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Analytical Determination of the Surface Area of a Threaded Fastener

ABSTRACT: Accurate determination of corrosion rates for threaded fasteners hinges on the ability to determine the surface area on which corrosion is occurring. Currently, no general analytical expression of surface area exists for the threaded fastener types. A recent voluntary withdrawal of chromated copper arsenate as the primary, long-standing preservative treatment for wood resulted in the need to determine the corrosion rates of threaded fasteners. This paper developed general analytical surface area equations for a wedge-shaped thread and the area between the threads for three cases: (1) an increasing thread root and crest diameter, (2) constant thread root and crest diameters, and (3) a constant thread root but a decreasing thread crest diameter. The expressions are applied, numerically verified, and compared to simplified models for a No. 10–2.54 cm (1-in.) long wood screw.

KEYWORDS: corrosion, analytical, fastener, screw, surface area

Nomenclature

A = total surface area
$a+b\theta$ = lower limit of integration for radius variable
B = head diameter
$c+d\theta$ = upper limit of integration for radius variable
D = crest diameter of fastener
E = shank diameter
H = head height
h_c = height of smooth cylindrical section
h = height of frustum of a right cone
K_c = corrosion rate constant
K = root diameter of fastener
k_D = slope of thread crest in tip region
k_K = slope of thread root in tip region
k_m = slope of thread crest during transition threaded to smooth shank section
k_t = thread slope
$k_\rho = \frac{\rho}{2\pi}$
l_m = length of reducing crest diameter into smooth shank section
l_{tip} = distance from tip of fastener to intersection of conical and cylindrical core sections
L_T = distance from tip of fastener to end of threaded fastener section
m_i = initial mass of corrosion specimen
m_f = final mass of corrosion specimen
R_c = corrosion rate
r_K = root radius of fastener
r_D = crest radius of fastener
r_c = radius of smooth cylinder
r_o = radius of larger end of a frustum of a right cone
r_i = radius of smaller end of a frustum of a right cone
$T = 1+k_t^2$
TR = length of smooth taper at end of threaded shank section
t_i = initial time when corrosion specimen entered given environment
t_f = final time when corrosion specimen entered given environment
t_w = thread width at root diameter
ρ = distance between thread crests
γ = density of corrosion specimen
θ_A = lower limit of integration for angle variable
θ_B = upper limit of integration for angle variable
Γ = surface area

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Introduction

In almost every timber engineering application, wood is in intimate contact with metal. Metallic fasteners embedded in wood are subject to corrosion by the presence of water and oxygen in the cellular structure of wood. Historically, the waterborne preservative chromated copper arsenate (CCA) had been used to extend the service life of outdoor wood structures. Due to the voluntary withdrawal of chromated copper arsenate (CCA) for residential use, many designers are now choosing to use alternatives to CCA such as alkaline copper quaternary (ACQ) and alkaline copper azole (CuAz). Limited published research exists detailing the effects alkaline-based preservatives have on the corrosion of fasteners in contact with wood, although it is believed that ACQ and other new preservatives are more corrosive than CCA [1].

While exposure and accelerated tests exist for the evaluation of corrosion in ACQ-treated wood, the accuracy of these tests is dependent on the ability of the researcher to precisely measure the quantities used to calculate the corrosion rate. The corrosion rate, R, in any weight loss test is commonly calculated from Eq 1 where m_i and m_f are the initial and final masses (g), t_i and t_f are the initial and final times (h), A is the surface area (cm^2), γ is the density (g/cm^3), and K_c is a constant (87 600 mm $\times$ cm^{-1} $\times$ h $\times$ year^{-1}).

$$R = K_c \frac{m_f - m_i}{A\gamma(t_f - t_i)} \tag{1}$$

Previous work for exposure tests [2,3] have highlighted the difficulty in determining the surface area of fasteners used in corrosion tests, and has shown that as the duration of the experiment is increased, the uncertainty in the corrosion rate measurement is dominated by uncertainties in the measurement of surface area. Currently electrochemical corrosion tests are being developed for fasteners in treated wood [4]. Since electrochemical tests can have lower experimental variation, accurate calculation of surface area is even more important.

Since the last half of the 20th century there has been an increase in the use of threaded fasteners, such as wood screws, deck screws, drywall screws, and annularly threaded nails. For accurate determination of the corrosion performance of these fastener types, generalized analytical surface area expressions are needed for use in Eq 1. This paper develops general mathematical expressions to determine surface area for a threaded fastener containing a wedge-like thread, applies these expressions to one specific fastener, and makes comparisons to estimates of surface areas by simplified models.

General Surface Area Equations

In general, a point on the surface of a body is described by the position vector

$$R = xi + yj + zk \tag{2}$$

to describe the entire surface we introduce two parameters u and v so that

$$x = x(u,v),\ y = y(u,v),\ z = z(u,v)$$

and

$$R = R(u,v) \tag{3}$$

Two vectors R_u and R_v can be defined at the intersecting point of R used in two contours. The surface area is the norm of the cross product of these two vectors, given as follows [5]:

$$\mathrm{A} = \int\int_{\Im} \mathrm{dA} = \int\int_{\Re} \| \bar{\mathrm{R}}_u \times \bar{\mathrm{R}}_v \| \, dudv \tag{4}$$

Taking the derivatives of R and applying the equations will result in the following expression

$$d\mathrm{A} = \sqrt{EG - F^2}\, dudv \tag{5}$$

where

$$\mathrm{E} = \left(\frac{\partial x}{\partial u}\right)^2 + \left(\frac{dy}{\partial u}\right)^2 \left(\frac{\partial z}{\partial u}\right)^2 \tag{6}$$

$$\mathrm{F} = \left(\frac{\partial x}{\partial u}\right)\left(\frac{\partial x}{\partial v}\right) + \left(\frac{dy}{\partial u}\right)\left(\frac{dy}{\partial v}\right) + \left(\frac{\partial z}{\partial u}\right)\left(\frac{\partial z}{\partial v}\right) \tag{7}$$

$$\mathrm{G} = \left(\frac{\partial x}{\partial v}\right)^2 + \left(\frac{dy}{\partial v}\right)^2 \left(\frac{\partial z}{\partial v}\right)^2 \tag{8}$$

This paper will discuss the application of Eqs 4–8 by breaking the fastener into three major regions: the thread, the root (the area between the threads), and the body surface (Fig. 1). The surface area of these three regions will be discussed separately starting with the thread. The total surface area of the entire fastener is the sum of these three individual surface area regions.

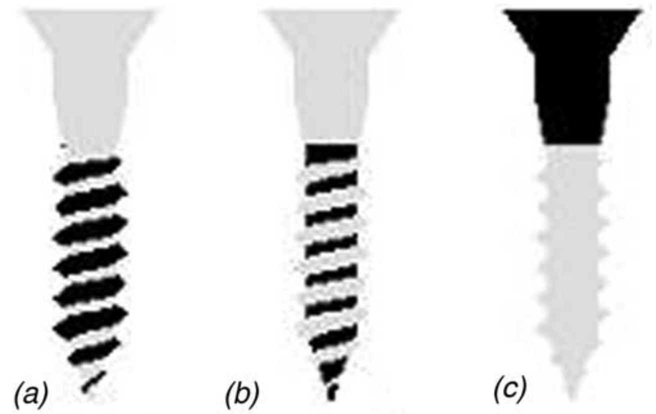

FIG. 1—*Sections of calculations of surface area (a) thread, (b) root, and (c) body.*

Thread Surface Equations

Before presenting the general analytical surface area expression we will present general equations that represent the thread geometry and the resulting integral surface area expression by application of Eqs 4–8. In general, a wedge-shaped thread on a fastener can be broken into three sections and modeled by a different thread geometry equation in each section to find the surface area. The three sections are: (1) fastener tip, where the thread root and crest diameter are increasing, (2) threaded shank, where the thread root and crest diameter are constant, and (3) mating section, where the root diameter is constant and crest diameter is decreased to mate with the smooth shank (Fig. 2).

Within the fastener tip region, two expressions will be needed to characterize the thread surface. The following expression with the plus sign represents the upper thread surface, while the negative sign represents the lower thread surface.

$$x = r\cos\theta \quad y = r\sin\theta \quad z = k_\rho \theta \pm k_t\left(\frac{k_\rho}{k_D}\theta - r\right) \tag{9}$$

where $k_\rho = \rho/2\pi$, $k_D = r_D/l_{tip}$, $k_t = thread\ slope = t_w/(D-K)$, t_w=the thread width at the root diameter, and the remaining variables are shown in Fig. 3.

Applying Eqs 4–8, along with the upper surface relationship generates the following expression for that surface area.

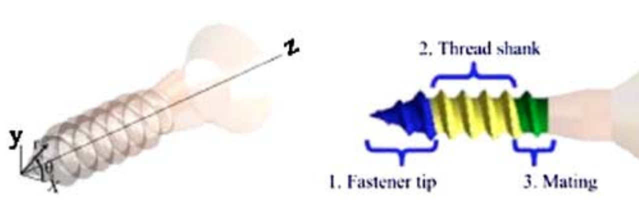

FIG. 2—*Thread coordinate system and analysis sections.*

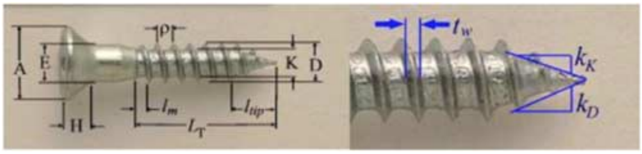

FIG. 3—*Geometry of threaded fastener.*

$$\Gamma^{upper\ thread} = \int\int \sqrt{r^2(1+k_t^2) + \left(k_\rho + \frac{k_t k_\rho}{k_D}\right)^2}\,drd\theta \qquad (10)$$

similarly, an expression for the lower surface can be generated using Eq 9 with the negative sign in Eqs 4–8

$$\Gamma^{lower\ thread} = \int\int \sqrt{r^2(1+k_t^2) + \left(k_\rho - \frac{k_t k_\rho}{k_D}\right)^2}\,drd\theta \qquad (11)$$

which is the same general expression for the upper thread surface area, as given in Eq 10.

For the upper and lower thread surface in the constant section the following expressions describe a point on the surface

$$x = r\cos\theta \quad y = r\sin\theta \quad z = k_\rho\theta \pm k_t(r_D - r) \qquad (12)$$

and the corresponding surface area using Eq 12 along with expressions 4–8 for both the upper and lower thread surface is

$$\Gamma = \int\int \sqrt{r^2(1+k_t^2) + k_\rho^2}\,drd\theta \qquad (13)$$

For the thread surface in the mating section the following expression applies for a point on the surface

$$x = r\cos\theta \quad y = r\sin\theta$$
$$z = k_\rho\theta \pm + k_t\left\{\left[r_D - \frac{1}{k_m}(k_\rho\theta - (L_T - l_m))\right] - r\right\} \qquad (14)$$

and corresponding surface area using Eq 14 with expressions 4–8 is

$$\Gamma^{upper\ thread} = \int\int \sqrt{r^2(1+k_t^2) + \left(k_\rho - \frac{k_t k_\rho}{k_m}\right)^2}\,drd\theta \qquad (15)$$

Similarly, an expression for the lower surface can be generated using Eq 14 with the negative sign in Eqs 4–8

$$\Gamma^{lower\ thread} = \int\int \sqrt{r^2(1+k_t^2) + \left(k_\rho + \frac{k_t k_\rho}{k_m}\right)^2}\,drd\theta \qquad (16)$$

In general the following expression represents the surface area of the threaded surface for all three regions and thread surfaces

$$\Gamma = \int\int \sqrt{Tr^2 + \Psi_i^2}\,drd\theta \qquad (17)$$

where

$$T = 1 + k_t^2 \qquad (18)$$

$$\Psi_1 = k_\rho + \frac{k_t k_\rho}{k_D} \qquad (19)$$

$$\Psi_2 = k_\rho - \frac{k_t k_\rho}{k_D} \qquad (20)$$

$$\Psi_3 = k_\rho \qquad (21)$$

$$\Psi_4 = k_\rho - \frac{k_t k_\rho}{k_m} \qquad (22)$$

$$\Psi_5 = k_\rho + \frac{k_t k_\rho}{k_m} \qquad (23)$$

in which Ψ_1 and Ψ_2 are valid in the cone region for the upper and lower thread surface area, respectively, Ψ_3 is valid in the straight region for both the upper and lower thread surface while Ψ_4 and Ψ_5 are valid in the mating region for the upper and lower thread surface area, respectively.

The surface area of the threaded portion of the fastener can be evaluated using the following three general cases:

$$\text{Case 1: } \Gamma = \int_{\theta_A}^{\theta_B}\int_{a+b\theta}^{c+d\theta} \sqrt{Tr^2 + \Psi_i^2}\,drd\theta \qquad (24)$$

$$\text{Case 2: } \Gamma = \int_{\theta_A}^{\theta_B}\int_{a}^{c+d\theta} \sqrt{Tr^2 + \Psi_i^2}\,drd\theta \qquad (25)$$

$$\text{Case 3: } \Gamma = \int_{\theta_A}^{\theta_B}\int_{a}^{c} \sqrt{Tr^2 + \Psi_i^2}\,drd\theta \qquad (26)$$

The only difference between these equations are the limits of integration, where $a+b\theta$ and $c+d\theta$ are, respectively, the lower and upper limits of integration for the radius, while θ_A and θ_B are, respectively, the lower and upper limits of integration for the angle. Depending on the region of interest different limits of integration are needed to properly describe the geometry. General limits for the develop expression are giving in Appendix A.

In general the thread surface area within the tip and mating regions (1 and 3) are evaluated using a combination of Eqs 24 and 25. For the thread shank region (2) only Eq 26 is needed.

Solving Eqs 24–26 for the general limits of integration yields the following expressions for each case

Case 1:

$$\Gamma = \int_{\theta_A}^{\theta_B}\int_{a+b\theta}^{c+d\theta} \sqrt{Tr^2 + \Psi_i^2}\,drd\theta$$
$$= \frac{1}{2}\left[\begin{array}{l}
\frac{\sqrt{T(c+d\theta_B)^2+\Psi_i^2}}{Td}\left(\frac{T(c+d\theta_B)^2+\Psi_i^2}{3} - \Psi_i^2\right) + \\
\frac{\Psi_i^2}{\sqrt{T}}\left(\theta_B + \frac{c}{d}\right)\ln\left(\sqrt{T}(c+d\theta_B) + \sqrt{T(c+d\theta_B)^2+\Psi_i^2}\right) \\
- \frac{\sqrt{T(a+b\theta_B)^2+\Psi_i^2}}{Tb}\left(\frac{T(a+b\theta_B)^2+\Psi_i^2}{3} - \Psi_i^2\right) \\
- \frac{\Psi_i^2}{\sqrt{T}}\left(\theta_B + \frac{a}{b}\right)\ln(\sqrt{T}(a+b\theta_B) + \sqrt{T(a+b\theta_B)^2+\Psi_i^2}) \\
- \frac{\sqrt{T(c+d\theta_A)^2+\Psi_i^2}}{Td}\left(\frac{T(c+d\theta_A)^2+\Psi_i^2}{3} - \Psi_i^2\right) \\
- \frac{\Psi_i^2}{\sqrt{T}}\left(\theta_A + \frac{c}{d}\right)\ln(\sqrt{T}(c+d\theta_A) + \sqrt{T(c+d\theta_A)^2+\Psi_i^2}) \\
+ \frac{\sqrt{T(a+b\theta_A)^2+\Psi_i^2}}{Tb}\left(\frac{T(a+b\theta_A)^2+\Psi_i^2}{3} - \Psi_i^2\right) \\
+ \frac{\Psi_i^2}{\sqrt{T}}\left(\theta_A + \frac{a}{b}\right)\ln(\sqrt{T}(a+b\theta_A) + \sqrt{T(a+b\theta_A)^2+\Psi_i^2})
\end{array}\right] \qquad (27)$$

Case 2:

$$\Gamma = \int_{\theta_A}^{\theta_B}\int_{a}^{c+d\theta} \sqrt{Tr^2+\Psi_i^2}\,dr\,d\theta$$

$$= \frac{1}{2}\left[\begin{array}{l} \frac{\sqrt{T(c+d\theta_B)^2+\Psi_i^2}}{Td}\left(\frac{T(c+d\theta_B)^2+\Psi_i^2}{3}-\Psi_i^2\right) \\ +\frac{\Psi_i^2}{\sqrt{T}}\left(\theta_B+\frac{c}{d}\right)\ln(\sqrt{T}(c+d\theta_B)+\sqrt{T(c+d\theta_B)^2+\Psi_i^2}) \\ -\frac{\sqrt{T(c+d\theta_A)^2+\Psi_i^2}}{Tb}\left(\frac{T(c+d\theta_A)^2+\Psi_i^2}{3}-\Psi_i^2\right) \\ -\frac{\Psi_i^2}{\sqrt{T}}\left(\theta_B+\frac{c}{d}\right)\ln(\sqrt{T}(c+d\theta_B)+\sqrt{T(c+d\theta_B)^2+\Psi_i^2}) \\ -\left[a\sqrt{Ta^2+\Psi_i^2}+\frac{\Psi_i^2}{\sqrt{T}}\ln(\sqrt{T}a+\sqrt{Ta^2+\Psi_i^2})\right](\theta_B-\theta_A) \end{array}\right] \tag{28}$$

Case 3:

$$\Gamma = \int_{\theta_A}^{\theta_B}\int_{a}^{c} \sqrt{Tr^2+\Psi_i^2}\,dr\,d\theta$$

$$= \frac{(\theta_B-\theta_A)}{2}\left[\begin{array}{l} c\sqrt{Tc^2+\Psi_i^2}+\frac{\Psi_i^2}{\sqrt{T}}(\sqrt{T}c+\sqrt{Tc^2+\Psi_i^2}) \\ -a\sqrt{Ta^2+\Psi_i^2}-\frac{\Psi_i^2}{\sqrt{T}}(\sqrt{T}a+\sqrt{Ta^2+\Psi_i^2}) \end{array}\right] \tag{29}$$

Root Surface Area

For the surface area between the threads, the root surface area, two general regions could be analyzed. The two regions are the fastener tip (labeled "1" in Fig. 2), where the root diameter is increasing, and a region where the root diameter is constant (labeled "2&3" in Fig. 2).

For the thread surface in the fastener tip section the following expression applies for a point on the surface

$$x = r\cos\theta \quad y = r\sin\theta \quad z = k_K r \tag{30}$$

where $k_K = r_K / l_{tip}$ and r_K is the root diameter and corresponding surface area using expressions 4–8 will lead to the following general expression

$$\Gamma = \int_{\theta_A}^{\theta_B}\int_{a+b\theta}^{c+d\theta} r\sqrt{1+k_K^2}\,dr\,d\theta \tag{31}$$

solving for the limit of integration gives

$$\Gamma = \frac{\sqrt{1+k_K^2}}{2}\left[\frac{(d^2-b^2)}{3}(\theta_B^3-\theta_A^3)+(cd-ab)(\theta_B^2-\theta_A^2)+(c^2-a^2)\times(\theta_2-\theta_1)\right] \tag{32}$$

For the constant root diameter region, in the fastener threaded shank section the following expression describes a point on the surface

$$x = r_K\cos\theta \quad y = r_K\sin\theta \quad z = z \tag{33}$$

and corresponding surface area using expressions 4–8 will lead to the following general expression

$$\Gamma = \int_{\theta_A}^{\theta_B}\int_{a+b\theta}^{c+d\theta} r_K\,dz\,d\theta \tag{34}$$

solving for the limit of integration gives

$$\Gamma = r_K\left[\frac{(d-b)}{2}(\theta_B^2-\theta_A^2)+(c-a)(\theta_2-\theta_1)\right] \tag{35}$$

Body Surface Area

Outside the regions where the threads exist, expressions for the body surface areas can be generated from classical expressions. Several expressions can be used (depending on the geometry of the fastener of interest) but it is likely that the following two geometric shapes will be utilized for all fasteners, a cylinder, and a frustum of a right circular cone. The surface area of the body of a cylinder is given by

$$\Gamma = 2\pi r_c h_c \tag{36}$$

where r_c is the radius and h_c is the height of the cylinder, and the surface area of the frustum is

$$\Gamma = \pi(r_o+r_i)\sqrt{(r_o+r_i)^2+h^2} \tag{37}$$

where r_o and r_i are the radii for each end of the frustum, and h is the distance between the parallel circles. By summing the surface area expression for the thread, surface, and body the entire surface area of the threaded fastener can be determined as shown by the specific example in the next section.

Application

In general the application of the above expression consists of defining the limits of integration relative to the tip of the fasteners. The following example applies developed surface area equations to a No. 10–2.54-cm (1-in. long) wood screw shown in Fig. 4. According to the ASME [6], the screw has the following geometric parameters for a cut thread type:

Body Diameter: E=0.483 cm (0.190 in.)
Thread Major Diameter: D=0.483 cm (0.190 in.)
Thread Minor Diameter: K=0.330 cm (0.130 in.)
Thread Thickness: t_w=0.124 cm (0.049 in.)
Thread Spacing: ρ=0.196 cm (0.077 in.)
Tip Length: l_{tip}=0.476 cm (0.188 in.)
Thread Length: L_T=1.667 cm (0.656 in.)
Reduction Length: l_m=ρ
Taper: TR=0.318 cm (0.125 in.)
Head Height: H=0.295 cm (0.116 in.)
Head Diameter: B=0.889 cm (0.350 in.)

For each three major regions (tip, shank, and mating) along with the transition for the tip to threaded shank, general analytical expressions for the limits of integration are given in Appendix A tables. Expressions are general to any fastener with a wedge shaped

FIG. 4—*Actual No. 10–2.54-cm long wood screw and mathematical model.*

fastener that satisfies the assumptions used to derive Eqs 27–29, 32, and 35. To model the root surface geometry during the transition between the tip and shank regions both Eqs 30 and 33 are utilized. Equation 30 is applied to model the geometry until $z=l_{tip}$, while 33 is valid for $z>l_{tip}$. Stated differently, Eq 30 is used to model the geometry until the inner diameter of the cone equals the thread minor diameter, which will occur at l_{tip} from the fastener end, and thereafter by Eq 33. Actual fasteners seem to have a gradual transition in this region. This simplified approach will slightly overestimate the actual surface area, as determined by surface area Eqs 32 and 35, but the overestimate will be small.

For the No. 10–2.54-cm long wood screw surface area equations, limits of integration (using Appendix A expression), and calculated surface area for each model part of the fastener is shown in Table 1.

Comparison to Numerical Analysis and Surface Area Estimates

The No. 10–2.54-cm (1-in.) long wood screw was modeled using same geometric Eqs 9, 12, 14, 30, and 33, to develop the general surface area integrals formulas, along with the expressions for the smoothed taper, smooth cylinder shank, and the fastener head. A

TABLE 1—*Surface area results for each location and total for No 10–2.54-cm (1-in.) long wood screw.*

Region	Calculation Surface	Eq[a]	Outer Integrand Limit θ_A	θ_B	Inner Integrand Limits $a+b\theta$: a	b	$c+d\theta$: c	d	Surface Area $\frac{\text{cm}^2}{1000}$
Tip	Upper thread	27(1)	0	13.926	…	0.0119	…	0.0157	64.71
	Lower thread	27(2)	0	15.341	…	0.0088	…	0.0157	133.03
	Root area	32	0	6.283	…	…	…	0.0088	9.82
	Root area	32	6.283	15.341	−0.0745	0.0119	…	0.0088	79.77
Tip–Shank Transition	Upper thread	28(1)	13.926	15.341	0.1651	…	…	0.0157	23.88
	Lower thread[b]	28(2)	15.341	17.340	0.2413	…	−0.0949	0.0150	46.09
	Root area–tip	32	15.341	17.340	−0.0745	0.0119	−0.0949	0.0150	25.52
	Root area–tip	32	17.340	20.209	−0.0745	0.0119	0.1651	…	22.96
	Root area–shank	35	17.340	20.209	0.4775	…	−0.0622	0.0311	21.15
Shank	Upper thread	29(3)	15.341	47.246	0.1651	0.2413	−0.0622	0.0311	642.36
	Lower thread	29(3)	17.340	47.246	0.1651	0.2413	−0.0622	0.0311	602.11
	Root area	35	20.209	47.246	−0.1334	0.0311	−0.0622	0.0311	317.47
Mating	Upper thread	28(4)	47.246	53.529	0.1651	…	0.8143	−0.0121	59.10
	Lower thread	28(5)	47.246	53.529	0.1651	…	0.8143	−0.0121	59.70
	Root	35	47.246	53.529	−0.1334	0.0311	−0.5302	0.0410	106.05
	Root	35	53.529	59.812	0.3968	0.0212	1.6662	…	69.17
							Total Thread Surface Area:		2282.88
Body		Eq.	r_i	r_o	h	r_c	h_c	…	
	Taper	33	0.165	0.242	0.318	…	…	…	416.88
	Shank	32	…	…	…	0.242	0.262	…	396.65
	Head	33	0.242	0.444	0.295	…	…	…	771.16
							Total Surface Area:		3867.56

[a]Number in parenthesis denotes the Ψ used in equations: $1=\Psi_1$, $2=\Psi_2$, $3=\Psi_3$, $4=\Psi_4$, and $5=\Psi_5$.
[b]$\int_a^b f(x)dx=-\int_b^a f(x)dx$ was utilized for calculation of surface area.

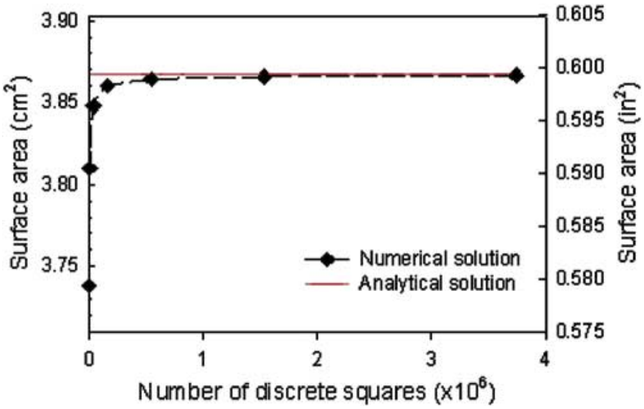

FIG. 5—*Comparison of analytical and numerical surface area calculations for No. 10–2.54-cm (1-in.) long wood screw.*

model generated from the equations and the actual fastener is shown in Fig. 4.

Using the geometric model data, the surface area was determined numerically by discretizing the model into a number of grids, taking the norm of the cross products of the vectors at each one of these grids and summing the results over the entire model. Figure 5 shows the results of the numerical analysis along with the analytical solutions. It is seen the numerical solution converges to the analytic solution as the number of grids increases and validates the general analytical expression for surface area.

Comparison of Surface Area Expression to Simplified Area Models

Because this method to calculate the surface area is so involved, it is worthwhile to examine how closely one can approximate the area with simplifying assumptions. The percent error will be calculated for four simplified models to determine the surface area. Since the only difference in these simplified models (estimates) and detailed expression is the treatment of the threaded portion of the fasteners, only these differences will be discussed. All estimates start with the minimum surface area of the root radius and idealize the tip as a cone, Eq 38. The first estimate approximates the threaded surface are right frustum placed back to back that extends the entire length of the threaded portion including the tip, Eq 39. The second estimate is the same as the first estimate but excludes threads in the tip region. Estimates three, Eq 41, and four, Eq 42, are the same as estimates one and two but increase the thread surface area using the ratio of the angled thread length to perpendicular threaded length. Equations for the base surface and four estimates are as follows:

$$\Gamma_b = \pi r_K \sqrt{r_K^2 + l_{tip}^2} + 2\pi r_K (L_T - l_{tip}) \tag{38}$$

$$\Gamma_1 = \Gamma_b + 2\pi \frac{(L_T - l_{tip})}{\rho}[(r_D + r_K)\sqrt{(r_D - r_K)^2 + t_w^2/4} - r_K t_w] \tag{39}$$

$$\Gamma_2 = \Gamma_b + 2\pi \frac{L_T}{\rho}[(r_D + r_K)\sqrt{(r_D - r_K)^2 + t_w^2/4} - r_K t_w] \tag{40}$$

TABLE 2—*Comparison of surface area expressions and simplified models.*

Estimate No.	Surface Area (cm^2) Simplified	Analytical	Difference	Percent Difference
1	4.120	3.869	0.251	6.49
2	3.822	3.869	−0.047	−1.22
3	4.129	3.869	0.260	6.71
4	3.828	3.869	−0.041	−1.06

$$\Gamma_3 = \Gamma_b + 2\pi \frac{(L_T - l_{tip})}{\rho}\left[(r_D + r_K)\sqrt{(r_D - r_K)^2 + t_w^2/4} - r_K t_w\right]\frac{\sqrt{(2\pi r_D)^2 + \rho^2}}{2\pi r_D} \tag{41}$$

$$\Gamma_1 = \Gamma_b + 2\pi \frac{L_T}{\rho}[(r_D + r_K)\sqrt{(r_D - r_K)^2 + t_w^2/4} - r_K t_w]\frac{\sqrt{(2\pi r_D)^2 + \rho^2}}{2\pi r_D} \tag{42}$$

Using these four estimates, a comparison to the analytical expression can be undertaken. The absolute difference and percent difference between estimates and analytical formulas are given in Table 2. From this table, the percent difference ranges between −1.06 and 6.71 % for the four models considered.

The authors are also developing electrochemical tests [4], similar to ASTM G 59 Standard Test Method for Conducting Potentiodynamic Polarization Resistance Measurements where instead of the standard cylindrical coupon, a portion of a threaded fastener is used. For these tests, the head and smooth shank of the fastener have been removed, only the threaded portion of the fastener is tested. Surface area comparisons between the analytical methods to estimates 3 and 4 resulted in a difference of 3.2 and −2.46 %, respectively. In all geometries studied percent difference is over 1 % and is dependent on the characteristics of the threaded fastener.

Conclusion

Knowledge of the surface area is essential to determine the rate of corrosion. Until this paper, the determination of the surface area of a threaded fastener has been one of the barriers to determining the corrosion rates of screws and threaded nails, quantitatively. For a wedge-shaped thread profile, general surface area expressions for both the threads and area between the thread roots were developed for the tip, threaded shank, and mating regions of the fastener. These analytical expressions were applied to a specific wood screw to highlight the surface area calculation procedure and to validate the expression by comparison to a numerical procedure. Use of these expressions along with an imaging system will allow for the determination of the surface area for a wide range of threaded fasteners.

Appendix A

Tables A1–A4

TABLE A1—*Equations and integration limits for tip region of fastener (UT=upper thread, LT=lower thread and RT=root).*

		Theta		Radius	
Surface	Eq[a]	Lower limit θ_A	Upper limit θ_B	Lower Limit $a+b\theta$	Upper limit $c+d\theta$
UT	35(1)	0	$[k_t r_K + l_{tip}] / k_\rho (1+k_t/k_D)$	$[k_\rho (1+k_t/k_D)/(k_K+k_t)]\theta$	$(k_\rho/k_D)\theta$
LT	35(2)	0	l_{tip} / k_ρ	$[k_\rho (1-k_t/k_D)/(k_K-k_t)]\theta$	$(k_\rho/k_D)\theta$
RT	41	0	2π	0	$[k_\rho (1-k_t/k_D)/(k_K-k_t)]\theta$
RT	41	2π	l_{tip} / k_ρ	$-[\rho(1+k_t/k_D)/(k_K+k_t)] + (k_\rho + k_t k_\rho/k_D) / (k_K+k_t)$	$k_\rho (1-k_t/k_D)/(k_K-k_t)\theta$

[a]Number in parenthesis denotes the Ψ used in equations: $1=\Psi_1$, $2=\Psi_2$, $3=\Psi_3$, $4=\Psi_4$, and $5=\Psi_5$.

TABLE A2—*Equations and integration limits for region connecting the tip to threaded shank of fastener (UT=upper thread, LT=lower thread and RT=root).*

		Theta		Radius	
Surface	Eq[a]	Lower limit θ_A	Upper limit θ_B	Lower Limit $a+b\theta$	Upper limit $c+d\theta$
UT	36(1)	$[k_t r_K + l_{tip}] / k_\rho (1+k_t/k_D)$	l_{tip} / k_ρ	r_K	$(k_\rho/k_D)\theta$
LT	36(2)	l_{tip} / k_ρ	$[r_K(k_K-k_t)+k_t r_D] / k_\rho$	$-k_t r_d / (k_k-k_t) + k_\rho / (k_k-k_t)$	r_D
RT	41	l_{tip} / k_ρ	$[r_K(k_K-k_t)+k_t r_D] / k_\rho$	$-\rho(1+k_t/k_D) / (k_K+k_t) + (k_\rho + k_t k_\rho/k_D) / (k_K+k_t)$	$-k_t r_d / (k_k-k_t) + k_\rho / (k_k-k_t)$
RT	41	$[r_K(k_K-k_t)+k_t r_D] / k_\rho$	$[k_t r_K + l_{tip}] / k_\rho (1+k_t/k_D) + 2\pi$	$-\rho(1+k_t/k_D) / (k_K+k_t) + (k_\rho + k_t k_\rho/k_D) / (k_K+k_t)$	r_K
RT	44	$[r_K(k_K-k_t)+k_t r_D] / k_\rho$	$[k_t r_K + l_{tip}] / k_\rho (1+k_t/k_D) + 2\pi$	l_{tip}	$-k_t(r_D-r_K)+k_\rho\theta$

[a]Number in parenthesis denotes the Ψ used in equations: $1=\Psi_1$, $2=\Psi_2$, $3=\Psi_3$, $4=\Psi_4$, and $5=\Psi_5$.

TABLE A3—*Equations and integration limits for the threaded shank region of fastener (UT=upper thread, LT=lower thread and RT= root).*

		Theta		Radius	
Surface	Eqn[a]	Lower limit θ_A	Upper limit θ_B	Lower Limit $a+b\theta$	Upper limit $c+d\theta$
Upper Thread	29(3)	l_{tip} / k_ρ	$(L_T-l_m) / k_\rho$	r_K	r_D
Lower Thread	29	$[r_K(k_K-k_t)+k_t r_D] / k_\rho$	$(L_T-l_m) / k_\rho$	r_K	r_D
Root	35	$[k_t r_K + l_{tip}] / k_\rho (1+k_t/k_D) + 2\pi$	$(L_T-l_m) / k_\rho$	$k_t(r_D-r_K)-\rho+k_\rho\theta$	$-k_t(r_D-r_K)+k_\rho\theta$

[a]Number in parenthesis denotes the Ψ used in equations: $1=\Psi_1$, $2=\Psi_2$, $3=\Psi_3$, $4=\Psi_4$, and $5=\Psi_5$.

TABLE A4—*Equations and integration limits for the region connecting the threaded shank to the smooth shank region of fastener (UT=upper thread, LT=lower thread and RT=root).*

Surface	Eqs[a]	Theta: Lower limit, θ_A	Theta: Upper limit, θ_B	Radius: Lower Limit, $a+b\theta$	Radius: Upper limit, $c+d\theta$
UT	28(4)	$(L_T-l_m)/k_\rho$	$[L_T-(k_t/k_m)(L_T-l_m)-(r_D-r_K)k_t]/k_\rho(1-k_t/k_m)$	r_K	$r_D+(L_T-l_m)/k_m-(k_\rho/k_m)\theta$
UT[b]	27(4)	$[L_T-(k_t/k_m)(L_T-l_m)-(r_D-r_K)k_t]/k_\rho(1-k_t/k_m)$	L_T/k_ρ	$r_D+(L_T-l_m)/k_m-L_T/k_t+(k_\rho/k_t)(1-k_t/k_m)\theta$	$r_D+(L_T-l_m)/k_m-(k_\rho/k_m)\theta$
LT	28(5)	$(L_T-l_m)/k_\rho$	L_T/k_ρ	r_K	$r_D+(L_T-l_m)/k_m-(k_\rho/k_m)\theta$
LT[b]	28(5)	L_T/k_ρ	$[L_T+(k_t/k_m)(L_T-l_m)+(r_D-r_K)k_t]/k_\rho(1+k_t/k_m)$	r_K	$r_D+(L_T-l_m)/k_m+L_T/k_t-k_\rho(1/k_m+1/k_t)\theta$
RT	35	$(L_T-l_m)/k_\rho$	$(L_T-l_m)/k_\rho+2\pi$	$k_t(r_D-r_K)-\rho+k_\rho\theta$	$-k_t(r_D+(L_T-l_m)/k_m-r_K)+k_\rho(1+k_t/k_m)\theta$
RT[b]	35	$(L_T-l_m)/k_\rho+2\pi$		$k_t(r_D+(\rho+L_T-l_m)/k_m-r_K)-\rho+k_\rho(1-k_t/k_m)\theta$	$-k_t(r_D+(L_T-l_m)/k_m-r_K)+k_\rho(1+k_t/k_m)\theta$
RT	(35)	$[L_T+(k_t/k_m)(L_T-l_m)+(r_D-r_K)k_t]/k_\rho(1+k_t/k_m)$	$[L_T-(k_t/k_m)(L_T-l_m)-(r_D-r_K)k_tk_\rho(1-k_t/k_m)+2\pi$	$k_t(r_D+(\rho+L_T-l_m)/k_m-r_K)-\rho+k_\rho(1-k_t/k_m)\theta$	L_T

[a]Number in parenthesis denotes the Ψ used in equations: $1=\Psi_1$, $2=\Psi_2$, $3=\Psi_3$, $4=\Psi_4$, and $5=\Psi_5$.

[b]Integration limit not used for No. 10–2.54 cm wood screw example since $l_m=\rho$. These expression are validate only if $l_m>\rho$.

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