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Material Variability and Repetitive Member Factors for the Allowable Properties of Engineered Wood Products

ABSTRACT: It has been argued that repetitive member allowable property adjustments should be larger for high-variability materials than for low-variability materials. We report analytic calculations and simulations that suggest that the order of such adjustments should be *reversed*, that is, given the manner in which allowable properties are currently calculated, as the coefficient of variation of the strength distribution of individual elements increases, the upward repetitive member adjustments (if any) of assemblies constructed from these elements should decrease.

KEYWORDS: assembly, reliability, redundancy

Introduction

In existing standards (e.g., Refs 1 and 2), upward adjustments in allowable properties are permitted for repetitive member assemblies of solid sawn wood. In the pre-2003 D5055 standard [3], this upward adjustment was larger for high-coefficient-of-variation (COV) material than for low-COV material. In section X1.4.1 of the pre-2003 D5055 standard a justification of the differing adjustments was given:

> The allowable bending stress increases for repetitive member use were derived taking into consideration the COV of the stiffness of various flange materials. The original theory justifying this type of increase seems to be based on the relative stiffness of the members and positive correlation between bending strength and stiffness. Logic indicates that as stiffness COV decreases so would the load sharing. That is, as stiffness COV tends to zero, lack of differential deflection eliminates load transfer.

For pragmatic reasons, the differing adjustments were removed in the 2003 revision of D5055 [4]. However, the intuitive basis for the differential adjustments is still found in section 5.1.3 of D6555 [5]:

> Load sharing tends to increase as member stiffness variability increases….

Based on the analytical calculations and computer simulations reported in this paper, we conclude that the differing repetitive member adjustments that have been based on this intuition (that is, upward adjustments that are larger for higher COV material) are flawed.

It is important to note that we are not arguing with the contention that "load sharing tends to increase as member stiffness variability increases." Instead we dispute the conclusion that is sometimes drawn from this intuition—that assemblies composed of higher variability material deserve larger upward adjustments in allowable properties. This conclusion is not valid as it makes the implicit assumption that high-variability assemblies and low-variability assemblies begin with the same probability of individual element failure. As we will see below, this assumption is false. Thus, higher variability assemblies begin with a handicap, and any advantage due to "better load sharing" does not make up for this initial disadvantage. In fact, for the assemblies that we consider, the probability that an assembly fails when subjected to a load equal to its allowable bending stress increases as COV increases. Given this basis, larger repetitive member upward adjustments for larger member strength COVs lead to assemblies with larger probabilities of failure and are thus unjustified. This is the essence of our argument. We provide the analytical and simulation details in the remainder of this paper.

For the purposes of this paper, we define assembly failure as the failure of any one of the assembly's members. A first approximation (not our final approximation) to the probability, P_F, of assembly failure is given by

$$P_F \approx 1-(1-p_S)^k \tag{1}$$

where:
p_S=probability that an individual element fails and
k=number of elements in the assembly.

Result 1 treats the elements of an assembly as independent members that all see the same load. As we will see below, this approximation overestimates P_F if weaker members tend to be less stiff and thus tend to see less of the load. However, it is a starting point for our analysis and emphasizes the fundamental role played by p_S.

In the section Probability of Failure of a Single Member, we obtain analytic formulas for p_S for normal and lognormal distributions. In the same section, we also point out the central fact that, as currently calculated, *allowable properties are not associated with a fixed probability of failure*. That is, the probability that a member fails when subjected to a load equal to its allowable bending stress is smaller for members that come from distributions with smaller coefficients of variation.

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In the section P_F for Normally Distributed, Perfectly Correlated Strength and Stiffness, we derive an expression for P_F in the case of perfectly correlated, normally distributed strengths and stiffnesses.

In the section Probability of Failure of an Assembly at an Unadjusted Allowable Stress Level, we discuss simulations of a particular hypothetical assembly. Our simulated assembly contains seven members, and we distribute loads in proportion to member stiffnesses. Our simulations produce estimates of P_F. In the same section, we discuss the tables that we produced from these simulations and the fact that they indicate that, at loads equal to allowable bending stresses, the probability of failure increases with increasing COV.

In the section Probability of Failure of an Assembly at an Adjusted Allowable Stress Level, we extend these simulations to cover the case in which our assemblies are subjected to loads that have been adjusted upward in accordance with the recommendations of the pre-2003 ASTM D5055. These simulations indicate that such a procedure leads to higher probabilities of failure for assemblies associated with higher member COVs.

Our simulations are based on normal and lognormal data (both “truncated” and untruncated), a simple model of seven member assemblies, and a particular definition of failure. Our model includes load sharing as defined in D6555 (in fact, because it assumes an infinitely stiff diaphragm it incorporates greater load sharing than that envisioned in the standard), but it does not include composite action or residual capacity. Thus we do not claim that our results hold in general, and we do not believe that our estimates of failure probabilities should be applied to any real world situations. However, these results certainly call into question the validity of adjusting high-variability materials more than low-variability materials. Such adjustments cannot be based on intuition. If permitted, they must be based on detailed theoretical calculations, simulations, or empirical evidence. See, for example, Section 8 of D6555.

Finally, we note that a reviewer of this paper speculated that our results might change if we focused on, for example, strength distribution fifth percentiles rather than allowable stress levels. We have performed additional simulations that compare the first, fifth, and tenth percentiles of the strength distributions of assemblies and single members. These simulations yield conclusions similar to those presented in the current paper. Details are provided in Ref 6.

Probability of Failure of a Single Member

Intuition

In the Introduction we asserted that the probability, p_S, that an individual element fails when subjected to a load equal to its allowable stress level is larger for high-COV materials than for low-COV materials. Because this fact is central to our argument and might be counterintuitive for some readers, we provide some additional explanation here.

In developing this intuition, we assume these strengths are normally distributed. However, in our simulations, we work with untruncated normal and lognormal distributions and with truncated normal and lognormal distributions.

For a normal strength distribution, the probability, p_S, that a particular specimen fails at its allowable stress level is the probability that the strength of the specimen lies below the fifth percentile of the strength distribution divided by 2.1 (the safety and duration of load factor). For a more variable material (of the same mean strength), the fifth percentile will be lower so the allowable stress level will be smaller. One might assume that this will imply that the strength of a specimen from a more variable material will be less likely to lie below its allowable stress level. However, the more variable material has a greater standard deviation, and it turns out that its allowable stress level is not as many standard deviations from its mean as is the allowable stress level of a less variable material. In particular, for a specimen from a normal distribution with mean μ and standard deviation σ, we have

TABLE 1—*For a normal population of mean one and a series of coefficients of variation, this table provides the fifth percentile of the distribution, the associated allowable stress level (fifth percentile divided by 2.1), the number of standard deviations that the allowable stress level lies below the mean of the distribution, and the probability p_s that a member of the normal population will have a strength value that falls below the allowable stress level.*

COV	Fifth Percentile	Allowable Stress	Standard Deviations Below Mean	p_S
0.05	0.918	0.437	11.26	$0.104E-28$
0.10	0.836	0.398	6.02	$0.864E-09$
0.15	0.753	0.359	4.28	$0.954E-05$
0.20	0.671	0.320	3.40	$0.334E-03$
0.25	0.589	0.280	2.88	$0.200E-02$

$$\text{allowable stress level} = (\mu - 1.645 \times \sigma)/2.1$$

and the number of standard deviations that the allowable stress level lies beneath the mean strength is

$$(\mu - (\mu - 1.645 \times \sigma)/2.1)/\sigma = (1.1 \times \mu + 1.645 \times \sigma)/(2.1 \times \sigma)$$
$$= 1.1/(2.1 \times \text{COV}) + 1.645/2.1$$

where:

$\text{COV} = \sigma/\mu$.

Thus the larger the COV, the closer (as measured in standard deviations) the allowable stress level is to the mean strength and the higher the probability that a specimen’s strength will fall below the allowable stress level. This is made precise in Table 1 and is illustrated in Fig. 1. In Fig. 1 both normal distributions have a mean of one. The broad distribution has a standard deviation of 0.25, and the narrow distribution has a standard deviation of 0.05. The dotted

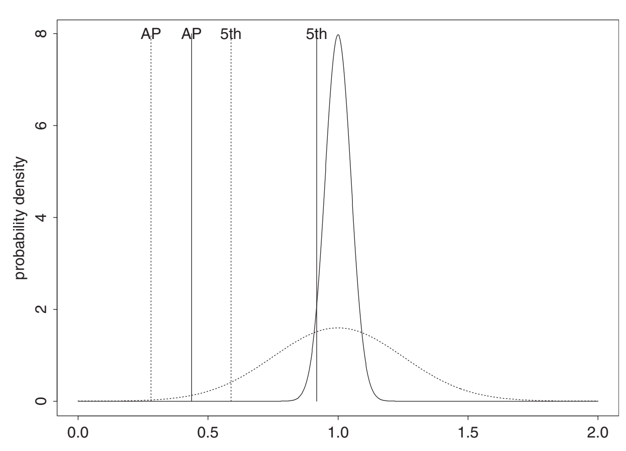

FIG. 1—*Both normal distributions have a mean of 1. The broad distribution has standard deviation 0.25, and the narrow distribution has standard deviation 0.05. The dotted vertical lines are at the allowable property level (AP) and fifth percentile (5th) of the broad distribution. The solid vertical lines are at the allowable property level (AP) and fifth percentile (5th) of the narrow distribution.*

vertical lines are at the allowable stress level and fifth percentile of the broad distribution. The solid vertical lines are at the allowable stress level and fifth percentile of the narrow distribution. Visually, it should be clear that the strength of a specimen from the broad distribution (COV=0.25) is more likely to fall below its allowable stress level than is the strength for a specimen from the narrow distribution (COV=0.05). In fact, as we can see from Table 1, the allowable stress level for the broad distribution lies only 2.88 standard deviations below the distribution's mean, and the probability that a specimen from this distribution fails at its allowable stress level is $2.0E-3$, while the allowable stress level for the narrow distribution lies 11.3 standard deviations below the distribution's mean, and the probability that a specimen from the narrow distribution fails at its allowable stress level is $1.0E-29$.

In the Normal Distribution and Lognormal Distribution subsections, we provide detailed formulas for p_S for normal and lognormal distributions in situations in which aging has reduced the original strength of the specimens.

Normal Distribution

Let μ be the mean of the population and σ the standard deviation. Let $\mathrm{COV} \equiv \sigma/\mu$ (rather than $\sigma/\mu \times 100$ as it is sometimes expressed). Suppose that a member has been in service for time t and that the strength reduction factor appropriate for this length of service is $r(t)$. Then, at time t the probability that a single member fails at its allowable stress level is given by

$$\begin{aligned} p_S &= \mathrm{prob}[N(\mu,\sigma^2)/r(t) < (\mu - 1.645 \times \sigma)/2.1] \\ &= \mathrm{prob}[N(0,1) < (r(t)(\mu - 1.645 \times \sigma)/2.1 - \mu)/\sigma] \\ &= \mathrm{prob}[N(0,1) < (r(t)/2.1 - 1)\mu/\sigma - r(t) \times 1.645/2.1] \\ &= \mathrm{prob}[N(0,1) < (r(t)/2.1 - 1)/\mathrm{COV} - r(t) \times 1.645/2.1] \end{aligned} \tag{2}$$

Note that for $r(t) < 2.1$, p_S increases as COV increases.

Lognormal Distribution

Let μ and σ denote the mean and standard deviation of the population *after* natural logs have been taken. Let COV be the standard deviation of the *original* population divided by the mean of the original population. It can be shown that for lognormal data, $\sigma = \sqrt{\ln(1+\mathrm{COV}\times\mathrm{COV})}$. Let $r(t)$ be as above. Then, at time t the probability that a single member fails at its allowable stress level is given by

$$\begin{aligned} p_S &= \mathrm{prob}[\mathrm{LN}(\mu,\sigma^2)/r(t) < \exp(\mu - 1.645\sigma)/2.1] \\ &= \mathrm{prob}[N(\mu,\sigma^2) < \mu - 1.645\sigma + \ln(r(t)) - \ln(2.1)] \\ &= \mathrm{prob}[N(0,1) < -1.645 + (\ln(r(t)) - \ln(2.1))/\sigma] \\ &= \mathrm{prob}[N(0,1) < -1.645 + (\ln(r(t)) \\ &\quad - \ln(2.1))/\sqrt{\ln(1 + \mathrm{COV} \times \mathrm{COV})}] \end{aligned} \tag{3}$$

Note that, again, for $r(t) < 2.1$, p_S increases as COV increases.

Truncated Distributions

Because real populations of boards sometimes have strength distributions that are left truncated (e.g., "number 2 and better"), we expanded our simulations to include truncated distributions. We did this as follows.

For a given correlation between modulus of elasticity (MOE) and modulus of rupture (MOR), we generated 100 000 samples from the bivariate normal distribution (in the lognormal case, the bivariate normal distribution was the joint distribution of the MOE and ln (MOR) values). We then identified the 60 000 boards with the largest MOEs (the top 60 % of the MOEs) and treated them as MSR (machine stress rated) number 2 and better. We treated the 40 000 of these 60 000 with the lowest MOEs (the bottom two-thirds of the number 2 and better) as number 2 boards. We then found the nonparametric estimate of the fifth percentile of the MORs of these 40 000 boards, divided this by 2.1, and took this as our allowable stress property. When we generated assemblies we only accepted boards with MOEs above the 40th percentile of the MOE distribution. Of course, because the correlation between MOE and MOR is not one, this led to boards with MORs that were sometimes below the 40th percentile of the MOR distribution. This led to MOR distributions that were not truly fully truncated. (For an MOE/MOR correlation of one, the MOR distribution would be fully truncated. For an MOE/MOR correlation of zero, the truncated MOR distribution would not differ from the original MOR distribution. Intermediate levels of correlation lead to intermediate levels of "truncation.")

In Fig. 2, we plot histograms of simulated MORs of MSR number 2 and better boards when MOR and MOE have bivariate normal distributions with COVs equal to 0.25 and correlations (top to bottom) of 1.0, 0.9, 0.6, and 0.0. In this figure we also mark allowable stress levels by solid vertical lines. In Fig. 3, we present the corresponding plots for COVs equal to 0.05. These figures suggest that for truncated distributions, a board is more likely (except in the case in which the correlation is 1.0) to have a strength value below the allowable property level for a high-COV distribution than for a low-COV distribution. We report simulations based on truncated distributions in the section Probability of Failure of an Assembly at an Unadjusted Allowable Stress Level.

P_F for Normally Distributed, Perfectly Correlated Strength and Stiffness

In the Normal Distribution and Lognormal Distribution subsections we performed simple calculations that demonstrated that in the normal and lognormal cases, the probability of failure of a *single member* that is subjected to a load equal to its allowable property increases as COV increases. In this section we perform simple calculations that establish that, for a special case (bivariate normal MOE/MOR distribution, perfect correlation between MOE and MOR), the probability that an *assembly* fails at an unadjusted property level increases with COV. These calculations are merely meant to be suggestive. The increase in probability of failure with COV that they suggest has, however, been confirmed by the simulations reported in the remainder of the paper.

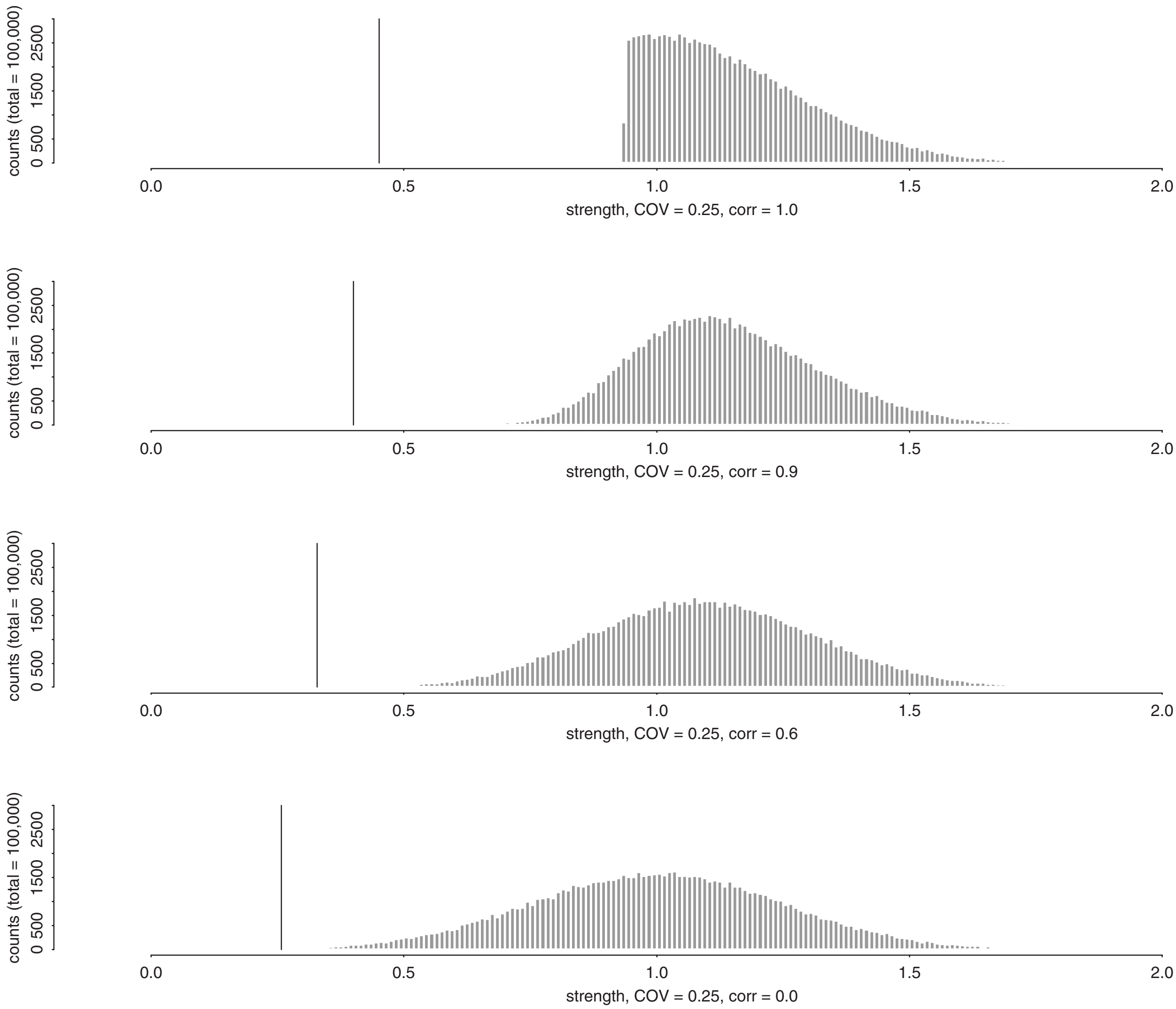

FIG. 2—*"Truncated normal" distributions. The COV is 0.25. The correlations between MOE and MOR are (from top to bottom) 1.0, 0.9, 0.6, 0.0. The vertical lines are at the allowable property levels.*

We have

$$P_F = \text{prob(one or more failures)}$$
$$= 1 - \text{prob(no failure of any member)}$$

There is no failure of any member if and only if

$$(\mu + \epsilon_i)/r(t) \geq \frac{\mu + \epsilon_i}{\mu + \epsilon_1 + \ldots + \mu + \epsilon_k} \times \frac{k \times (\mu - 1.645 \times \sigma)}{2.1} \tag{4}$$

holds for $i=1,\ldots,k$. Here, k is the number of members in the assembly, μ is the mean of the strength distribution of the members, the ϵ's are normally distributed with mean zero and standard deviation σ, $(\mu+\epsilon_i)/r(t)$ is the strength of the ith member, $(\mu+\epsilon_i)/(\mu+\epsilon_1+\ldots+\mu+\epsilon_k)$ is the fraction of the load on the assembly that the ith element sees (under the assumption that strength and stiffness are perfectly correlated), and $k\times(\mu-1.645\times\sigma)/2.1$ is the total load on the assembly.

The k inequalities of Eq 4 hold if and only if

$$1 \geq \frac{r(t)}{(\mu + \bar{\epsilon})} \times \frac{\mu - 1.645 \times \sigma}{2.1}$$

where:

$\bar{\epsilon}=(\epsilon_1+\ldots+\epsilon_k)/k$.

Thus,

$$\begin{aligned}
\text{prob(no failure of any member)} &= \text{prob}(\mu + \bar{\epsilon} \geq \mu \times r(t) \times (1 - 1.645 \times \text{COV})/2.1) \\
&= \text{prob}(N(\mu,\mu^2\text{COV}^2/k) \geq \mu \times r(t) \times (1 - 1.645 \times \text{COV})/2.1) \\
&= \text{prob}(N(0,\text{COV}^2/k) \geq r(t) \times (1 - 1.645 \times \text{COV})/2.1 - 1) \\
&= \text{prob}\left(N(0,1) \geq \frac{\sqrt{k}}{\text{COV}} \times \left(r(t) \times \frac{1 - 1.645 \times \text{COV}}{2.1} - 1 \right) \right) \\
&= \text{prob}\left(N(0,1) \geq \sqrt{k} \times \left(\frac{1}{\text{COV}} \left(\frac{r(t)}{2.1} - 1 \right) - \frac{r(t) \times 1.645}{2.1} \right) \right)
\end{aligned} \tag{5}$$

and

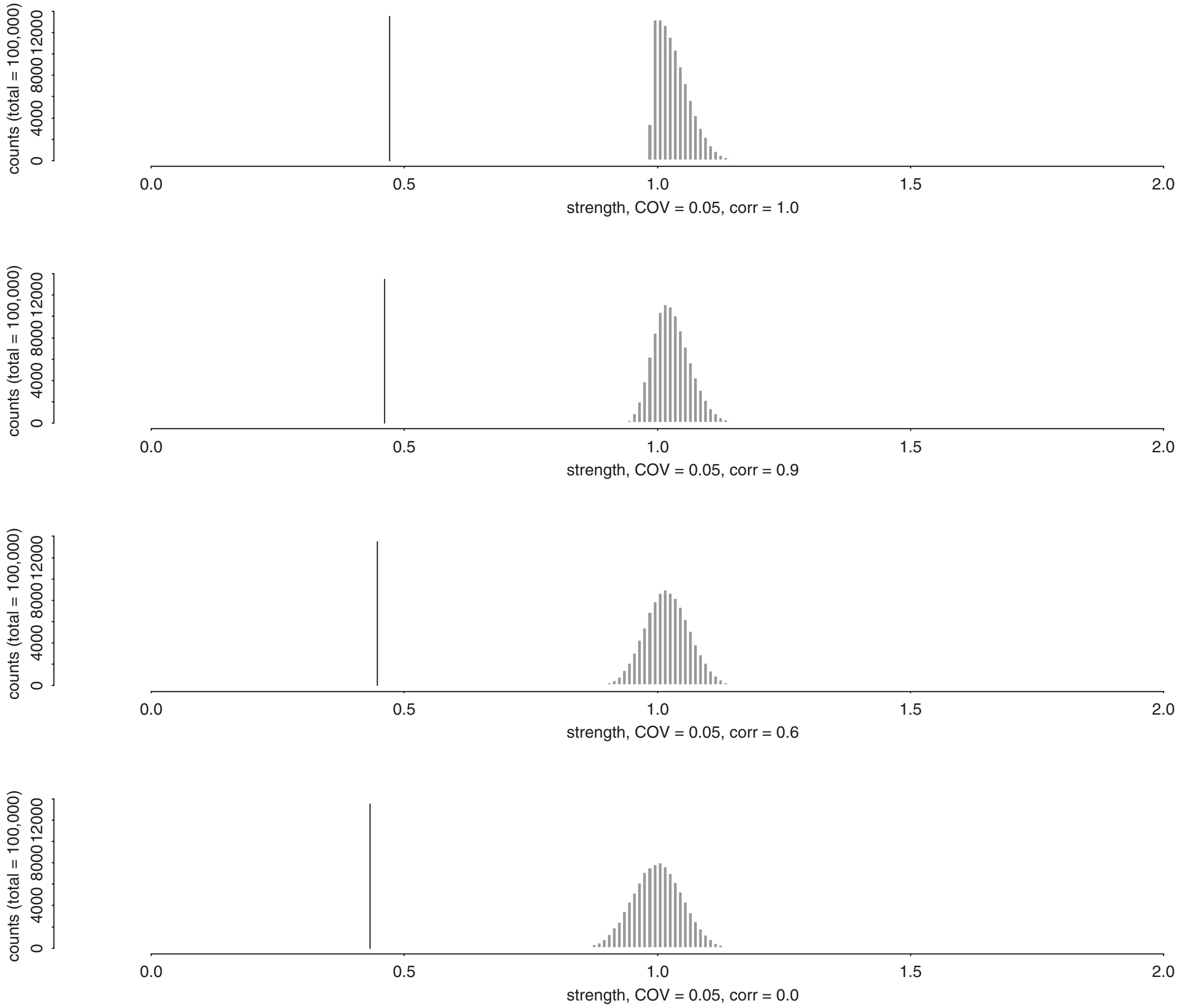

FIG. 3—*Truncated normal distributions. The COV is 0.05. The correlations between MOE and MOR are (from top to bottom) 1.0, 0.9, 0.6, 0.0. The vertical lines are at the allowable property levels.*

$$\text{prob(failure of at least one member)} = \text{prob}\left(N(0,1) < \sqrt{k} \times \left(\frac{1}{\text{COV}}\left(\frac{r(t)}{2.1} - 1\right) - \frac{r(t) \times 1.645}{2.1}\right)\right) \quad (6)$$

Provided that $r(t) < 2.1$, this probability will increase as COV increases.

Probability of Failure of an Assembly at an Unadjusted Allowable Stress Level

For the purposes of this paper, we consider a simple model of seven member assemblies. No claim is made that this is a highly realistic model or that the results can be extrapolated to arbitrary assemblies. However, the results from our simulations do suggest that assemblies of higher COV material do not deserve higher upward repetitive member adjustments.

In an associated Forest Products Laboratory technical report [7], we consider four cases—normal, lognormal, truncated normal, and truncated lognormal. For the purposes of brevity, here we only consider the truncated lognormal case. However, the conclusions that can be drawn from all four simulations are the same.

Truncated Lognormal Distribution Version of the Model

1. Member strengths are drawn from a truncated (see Truncated Distributions subsection) lognormal distribution.
2. Member strengths are reduced by a factor of $r(t)$. In this paper we consider $r(t)=2.1$, 2.0, 1.9, 1.8, 1.7, 1.6, 1.5, and 1. The mean 10 year reduction factor due to duration of load is believed to be 1.6 [8]. We considered the other values in our simulations to establish the trends in P_F as a function of COV as the material ages.
3. The assemblies contain seven members.
4. Each assembly is subjected to the load 7 $\times$(fifth percentile of number 2 strengths)/2.1.
5. Because the assembly elements are connected by a diaphragm, they each bend the same distance, so loads are proportional to MOEs. In particular

$$\text{load}_i = (\text{assembly load}) \times \text{MOE}_i/(\text{MOE}_1 + \dots + \text{MOE}_7).$$

6. The correlation between In(MOR) and MOE is 0.5 or 0.7.
7. The failure of the assembly occurs when any member fails.

The FORTRAN code for this simulation can be obtained in Ref 9.

The results from this simulation depend on $r(t)$. Results for $r(t)=2.1, \dots, 1.5$ and $r(t)=1$ are provided in Table 2. For correlations 0.5 and 0.7 between ln(strength) and MOE, Table 2 contains the COV values, the corresponding probability of failure of a single member, p_S, the weakest link approximation to P_F in Eq 1, and a simulation estimate of P_F that takes into account load sharing.

Here are the relevant facts to note from Table 2. (In Fig. 4 we plot the log of the simulation estimate of P_F versus COV for $\rho = 0.7$ and $r(t)=2.1, 2.0, \dots, 1.5, 1.0$.)

1. As we would expect, due to load sharing, the simulation estimates of P_F are generally lower than the weakest link approximation to P_F given by Eq 1.
2. Also, as we would expect, P_F decreases as the correlation between MOE and MOR increases (as load sharing becomes more effective).
3. Except in the $r(t)=2.1$ case, the simulation estimate of P_F increases with COV, so we would expect that any upward adjustments of allowable stress level should be smaller for larger COVs.

The results from the other three simulations are similar. These results do not support the assumption that higher COVs justify larger upward repetitive member adjustments.

Probability of Failure of an Assembly at an Adjusted Allowable Stress Level

The fact that higher COV material is associated with a higher probability of failure at an unadjusted allowable stress level does not necessarily imply that it does not deserve a higher upward adjustment than a lower COV material. A priori, it is conceivable that the probability of failure of a lower COV material might increase more rapidly as load is increased than the probability of failure for a higher COV material. To check this in a practical situation we ran our model under the three conditions discussed in the pre-2003 ASTM D5055. In particular the authors of that standard permitted a 15 % upward adjustment for material with a 25 % COV, a 7 % upward adjustment for material with an 11 % COV, and a 4 % upward adjustment for material with a 7 % COV. Our results for the truncated lognormal strength distribution are presented in Table 3. Corresponding tables for the other three cases that we considered can be found in Ref 7. The tables do not support the larger upward adjustments for larger COV material that were specified in the standard. Such adjustments lead to probabilities of failure that are often much larger for the high-COV material than for the low-COV material.

The FORTRAN code for these simulations can be obtained in Refs 10–13.

An Objection

A reader who is wedded to the idea that repetitive member adjustments should be larger for materials with a larger COV might make the following argument.

Suppose that individual elements have normally distributed bending moduli with mean μ and variance σ^2. Further suppose that the bending modulus of an assembly is the average of the bending moduli of its elements. Then the fifth percentile of the modulus distribution of an element is

$$\mu - 1.645 \times \sigma = \mu(1 - 1.645 \times \text{COV})$$

and the fifth percentile of the modulus distribution of an assembly of n elements is

$$\mu - 1.645 \times \sigma/\sqrt{n} = \mu(1 - 1.645 \times \text{COV}/\sqrt{n})$$

Thus the ratio of the fifth percentile of the modulus distribution of the assembly to the fifth percentile of the modulus distribution of an element is

$$(1 - 1.645 \times \text{COV}/\sqrt{n})/(1 - 1.645 \times \text{COV})$$

and a little calculus demonstrates that this ratio increases as COV increases. Thus, the argument goes, the repetitive member "adjustment" from the modulus of an element to the modulus of an assembly increases with COV.

There are two problems with this argument. The first and possibly more minor problem is that the modulus of an assembly is not in general the average of the moduli of its elements. In fact, if load transfer to stronger elements is limited (if the correlation between MOE and MOR is not high), the modulus of the assembly can be closer to the modulus of its weakest element than to the average modulus of its elements. The second and more fundamental problem with this argument is that allowable properties are not fifth percentiles. Instead they are currently calculated as fifth percentiles divided by 2.1. Thus, since equivalent assembly designs and repetitive member adjustments should lead to equivalent probabilities of failure under equal loads, we must approach the problem as in the current paper, and as we saw in the Intuition subsection and its sequel, the division by 2.1 leads to individual element probabilities of failure that are higher for more variable material. This in turn leads to assemblies that have higher probabilities of failure for more variable material, and thus we are not justified in awarding such assemblies higher repetitive member adjustments.

A Second Objection

It can be argued that the only age-related strength reduction that is relevant in our tables is $r(t)=2.1$, and for this reduction, individual element probabilities of failure are the same for all COVs, and assembly probabilities of failure do go down as COVs increase. One response to this argument is that we do not actually expect 2.1 reductions due to duration of load. Instead the mean 10 year reduction is believed to be 1.6 [8]. The 2.1 is achieved by multiplying by an additional "factor of safety" of 1.3. A second response is that we do not see failures with the frequency that would be predicted if we really had reductions on the order of 2.1. A third response is that even if we really believed that lifetime reductions in strength were of the order of 2.1, design engineers typically apply an additional 1.3 factor of safety (that does not take into account differences in COV) to the allowable property. In this case, even if we assume a 2.1 reduction in strength due to duration of load, we have (here we use the normal distribution for purposes of illustration)

TABLE 2—*Failure probabilities of single members (p_S) and seven member assemblies (P_F) when the strengths of the single members have been reduced by the factor r(t), the single members are subjected to a load equal to their allowable property level, and the assemblies are subjected to a load equal to seven times the allowable property level for a single member, "truncated lognormal" case.*

		ρ=0.5			ρ=0.7		
			P_F			P_F	
$r(t)$	COV	p_S [a]	Weakest Link[b]	Load Sharing[c]	p_S [a]	Weakest Link[b]	Load Sharing[c]
2.1	0.05	0.381E−01	0.238E+00	0.235E+00	0.344E−01	0.217E+00	0.145E+00
	0.10	0.382E−01	0.239E+00	0.225E+00	0.344E−01	0.217E+00	0.139E+00
	0.15	0.383E−01	0.239E+00	0.218E+00	0.344E−01	0.217E+00	0.134E+00
	0.20	0.382E−01	0.238E+00	0.211E+00	0.344E−01	0.217E+00	0.130E+00
	0.25	0.381E−01	0.238E+00	0.206E+00	0.344E−01	0.217E+00	0.128E+00
	0.30	0.382E−01	0.239E+00	0.202E+00	0.344E−01	0.217E+00	0.126E+00
	0.35	0.381E−01	0.238E+00	0.198E+00	0.344E−01	0.217E+00	0.124E+00
	0.40	0.381E−01	0.238E+00	0.195E+00	0.344E−01	0.217E+00	0.122E+00
2.0	0.05	0.211E−02	0.147E−01	0.153E−01	0.105E−02	0.730E−02	0.323E−02
	0.10	0.103E−01	0.701E−01	0.659E−01	0.718E−02	0.492E−01	0.253E−01
	0.15	0.164E−01	0.109E+00	0.979E−01	0.125E−01	0.844E−01	0.448E−01
	0.20	0.205E−01	0.135E+00	0.117E+00	0.164E−01	0.109E+00	0.578E−01
	0.25	0.232E−01	0.152E+00	0.128E+00	0.190E−01	0.126E+00	0.670E−01
	0.30	0.253E−01	0.164E+00	0.136E+00	0.210E−01	0.138E+00	0.734E−01
	0.35	0.268E−01	0.173E+00	0.141E+00	0.226E−01	0.148E+00	0.772E−01
	0.40	0.278E−01	0.179E+00	0.144E+00	0.238E−01	0.155E+00	0.814E−01
1.9	0.05	0.303E−04	0.212E−03	0.273E−03	0.343E−05	0.242E−04	0.800E−05
	0.10	0.193E−02	0.134E−01	0.128E−01	0.894E−03	0.624E−02	0.263E−02
	0.15	0.595E−02	0.409E−01	0.359E−01	0.363E−02	0.252E−01	0.112E−01
	0.20	0.979E−02	0.665E−01	0.569E−01	0.676E−02	0.464E−01	0.217E−01
	0.25	0.131E−01	0.882E−01	0.733E−01	0.959E−02	0.653E−01	0.311E−01
	0.30	0.158E−01	0.106E+00	0.850E−01	0.120E−01	0.811E−01	0.392E−01
	0.35	0.179E−01	0.119E+00	0.943E−01	0.140E−01	0.937E−01	0.457E−01
	0.40	0.197E−01	0.130E+00	0.102E+00	0.157E−01	0.105E+00	0.512E−01
1.8	0.05	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
	0.10	0.230E−03	0.161E−02	0.159E−02	0.603E−04	0.422E−03	0.132E−03
	0.15	0.176E−02	0.122E−01	0.107E−01	0.792E−03	0.553E−02	0.211E−02
	0.20	0.422E−02	0.292E−01	0.241E−01	0.238E−02	0.165E−01	0.673E−02
	0.25	0.684E−02	0.469E−01	0.378E−01	0.431E−02	0.298E−01	0.126E−01
	0.30	0.926E−02	0.631E−01	0.501E−01	0.631E−02	0.434E−01	0.191E−01
	0.35	0.115E−01	0.779E−01	0.606E−01	0.812E−02	0.555E−01	0.245E−01
	0.40	0.133E−01	0.896E−01	0.689E−01	0.974E−02	0.662E−01	0.298E−01
1.7	0.05	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
	0.10	0.186E−04	0.130E−03	0.117E−03	0.229E−05	0.159E−04	0.300E−05
	0.15	0.400E−03	0.280E−02	0.244E−02	0.126E−03	0.883E−03	0.276E−03
	0.20	0.159E−02	0.111E−01	0.907E−02	0.676E−03	0.472E−02	0.166E−02
	0.25	0.321E−02	0.222E−01	0.177E−01	0.167E−02	0.116E−01	0.436E−02
	0.30	0.505E−02	0.348E−01	0.272E−01	0.302E−02	0.209E−01	0.834E−02
	0.35	0.688E−02	0.472E−01	0.360E−01	0.439E−02	0.304E−01	0.126E−01
	0.40	0.869E−02	0.593E−01	0.445E−01	0.579E−02	0.398E−01	0.167E−01
1.6	0.05	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
	0.10	0.571E−06	0.417E−05	0.800E−05	0.000E+00	0.000E+00	0.000E+00
	0.15	0.714E−04	0.500E−03	0.474E−03	0.133E−04	0.930E−04	0.280E−04
	0.20	0.501E−03	0.350E−02	0.268E−02	0.163E−03	0.114E−02	0.294E−03
	0.25	0.134E−02	0.936E−02	0.730E−02	0.561E−03	0.392E−02	0.127E−02
	0.30	0.255E−02	0.177E−01	0.133E−01	0.127E−02	0.886E−02	0.309E−02
	0.35	0.396E−02	0.274E−01	0.203E−01	0.217E−02	0.151E−01	0.564E−02

TABLE 2— *(Continued.)*

		$\rho=0.5$			$\rho=0.7$		
			P_F			P_F	
$r(t)$	COV	p_S [a]	Weakest Link[b]	Load Sharing[c]	p_S [a]	Weakest Link[b]	Load Sharing[c]
	0.40	0.532E−02	0.367E−01	0.267E−01	0.315E−02	0.218E−01	0.839E−02
1.5	0.05	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
	0.10	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
	0.15	0.101E−04	0.709E−04	0.500E−04	0.100E−05	0.709E−05	0.100E−05
	0.20	0.121E−03	0.847E−03	0.663E−03	0.281E−04	0.197E−03	0.570E−04
	0.25	0.510E−03	0.357E−02	0.265E−02	0.149E−03	0.104E−02	0.299E−03
	0.30	0.118E−02	0.822E−02	0.589E−02	0.474E−03	0.331E−02	0.103E−02
	0.35	0.204E−02	0.142E−01	0.105E−01	0.963E−03	0.672E−02	0.221E−02
	0.40	0.307E−02	0.213E−01	0.153E−01	0.158E−02	0.110E−01	0.378E−02
1.0	0.05	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
	0.10	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
	0.15	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
	0.20	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
	0.25	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
	0.30	0.100E−05	0.709E−05	0.100E−04	0.143E−06	0.834E−06	0.000E+00
	0.35	0.153E−04	0.107E−03	0.510E−04	0.171E−05	0.121E−04	0.300E−05
	0.40	0.424E−04	0.297E−03	0.195E−03	0.586E−05	0.409E−04	0.120E−04

[a]Monte Carlo result based on 7 000 000 simulated members.

[b]Weakest link—$P_F = 1-(1-p_S)^7$. This assumes that all members see the same load.

[c]Load sharing—Monte Carlo result based on 1 000 000 simulated assemblies. Due to load sharing, these simulation estimates of P_F are generally smaller than the weakest link estimates.

$$p_S = \text{prob}[N(\mu,\sigma^2)/2.1 < ((\mu - 1.645 \times \sigma)/2.1)/1.3]$$
$$= \text{prob}[N(\mu,\sigma^2)(1.3/2.1) < (\mu - 1.645 \times \sigma)/2.1]$$
$$\approx \text{prob}[N(\mu,\sigma^2)/1.6 < (\mu - 1.645 \times \sigma)/2.1]$$

so the appropriate sections of our tables are those in which $r(t) = 1.6$. In these sections of the tables, P_F increases as COV increases.

The Wrong Lesson

In questioning the belief that higher COV material deserves a higher repetitive member upward adjustment, we are *not* advancing an argument for upward adjustments in low-COV material that are higher than allowed under current standards. As we noted earlier, our simulations are based on a simple model of an assembly, and although they are sufficient to cast doubt on the belief that higher upward adjustments are justified for higher COV material, they are not sufficient to identify proper upward adjustments (if any).

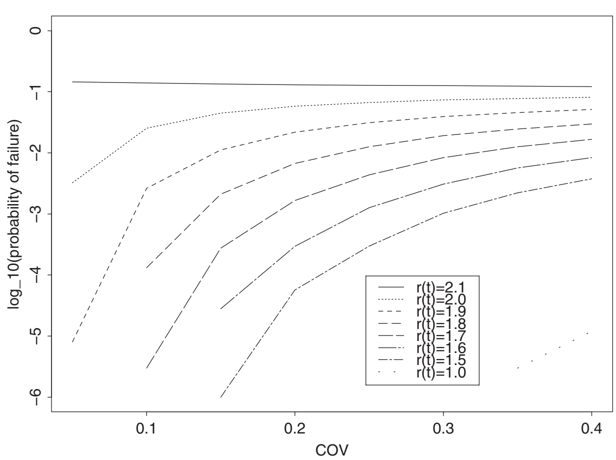

FIG. 4—*Truncated lognormal case. Here, for* $\rho = 0.7$*, we have plotted the logs (base 10) of the simulation* P_F *data in Table 2.*

Summary

We have demonstrated that, for at least one hypothetical assembly, from a reliability[3] standpoint, upward repetitive member adjustments that are larger for more variable material are not justified. On the contrary, our simulations suggest that given the manner in which individual element allowable stress levels are currently calculated, as the COV of the strength distribution of individual elements increases, the upward repetitive member adjustments (if any) of assemblies constructed from these elements should decrease.

These results stem from the fact that under current standards, individual element allowable stress levels associated with higher COV material correspond to larger probabilities of assembly failure. This, in turn, follows from the fact that allowable stress levels are not as many standard deviations below mean strength for a high-COV material as for a low-COV material. Thus, the probabil-

[3]In the simulations reported here, we assumed that the load was fixed at the allowable property. More complete simulations would replace the fixed load with a load distribution whose 99th (for example) percentile was set equal to the allowable property.

TABLE 3—*Failure probabilities of seven member assemblies* (P_F) *when strengths of single members have been reduced by the factor* $r(t)$ *and the assemblies are subjected to loads adjusted in accordance with pre-2003 ASTM D5055, truncated lognormal case.*

		P_F [a]	
$r(t)$	COV	$\rho=0.5$	$\rho=0.7$
2.1	0.07	$0.584E+00$	$0.521E+00$
	0.11	$0.614E+00$	$0.558E+00$
	0.25	$0.556E+00$	$0.497E+00$
2.0	0.07	$0.167E+00$	$0.937E-01$
	0.11	$0.312E+00$	$0.225E+00$
	0.25	$0.416E+00$	$0.339E+00$
1.9	0.07	$0.205E-01$	$0.518E-02$
	0.11	$0.108E+00$	$0.524E-01$
	0.25	$0.284E+00$	$0.204E+00$
1.8	0.07	$0.115E-02$	$0.980E-04$
	0.11	$0.254E-01$	$0.715E-02$
	0.25	$0.177E+00$	$0.107E+00$
1.7	0.07	$0.260E-04$	$0.000E+00$
	0.11	$0.388E-02$	$0.556E-03$
	0.25	$0.982E-01$	$0.486E-01$
1.6	0.07	$0.000E+00$	$0.000E+00$
	0.11	$0.394E-03$	$0.200E-04$
	0.25	$0.483E-01$	$0.189E-01$
1.5	0.07	$0.000E+00$	$0.000E+00$
	0.11	$0.280E-04$	$0.100E-05$
	0.25	$0.211E-01$	$0.601E-02$
1.0	0.07	$0.000E+00$	$0.000E+00$
	0.11	$0.000E+00$	$0.000E+00$
	0.25	$0.120E-04$	$0.000E+00$

[a]Monte Carlo result based on 1 000 000 simulated assemblies. The simulation takes into account both load sharing and the upward adjusted loads.

ity, p_S, that an individual element fails when subjected to a load equal to its allowable stress level is larger for members from strength distributions with larger COVs. This leads to a larger P_F, the probability of at least one member failure in an assembly.

Our results should not be used to argue for a particular upward adjustment in the standards. Improved analytical models and additional empirical testing would be required to establish appropriate (if any) upward repetitive member adjustments.

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