2 grade material only, whereas we used No. 1, 2 and 3 grade material. In a subsequent paper, Yang et al. (2017) reported weak relationships for static MOR and NDT using the Hitman ($R^2 = 0.28$, RMSE = 14.4 MPa), the A-Grader (Falcon Engineering Ltd.; $R^2 = 0.27$, RMSE = 14.9 MPa) and the E-Computer ($R^2 = 0.26$, RMSE = 14.7 MPa). In the current study, we found similar results for $F_b$, confirming the somewhat poor predictive performance of NDT methods for strength properties, as compared with MOE.

In a study on 2 x 4 southern pine lumber, Wang et al. (2008) reported strong correlations between static MOE and MOE$_{dyn}$ determined from density and acoustic velocity using the Sylvatest instrument ($R^2 = 0.82$) and the E-Computer ($R^2 = 0.87$). Wessels et al. (2011a) found relatively strong relationships with MOE ($R^2 = 0.72$) and weak relationships with MOR ($R^2 = 0.20$) for hybrid *Pinus elliottii* x *Pinus caribaea* standing timber and boards, using a resonance instrument. Overall, our results are similar to those from the studies discussed above and collectively show that NDT tools give reasonable predictions of lumber MOE. Note that the high data variability in lumber is not unique and is also observed when using NDT to predict mechanical properties of small clearwood specimens (Auty and Achim, 2008).

The current study is set apart from previous studies in that besides quantifying the relationships between static wood properties (MOE and $F_b$) and those derived from NDT methods, we also estimated the probabilities for given NDT values of meeting specific design value thresholds utilizing two different statistical approaches based around the normal and the binomial distributions. Although results from both approaches were similar, the binomial distribution approach was simpler when calculating the probabilities. The binomial approach requires new regression parameters ($\beta_0$ and $\beta_1$) to be calculated for each threshold-dependent variable, whereas with the normal distribution approach the regression parameters and the standard error of
the estimate (σ) remain the same for a given dataset. Because the parameters for the binomial distribution are unique for each threshold value, the results may be slightly more reliable than the normal distribution approach.

When predicting the mean response at the 50 per cent confidence level using the normal distribution approach, it follows that 50 per cent of the pieces will not meet the MOE design value. For each NDT measurement, the values needed to meet the design value at 95 per cent confidence, compared with 50 per cent confidence, were on average 28 per cent higher. An interesting question here is what level of confidence is required for a specific scenario? Because the MOE design values are at the mean level of the population, not every piece needs to meet the design value for each grade and thus for some applications (e.g. a floor system with multiple members), a 50 per cent confidence is likely sufficient. The MSR grading systems require that 95 per cent of the pieces exceed 82 per cent of the mean value (ALSC, 2013, 2014), and thus here a 95 per cent confidence level is appropriate. An important consideration when using NDT values are the differences between static and dynamic values. Divós and Tanaka (2005) found that MODyn properties were 10 per cent higher than static MOE properties in clearwood spruce samples. We found similar results in our study, where MODyn_HM was 15 per cent higher and MODyn_PLG and MODyn_TV were both 16 per cent higher than static MOE.

For all the three NDT instruments used in this study, the calculation of MODyn depends on the density of the material. An interesting point is the relative importance of predicting MOE using density as compared with acoustic velocity or frequency. We found that SG had a higher correlation with MOE (R = 0.72)
and $F_b$ ($R = 0.58$) than acoustic velocity or frequency. Wessels et al. (2015b) also found that density was a better predictor of MOE and MOR than acoustic velocity in Pinus patula; however, in that study, acoustic velocity was determined using a time-of-flight instrument and not a resonance instrument. Acoustic velocity using resonance tools is generally considered more accurate than the time-of-flight approach because the former measures multiple waves (Mora et al., 2009; Wang, 2013). From an anatomical perspective, increasing acoustic velocity from pith to bark is attributed to an increase in density and tracheid length, and a decrease in microfibril angle (Hasagawa et al., 2011). Although few studies have directly assessed the effect of microfibril angle on MOE in lumber, similar correlations with those found in this study have been reported between MOE and MFA ($R = -0.73$), and MOE and density ($R = 0.69$) for Pinus patula (Wessels et al., 2015).

Each NDT tool used in this study showed similar accuracy in predicting mechanical properties, although the transverse vibration tool was slightly more accurate than the stress wave tools. These differences could be the result of measurements made in the axial vs edgewise direction. In Norway spruce (Picea abies L. Karst.), Olsson et al. (2012) demonstrated that MOE$_{dyn}$ calculated from acoustic velocity measured in the edgewise orientation was slightly more accurate at predicting static MOE than velocity measured in the axial orientation ($R^2 = 0.89$ vs 0.84). They concluded that dynamic measurements of edgewise stiffness are better able to capture low stiffness areas within lumber than dynamic measurements of axial stiffness, which are more a function of stiffness across the entire cross section (Olsson et al., 2012).

Each NDT tool also has advantages and disadvantages regarding its setup and use. Although the Hitman and PLG stress wave tools yielded similar results, they are deployed differently. The Hitman is a portable system designed for use on logs but can also be used to evaluate lumber, whereas the PLG system requires a computer connection, and while designed for lumber, it can also be used on logs. For the accelerometer in the Hitman system to record the resonant frequency, it needs to be in direct contact with the wood (Achim et al., 2011). Thus, care is needed when using the system to maintain contact between the unit and the sample during a measurement reading. For logs, this is not a challenge given their mass, but for smaller dimension lumber, this can pose challenges when generating the stress wave using a hammer. In the case of the PLG, the microphone is placed on a stand near the end of the lumber but not in direct contact with the piece, thereby making it easier to use. Compared with the stress wave tools, the E-Computer system is slightly more accurate but has the disadvantage of requiring more care in the calibration and initial setup of the instrument. The E-Computer is traditionally used for testing lumber flatwise (ASTM, 2012); however, edgewise testing is also possible, but specimen placement, particularly with warped lumber, is more challenging (Yang et al., 2015).

Either of the statistical frameworks presented here can be applied by industry in quality control systems once stable, mill-validated relationships have been developed between static and NDT measurements. At the same time, further work is needed to decrease the prediction errors from the NDT techniques and consequently increase the accuracy in the segregation of material between grades. Our data were collected under ambient conditions, but with reasonable control over lumber moisture content. Since acoustic velocity and MOE$_{dyn}$ both decrease, and density increases, with increased moisture content (Chan et al., 2011), consistency in mill operations would be important for the accuracy of the prediction models. Mills with inline moisture measurement systems could likely improve prediction accuracy by adjusting the NDT readings to a specific moisture content.

An important factor observed in this, and other studies, is the difficulty in predicting bending strength (MOR and $F_b$). Here, we specifically did not address the impact of knots on the $F_b$ of lumber, but unsurprisingly, accounting for knots improves predictions.

![Normal Distribution Linear Regression](image1)

![Binomial Distribution Logistic Regression](image2)
of bending strength (Olsson et al., 2012, 2013; Oscarsson et al., 2014). Although previously unavailable in commercial facilities, new systems have been implemented that enable real-time quantification of knots on sawn lumber (Baillères et al., 2012; Hitaniemi et al., 2014), usually by measuring localized grain angle using the ‘tracheid effect’ (Viguier et al., 2017). These systems are frequently combined with inline measurements of SG and/or MOE_dyn, and can provide much more accurate predictions of mechanical properties by simultaneously combining several wood characteristics into the models (Baillères et al., 2012). A recent study by Wong et al. (2016) used Bayesian methods to develop prediction models for Spruce-Pine-Fir lumber bending and tensile strength that included parameters accounting for the number of large knots, the presence or absence of shake and a miscellaneous category to cover other defects (e.g. grain deviations and wane). Inclusion of this information for each piece of lumber would improve predictions of lumber performance characteristics.

**Conclusions**

The results from this study show that while NDT methods can accurately predict the confidence limits surrounding mean design values for MOE, the range of expected values poses challenges for the forest industry due to the uncertainty surrounding measured static values. Our study highlights the value of calculating the probability of meeting specific design thresholds. We incorporated the standard error of the estimate to improve inferences about the desired population by integrating the likelihood function from a linear or power model to calculate the probability of meeting a certain design value threshold. The normal distribution predictions yielded similar probability values to those obtained using the more traditional binomial distribution approach using logistic regression. The advantage of the normal distribution approach over the binomial approach is that the regression parameters do not need to be re-estimated for each threshold value. Either approach used here could be incorporated into prediction equations used by industry to determine, with increased confidence, whether lumber properties predicted using NDT methods meet design value requirements.

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**Conflict of interest statement**

None declared.

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