PHOTOMECHANICAL STRESS ANALYSIS OF CELLULOSIC-MANURE COMPOSITES

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ABSTRACT

Being able to stress analyze green materials such as cellulosic-manure composites addresses several societal issues. These include providing engineering members fabricated from materials which can be particularly suitable for developing nations, relieving a troubling by-product of agricultural regions and reducing the demands on our landfills. However most engineering applications of these materials necessitate knowledge of their structural integrity. This implies ability to evaluate stresses in such materials. Motivated by the above, this paper describes the ability to photomechanically stress analyze engineering members produced from such green materials.

1. Introduction

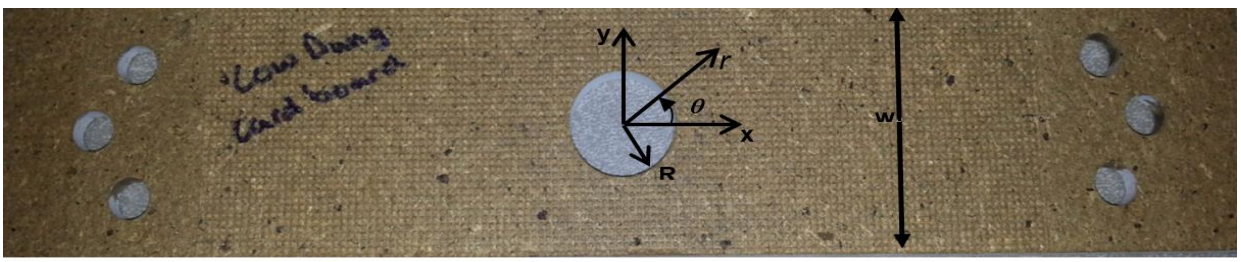
Candidate methods to experimentally stress analyze members fabricated from green materials include strain gages, moiré, digital image correlation (DIC), photoelasticity and TSA (thermoelastic stress analysis). Strain gages provide only point-by-point information so such analyses can involve bonding, electrically exiting and monitoring many gages. Moiré and DIC require applying a pattern to the component. Although displacements have been measured in green materials using electronic speckle (ESPI) [1], the set-up needed to record both in-plane displacements is cumbersome. Moiré, DIC and speckle also require differentiating the measured data, which has its own perils. Birefringent photoelasticity can be very laborious and time-consuming. In addition to being applicable for actual structures in their operating environment, TSA provides the stresses without having to either differentiate the measured information or know the constitutive properties. This paper demonstrates the ability to thermoelastically stress analyze members fabricated from a cellulosic-manure composite. The present illustrative example is a tensile loaded plate containing a central circular hole. The authors are unaware of any previous application of TSA to manure composites.

2. Material and Specimen Preparation

The initial panel was fabricated from two cellulosic fiber sources: anaerobic digested bovine manure and recycled old corrugated containers. The manure was collected from a farm and processed through a digester, where it was diluted to 6% solids at constant temperature of between 38.0-39.3C for 22 to 25 days. During this time the bacteria digests the cellulosic material, while producing methane. The digested manure was screen separated from the bulk of the liquid, leaving a separated solids or fiber content of about 25 percent. This remaining non-digested cellulosic solids content is used to form the fibrous mats. The corrugated material came from a local box plant and contained excess cut-offs and scrap material. That received material was hydropulped at approximately 10% solids for 20 minutes to produce a fibrous slurry. The processed manure and paper were combined 1:1 by dry-weight and mixed with water in a forming box to achieve an approximate 2% solids slurry. A mat of randomly oriented fibers was produced by pouring the mixture onto a screen and applying a vacuum beneath the screen to pull the water/fiber mixture toward the screen to form a loose fibrous mat. The mat was then wet-pressed to remove much of the residual water to produce a relatively thick mat, Fig. 1. This mat was subsequently hot-pressed at 160ºC and 1.0 MPa until dry. Without any resin, the two fibrous materials combined to produce a naturally bonded in-plane randomly oriented cellulose-fiber panel whose properties are comparable with those of commercial hardboard.

Figure 2 shows the present central circularly perforate plate as machined from the panel. The face of the plate to be analyzed was sanded to provide a smooth surface after which that surface was sprayed with flat black paint.

Fig. 1 Composite mat ready for application of heat and pressure.



R = 12.7 mm, w = 62 mm, t = 7.2mm

Fig. 2 Perforated cellulosic-manure composite plate.

3. Analytical Tools

A relevant stress function was used to process the recorded temperature information. For plane-stress

$$S^* = K\Delta S \tag{1}$$

in which the recorded TSA signal, S^*, is related to the change in the sum of the normal stresses, $S = \sigma_1 + \sigma_2 = \sigma_{rr} + \sigma_{\theta\theta} = \sigma_{xx} + \sigma_{yy}$, and K is a thermo-mechanical coefficient. Differentiating the stress function gives

$$\sigma_{rr} = \frac{b_0}{r^2} + 2c_0 - \sum_{n=2,4,6}^{N} \begin{bmatrix} a_n n(n-1)r^{(n-2)} + b_n(n+1)(n-2)r^{(n)} \\ +c_n n(n+1)r^{-(n+2)} + d_n(n-1)(n+2)r^{-(n)} \end{bmatrix} cos(n\theta) \tag{2}$$

$$\sigma_{\theta\theta} = \frac{b_0}{r^2} + 2c_0 + \sum_{n=2,4,6}^{N} \begin{bmatrix} a_n n(n-1)r^{(n-2)} + b_n(n+1)(n+2)r^{(n)} \\ +c_n n(n+1)r^{-(n+2)} + d_n(n-1)(n-2)r^{-(n)} \end{bmatrix} cos(n\theta) \tag{3}$$

$$\sigma_{r\theta} = \sum_{n=2,4,6}^{N} \begin{bmatrix} a_n n(n-1)r^{(n-1)} + b_n n(n+1)r^{(n)} \\ -c_n n(n+1)r^{-(n+2)} - d_n n(n-1)r^{-(n)} \end{bmatrix} sin(n\theta)$$

(4)

$$S = (\sigma_{rr} + \sigma_{\theta\theta}) = (\sigma_{xx} + \sigma_{yy}) = (\sigma_1 + \sigma_2) = 4c_0 + \sum_{n=2,4,6}^{N} \begin{bmatrix} 4b_n(n+1)r^{(n)} \\ -4d_n(n-1)r^{-(n)} \end{bmatrix} cos(n\theta)$$

(5)

R is the radius of the hole, radius *r* is measured from the center of the hole, angle θ goes counterclockwise from the longitudinal *x*-axis, Fig. 2, *N* is the finite upper limit on the summations and b_o, c_o, b_n, c_n and d_n are the Airy coefficients. By imposing the traction-free conditions analytically on the edge of the hole, eqn (3) can be re-written as

$$\sigma_{\theta\theta} = \left(\frac{2 \cdot R^2}{r^2} + 2\right) c_0 + \sum_{n=2,4,6}^{N} \left\{ \begin{matrix} \begin{bmatrix} (n-1)r^{(n-2)} \cdot (-1)(n+1)R^2 \\ +(n+1)(n+2)r^n + R^{2(n+1)}(n+1)r^{-(n+2)} \end{bmatrix} b_n \\ + \begin{bmatrix} (n-1)r^{(n-2)} \cdot (-1)R^{2(1-n)} \\ +(1-n)R^2(n+1)r^{-(n+2)} + (n-1)(n-2)r^{-(n)} \end{bmatrix} d_n \end{matrix} \right\} cos(n\theta)$$

(6)

Expressions corresponding to eqn (6) can also be written for σ_{rr} and $\sigma_{r\theta}$. Using 4500 thermoelastically determined values of *S* with eqn (5) throughout the plate and seven Airy coefficients, the values of these coefficients were evaluated for the over-determined system of equations using least squares. Knowing the magnitude of the Airy coefficients, the individual stress components can be evaluated everywhere, including on the edge of the hole from eqn (6).

4. Experimental Details

Figure 3 is a photograph of the experimental arrangement, including the loaded specimen and TSA recording camera while Fig. 4 is a representative TSA image recorded at 20 Hz. The recorded TSA data of Fig. 4 were averaged about the horizontal and vertical axes. The value of the thermo-mechanical coefficient, *K*, was evaluated from a uniaxial tensile strip which was machined from the same composite sheet, and similarly painted and also tested at 20 Hz.

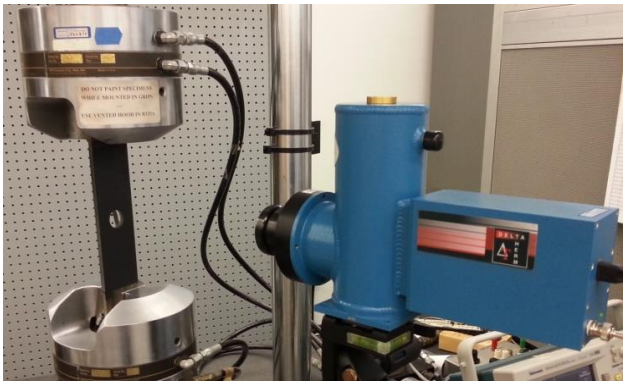


Fig. 3 Experimental setup with flat black paint on the surface of the specimen.

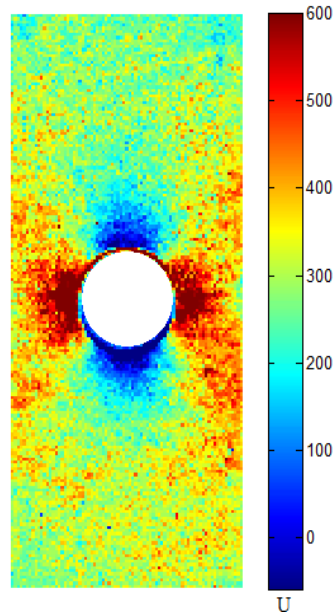


Fig. 4 Recorded Thermogram of vertically loaded tensile plate.

5. Results

Figure 5 plots the normalized TSA-determined stress along the edge the hole, where σ_o is the averaged stress based on the net area. The TSA-evaluated stress concentration factor of 2.1 compares with 2.2 reported by Roark [2].

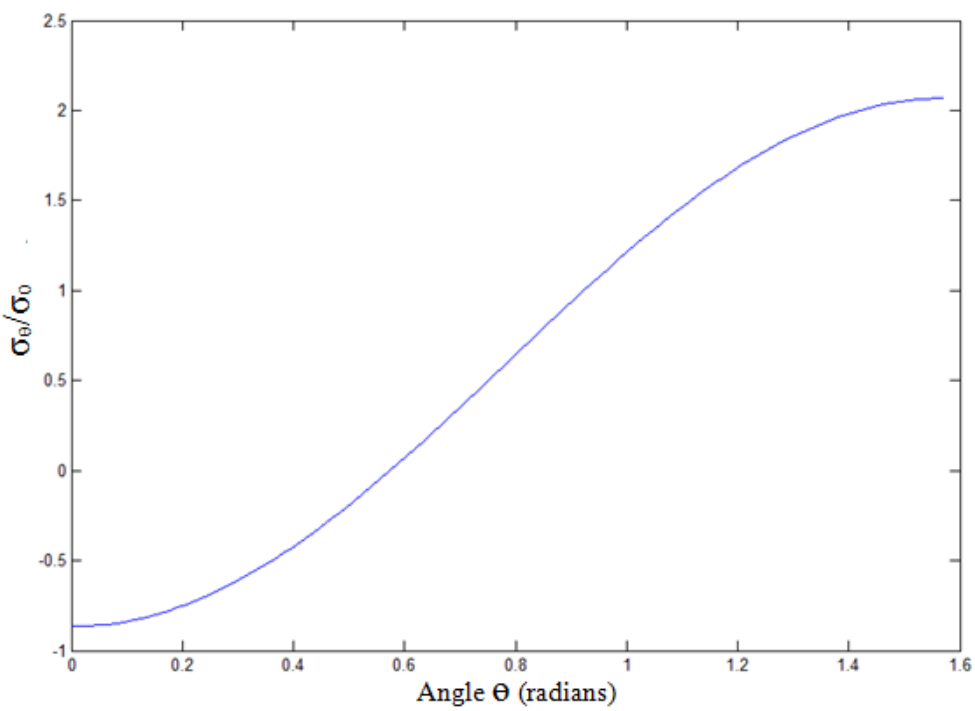


Fig. 5 TSA-determined stress along edge of hole.

Summary, Discussion and Conclusions

The ability to extend TSA to obtain the stresses in a cellulosic-manure composite is demonstrated. This capability address several major societal problems, *e.g.,* being able to assess engineering reliability based on known stresses of items such as domestic and/or infrastructure structures fabricated from these materials [which can be particularly suitable for developing nations], relieving a troubling by-product of highly agricultural regions and reducing the demands on our landfills. Engineering applications invariably involve cutouts or reentrant corners, resulting in stress concentrations. Of the potential methods of evaluating stresses in such materials, TSA is applicable to actual structures in their operating environment, and necessitates neither differentiating the measured information nor knowing the constitutive properties.

References

1. Kozlosky, A. (2008) Confidence Building of Contemporary Green Engineering Materials, MS Thesis, UW-Madison, Madison, WI.
2. Young, W. C. and Budynas, R. G. (2002) Roark's Formulas for Stress and Strain, 7th Edition, McGraw-Hill, New York.