Cantilever-Beam Dynamic Modulus for Thin Fiberboard Composite Products: Part 2 Property Comparison with Bending Modulus

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Abstract

A cantilever beam vibration testing apparatus has been developed to provide a means of dynamic non-destructive evaluation of modulus of elasticity for small samples of wood or composite materials. The apparatus applies a known displacement and records displacement as a function of time to determine frequency and calculate its dynamic modulus. An extensive set of fiberboard panels were made from recycled paper fiber with resin contents of 0.0, 0.5, 1.5, 3.0, and 4.5%; thicknesses of 1.27, 1.90, 2.54, and 4.76mm; and nominal densities of 400, 600, 800, 1000, and 1 150 kg/m^3. Comparisons were made between dynamic-modulus and modulus of elasticity obtained from mid-point bending. Initial analyses showed boards conditioned to 50% RH had an initial slope of 1.04 for the dynamic-to-bending relationship with a correlation of 0.77 R^2. Boards conditioned to 90% RH showed a linear slope of 1.4 dynamic-to-bending relationships with 0.80 R^2 correlations. Further analysis of the data showed linear slopes of 1.2 and 1.44 between DMOE and BMOE after conditioned at 50% RH and 90% RH, respectively. Both had correlation coefficients of 0.97. The dynamic apparatus was able to further differentiate between panels without or with resin when comparing panels conditioned at 50% RH and 90% RH. The correlations for these last data sets were 0.99. This presentation will discuss the comparisons between dynamic and static testing being done at the Forest Products Laboratory and Beijing Forestry University.

Introduction

Evaluation of wood and wood composites properties through vibrational methods have been used with good success for several decades (Moslemi, 1967, Ross and Pellerin, 1994, and Ilic, 2003). They have shown that non-destructive vibrational properties correlate with static bending or tensile moduli as well as being used to obtain damping coefficients. Most of these analyses focused on measuring frequency response using relative amplitude variation through indirect methods such as accelerometers, load cells, piezoelectric films, capacitance rather than direct displacement of the wood or structure in response to a direct load or displacement input.

The USDA Forest Products Laboratory (FPL) has conducted numerous research studies on the use of vibrational methods to determine structural material properties on wood. These studies have focused on solid large wood pieces and structures using either longitudinal stress-wave or simply-supported transverse beam vibration techniques (Ross et.al., 1991; Schad, et.al., 1995; Murphy, 1997; Ross et.al., 2005). For large pieces of wood or structures it is relatively easy to use a simply supported beam method when transverse vibration is used. However, as specimen size is reduced it becomes more difficult to simply support a small light-weight beam and measure vibration. Therefore, as specimen size is reduced, the cantilever beam becomes the preferred method to obtain vibrational characteristics. This is seen in other industries where cantilever beams are used to test material properties of small flat beams (Weihs et.al., 1988; Yam et.al., 2004). The Dynamic Modulus Analyzer (DMA) also uses this principle to analyze small sample beams in forced

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vibration/thermal analyses. The DMA is limited to specimen sizes of 5 mm (t) x 15 mm (b) x 35 mm (L). There has been little development of transverse vibration analysis for traditional composite materials such as veneer, plywood, medium density fiberboard, hardboard, etc.

A new cantilever beam vibration (CBV) apparatus for thin composite materials has been developed that uses non-contact laser measurement methods to directly measure tip displacement. With direct displacement it is possible to determine dynamic modulus of elasticity, damping characteristics, creep, static bending modulus, and shear modulus. As new composite products continue to be developed that have increasing demands on performance, there is a need to have better analysis tools to differentiate products or to describe enhanced performance characteristics. We developed the CBV apparatus to continue work on testing thin to moderately thick wood-fiber composite materials. Though much vibration theory treats the static and dynamic moduli as equivalent (Harris, 2002), differences have been observed in the comparison of static bending and dynamic vibration data for simply-supported beams (Ross et.al., 1991).

The goal of the research is to provide a means to measure transverse vibrational properties of cantilever beams both qualitatively and quantitatively to better understand fundamental wood-composite material properties and their relationship with processing conditions. This development process has included the design of apparatus, development of software, and evolution of a test method (Turk et.al, 2008). This paper focuses on the comparing initial cantilever beam vibration dynamic modulus of elasticity (DMOE) with static bending modulus of elasticity (BMOE).

Cantilever Beam Vibration Theory

The frequency of the first mode of free vibration of a cantilever beam is given by Equation. [1] (Harris, 2002).

$$\omega_{n1} = 2\pi f = \left(\frac{1.875^2}{l^2}\right)\sqrt{\frac{EI}{m_u}} \qquad [1]$$

Where:

ω_{n1} is Frequency of the first natural mode of vibration (radians/s)

f is Detected frequency of the first natural mode of vibration (Hz)

l is Unclamped or "free" length of the cantilever beam (m)

E is Bulk modulus of elasticity (N/m^2)

I is Area moment of inertia of the beam cross-section (m^4)

m_u is Mass per unit length (kg/m)

Equation [1] can be rearranged and written in terms of known values to provide the bulk modulus of elasticity, Equation [2].

$$E = \frac{m_u}{I}(2\pi f)^2\left(\frac{l^2}{1.875^2}\right)^2 = \frac{M}{L}\frac{12}{bt^3}(2\pi f)^2\left(\frac{l^2}{1.875^2}\right)^2 \qquad [2]$$

Where:

M is Mass of the specimen (kg)

L is complete length of the specimen (m)

b is Base width of the specimen (m)

t is Thickness of the specimen (m)

Equation [2] is an idealized equation of vibration that neglects the effects of shear force and rotary motion in the specimen. To determine the relative magnitude of the shear force terms to the idealized solution, Timoshenko (Harris, 2002) introduced a frequency correction factor, **Figure 1,** that was influenced by the mode of vibration (n = 1, 2, 3, ...) and the geometry of the beam as defined by the radius of gyration divided by free length.

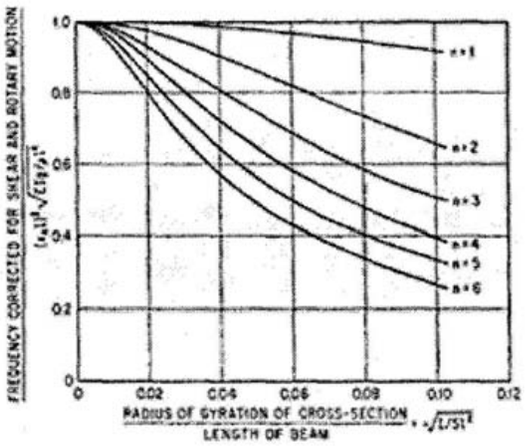

Figure 1. *Influence of shear force and rotary motion on natural frequencies of uniform cantilever beams ($E/_KG$=3.2) The curves relate to the corrected frequency of that given by Equation [2] (Harris, 2002).*

$$Frequency_Correction_Factor = (k_n l)^2 \sqrt{\frac{EIg}{\rho l^4}} \qquad [3]$$

$$\frac{Radius_of_Gyration}{Free_Length} = \frac{\sqrt{\frac{I}{Area}}}{l} = \sqrt{\frac{t^2}{12l^2}} = 0.288\frac{t}{l} \qquad [4]$$

For free-vibration of a cantilever beam (n=1) shear correction factor (Eq.3) approaches 1 when radius of gyration divided by the free length (Eq.4) is approximately less than 0.02 **(Fig.1).** Therefore, the free length of the sample should be adjusted so that the ratio of thickness: length *(t/l)* is less than 0.07 for the shear and torsion effects to be negligible and for the frequency correction factor to approach 1.0.

Bending Beam Theory

The deflection of a simply supported beam with a mid-point load is given by Equation. [5].

$$y = \frac{Pl^3}{48EI} \qquad [5]$$

Where:

y is Mid-point deflection (m)

P is Mid-point load (N)

l_s is Span, simply supported beam length (m)

E is Bulk modulus of elasticity (N/m^2)

I is Area moment of inertia of the beam cross-section (m^4)

$$E = \frac{P}{y}\frac{l_s^3}{48I} = \frac{P}{y}\frac{l_s^3}{4bt^3} \qquad [6]$$

Equation [5] can be rearranged to determine the BMOE **(Eq.6)** based on the load/deflection *(P/y)* curve for a simply supported beam with a constant cross-section.

Experimental

Fiber furnish

The fiber furnish was obtained from hydropulped recycled old corrugated containers (OCC). In previous studies it had been shown to form well and produce composites with excellent properties (Hunt and Vick 1999, Hunt and Supan 2006). The OCC pulp was produced from commercially available corrugated paper box flats by hydropulping to an approximate freeness of 600 CSF (TAPPI 2004). The OCC fiber furnished also had sufficient surface energy that allowed the production of wet-formed mats with low-density without resin ("self-bonded" mat).

Panel fabrication

All test panels were wet-formed at 1% consistency (consistency = dry fiber weight/ (water + fiber weight)). Fiber amounts for each panel were adjusted to achieve a thickness of 1.27, 1.90, 2.54, or 4.76 mm at each of the given density levels. Wet-formed phenol-formaldehyde hardboard resin was used as indicated at 0%, 0.5%, 1.5%, 3.0%, and 4.5% based on dry fiber weight. Aluminum sulfate powder was added to the pulp slurry to drop the pH to 4 to precipitate the resin onto the fibers in the pulp slurry. The slurry was blended for 15 min before forming. After forming and vacuum pressing, the formed sheets had approximately 80% MC, wet basis (400% MC dry basis). Each mat was placed between two screens and two 3.2-mm-thick stainless steel cauls (0.125-in.). The wet mats were hot-pressed at 175°C with continuous pressure to target densities of 400, 600, 800, 1000, and 1150 kg m^{-3} until the panels were dry (approximately 1% to 2% MC). The experimental design included 5 target density levels, 4 target thicknesses, and 5 resin levels **(Table 1).** The number of panels fabricated from each fiber type and condition are also listed in **Table 1.**

Testing

Test specimens were cut from the flat panels to be evaluated for their bending as outlined in the American Society for Testing and Materials Standard D- 1037 (ASTM, 1996). Four bending specimens were cut from each panel. The parameter and data of specimens are listed in **Table 2.**

Two from each group, oriented perpendicular to one another, were placed in a conditioned room held at 22°C (72°F), 50% relative humidity. The other two specimens, also oriented perpendicular to one another and were placed in a conditioned room held at 26.57°°C (80°F), 90% RH.

The CBV apparatus (Turk et. al., 2008) was used to obtain DMOE from specimens different those used for bending. The dynamic vibration specimens were 50.8 mm (2.0 inches) wide and 254 mm (10.0 inches) long. According to Timoshenko's theory, the lengths (beam length minus the length under the clamp of 50.8 mm) resulted in a *t:l* ratio ranged from 0.006 to 0.023 which are significantly less than the 0.07 ratio, where shear or rotation effects may influence the determination of DMOE. The specimens were clamped using a plate and screw gripping mechanism, **Figure 2.**

A torque wrench was used to apply a consistent clamping force through a ¾ inch -16 threaded bolt to the plate that covers 50.8 cm (2.0 inches) on one end of the specimen. On the free end of the

specimen, an adjustable height laser-displacement measuring assembly was used to apply a consistent displacement to the beam specimen. Once the displacement was applied by the triggering mechanism, the operator retracted a spring pin that caused a displacement deck to rapidly rotate away from the specimen, allowing the beam to enter free vibration. The laser measured displacement of the beam tip as a function of time. The CBV software controlled sampling rate at 1000Hz and took a total sample count of between 1000 and 3000 counts. These software parameters provided good data for the specimens, yielding at least 25 sample points per waveform similar to that shown in **Figure 3.**

Table 1. —*Number of panels per target density, thickness, and resin content for recycled old corrugated containerpanels*

Target density (kg m^{-3})	Hot press board pressure (kPa)	Target thickness (mm)	Recycled old corrugated containers with resin contents of				
			0.0 (%)	0.5 (%)	1.5 (%)	3.0 (%)	4.5 (%)
400	103	1.27	3	3	3	3	3
		2.54	3	3	3	3	3
600	206	1.27	3	3	3	3	3
		2.54	3	3	3	3	3
800	483	1.27	3	3	3	3	3
		2.54	3	3	3	3	3
1,000	1,380	1.27	3	3	3	3	3
		1.90	3	—	—	—	—
		2.54	3	3	3	3	3
		4.76	3	—	—	—	—
1,150	2,070	1.27	3	3	3	3	3
		2.54	3	3	3	3	3

Table 2.—*Parameter and data of panel production relevant for bending*

Bending settings	Panel thickness			
	1.27 (mm)	1.90 (mm)	2.54 (mm)	4.76 (mm)
Span (mm)	30.5	45.7	61	114
Length (mm)	81.3	96.5	111.7	165.1
Width (mm)	50.4	50.4	50.4	50.4
Load rate (mm/min)	0.61	0.91	1.22	2.29
Number of specimens	168	24	168	24

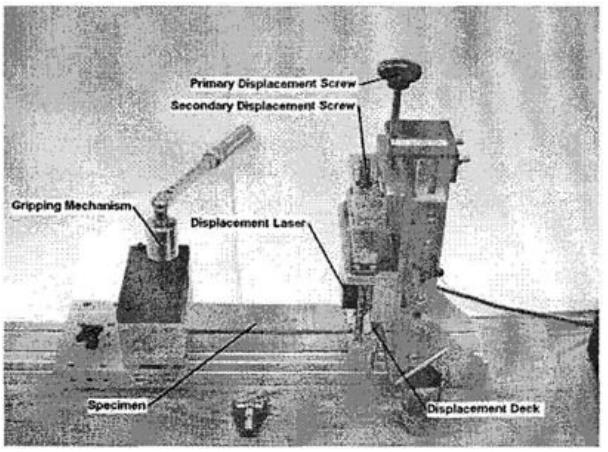

Figure 2. *Cantilever Beam Vibration tester shown with a specimen in a horizontal position.*

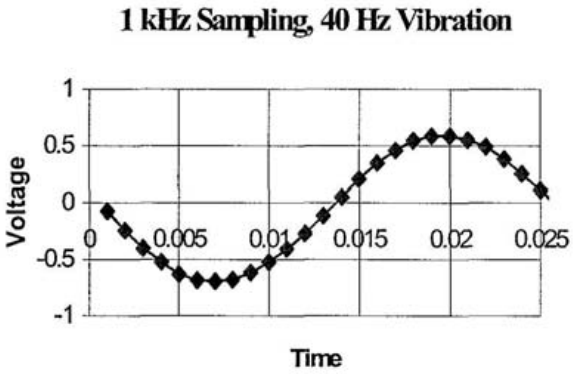

Figure 3. *Data acquisition of approximately 25 data points per wave form yields good representation of the vibrational beam response.*

The specimen's mass (M), total length (L), width (b), and thickness (t) were measured and input into the software. The specimen was inserted 50.8 mm into the grip mechanism and centered with a special plate. The 50.8 mm grip length was subtracted from L to obtain the free beam length (l). The specimen was clamped to obtain no more than 10% compression deflection applied by the clamp screw or a maximum pressure of approximately 689 kPa (100 PSI), whichever is less. An example of a typical specimen (2.3 mm (t) x 50.8 mm (b) x 254 mm (L)) response is shown in **Figure 4.** The free end of the specimen was depressed, and then released into free vibration. The resulting displacement over time was recorded. The bulk modulus was determined according to Equation [2].

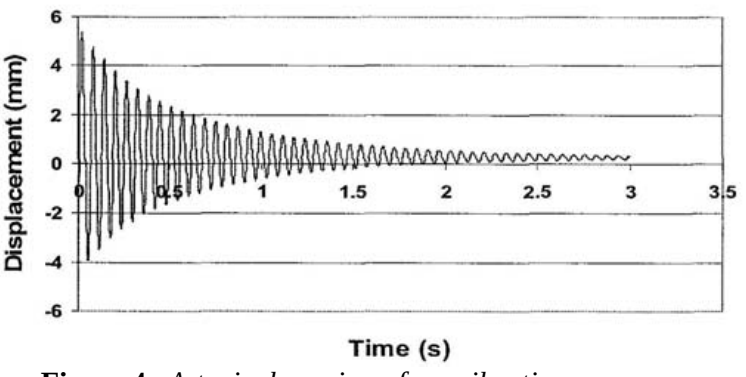

Figure 4. *A typical specimen free vibration response.*

Results and Discussion

The dynamic cantilever beam vibration set-up is shown in **Figure 2** and the bending test set-up is shown in **Figure 5.** Comparisons were made between DMOE and BMOE data sets tested at 50% RH and at 90% RH.

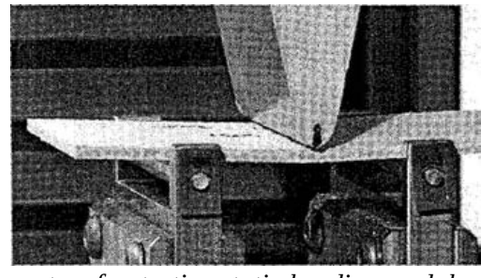

Figure 5. —*Bending set-up for testing static bending modulus of elasticity for hardboard.*

Figure 6 plots the corresponding data between the two test methods. The slopes for the regression lines for 50% RH and for 90% RH were 1.06 and 1.45, respectively. The correlation for the data tested at 50% RH was 0.78 for and the correlation at 90% RH test conditions was 0.80. From **Figure 6,** we might conclude that at 50% RH the DMOE and BMOE are basically equal but at higher moisture contents DMOE is less affected by the increase in moisture resulting in a higher correlation slope. However, there was significant scatter in both data sets and further data analyses follows.

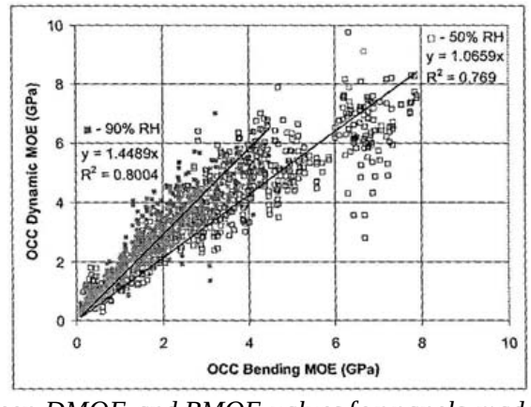

Figure 6. —*Comparison between DMOE and BMOE values for panels made from recycled old corrugated containers tested at 50% RH and 90% RH.*

From Equation [2] and Equation [6], it can be seen that the panel physical dimensions (Length, width, and thickness) were used to determine both DMOE and BMOE. The physical dimensions were measured from different samples, using different equipment, and by different people. This introduces the potential for significant comparison differences. In addition, if there is an error and that measurement is either squared or cubed, the effect on the calculation can have a major impact on the results. Of the physical measurements, thickness has the greatest potential for introducing the greatest error than for either specimen width or length measurement. For example, if the measurement method used to measure these parameters is the same, then the ratio of the accuracy-to-actual will be:

1. Thickness - accuracy (+/- 0.127 mm)/measurement (1.27 to 4.8 mm) = +/- 0.1 to 0.026
2. Width - accuracy (+/- 0.127 mm)/measurement (50.8 mm) = +/- 0.0025
3. Length - accuracy (+/- 0.127 mm)/measurement (250 mm) = +/- 0.0005

The higher this ratio, the higher the potential for error and scatter in the data. The thickness term is used to determine DMOE and BMOE **(Eq.2** and **Eq.6)** and are cubed so it has even greater influence on the calculated results. To remove this potential error effect of thickness *(l/t^3)* from the comparisons, both data sets of DMOE and BMOE were multiplied by the thickness cubed *(t^3)*. **Figure 7** shows the DMOE*t^3 vs. BMOE*t^3 comparison after removing the thickness effect. There was a significant improvement in correlation coefficients. The linear regression slopes were 1.2 for 50% RH and 1.44 for 90% RH. Both had correlation coefficients of 0.97. Without the potential of thickness errors, the plots show that DMOE*t^3 had a significantly higher value than BMOE*t^3. It is also again evident that moisture content had an effect on modulus of elasticity.

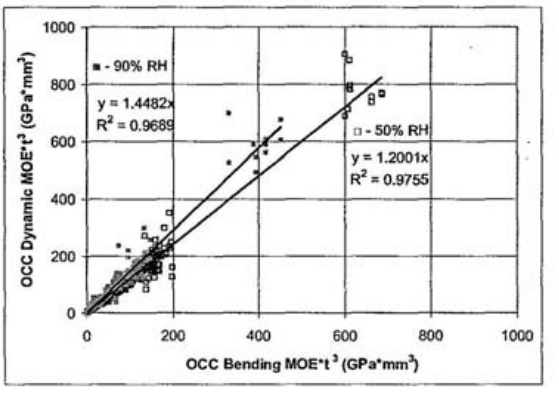

Figure 7. *—Comparison between DMOE and BMOE values multiplied by measured thickness cubed (t^3) values for panels made from recycled old corrugated containers tested at 50% RH and 90% RH.*

A newer apparatus is being built through a joint cooperation effort between Beijing Forestry University and the Forest Products laboratory (Zhang et.al. 2009). The new apparatus places the sample vertical and sequentially measures both static MOE and DMOE. Preliminary data shows a similar slope of 1.18 at 50% RH with a correlation coefficient of 0.96 for wood fiber composite panels.

In another paper and using the same panels as for this paper (Hunt et. al., 2008), it was shown that equilibrium moisture contents for the panels were reduced with the addition of as little as 0.5% resin **(Table 3).** Further comparisons were investigated to determine if resin (or equilibrium moisture content) had any effect on DMOE*t^3 and BMOE*t^3 data.

Table 3. *—Equilibrium moisture content of the lodgepole pine (LP) and old corrugated carton (OCC) panels after conditioning at either 50% RH and 90% RH.*

Resin content (%)	Equilibrium moisture content (%) [Standard deviation]	
	50% RH	90% RH
0.0	6.31 [0.17]	16.6 [0.30]
0.5	5.34 [0.19]	14.8 [0.40]
1.5	5.31 [0.16]	14.5 [0.42]
3.0	5.35 [0.19]	14.7 [0.50]
4.5	5.27 [0.15]	14.3 [0.35]

Figure 8 and **Figure 9** shows DMOE*t^3 vs. BMOE*t^3 data sets plotted without or with resin at either 50% or 90% RH conditions.

These comparisons did not show any significant difference between panels without or with resin in either the 50% or 90% RH test conditions because there was too much scatter in the data. Similarly, when you compare only BMOE*t^3 data for values between 50% RH vs. 90% RH conditions, **Figure 10,** there is still too much scatter in the data to show any significant difference between panels without or with resin.

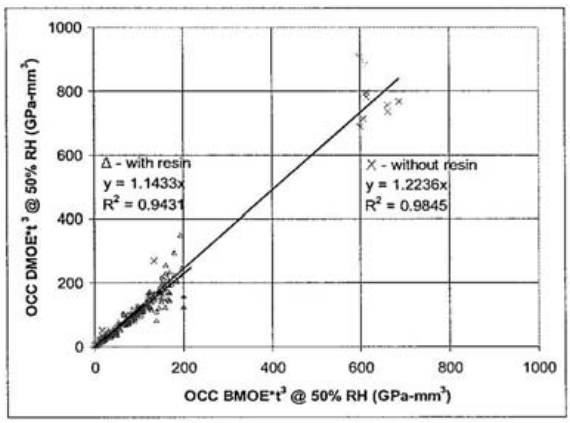

Figure 8. *—Comparison between DMOE and BMOE values multiplied by measured thickness cubed (t3) values for panels made from recycled old corrugated containers tested at 50% RH without and with resin.*

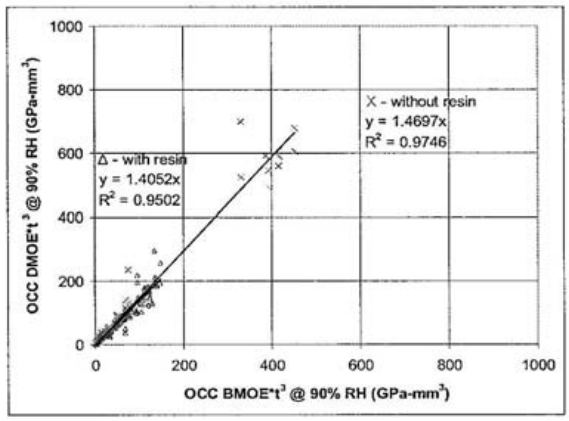

Figure 9. *—Comparison between DMOE and BMOE values multiplied by measured thickness cubed (t3) values for panels made from recycled old corrugated containers tested at 90% RH without and with resin.*

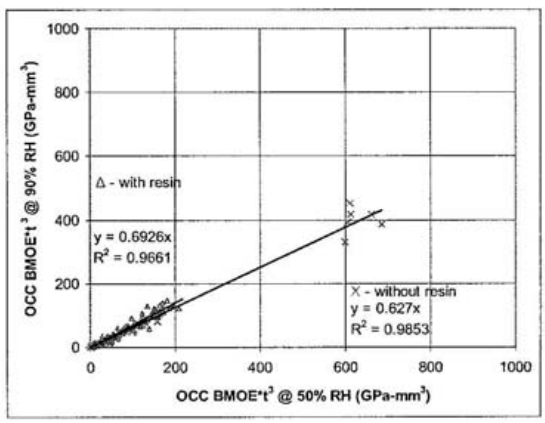

Figure 10. *—Comparison of BMOE values multiplied by measured thickness cubed (t3) values for panels made with or without resin from recycled old corrugated containers tested either at 50% RH and 90% RH.*

In **Figure 11** however, DMOE*t^3 data only shows significant distinguishable differences when compared at 50% RH and 90% RH. The DMOE test apparatus was able to distinguish differences in the data showing that adding resin (or reducing the equilibrium moisture content) does influence modulus of elasticity when exposed to higher relative humidity environments. This last DMOE*t^3 data sets had the best correlation coefficients of 0.998 and 0.992.

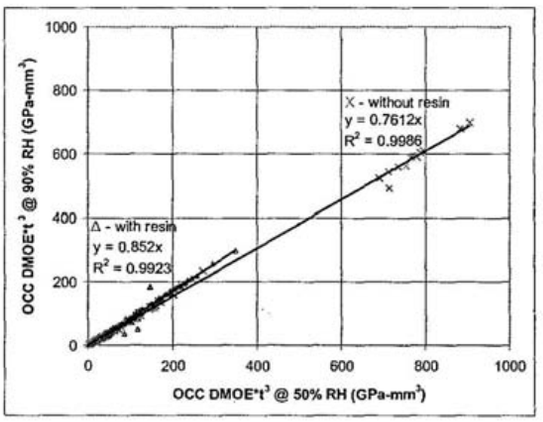

Figure 11. *–Comparison of DMOE values multiplied by measured thickness cubed (t3) values for panels made with or without resin from recycled old corrugated containers tested either at 50% RH and 90% RH.*

The data comparisons between DMOE*t^3 and BMOE*t^3 indicates that the dynamic modulus of elasticity yields higher values than those determined using the static simply supported method. In the literature, there is discussion on the influence of both strain-rate and moisutre on the MOE especially for visco-elastic materials such as wood, paper, and cellulosic composites. Zauscher (Zauscher et.al. 1997) discussed his H-bond theory to describe why the modulus of elasticity for paper increased as strain-rate increased and decreased as moisture content increased. It is possible that the dynamic strain rate caused by the cantilever beam vibration is sufficiently fast to cause an increase in MOE than the much slower strain rate used for the static bending test. It may be necessary to factor in strain-rate and equilibrium moisture content when determining MOE. The clamped-end of the cantilever beam method also introduces different boundary conditions than the free-ends of the simply support beam. The effects of clamped-end of cantilever beam method needs to be further determined. While these questions need further analyses, the cantilever beam vibration method has the potential to provide additional insight into characterizing composite performance not possible with static bending. Further study of these and other effects using the cantilever beam

Summary

- Correlating DMOE vs. BMOE first requires careful measurement of all input values used to calculate the modulus of elasticity. The larger the ratio between measurement accuracy and the measured input value the more care needs to be taken to minimize errors. This is even more critical when those terms are squared or cubed. The potential of errors in the measurement process can lead to wide scatter of data and poor correlations.
- DMOE has a higher calculated modulus of elasticity than that obtained from BMOE for both 50% and 90% RH conditions.
- There is a greater difference between DMOE and BMOE at 90% RH than at 50% RH.
- DMOE values were less affected by moisture content than BMOE.

- DMOE can be used to distinguish between small differences in modulus of elasticity when comparing effects at different resin contents (or equilibrium moisture contents).
- DMOE may provide additional insights into composite panel differences not possible with BMOE.

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