# Cantilever Beam Static Bending and Vibration Test Apparatus Developed for Thin Composite Products

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## Abstract

A new cantilever beam apparatus is being cooperatively developed to measure static and vibration properties of small and thin samples of wood or composite materials. The apparatus applies a known displacement to a cantilever beam, measures static load, then releases the beam into its natural first mode of transverse vibration. The apparatus then records free vibrational displacement as a function of time. The static bending modulus is determined and then the free vibration frequency is used to calculate its dynamic modulus. This presentation will discuss the cooperative research work to develop the apparatus and preliminary results.

## Introduction

There has been much work in the area of non-destructive evaluation (NDE) of large timbers, thick dimensional wood boards, paper, as well as electronics and high-end composite materials, but there has not been much work with wood-fiber composite boards in the range of 1 to 10 mm thick. These composite materials include those from pulp molding, wood-fiber/plastic, plywood, medium density fiberboard, etc.

Evaluation of wood and wood composites properties through vibrational methods have been used with good success for several decades (Moslemi, 1967, Ross and Pellerin, 1994, and Ilic, 2003). In the literature, most if not all the studies that have focused on non-destructive testing of solid large wood pieces use either longitudinal stress-wave or simply-supported transverse beam vibration techniques (Ross et.al., 1991; Schad, et.al., 1995; Murphy, 1997; Ross et.al., 2005). For large pieces of wood it's easy to use a simply supported beam method when transverse vibration is used. However, as specimen size is reduced it becomes more difficult to simply support a small light-weight beam and measure its free vibration without the beam vibrating off the supports. Therefore, as specimen size is reduced, the cantilever beam becomes the preferred method to constrain the specimen on one end to obtain vibrational characteristics from the other.

The USDA Forest Products Laboratory (Turk et.al, 2008) developed a dynamic cantilever beam vibration (CBV) apparatus to test thin to moderately thick wood-fiber composite materials. Based on this initial apparatus, the authors are working cooperatively to develop an improved cantilever beam test apparatus that measures both static bending and vibration properties for thin composite products from one test. This new apparatus combines the two tests into one test sequence This has the advantage of being able to obtain both static and dynamic properties from the same specimen at the same test conditions. The goal of the research is to provide a means to measure

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static bending and transverse vibrational properties of cantilever beams for thin composite materials. As a NDE tool, this apparatus could be used as a standard test method as well as part of quality of cantilever beams for thin composite materials. As a NDE tool, this apparatus could be used as a standard test method as well as part of quality control to quickly assess material properties and their relationship with processing conditions. To date, this development work has included the design of apparatus, development of software, evolution of a test method, and some test results.

## Static Bending and Dynamic Vibration Theory

### Static modulus of elasticity (SMOE) test

For static bending of a cantilever beam, as shown in **Figure 1,** the equation that describes deflection is as follows **(Eq.1):**

$$\delta = \frac{Pl^3}{3EI} \qquad [1]$$

Where:

$P$ = static load (N)

$\delta$ = deflection of static load point (m)

$l$ = unclamped or "free" length of the cantilever beam (m)

$E$ = static modulus of elasticity (N/m$^2$)

$I$ = area moment of inertia of the beam cross-section (m$^4$)

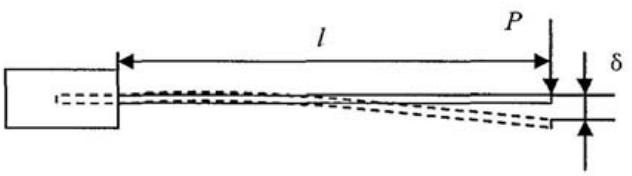


**Figure 1.** *Static bending of a cantilever beam*

In order to calculate the SMOE, we can rewrite Equation **[2]** as follow:

$$E = \frac{Pl^3}{3\delta I} = \frac{4Pl^3}{\delta b t^3} \qquad [2]$$

Where:

$b$ = base width of the beam (m)

$t$ = thickness of the beam (m)

### Dynamic modulus of elasticity (DMOE) test

The frequency of the first mode of free vibration of a cantilever beam is given by Equation **[3]** (Harris, 2002).

$$\omega_{nl} = 2\pi f = \left(\frac{1.875}{l}\right)^2 \sqrt{\frac{EI}{m_u}} \qquad [3]$$

Where:

$\omega_{n1}$= Frequency of the first natural mode of vibration (radians/s)

$f$ = Detected frequency of the first natural mode of vibration (Hz)

$l$ = Unclamped or "free" length of the cantilever beam (m)

$E$ = Bulk modulus of elasticity (N/m$^2$)

$I$ = Area moment of inertia of the beam cross-section (m$^4$)

$m_u$ = Mass per unit length (kg/m)

Equation **[3]** can be rearranged and written in terms of known values to provide the bulk modulus of elasticity, Equation **[4].**

$$E = \frac{m_u}{I}(2\pi f)^2\left(\frac{l^2}{1.875^2}\right)^2 = \frac{M}{L}\frac{12}{bt^3}(2\pi f)^2\left(\frac{l^2}{1.875^2}\right)^2 \qquad [4]$$

Where:

$M$ = Mass of the specimen (kg)

$L$ = Complete length of the specimen (m)

$b$ = Base width of the specimen (m)

$t$ = Thickness of the specimen (m)

Equation **[4]** is an idealized equation of vibration that neglects the effects of shear force and rotary motion in the specimen. To determine the relative magnitude of the shear force terms to the idealized solution, Turk and Hunt et al (Turk, 2008) discussed this and suggested that if care is taken to control the ratio of thickness and length such that the radius of gyration divided by the free length is < 0.005 (dimensionless), the frequency correction factor approaches 1.0 such that the shear and rotary effects are negligible, This works out so that the *l/t* ratio needs to be > 58 for the effects to be negligible.

## Experimental Apparatus

The new CBV apparatus **(Fig.2)** consists of a base, a bracket, a clamp, a laser sensor and a primary displacement mechanism. The specimen clamp can be positioned at 4 different heights along the bracket to accommodate different specimen lengths. While larger apparati could be built, the maximum specimen size the current apparatus can accommodate is 8.6 mm (*t*) x 50 mm (*b*) x 550 mm (*L*). The specimen is clamped using a plate and screw assembly. A torque wrench is used to apply a consistent force through a threaded bolt to a plate that covers 50 mm of one end of the specimen. On the other end of the specimen, an adjustable position laser-displacement measuring assembly is mounted perpendicular to the specimen. The laser is positioned at the end of the specimen and can be adjusted to "zero" input signal at mid-line or zero load position. The primary displacement mechanism consists of a hook connected to a load cell. The hook can be used to apply a pre-load or a consistent initial displacement to the end of the specimen. The hook mechanism also serves as a mount for the triggering mechanism. The initial displacement is adjustable. When the specimen is positioned by the hook at the end of the specimen, the load cell senses the load and the laser measures the displacement. Both measurements are used to calculate the static MOE. A torsion spring keeps the hook horizontal until the operator pushes the hook down to release the beam to free vibration. The resultant free vibration is the first mode of vibration. The laser measures displacement of the beam tip as a function of time. The specimen hangs vertical to minimize gravitational affects during transverse vibration.

The CBV control and analysis software is written in Labview. Software inputs are used to control sampling rate, total sample count, and can be adjusted during each calibration. Once the

software detects an upward deflection, the triggering routine assumes the sample end has been released and the data acquisition routine begins.

The next step in the data analysis is the determination of average pre-load and the frequency of vibration. The software uses multi-average with the input pre-load data and Fast-Fourier Transformation algorithm for the vibration displacement data. A data sampling rate approximately 25 times the vibrational frequency provides sufficient characterization of the vibration for the Fast-Fourier Transformation algorithm to extract a single frequency.

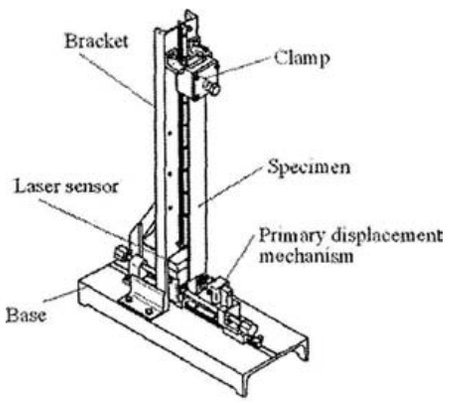


**Figure 2.** —*Cantilever Beam Vibration tester shown with a specimen in position*

After detecting the average pre-load and the vibration frequency, the program has all the necessary information needed in order to determine the static MOE and the theoretical dynamic bulk MOE.

## Testing Procedure

The specimen's mass ($M$), total length ($L$), width ($b$), and thickness ($t$) are measured and input into the software. The specimen is then inserted 50 mm into the clamp and centered with a special plate. The 50 mm grip length is subtracted from $L$ to obtain the free beam length ($l$). The specimen is next clamped using a torque wrench to obtain no more than 10% compression deflection applied by the clamp screw or a maximum pressure of approximately 689 kPa, whichever is less.

An example of a typical specimen (2.6 mm ($t$) x 50 mm ($b$) x 340 mm ($L$)) vibration response is shown in **Figure 3.** A primary displacement of 10.8 mm was applied to the unsupported specimen edge, and then the specimen was released into free vibration. The resulting displacement (voltage) over time is recorded. The first CVB (Turk et al, 2007), held the specimen horizontally, the slight downward drift in the curve took place, due to a small amount of creep due to gravity of the thin-cantilevered fiberboard specimen. This (second) CVB holds specimen vertically, there is no downward drift in the curve.

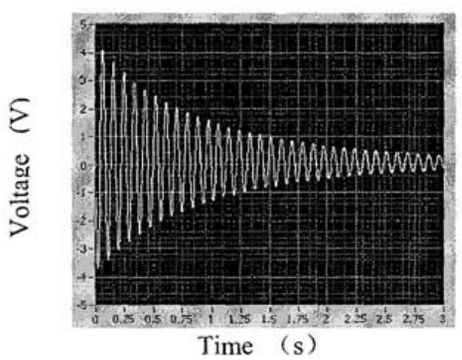


**Figure 3.** —*A typical specimen free-vibration response*

# Testing Results

## Repeatability

To verify the repeatability and non-destructive nature of the testing procedure, a random specimen (2.6 mm (*t*) x *50* mm (*b*) x 340 mm (*L*)) was loaded and tested *5* consecutive times without removing it from the specimen grip or re-adjusting the positioning screws. The results show excellent repeatability, with a maximum variation in recorded frequency of 0.02 Hz **(Table 1).** Similar observations were made with other samples evaluated multiple times. We believe the repeatable response is specifically related to starting at the same initial displacement before release into the beams free vibration. Also, it can be seen that the dynamic MOE is slightly higher that the static MOE.

**Table 2.** *Repeated testing results for a single specimen without repositioning*

| Specimen ID | Static MOE (GPa) | Dynamic MOE (GPa) | Initial Displacement (mm) | Frequency *f* (Hz) |
|---|---|---|---|---|
| ASB2.6-44 | 4.1 1 | 4.88 | 11.1 | 11.68 |
| ASB2.6-44 | 4.16 | 4.50 | 11.1 | 11.70 |
| ASB2.6-44 | 4.17 | 4.49 | 11.1 | 11.69 |
| ASB2.6-44 | 4.18 | 4.49 | 11.1 | 11.69 |
| ASB2.6-44 | 4.13 | 4.50 | 11.1 | 11.70 |
| Average | 4.15 | 4.49 | 11.1 | 11.69 |

## Dynamic MOE and static MOE

**Figure 4** shows results of dynamic MOE and static MOE of 5 MDF types, total 255 specimens, tested by the apparatus. It can be seen that the dynamic MOE is slightly higher than the static MOE, and there is a good relationship between the two Moes. Future research will compare strain rates to determine if the increaseed strain rates due to dynamic vibration result in higher MOE values.

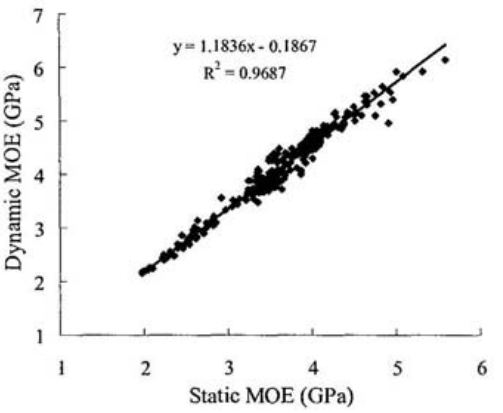


**Figure 4.** *Dynamic MOE and static MOE tested by the apparatus*

# Summary

The cantilever beam static bending and vibration test apparatus provides a quick method to measure pre-load and end displacement of a fiberboard composite beam for both static and dynamic response. The apparatus applies a known displacement to the beam before it is released to free vibration. The testing is non-destructive and highly repeatable in determining static MOE, dynamic MOE. Software has been developed that rapidly and successfully processes the test data. The goal is to develop an apparatus and analysis tools to quickly produce comparable static bend data with

that of standard bending values while also producing additional information about a specimens dynamic properties. We believe that the new apparatus will provide both qualitatively and quantitatively information to better understands the fundamental material properties of thin fiber-based composites.

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