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SIMULATENEOUS HIERARCHICAL AND MULTI-LEVEL OPTIMIZATION FOR MATERIAL CHARACTERIZATION AND DESIGN OF EXPERIMENTS

John G. Michopoulos
Naval Research Laboratory
Center of Computational
Material Science
Washington DC 20375, USA
john.michopoulos@nrl.navy.mil

Tomonari Furukawa
University of New South Wales
School of Mechanical & Manufacturing Engineering
New South Wales, Australia
t.furukawa@unsw.edu.au

John C. Hermanson
USDA Forest Products Laboratory
Wood Based Materials and Structures
Madison, WI, USA
jhermans@wisc.edu

Samuel G. Lambrakos
Naval Research Laboratory
Center of Computational
Material Science
Washington DC 20375, USA
sam.lambrakos@nrl.navy.mil

ABSTRACT

A hierarchical algorithmic and computational scheme based on a staggered design optimization approach is presented. This scheme is structured for unique characterization of many continuum systems and their associated datasets of experimental measurements related to their response characteristics. This methodology achieves both online (real-time) and offline design of optimum experiments required for characterization of the material system under consideration, while also achieving a constitutive characterization of the system. The approach assumes that mechatronic systems are available for exposing specimens to multidimensional loading paths and for the acquisition of data associated with stimulus and response behavior. Material characterization is achieved by minimizing the difference between system responses that are measured experimentally and predicted based on model representation. The performance metrics of the material characterization process are used to construct objective functions for the design of experiments at a higher-level optimization. The distinguishability and uniqueness of solutions that characterize the system are used as two of many possible measures adopted for construction of objective functions required for design of experiments. Finally, a demonstration of the methodology is presented that considers the best loading path of a two degree-of-freedom loading machine for characterization of the linear elastic constitutive response of anisotropic materials.

Keywords: design optimization, material characterization, design of experiments, mechatronic systems, constitutive response, anisotropic materials, composites.

INTRODUCTION

Automated mechatronic systems capable of applying multidimensional loading and collecting specimen response data present two unique opportunities with respect to data driven inverse modeling. The first and more well-known opportunity is the exploitation of experimental data for parameter estimation associated with models describing material constitutive behavior. The second and certainly less explored opportunity is the dynamic-data-driven identification of the optimum design of experiments required for achieving best exploitation of data for parameter estimation. The goal of this paper is to describe a methodology that is structured for consideration of both opportunities. Accordingly, the ultimate goal of the effort described here is to demonstrate online and offline exploitation of data relative to its effect on model formation and design of experiments within the context of dynamic data driven application systems (DDDAS) adopted for

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structural health monitoring and critical event prediction [1-3]. It should be mentioned here that the present effort is different from all other DDDAS efforts in that is formulated for a multidimensional loading (data-space) and is the only one known to the authors that is presented for material systems and their corresponding parameter identification experiments.

Utilization of data-driven design optimization practices in order to determine constitutive behavior parameters of materials under mechanical loadings has been based traditionally on experimental procedures having rigid architectures and no consideration of the influence of experimental design on the quality of the material parameter estimation. The advent of mechatronic systems, however, characterized by multiple degrees of freedom, and thus capable of multidimensional mechanical loading [4-5], has introduced the potential of multiple designs of experiments for the acquisition of behavioral data essential for parameter estimation.

The work presented in this paper is motivated by the goal of demonstrating that it is possible to dynamically affect the manner by which data are gathered in multidimensional data and model spaces. The contribution of the present work that departs from other DDDAS efforts is based on the presentation of a hierarchical design optimization methodology that interrelates two successive design optimization subprocesses for the case of material property identification and its corresponding optimal experiments. One of these subprocesses is responsible for the traditional parameter estimation associated with either linear or nonlinear material constitutive behavior; the other subprocess is responsible for the nontraditional parameter estimation associated with the characterization of the loading path followed by a multidimensional loading frame. In particular, this approach allows for the development of a DDDAS that adapts such that two sets of objectives are satisfied. The first set of objectives contains those related to determining the material parameters and is based solely on physical performance measures of the parameterization or model selected to represent the constitutive behavior of a given material. The second set of objectives contains those related to determining the online parametric characteristics of an experimental sequence as controlled by a multiple degree of freedom loading machine. It is significant to note that one can extend this optimization methodology to include determination of offline characteristics of experiments.

The concept of a meta-objective function is constructed to determine performance of a given constitutive model having been employed in the previous optimization cycle iteration. Thus, the experimental design is generated dynamically as data are being acquired in a fashion that optimizes the performance of the lower level optimization employed for the material parameter estimation.

The paper continues with a section that defines the methodology presented here. Subsequently, an application related to characterizing the elastic response of a composite material is described where the performance of the characterization process is defined in terms of the uniqueness and distinguishability of the parameter set that has been deduced as solution of a singular value decomposition (SVD) problem. Finally, an example of the methodology is described, which is followed by a discussion of results and future research.

HIERARCHICAL FRAMEWORK

The hierarchical nature of the methodology presented here is based on the observation that there are at least two layers of design optimization activities that can be involved in using data obtained from experiments for the characterization of a system.

As shown in Fig. 1 the lower level (i.e., level-1) is assigned with the more traditional task of identifying the parameters associated with the behavior of a model in general, and the material constitutive model in particular. A performance specification for that model and an instantaneous snapshot of its behavior, as instantiated from the previous set of material model parameters, are used for specification of the optimizer's objectives (in terms of the related objective functions) and associated equality or inequality constraints. In this

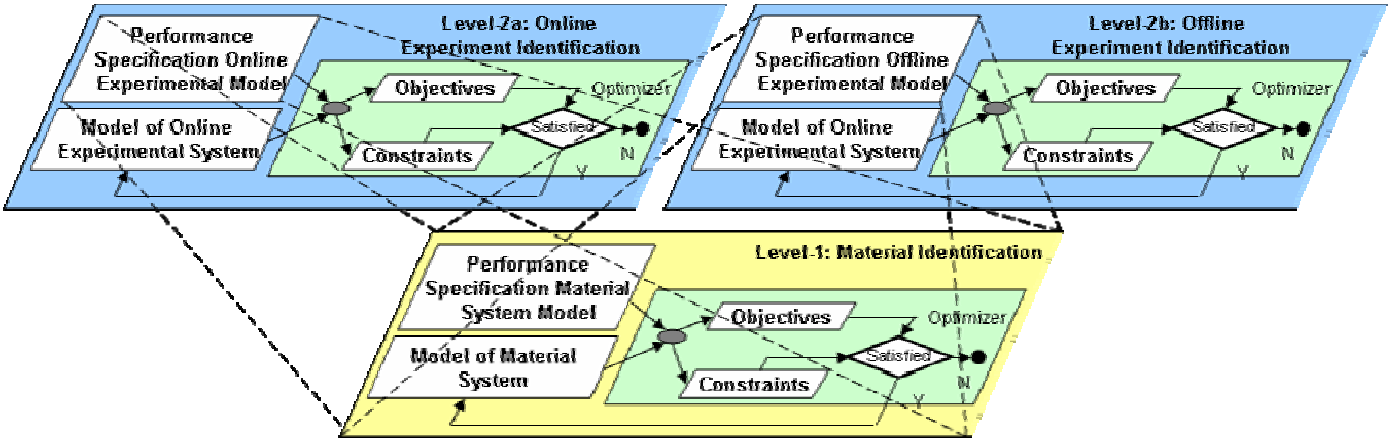

Figure 1: Design Optimization Hierarchy

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performance specification of the material system model, it is usually required that the output of the tentative parameterization or model be within a given tolerance relative to experimental data characterizing the behavioral response of the system.

At the higher level (i.e., level 2) of this hierarchy, essentially all the components discussed in the lower level still exist. For this level, however, instead of designing the material model, optimization is utilized for the design experimental procedures. This optimization is achieved by determining parameters that characterize design of experiments in particular. This second level can be further subdivided into sublevels. One sublevel involves characterizing the experiment dynamically online with the material characterization itself (i.e., level 2a). The other involves characterizing the features of the experiment in an offline fashion prior to execution of the experiment (i.e., level 2b). The blocks shown in Fig. 1 designated by the labels "model of online experimental system" do in fact correspond to the model approximating the experimental procedure itself. The blocks in Fig. 1 designated by the labels including "performance specification" correspond to the performance specification of this experimental model for both online and offline cases.

The performance specifications of level-2a and level-2b can be defined in terms of objectives that maximize the quality of the determined model at the lower level and also maximize its computational performance. The necessary and sufficient condition for this to be possible, requires the creation of objective functions at this level, that are expressed in terms of design variables that express quality features of model determination at level-1. These objective functions can further be endowed with measures of computational performance such as speed of calculation, computational cost of algorithm, level of accuracy, etc. Thus multi-objective function determination is implied here with at least two different partitions. One partition expresses the numerical performance of the lower level model while the other expresses its computational efficiency. For each acceptable determination of a model (establishment of a set of material model parameters) there is a set of values expressing the quality of the numerical and computational operations that depend on various decisions made (codified as design variable instantiation) directly related to the procedure according to which experiments are to be conducted. The fact that the overall performance of the lower level is adopted for performance specification at the higher level implies that indeed the higher level represents the design meta-level of the lower level.

Figure 2 shows a restructuring of the design optimization levels such that the ordering is from the perspective of applying initially the offline design optimization and subsequently the online optimization. In the case of this restructuring the online optimization includes the determination of both the material constitutive parameters and the experimental parameters. This representation utilizes information theoretic semantics and clearly indicates that the offline optimization must be performed by means of finite element analysis (FEA) of the specimen. Accordingly, it is necessary to adopt a parameterized model representation of the constitutive response.

GENERAL SYSTEM PRELIMINARIES

In the general case of the lower level plane on Fig. 1, it is expected that a system under identification is described by some general form of the type:

$$\mathbf{y} = \boldsymbol{f}(\mathbf{p};\mathbf{x}) \tag{1}$$

where $\mathbf{y} \in Y^{q_y} \subseteq \mathbb{R}^{q_y}, \mathbf{x} \in X^{q_x} \subseteq \mathbb{R}^{q_x}$ represent q_y output, q_x input state variables, respectively. The vector $\mathbf{p} \in P^p \subseteq \mathbb{R}^p$ represents p unknown parameters characterizing the system (i.e., design variables within a design optimization context). The vector function $\boldsymbol{f}(\mathbf{p};\mathbf{x}) \in Y^{q_y} \subseteq \mathbb{R}^{q_y}$ represents the behavior of the system.

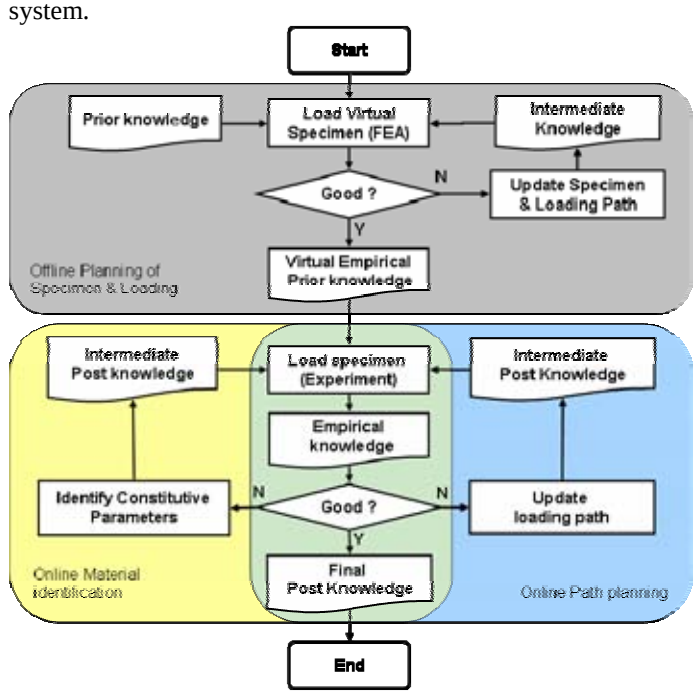

Figure 2: Design Optimization Hierarchy

Determination of this vector function is equivalent to a determination of all q_y components y_u of the vector $\mathbf{y}$. This is equivalent to the identification of q_y systems $y_u = f_u(\mathbf{p};\mathbf{x})$. Assuming that one exercises the corresponding physical system l times, incrementally, one is then able to construct the experimental pairs $(y_u, \mathbf{x})_k^{\mathrm{E}} = (y_{u,k}^{\mathrm{E}}, \mathbf{x}_k^{\mathrm{E}})$, where $k = 1,\ldots,l$, and the superscript "E" indicates the experimental character of a given quantity. At this stage it is advantageous to construct a decomposable factorization of $f_u(\mathbf{p};\mathbf{x})$. This can be achieved in general using a formalism that follows a Taylor-series expansion [7] according to

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$$f_u(\mathbf{p};\mathbf{x}) = \sum_{\mathbf{m}} p_u[\mathbf{m}] x_1^{m_1} x_2^{m_2} \ldots x_{q_x}^{m_m} \ . \qquad (2)$$

The index vector $\mathbf{m} = [m_1, m_2, \ldots, m_m]$ is an m-tuple of nonnegative integers that identifies the term in the series, or equivalently, the order of each variable in each of the monomial terms of the series. This implies that the total order of each monomial term will be $m = |\mathbf{m}| = m_1 + m_2 + \ldots + m_m$. Thus, the number of terms in the series is computed by the binomial coefficient

$$p_{up} = \binom{m + q_x}{m}, \qquad (3)$$

which essentially defines the dimensionality of the column parameter vector $\mathbf{p}$. The components of this vector or the coefficients of the series in Eq. (2) can now be considered as design variables of an optimization problem that requires the minimization of the quantity $\|\mathbf{A}_u \mathbf{p}_u - \mathbf{b}_u\|$ that expresses the error between the experimentally observed output behavior $\mathbf{b}_u$ and that which is estimated analytically as expressed by the product $\mathbf{A}_u \mathbf{p}_u$. Here $\mathbf{b}_u^T = (y_{u1}, \ldots, y_{uk}, \ldots, y_{ul})$, and $\mathbf{A}_u$ is an $l \times p_{up}$ array whose elements are evaluations of the combinations $x_1^{m_1} x_2^{m_2} \ldots x_{q_x}^{m_m}$ of terms from the series given in Eq. (2). When this minimization is defined with respect to the L_2 norm, one is able to construct [8] an objective function for minimization of the form

$$\left\|\mathbf{A}_u \mathbf{p}_u - \mathbf{y}_u^E\right\|_{L_2}^2 = (\mathbf{A}_u \mathbf{p}_u - \mathbf{y}_u^E)^T (\mathbf{A}_u \mathbf{p}_u - \mathbf{y}_u^E) = \sum_{k=1}^{l} (\mathbf{A}_{uk} \mathbf{p}_u - \mathbf{y}_{uk}^E)^2 \qquad (4)$$

This formulation suggests that linear least squares methods can be used to determine the vector of the system parameters $\mathbf{p}_u$ that represent the coefficients of the generally non-linear system model with respect of the input vector $\mathbf{x}$. This fact has generated confusion within the literature and therefore requires some emphasis for purposes of clarity. Consequently, $\mathbf{f}(\mathbf{p};\mathbf{x})$ can be determined through a determination of its components $f_u(\mathbf{p}_u;\mathbf{x})$. This is equivalent, however, to a determination of $p \times q_y$ parameters, and is therefore equivalent to solving q_y optimization problems, where each problem is associated with p system parameters.

In order to reduce the complexity of the general problem, as stated until this stage of our development, and to ensure that each $f_u(\mathbf{p}_u;\mathbf{x})$ is evaluated using a formalism that is consistent with respect to all components, we focus on a class of problems that is characterized by a particular mathematical representation that has its foundation in potential theory and continuum mechanics [8]. This particular representation of systemic behavior is in fact popular within continuum mechanics and has its origins in the development of hyperelasticity. Accordingly, it is postulated that there exists a potential function $\Psi(\mathbf{p};\mathbf{x})$ such that

$$y_u = f_u(\mathbf{p};\mathbf{x}) = \frac{\partial \Psi(\mathbf{p};\mathbf{x})}{\partial x_u} \ . \qquad (5)$$

This formulation effectively equips the systemic representation with a structure for determining all components y_u of the vector $\mathbf{y}$ from a single scalar potential function $\Psi(\mathbf{p};\mathbf{x})$. An often forgotten assumption enabling this formulation is that the input and output variables can actually form a correspondence through interrelationship as conjugate pairs $\{y_u, x_u\}$, $u = 1, \ldots, q$, where $q = q_x = q_y$ (and therefore, this approach is not applicable for systems with mismatching cardinality of the input and output sets.) In this case the design optimization problem for level-1 optimization is reduced to that of a determination of the function $\Psi(\mathbf{p};\mathbf{x})$. A standard technique for determining this function involves its construction as an additive linear combination of basis functions $\boldsymbol{\beta}(\mathbf{x})$, weighted by the unknown coefficients $\mathbf{p}$ according to

$$\Psi(\mathbf{p};\mathbf{x}) = \mathbf{p} \cdot \boldsymbol{\beta}(\mathbf{x}) \ , \qquad (6)$$

where $\mathbf{p} = [p_1, p_2, \ldots, p_p]$ and $\boldsymbol{\beta} = [\beta_1(\mathbf{x}), \beta_2(\mathbf{x}), \ldots, \beta_n(\mathbf{x})]^T$. Another approach for construction of the function $\Psi(\mathbf{p};\mathbf{x})$ is based on thermodynamics. This approach, which assumes that $\Psi(\mathbf{p};\mathbf{x})$ represents an internal energy density function for many continuum systems, permits a Taylor-series expansion about the origin $\mathbf{x} = \mathbf{0}$ with respect to the state variables represented by the components forming the basis set of the input state subspace X^q. Accordingly, a second order expansion with respect to the variables x_u results in a first order constitutive theory following Eq. (5). Clearly when terms of higher than second order are employed, the resulting systemic behavior will be nonlinear. Another important and frequently forgotten fact is that Eq. (6) is actually Eq. (2) expressed in vector notation, with the subtle difference, however, that the basis functions $\boldsymbol{\beta}(\mathbf{x})$ are arbitrary and therefore can be selected such that their structure is more convenient for a particular system analysis. Accordingly, the structure of the basis functions $\boldsymbol{\beta}(\mathbf{x})$ can be selected such that Eq. (6) is expressed by fewer terms relative to Eq. (2).

Substitution of Eqs. (5) and (6) into Eq. (1) yields the systemic behavior model

$$\mathbf{y} = \mathbf{p} \bullet \nabla_{\mathbf{x}} \boldsymbol{\beta}(\mathbf{x}) \ . \qquad (7)$$

Within the context of continuum systems and their corresponding constitutive responses, Eqs. (1,5-7) represent the behavior of the medium for all random volume elements (RVEs) within the geometry that encloses it, and is independent of shape. However, for the sake of identifying the components of the parameter vector $\mathbf{p}$, experimental measurements are to be made at discrete locations $i \in [1, \ldots, l]$ on the specimen, or in general, the system. At the same time excitation and response is measured in terms of input-output pairs for various magnitudes of excitation indexed by $k \in [1, \ldots, m]$ for a total of m different magnitudes. Accordingly, one can construct a vector expressing the behavior of the system as calculated analytically according to

$$\mathbf{Y}_k(\mathbf{p};\mathbf{x}) = [(y_1(\mathbf{p};\mathbf{x}), \ldots, y_n(\mathbf{p};\mathbf{x}))_1, \ldots, \ (y_1(\mathbf{p};\mathbf{x}), \ldots, y_n(\mathbf{p};\mathbf{x}))_l]_k^{\ T} \ , \qquad (8)$$

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and correspondingly, the behavior of the system as measured experimentally according to

$$\mathbf{Y}^e{}_k = [(y^e{}_1,\ldots,y^e{}_n)_1,\ldots,\ (y^e{}_1,\ldots,y^e{}_n)_l]_k{}^T \tag{9}$$

The quantity

$$\left\|\mathbf{Y}_k - \mathbf{Y}^e{}_k\right\|^2 = \sum_{j=1}^{m}(\mathbf{Y}_{kj}(\mathbf{p};\mathbf{x}) - \mathbf{Y}^e{}_{kj})^2 \ , \tag{10}$$

which expresses the square of the L_2 norm of the residual difference of Y_k and $Y^e{}_k$ in terms of the least square difference of their respective magnitudes for each excitation increment.

The equality expressed by Eq. (10), however, must be satisfied for all excitation levels *m* and must be extended to include all discrete measurement positions *l*. This condition combined with the substitution of Eqs. (4-6) into Eq. (10) yields the generalized form,

$$J_0{}^{lev-1} = \sum_{t=1}^{l}\sum_{j=1}^{m}(y_{jt}(\mathbf{p};\mathbf{x}) - y^e{}_{jt})^2 = \sum_{t=1}^{l}\sum_{j=1}^{m}(p_p \cdot \nabla_{\mathbf{x}}\boldsymbol{\beta}(\mathbf{x}_{jt}) - y^e{}_{jt})^2 \tag{11}$$

Since this expression provides the definition of the residual error it can be used to define the objective function $J_0{}^{lev-1}$ that when minimized yields the unknown parameter vector $\mathbf{p}$, and therefore, the design optimization process that is performed at level-1. As expected, the individual objectives folded in Eq. (11) that are related with the each individual output are satisfied as they are affected by the simultaneous presence of the rest of them.

If one assumes that linear constitutive behavior can approximate the behavior of a given system, it then follows from Eq. (4) that $\boldsymbol{\beta}(\mathbf{x})$ must be a second order function of the components of $\mathbf{x}$. In that case it is trivial to show that determination of Eq. (8) is reducible to a problem involving the determination of the scalar function $\Psi(\mathbf{p};\mathbf{x})$ according to

$$J_0{}^{lev-1} = (\mathbf{Bp} - \overline{\mathbf{y}}^e)^T(\mathbf{Bp} - \overline{\mathbf{y}}^e) = \sum_{s=1}^{m+l}(\sum_{t=1}^{n=2}B_{st}p_t - \overline{y}_s^e)^2 \ , \tag{12}$$

where the overbar quantities correspond to the experimental values of the generalized work function corresponding to the inner product of the input and output vectors of the original system.

The problem expressed by Eq. (12) can be solved by methods based on the solution of normal equations (Normal-Equations method), QR factorization, or Singular Value Decomposition (SVD) [9]. Selecting one of these methods to determine the unknown model parameters can be a process that depends on ranking these methods with regard to their performance in terms of attributes or metrics that are important to the user. It has been documented for example that the Normal Equation method is computationally fast and requires less resources, but is less accurate. In contrast, it is well known that SVD requires substantial computational resources, but is more reliable than other methods. It is natural therefore, to ask the question of how these attributes might vary given that there exists user control of the characteristics that determine the particular choice of experimental data adopted for population of all column arrays having superscript "*e*", denoting experimental value, in the relations presented above.

COMPOSITE MATERIAL SYSTEM

For demonstration purposes we consider a linear anisotropic material with the four moduli representing its constitutive parameters. We have already demonstrated [10,11] that this problem can be reduced to the following linear (with respect to the unknown parameter vector) relation

$$\mathbf{G}(\theta)\mathbf{q}_M = \mathbf{w}_k \ , \tag{13}$$

where $\mathbf{G}(\theta) = \left[\hat{\mathbf{g}}_1(\theta),...,\hat{\mathbf{g}}_n(\theta)\right]^T$ is a $m \times 4$-dimensioned array that contributes to the finite element approximation of the internal energy stored in the system from an increment of strain from point $k-1$ to point k in a manner that does not contain the material moduli since this quantity is contributed by the 4x1 array $\mathbf{q}_M$ of unknown parameters on the left hand side of Eq. (13). The right side the *m*-dimensional array $\mathbf{w}_k = \left[W_1/t,...,W_n/t\right]^T$ contains the external work that is applied as excitation into the system for all loading increments. In Eq. (13) the right side represents the measured output of the system, while the left side represents the corresponding change inside the system due to all possible excitation inputs. This equation is a special case for application of the more general Eq. (7). Its solution can be approached as a special case of the problem presented by Eq. (12) and can be achieved by using any of the three methods available for implementation of least squares approximation.

While selecting among the various methods may implicitly suggest the idea of yet another optimization level, here we will focus on SVD for the purpose of determining the parameters of the material model associated with level-1. We will also neglect additional computational performance criteria and focus only on potential measures of performance of SVD implementation from an algorithmic perspective.

Thus, the solution containing the identified parameters according to Eq. (13) can be written in the form

$$\mathbf{q}_M = \mathbf{G}(\theta)^+\mathbf{w}_k \tag{14}$$

where

$$\mathbf{G}(\theta)^+ = \left\{\mathbf{G}(\theta)^T\mathbf{G}(\theta)\right\}^{-1}\mathbf{G}(\theta)^T \tag{15}$$

is the pseudoinverse of $\mathbf{G}(\theta)$ and it exists uniquely only when $n \geq 4$ or when the system of linear equations represented by Eq. (14) is overdetermined.

LEVEL-1 PERFORMANCE MEASURES

What determines the quality of solving Eq. (13) is now reduced to determining the quality of applying Eq. (14) and therefore the quality of the process associated with establishing the pseudoinverse array defined by Eq. (15). We have identified in the past [10,11] that the concepts of "uniqueness" and "distinguishability" of the obtained solution can be used as performance metrics for the determination of the parameter column array $\mathbf{q}_M$. In order to define these two concepts it is necessary to focus on a few preliminaries relating the singular values of the SVD process to the problem at hand. In particular,

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the uniqueness of the solution depends not only on the size of $\mathbf{G}(\theta)$ but also on whether $\mathbf{G}(\theta)^T\mathbf{G}(\theta)$ in Eq. (12) has an inverse matrix (or this matrix is fully ranked ($r=4$)). In order to guarantee the uniqueness and further prepare for designing optimal experiments via level-2 optimization, the proposed technique obtains the singular values of the matrix as a result of singular value decomposition (SVD) [9] as they are expressed by the factorization:

$$\mathbf{G}(\theta)=\mathbf{USV}^T \tag{16}$$

where $\mathbf{U}\in\mathbb{R}^{n\times n}$ and $\mathbf{V}\in\mathbb{R}^{4\times 4}$ are orthogonal to each other and $\mathbf{S}\in\mathbb{R}^{n\times 4}$ is a diagonal array with real, non-negative singular values $s_i, \forall i\in\{1,...,4\}$. These singular values can be used to define two measures characterizing parameter identification, i.e., distinguishability and uniqueness of the solution, which are described as follows.

From a conceptual perspective distinguishability can be defined as the property of the obtained solution to provide the largest possible variation of the measured response of two systems when their material parameters are very close to each other. It has been demonstrated that when two material systems exhibit a small difference in their properties, then the difference in the values of their corresponding responses (observed experimentally) depend linearly on **S** [9]. Accordingly, any expression of the combined effect of the elements of **S** as it increases has the ability to distinguish two materials that are seemingly close to each other from a properties perspective, by producing exaggerated energy responses that are scaled values of these property variations.

From a quantitative perspective we have defined distinguishability as the product of all singular values as,

$$F^d=\prod_{i=1}^{n}s_i\ . \tag{17}$$

From a conceptual perspective uniqueness has been defined as the measure of whether $\mathbf{G}(\theta)^T\mathbf{G}(\theta)$ in Eq. (15) has an inverse matrix or not, is equivalent to the existence of $\left(\mathbf{S}^T\mathbf{S}\right)^{-1}$ as shown by substituting Eq. (16) into Eq. (15) that yields

$$\mathbf{G}(\theta)^+=\mathbf{V}\left\{\mathbf{S}^T\mathbf{S}\right\}^{-1}\mathbf{S}^T\mathbf{U}^T \tag{18}$$

The necessary and sufficient condition for this to occur is given by

$$\left|\mathbf{S}^T\mathbf{S}\right|=s_1^{\,2}s_2^{\,2}s_3^{\,2}s_4^{\,2}\neq 0\ , \tag{19}$$

i.e., $s_i\neq 0, \forall i\in\{1,...,4\}$. In addition to helping identify the uniqueness, singular values can also be used to quantify the degree of uniqueness of the solution. This is because a non-zero but near-zero singular value, if it exists, dominates the elements of the pseudoinverse matrix given by Eq. (18) and makes the parameters having the other singular values difficult to identify uniquely. Accordingly, the degree of uniqueness can be quantified by the fact that the greater the differences of the singular values, the more unique (i.e. higher uniqueness) the solution.

One way to define uniqueness quantitatively requires the introduction of the concepts of maximum and minimum singular values

$$\begin{aligned} s_{\max}&=\max\left\{s_i \mid \forall i\in\{1,...,4\}\right\}\\ s_{\min}&=\min\left\{s_i \mid \forall i\in\{1,...,4\}\right\}\end{aligned} \tag{20}$$

and evaluation of deviation from uniqueness in terms of the condition number *c*, which is commonly used in sensitivity analysis as:

$$F^u\equiv\frac{1}{c}=\frac{s_{\min}}{s_{\max}}\leq 1 \tag{21}$$

It is important to note that distinguishability increases as any of the s_i increase, while uniqueness increases as the condition number decreases to unity.

LEVEL-2 OPTIMIZATION

If in addition to determining the material parameters we require that this is achieved such as distinguishability and uniqueness are as high as possible, then we have instantly defined the goals of the level-2 optimization regardless of whether we are referring to the online (level-2a) or offline (level-2b) versions. The design variables at level-2 have to therefore be connected with what is controllable in an experimental setup used to acquire experimental data for identification of the material parameters at level-1. Such parameters can be those that define the evolution of the loading path, such as total number of increments, loading path increment magnitude, and loading path increment orientation. For the case of a displacement controlled two degree of freedom (2-DoF) testing machine, used for experimentation, the parameter vector to be identified per loading increment could be formed by the measure of displacement increment

$$\Delta u_k=\left\|\mathbf{u}_{k+1}-\mathbf{u}_k\right\| \tag{22}$$

and the angle denoting the change in orientation of the loading path between to successive increments defined as

$$\Lambda_{k,k+1}=\tan^{-1}\left(\frac{u_{y|k+1}-u_{y|k}}{u_{x|k+1}-u_{x|k}}\right)$$

(23)

where the total boundary displacement vector is defined by its components along the two axes according to the usual definition $\mathbf{u}_k\equiv[u_x,u_y]_k=[u_{x|k},u_{y|k}]$. In the subsequent analysis we will assume constant displacement increment of a chosen magnitude and the parameter to be optimized is the load path directional change $\Lambda_{k,k+1}$ for all increments. Since increased uniqueness and distinguishability both express a sense of reliability of the SVD process used for determining the material parameters in level-1, we can define a vectorial objective function that needs to be maximized for maximum reliability. It is constructed such as

$$\mathbf{J}_{K,K+1}=\left[J^d_{K,K+1},J^u_{K,K+1}\right]\rightarrow\max_{\Lambda_{K,K+1}}\ , \tag{24}$$

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where the objective function components $J^d_{K,K+1}, J^u_{K,K+1}$ represent the corresponding increments of distinguishability and uniqueness along an increment of loading according to

$$J^d_{K,K+1} = \bar{F}^d_{0,K+1}(\Lambda_{K,K+1}) - \breve{F}^d_{0,K}$$
$$J^u_{K,K+1} = \bar{F}^u_{0,K+1}(\Lambda_{K,K+1}) - \breve{F}^u_{0,K} \qquad (25)$$

Distinguishability and uniqueness are computed through the derivation of the matrix $\mathbf{G}(\theta)$ in Eq. (18). In order to compute $F^{(\cdot)}_{0,K}$, the matrix $\mathbf{G}(\theta)$ is assembled for the sensor readings at load increment *k*, through the derivation of the alternate form of Eq. (13) [10,11]:

$$t\breve{\mathbf{g}}_k(\theta)^T \mathbf{q}_M = \Delta\breve{W}_k, \ \forall k \in \{1,\ldots,K\} \qquad (26)$$

Distinguishability and uniqueness of $\bar{F}^{(\cdot)}_{0,K+1}(\Lambda_{K,K+1})$ for steps less than $K+1$ are predicted from the matrix with the additional row:

$$t\hat{\mathbf{g}}_{K+1}(\theta)^T \mathbf{q}_M = \Delta\hat{W}_{K+1}(\Lambda_{K,K+1}), \qquad (27)$$

where $\hat{\mathbf{g}}_{K+1}(\theta)^T$ and $\Delta\hat{W}_{K+1}(\Lambda_{K,K+1})$ are computed via Finite Element Analysis (FEA) with the controllable boundary displacements/forces governed by $\Lambda_{K,K+1}$ and using the material parameters (elastic moduli for this case) identified up to this stage. This represents a 1-step look-ahead computation relative to the activities of level-1 as they relate to those of level-2.

Throughout the paper the symbolic representation $(\breve{\cdot})$ is used to represent the quantity $(\cdot)$ as it is associated with FEA.

As expected, the solution of the two objective functions problem is not given by a single point but by a space satisfying the Pareto-optimality, which is often referred to as Pareto-optimal front [10-12]. This formulation requires the derivation of a Pareto-optimal front prior to the determination of a single solution and this is extremely time-consuming from a computational perspective. For this reason and assuming that the Pareto-optimal front is convex or near-convex, the problem can be reformulated in a manner such that a single scalar objective function is constructed according to the generalized form

$$J_{K,K+1} \equiv (1-\mu)J^d_{K,K+K_n} + \mu J^u_{K,K+K_n} \rightarrow \max_{\Lambda_{K,K+K_n}} \qquad (28)$$

where each objective function is given by the scaled increment:

$$J^\alpha_{K,K+K_n} \equiv \frac{\bar{F}^\alpha_{0,K+K_n} - F^\alpha_K}{F^\alpha_K}, \ \forall \alpha \in \{d,u\} \qquad (29)$$

and $\mu \in [0,1]$ is a weighting factor that controls the bias towards one or the other component of the objective function. The formulation expressed by Eqs. (28-29) avoids the derivation of the Pareto-front.

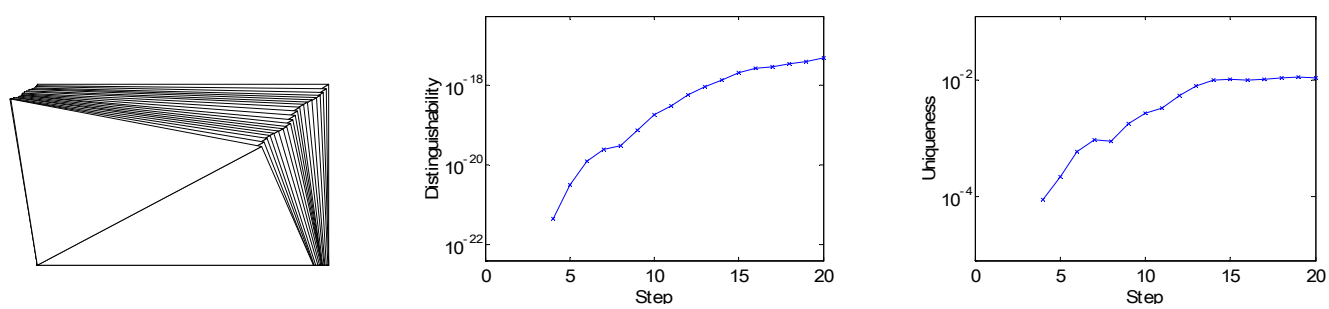
NUMERICAL EXAMPLES

For the sake of numerical demonstration of the proposed concepts the material selected for generating the necessary simulated experimental data is a typical laminate constructed from an epoxy resin/fiber laminae system of type AS4/3506-1 with a balanced +/- 30 degrees stacking sequence. The elastic moduli of this material are listed in Table 1.

Table 1 Moduli of AS4/3506-1 laminae

Lamina		Laminate	
Prop.	Value	Parm.	Value
E_{11}	114x10^9 Pa	Q_{11}	1.1485x10^8 Pa
E_{22}	9.6x10^9 Pa	Q_{22}	9.7452x10^6 Pa
G_{12}	5.99x10^9 Pa	Q_{66}	5.9986x10^6 Pa
ν_{12}	0.334	Q_{12}	3.2549x10^6 Pa

All subsequent computational results have been produced by the implementation of the analysis presented earlier within MATLAB [13]. Figure 3 shows the deformed stages of a simple rectangular plate made from the specified material and displaced under the influence of an undulating loading path for a sequence of 20 increments in this path (a). Distinguishability and uniqueness are increasing with increasing load step increments in Fig. 3(b) and 3(c) respectively. We have already discussed elsewhere [10-12] the fact that an undulating path (u_x displacement component is non monotonic), maximizes distinguishability and uniqueness more efficiently than a uniaxial loading path along the y-direction (that cannot actually determine all unknown material parameters) or a linear path with monotonic u_x and u_y displacement components. This represents the results achieved during level-1 optimization.



(a) Deformation by FEA (b) Distinguishability (c) Uniqueness

Figure 3. Undulating loading path

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Figure 4 presents the results of performing the level-2 optimization as described earlier, in terms if the evolution of the pareto front as defined in terms of distinguishability and uniqueness for a the entire range of the weighting factor $\mu \in [0,1]$. Figure 5 depicts the results of the material moduli determination as a function of loading steps for the loading path represented by Fig. 3(a). The choice of the optimal solution has no physical basis and corresponds to setting $\mu = 0.5$ because this

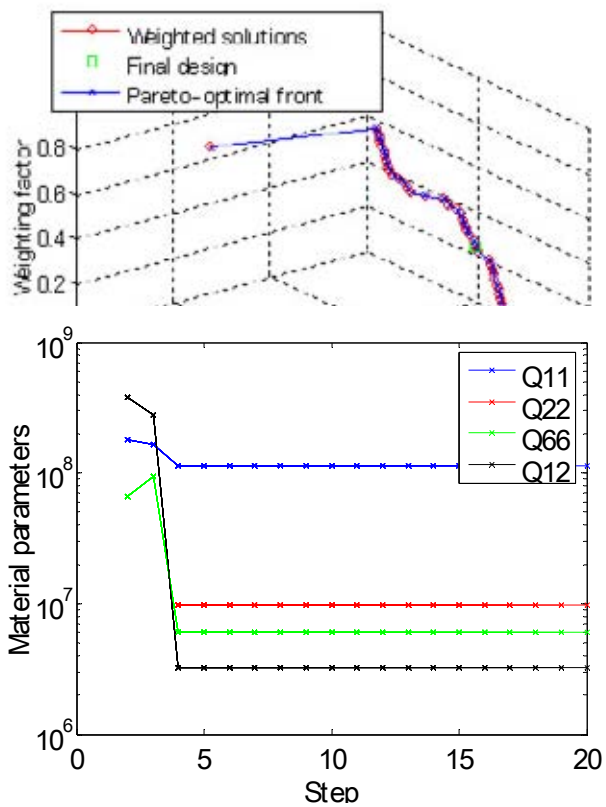

Figure 5. Evolution Material Properties Characterization

choice balances the contribution of each one of the original objective functions on the global one as expressed by Eq. (25). The resulting solution for level-2 optimization is expressed in terms if the loading path defined on the u_x- u_y plane as shown in Fig. 6.

CONCLUSIONS AND DISCUSSION

The preliminary framework of a general hierarchical methodology was proposed. This methodology can succeed in both the determination of parameters characterizing the response of a system, and characterizing the design parameters of an experiment required to collect data necessary for the systemic characterization. The approach was applied in the context of an anisotropic material system. The systemic constitutive response of linear anisotropic behavior to be identified was selected to be that of an elastic system defined from its four elastic moduli. These were the design variables

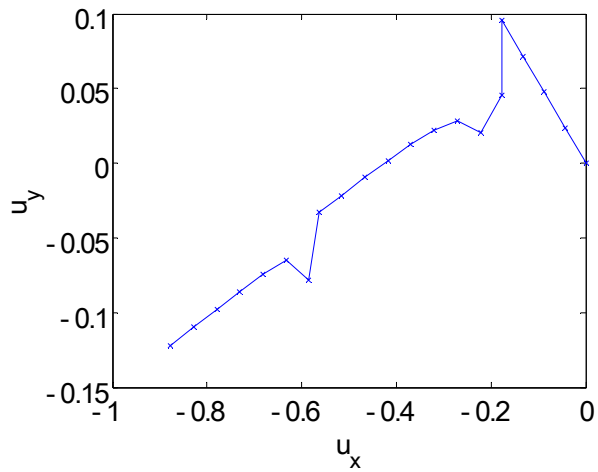

Figure 6. Final design of loading path chosen for $\mu = 0.5$

adopted for the first hierarchical level of optimization (level-1). The experimental model that was required for the second level of optimization (level-2) was chosen to represent the loading path within a 2-dimensional loading space. Implied here is the existence of a 2-degree of freedom loading frame, capable of applying such a loading path and of measuring both the path and mechanical load characteristics for each increment.

In order to achieve a definition of the objective function at level-2, the quantities of distinguishability and uniqueness were introduced as performance metrics of the design optimization process at level-2, thus quantifying the performance of the SVD process employed. Accordingly, a two-component meta-objective function was constructed for maximization. Maximization of this dual objective function leads to the creation of a Pareto-optimal front that ultimately contains the loci of all acceptable solutions that can be used to determine dynamically the experimental design specification in terms of a loading-path direction parameter. Numerical simulation of the entire process was performed in order to demonstrate its feasibility. We demonstrated that the material moduli unknowns can be determined in conjunction with the loading path characteristics needed to design an appropriate experiment for collecting data required for level-1. Clearly, the question of "how good the design optimization for level-n is" from the perspective of the optimization of level-(n+1) is valid for all subsequent levels a user wishes to employ. This effectively extends the hierarchy upward. Practicality and total computational cost will eventually have to appear in these objective functions and the hierarchy's extension will eventually have to stop. This will also determine the throughput capability of the entire process from a DDDAS perspective. Various extensions of this type will be considered for further investigation, while simulation as the activity of exercising the determined model, will also be added for completion the triad (dynamic and simultaneous physical model identification, design of experiments and design of simulation) of activities

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associated with a DDDAS [14]. Finally it is planned that actually experimental data with their natural uncertainty sources will be used to reinforce the validity of the proposed procedure.

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