Principles of Minimum Cost Refining for Optimum Linerboard Strength

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ABSTRACT

The mechanical properties of paper at a single basis weight and a single targeted refining freeness level have traditionally been used to compare papers. Understanding the economics of corrugated fiberboard requires a more global characterization of the variation of mechanical properties and refining energy consumption with freeness. The cost of refining energy to increase paper performance is of the same magnitude as the cost of adding fiber to obtain an equal increase in performance; the costs of energy and fiber vary dramatically with geography and market conditions. We provide formulas that can be programmed into a spreadsheet to graphically examine market condition and performance scenarios. We characterize a recycled linerboard and determine the market conditions that favor increased refining compared to increased fiber.

KEYWORDS

Cost optimization, Economics, Linerboard strength, Recycling.

INTRODUCTION

An important step in papermaking is the mechanical refining of pulp fibers (conventionally via disc refiners) to generate optimum mechanical properties of the paper. Researchers have improved the strength properties of paper made from recycled fiber by blending the fiber with virgin fiber and chemical additives (1), using chemical treatment, and modifying the paper machine. Refining is the most practical process and has been adopted for mill production. However, the more the initial pulp is refined, the more difficult it is to restore the initial strength of recycled pulp through refining (2). Tensile strength has been the property most investigated (2,3). Data are limited on how refining affects paper compressive strength. stiffness properties, or caliper.

The Forest Products Laboratory has researched the feasibility of using small-diameter trees, waste wood, and unconventional wood species for papermaking using an experimental refiner and formed handsheet specimens. The laboratory relationship between refining variables and paper performance provides insight into a process that is undoubtedly more complicated on an industrial scale. Mechanical refining is an energy-intensive process at both the laboratory and industrial scale. As we shall demonstrate, the cost of refining energy to increase paper performance is of the same magnitude as the cost of adding fiber to obtain an equal increase in performance.

In a study of important variables affecting corrugated fiberboard. Koning and Haskell (4) explored the effects and interactions of seven typical papermaking factors (wood species, pulp yield, type of refiner, refining consistency, amount of refining, wet-press pressure, and surface) on various strength properties of 205-g/m^2 linerboard handsheets. Among the properties investigated, compressive strength, modulus of elasticity, and thickness, which are known to relate to the compressive strength of corrugated fiberboard boxes, were consistently and significantly affected by the freeness level achieved by refining. In general. the authors found that the lower of two freeness levels investigated resulted in better linerboards by virtue of increasing overall strength.

Greater linerboard strength can thus be obtained at a lower freeness by more refining, but at a higher energy cost. Greater linerboard strength can also be obtained at a higher basis weight with a proportionally higher fiber cost. When the linerboard is considered as a component in corrugated fiberboard and the compressive strength of a box is important, the most economical linerboard in terms of fiber

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and refining energy inputs is one with the minimum total cost of fiber and energy that yields the required box compressive strength. The generalization that lower freeness is better is not as evident.

Linerboard in corrugated fiberboard specified by edgewise crush strength and recycled corrugated medium are produced through the use of higher process refining levels compared to that used for conventional containerboards. Refining energy is among the most costly process variables in containerboard production. The benefits of process refining in combination with the basis weight of the components is important to understanding the economics of corrugated fiberboard.

OBJECTIVE

We compare the cost and performance of two kinds of linerboard: (a) expensive-to-process but lightweight linerhoard made at a low hasis weight from highly refined pulp and (b) cheaper-to-process but heavier linerhoard made from less refined pulp and more fiber. The empirical variations of refining energy consumption with freeness and variation in mechanical properties with freeness are used to examine theoretically how the performance of the linerboard facing in Corrugated fiberboard relates to the total cost of energy and fiber. Results quantify the optimum refining level that yields fiberhoard with adequate performance at the lowest cost of energy plus fiber.

Understanding the effect of refining on paper properties at the laboratory scale is a necessary first step in optimizing industrial production. Production costs emanating from paper machine speed, scheduling efficiency, inventory and distribution, etc., are ultimately important but are not examined in this study. Refining to a low freeness level reduces the drainage rate on the paper machine and might be constrained by a practical cost in actual production. Increasing a paper's hasis weight increases its drying time and shipping costs per unit area of paper. The final economics are a function of the effects and interactions among refining, basis weight, and production variables. The results of this study provide a methodology for quantifying the economic feasibility of new materials and for establishing boundaries in a more broadly designed industrial experiment.

MATERIALS AND TESTS

A single commercial linerboard (94% kraft pulp and 6% recycled cormgated containers) was recycled into pulp and refined at three levels of consistency: 5%, 8%, and 12%. Linerboard material was first pulped in a 50 liter Voith (Heidenheim, Germany) laboratory pulper at 10% consistency for 30 minutes and with nominal hot water. The pulp slurry was thickened and refrigerated for later use.

Atmospheric refining was later carried out on a 305-mm Sprout–Waldron(Muncy, Pennsylvania) single-disc refiner with C2976 plates. Refining at each level of consistency combined with each of three levels of refiner plate gap (102, 178, and 254 mm) was done at up to three freeness levels. The processed pulps at 5% and 8% consistency were refined with one to three passes through the refiner. The processed pulp at 12% consistency was refined with one or two passes through the refiner. Canadian standard freeness (CSF) measured according to TAPPI Test Method T 227 (5) and cumulative specific energy consumption were determined for the pulp after each pass through the refiner.

Handsheets at three levels of basis weight (BW) (nominally 126, 185, and 205 g/m^2) were made from each of the 24 refined pulps. For comparison, the freeness of a control material was determined from unrefined pulp, and control handsheets were made from unrefined pulp at the three BW levels. Freeness and specific energy consumption levels determined for the pulps and actual average BW level for the respective handsheets are given in Table 1.

Compression load–strain curves and bending stiffness data that could be used to compute the compressive strength and buckling strength of a facing microplate in corrugated fiberboard were determined from handsheet specimens. Compression load–strain curves up to failure were produced with the apparatus described by Gunderson (6). Bending stiffness in a Taber instrument was determined following TAPPI T 489 (7). The surface-to-surface thickness (caliper) was determined by TAPPI T411 (8).

Prior to testing, materials were preconditioned in a dry environment below 30% RH and then conditioned at 50% relative humidity (RH). All tests were conducted in a conditioned environment at 23°C and 50% RH in accordance with ASTM Standard D 685–93(9).

RESULTS AND DISCUSSION

Energy Model

Our data on freeness were found to vary linearly with specific energy consumption and with interactions among consistency, plate refiner gap, and specific energy consumption according to the statistical equation

$$F = \alpha + \beta_0\,SEC + \beta_1 C\,SEC + \beta_2 C\,G\,SEC \quad (1)$$

where

F is freeness (mL CSF),
SEC specific energy consumption (W·h/kg),

C consistency (%), and
G plate refiner gap (µm).

Statistically fitted coefficients obtained via a least squares regression were determined as α = 732, β_0 = -0.179, b_1 = -0.0250, β, = 7.37 x 10^{-5} The standard errors associated with α, β_0, β_1, and β_2 are 0.6%, 4.0%, 4.2%, and 5.8%, respectively, as a percent magnitude of the fitted coefficients. Expanding Eq. (1) with additional coefficients applied to *C, G, CG,* and *G* SEC was not found to improve the fit to data significantly.

The fit of Eq. (1) to our data (Table 1) is shown in Figure 1 in terms of the inverse expression

$$\mathrm{SEC} = \frac{F-\alpha}{\beta}$$

where

$$\beta = \beta_0 + \beta_1 C + \beta_2 C\,G \qquad (3)$$

Equation (2) with Eq. (3) as input expresses the behavior that for each combination of fixed *C* and *G,* the respective SEC-*F* plot passes through a common abscissa a and has a slope 1/β depending on *C* and *G* (Fig. 1). The fitted level of a predicts the pulp freeness prior to refining and is within

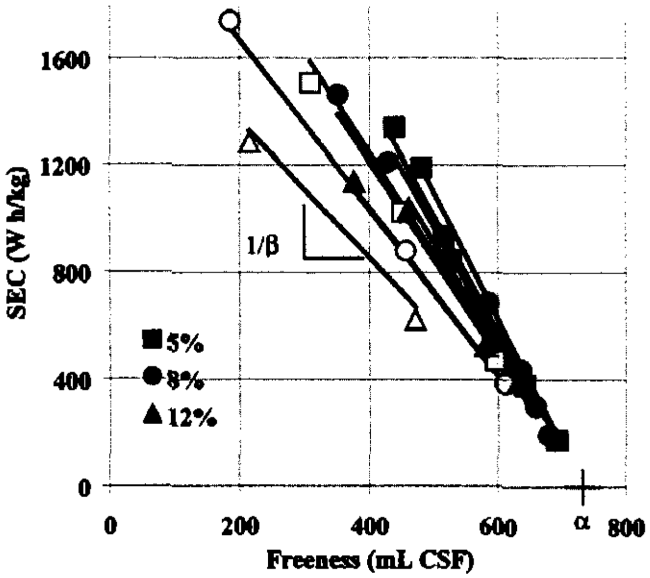

Figure 1. **Variation of total specific energy consumption (SEC) with freeness for linerboard pulp refined at nine combinations of consistency at 5%, 8%, or 12% and refining plate gap at 102, 178, or 254 µm. The legend shown is for the three consistency levels and a 254 µm plate gap. Points on the graph are shaded white (102 µm), gray (178 µm), or black (254 µm) to indicate other levels of plate gap. The linear fits to data pass through a common abscissa a and have slopes 1/β that depend on consistency and plate gap according to Eq. (3). r^2 = 0.986.**

4% of the actual determination of 704 mL CSF for the control material (Table 1).

Although Eq. (2) provides a way to accurately interpolate among the data of this study, it does not predict an increasing energy consumption approaching infinity as a zero freeness level is sought. Equation (2) is sufficient for understanding the performance of our experimental refiner within the scope of this study. For commercial refiners and other input variables that affect refining energy consumption, more mechanistic equations' would need to be considered.

Mechanical Properly Model

Data on the variation of compression load *P* with compression strain ε were reduced to average parameter values c_1 and c_2 in a fit of the equation

$$P = c_1 \tanh(c_2 / c_1\, \varepsilon) \qquad (4)$$

up to the ultimate strength P_u at failure. The initial slope of Eq. (4) is given by c_2. Parameter c_1 is an asymptotic level of *P* if Eq. (4) is extrapolated to an infinite E. The fitted values of c_1 and c_2 along with the determinations of P_u, bending stiffness *D,* and caliper t_c are given in Table 1. Strength and stiffness are given in terms of a unit width of specimen.

Variation of each mechanical property Q with BW and *F* (Table 1) was statistically analyzed with the empirical formula

$$Q = b_0 + b_1\left(\frac{BW}{BW_0}\right)^{b_?} + b_2\left(\frac{F}{\alpha}\right)^{b_f} + b_3\left(\frac{BW}{BW_0}\right)^{b_?}\left(\frac{F}{\alpha}\right)^{b_f} \qquad (5)$$

Fitted coefficients for each case of Q represented by c_1, c_2, P_a, *D,* and t_c are given in Table 2 with normalizing inputs in terms of the lowest nominal basis weight, BW_0 = 126 and α = 732 mL CSF.

Equation (5) is a statistical linear combination of variables BW, *F,* and interaction BW *F* with optimum exponential powers applied. The number of coefficients for each case of *Q* in Table 2 is for the most accurate fit of Eq. (5) that is statistically significant based on the statistical *F*-test at the

[1] Note that Eq. (2) should not be extrapolated below the freeness levels of this study. For a more mechanistic relation between SEC and F including lower freeness levels, the equation given by

$$\mathrm{SEC} = \frac{\alpha}{2\beta}\ln\left(\frac{F}{2\alpha - F}\right)$$

is noteworthy. This example of a more general equation approximates Eq. (2) at our experimental levels of F and also approaches infinity as F approaches zero.

Table 1. **Mechanical properties of handsheets made at three nominal BW levels from pulps refined at various levels of consistency and refiner plate gap (*G*), measured freeness level (*F*), and total specific energy consumption (SEC) resulting from one to three refiner passes**[a]

5% consistency

G μm	*F* mL	SEC W·h/kg	BW g/m²	c_1 kN/m	c_2 kN/m	P_u kN/m	*D* mNm	t_c μm
254	691	176	129	3.04	811	2.40	6.91	257
			185	4.46	1,096	3.55	11.49	363
			207	5.12	1,157	4.01	13.79	406
	528	843	128	3.63	890	2.99	6.70	229
			188	5.69	1,311	4.57	11.18	251
			206	6.31	1,394	5.36	12.62	347
	481	1,189	127	3.97	948	3.04	6.62	216
			183	6.13	1,339	4.92	10.61	300
			205	6.33	1,420	5.33	12.47	343
178	642	379	129	3.28	825	2.61	6.92	249
			187	5.01	1,136	4.04	11.32	353
			208	5.49	1,242	4.59	13.69	399
	517	933	126	3.77	896	3.08	6.55	221
			189	5.88	1,319	4.81	11.18	307
			202	6.42	1,483	5.45	12.52	340
	438	1,341	125	3.90	938	3.03	6.47	206
			183	6.07	1,350	5.01	10.80	297
			197	6.77	1,491	5.45	11.75	318
102	596	471	128	3.26	809	2.68	6.65	236
			185	4.97	1,174	4.07	11.02	335
			203	5.51	1,296	4.62	12.69	361
	451	1,024	128	3.95	947	3.15	6.59	211
			185	6.19	1,383	4.95	10.61	302
			203	7.09	1,525	5.68	12.47	326
	309	1,506	127	4.28	991	3.24	6.31	201
			185	6.73	1,511	5.09	10.82	284
			208	7.43	1,574	5.95	12.75	323

8% consistency

G μm	F mL	SEC W·h/kg	BW g/m²	c_1 kN/m	c_2 kN/m	P_u kN/m	*D* mNm	t_c μm
254	678	196	121	2.84	766	2.20	6.82	257
			180	4.38	1,116	3.62	11.80	368
			205	5.01	1,232	4.18	14.07	406
	636	436	128	3.32	851	2.71	7.13	249
			187	5.27	1,269	4.47	11.98	347
			208	5.84	1,369	4.76	13.62	373
	428	1,210	127	4.10	984	3.00	6.71	213
			184	5.89	1,337	4.57	10.25	297
			204	6.58	1,474	5.41	13.19	338
178	659	301	124	3.34	838	2.55	6.66	244
			185	4.81	1,184	4.08	11.96	361
			202	5.31	1,270	4.61	13.47	389
	585	685	124	3.40	832	2.72	6.70	234
			184	5.23	1,224	4.53	11.29	328
			201	5.97	1,356	5.11	12.99	358
	351	1,463	127	4.27	999	3.07	6.40	201
			184	6.34	1,396	4.91	10.67	287
			201	7.01	1,525	5.77	12.64	323
102	611	386	125	3.24	838	2.71	6.78	244
			184	5.06	1,219	4.41	11.36	340
			205	5.54	1,303	4.77	13.38	376
	456	879	128	3.39	848	2.56	5.02	206
			196	5.77	1,289	4.81	11.03	307
			220	6.66	1,505	5.49	13.46	348
	185	1,739	125	4.25	1,004	2.99	6.40	198
			186	7.04	1,588	5.17	10.47	287
			202	8.00	1,731	5.89	12.03	310

12% consistency

G μm	*F* mL	SEC W·h/kg	BW g/m²	c_1 kN/m	c_2 kN/m	P_u kN/m	*D* mNm	t_c μm
254	630	390	127	3.48	835	2.69	6.83	246
			185	5.76	1,286	4.54	10.73	325
			201	6.01	1,390	4.96	12.41	361
	461	1,038	129	4.11	932	3.10	6.60	213
			184	6.26	1,384	5.04	10.50	295
			203	6.62	1,439	5.20	12.04	325
178	580	526	125	3.81	908	2.83	6.39	224
			190	5.97	1,378	4.78	10.96	318
			203	6.00	1,367	5.04	12.34	340
	377	1,139	126	4.18	993	3.11	6.42	203
			182	6.22	1,324	5.01	10.27	287
			203	6.79	1,520	5.60	12.36	315
102	471	625	126	3.97	939	2.97	6.34	208
			185	6.12	1,350	4.88	10.45	310
			203	6.63	1,500	5.56	11.81	335
	215	1,285	125	4.35	1,005	2.99	6.35	193
			185	6.73	1,461	5.20	10.74	284
			204	7.35	1,522	5.94	12.57	320

Control

F mL	BW g/m²	c_1 kN/m	c_2 kN/m	P_u kN/m	*D* mNm	t_c μm
704	124	2.82	766	2.02	6.59	264
	186	3.99	1,030	3.20	11.47	386
	206	4.95	1,202	4.14	12.77	396

[a]BW is basis weight; c_1 and c_2 are parameters; P_u is ultimate strength; *D*, bending stiffness; t_c, surface-to-surface thickness.

Table 2. **Evaluations of coefficients obtained from least squares regression tit of Eq. (5) to mechanical property data in Table 1 using BW_0 = 126 and α = 732 mL CSF.**

	Mechanical property				
Eq. (5)	c_1	c_2	P_u	D	t_c
coefficient	(kN/m)	(kN/m)	(kN/m)	(mN·m)	(µm)
b_0	−0.50	63.0	−1.20	2.72	21.9
b_1	5.14	991	4.34	3.66	181
b_2	0	0	0.95	0	0
b_3	−1.84	−301	−1.86	0.92	78.1
b_g	1	1	1	2	1
b_f	2	2	4	7	5
r^2	0.966	0.961	0.973	0.985	0.972

95% confidence level. Mechanical properties c1, c2, and tc were found to depend on b0, b1, b3, and b1; Pu depends on b0, b1, b2, b3, and b1; and D depends on; b0, b1, b3, bg, and b1.

Data on each mechanical property acquired at the actual BW level and respective predictions at the nominal BW levels are shown in Figures 2 to 6. For each fit of Q, it was further verified that expanding Eq. (5) to include coefficients related to C and G did not result in a significant improvement.

Cost Determination

For purposes of this study, we consider the recycled linerboard cost as the sum of the costs attributable

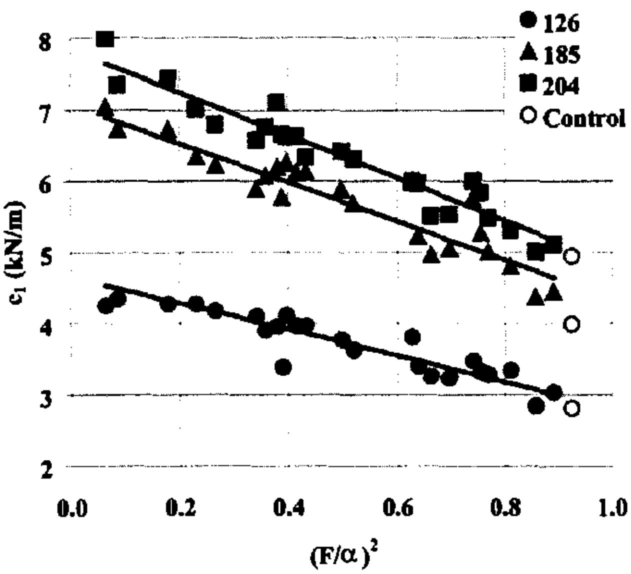

Figure 2. **Variation of compressive load–strain constant c_1 with freeness ratio F/α among data in Table 1. Points represent respective mechanical property determined from handsheet at actual basis weight (BW). Lines are predicted property via Eq. (5) based on nominal BW. Legend refers to nominal BW (g/m^2) and control material (Table 1).**

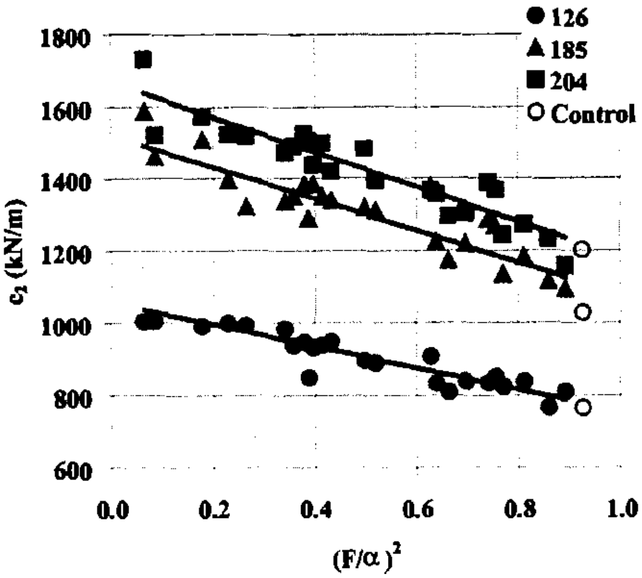

Figure 3. **Variation of compressive load-strain constant c_2 with freeness ratio F/α among data in Table 1. See Figure 2 for explanation of points and legend.**

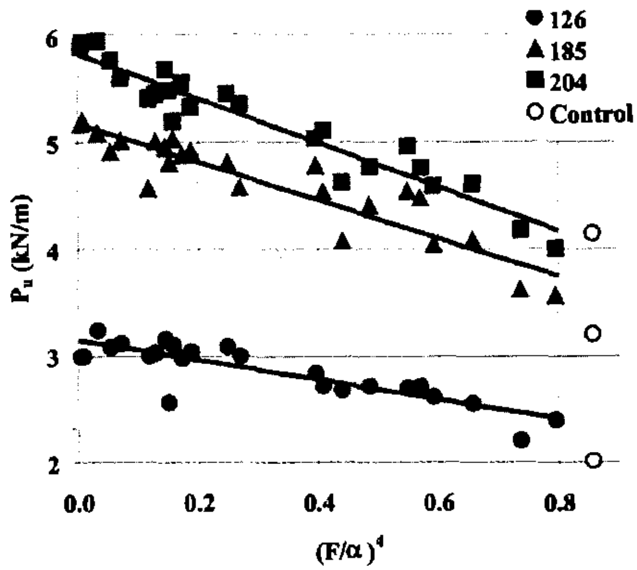

Figure 4. **Variation of ultimate compressive strength P_u with freeness ratio F/α See Figure 2 for explanation of points and legend.**

exclusively to fiber and the specific energy consumption of refining. The best example of a relevant commercial raw material representing the fiber would probably be the recycled pulp that enters the head box on the paper machine. The ultimate purchase price of linerboard by the corrugating converter will include other costs of marketing and production beyond the scope of this investigation.

Costs of recycled fiber and energy vary dramatically with geography and market conditions. For instance, one measure of unprocessed fiber cost would be the price of old

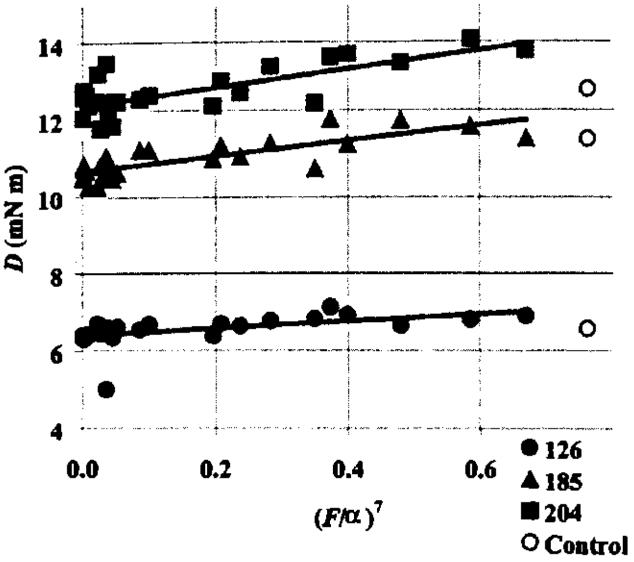

Figure 5. **Variation of bending stiffness *D* with freeness ratio *F*/α See Figure 2 for explanation of points and legend.**

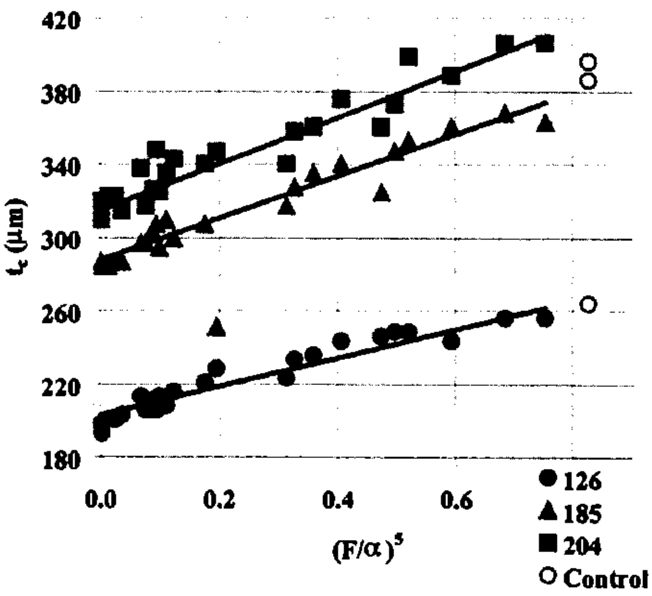

Figure 6. **Variation of surface-to-surface thickness t_c with freeness ratio *F*/α See Figure 2 for explanation of points and legend.**

corrugated containers. Prices reported in the North *American Factbook* (10) vaned throughout the year 2000 from $42.25 per metric ton (t) in New York to $161.67/t in the San Francisco/Los Angeles area. Electrical energy rates during the winter of2001 were reported by the Edison Electric Institute (11) to range from 4.4 cents/kW·h for a 1000-kW demand in Missouri to 13.7 cents/kW·h for a 500-kW demand in New York.

In adding these costs, it is helpful to express them in terms of a unit area of linerboard:

$$C_t\left(\frac{\$}{10^3\,\mathrm{m}^2}\right) = C_f\left(\frac{\$}{10^3\,\mathrm{m}^2}\right) + C_e\left(\frac{\$}{10^3\,\mathrm{m}^2}\right) \quad (6)$$

C_f is the cost of fiber in linerboard at a specific basis weight and is computed from the conventional price C_F reported per ton.

$$C_f\left(\frac{\$}{10^3\,\mathrm{m}^2}\right) = C_F\left(\frac{\$}{\mathrm{t}}\right)\mathrm{BW}\left(\frac{\mathrm{g}}{\mathrm{m}^2}\right)\left(\frac{\mathrm{t}}{1000\,\mathrm{kg}}\right)\left(\frac{\mathrm{kg}}{10^3\,\mathrm{g}}\right) = 10^{-3} C_F\,\mathrm{BW} \quad (7)$$

C_e is the cost of refining energy in linerboard at a specific basis weight and is computed from the conventional energy rate C_E.

$$C_e\left(\frac{\$}{10^3\,\mathrm{m}^2}\right) = C_E\left(\frac{\mathrm{cents}}{\mathrm{kW\cdot h}}\right)\left(\frac{\$}{100\,\mathrm{cents}}\right)\mathrm{SEC}\left(\frac{\mathrm{W\,h}}{\mathrm{kg}}\right)\mathrm{BW}\left(\frac{\mathrm{g}}{\mathrm{m}^2}\right)\left(\frac{\mathrm{kW\,h}}{10^3\,\mathrm{W\,h}}\right)\left(\frac{\mathrm{kg}}{10^3\,\mathrm{g}}\right) = 10^{-2} C_E\,\mathrm{SEC}\ \mathrm{BW} \quad (8)$$

C_t is the combined cost to consider. For example, with fiber at $100/t and energy at $0.09/kW·h, the cost components of a 200-g/m^2 linerboard made from pulp refined at 1,000 W·h/kg would he %20/10^3 m^2 + $18/$10^3$ m^2 = $38/$10^3$ m^2.

Performance Criteria

The compressive strength of a corrugated box is related to the edgewise crush strength of the combined board, which in turn depends on the localized compressive strength of the linerboard facings. If a facing component in cormgated fiberboard is adequately supported against failure by local buckling, its load-carrying capacity approaches P_u of the linerboard material. P_u provides an upper bound on strength. A lower hound is the buckling strength B_u of a facing microplate under simple support conditions and can be determined from the theory by Johnson and Urbanik (12).

For conciseness, we provide equations sufficient for producing the plots in Figures 7 to 9. The complete derivations are provided by Johnson and Urbanik (12, 13). We also limit our analysis to our isotropic handsheet data at standard test conditions as inputs, although the Johnson and Urbanik (12) theory is appropriate to anisotropic properties at arbitrary relative humidity if such data are available.

Buckling strength is given by

$$\mathrm{B}_u = c_1 \tanh \hat{\varepsilon} \quad (9)$$

where $\hat{\varepsilon}$ is a dimensionless buckling strain computed iteratively from

$$\hat{\varepsilon} = \frac{\pi^2 S}{6(1-\nu^2)}\left(1+\sqrt{1-(1-\nu^2)\left(1-\frac{2\hat{\varepsilon}}{\sinh(2\hat{\varepsilon})}\right)}\right) \quad (10)$$

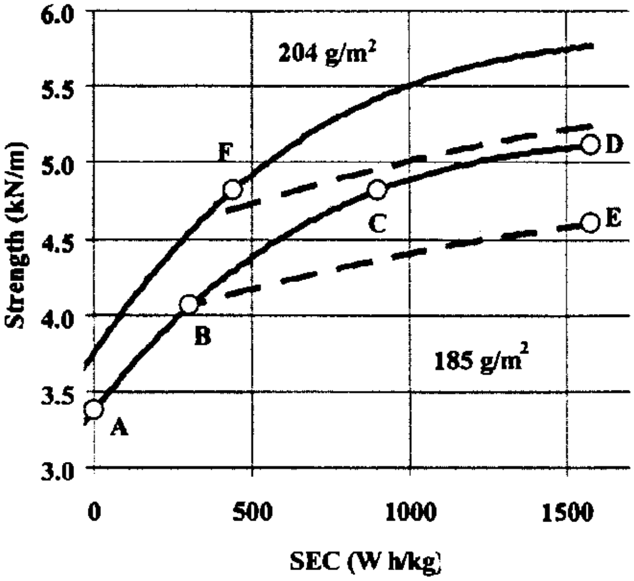

Figure 7. **Variation of microplate strength with SEC of refining and dependence on flute size for linerhoard at two BW levels.** Lower plot (points A–E), 185-g/m^2 linerboard; upper plot (point F), 204-g/m^2 linerboard. Solid lines, predicted P_u in B- or C-flute corrugated structure; dashed lines, predicted B_u in A-flute structure.

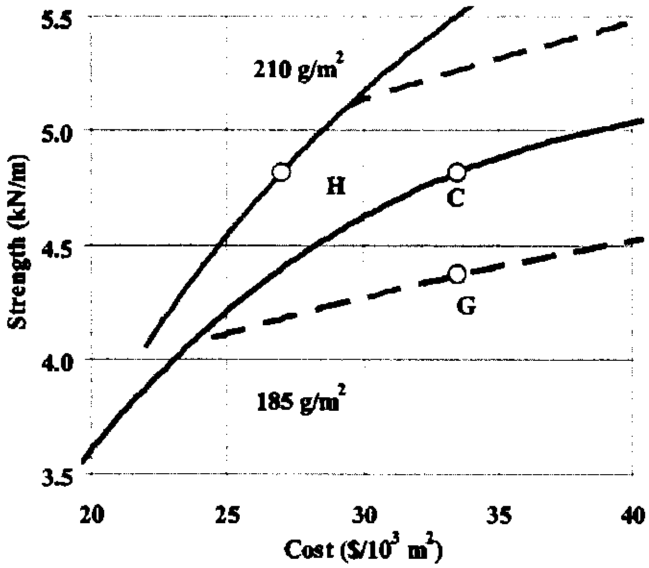

Figure 8. **Variation of microplate strength with combined cost components of fiber at C_F = $100/t and energy at C_E = 9 cents/kW·h and dependence on flute sue for linerboard at two BW levels.** Lower plots (points C, G), 185-g/m^2 linerboard; upper plot (point H), 210-g/m^2 linerboard. See Figure 7 for explanation of the line patterns.

An initial estimate of $\hat{\varepsilon}$ is given by

$$\hat{\varepsilon} = \frac{\pi^2 S}{3(1-\nu^2)} \qquad (11)$$

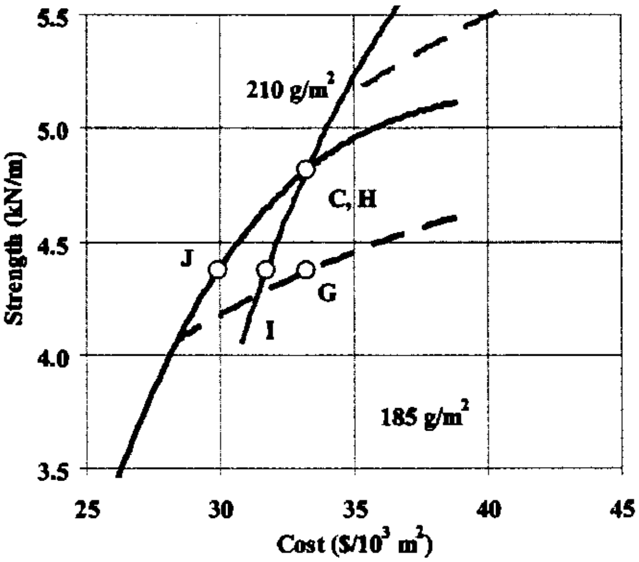

Figure 9. **Variation of microplate strength with combined cost components of fiber at C_F = $140/t and energy at C_E = 5 cents/kW·h and dependence on flute sue for linerboard at two BW levels.** Lower plot (points C, J, G), 185-g/m^2 linerboard; upper plot (points H, I), 210-g/m^2 linerboard. See Figure 7 for explanation of line patterns.

Dimensionless microplate stiffness (S) in terms of mechanical properties of the paper is inputted from

$$S = \frac{c_2}{c_1}\left(\frac{t}{l}\right)^2\sqrt{\frac{\nu_1}{\nu_2}} = \frac{12D}{c_1 l^2} \qquad (12)$$

For simplicity, the second form of S in Eq. (12) is appropriate to handsheet data only.

Input Sand iteration of â involves the material Poisson's ratios ν_1, ν_2, their geometric mean ν, thickness t appropriate to homogeneous material, and microplate width l. When ν is unknown, the average ν = 0.268 considered in the calculations of Johnson and Urbanik (13) is useful to apply. Using the corrugated flute pitch[1] as input l, the upper bound on strength at P_u is apt to apply to corrugated fiberboard having a narrow flute pitch, as in B-flute, or high stiffness facing relative to the linerboard thickness. The lower bound on strength at B_u is apt to apply to a wider flute pitch, as in A-flute, or low stiffness facing.

The linerboard performance of interest is the strength of a facing microplate in a corrugated fiberboard structure. The strength depends further on the support conditions provided by the corrugated medium but lies somewhere between P_u and B_u. P_u is to be calculated directly from Eq. (5) using

[1] Typical values given by Urbanik (14) are 6.35, 7.21, and 8.47 mm for B-, C-, and A-flute, respectively.

the parameters of mechanical property P_u from Table 2 as inputs. B_u is to be calculated from Eq. (9) as an iterative function of Eqs. (10–12) using the parameters of mechanical properties c_1 and D from Table 2 as inputs and values of v and l.

Relation of Performance to Energy Consumption

As Figure 1 shows, the specific energy consumption of refining varied with the refining freeness level for our specific linerboard pulp. As Figures 2 to 6 show, mechanical properties varied with basis weight and freeness for handsheets made from the refined pulp. If the facing components of corrugated fiberboard have properties represented by handsheets, performance would be expected to vary with consumed energy, as shown in Figure 7.

The predicted performance of linerboard facings at two BW levels is plotted in Figure 7. In addition. two ranges of flute size are analyzed. The conditions $C = 8\%$ and $G = 178$ µm are fixed. If a linerboard facing is made to 185 g/m^2 BW, its maximum strength from pulp refined at various levels would follow the path shown by points A, B, C, and D (Fig. 7) corresponding to calculated freeness levels of 732, 648, 485, and 300 mL CSF, respectively. Increasing levels ofrefining reduce pulp freeness, increase energy consumption (Fig. 1), and increase strength (Fig. 4). Path A–B–C–D (Fig. 7) quantifies maximum strength in terms of P_u for a 185-g/m^2 linerboard.

Path A–B–C–D (Fig. 7) is the predicted linerboard performance in a B-flute or C-flute corrugated fiberboard. If the same linerboard is fabricated into an A-flute structure, strength is predicted to follow the path A–B–E (Fig. 7). Path B–E results from a condition where B_u (Eq. (9)) is less than P_u. The actual intersection at point B depends on the value of l = 8.47 mm assumed to apply.

A curve predicting the performance of a 204-g/m^2 linerboard is also shown in Figure 7. Point F for a 204-g/m^2 linerboard with a calculated freeness level of 611 mL CSF is at the same strength level as point C for a 185-g/m^2 linerboard. In each case, P_u = 4.82 kN/m. However, point F results from pulp refined to a freeness level consuming 440 W·h/kg, whereas point C results from pulp refined to a freeness level consuming 900 W·h/kg. Strength at point F, which represents more fiber and less consumed energy, is the same strength as that at point C, which represents less fiber and more consumed energy. The better choice for corrugated fiberboard depends on the relative costs of fiber and energy.

Optimization

The quantification of point C as presented in Figure 7 is shown again in Figure 8 in terms of the cost C_t, assuming a fiber cost C_F of \$100/t and an energy cost C_E of 9 cents/kW·h. Point C shows that the 185-g/m^2 linerboard would have a strength P_u of 4.82 kN/m at a cost C_t of \$33.49/10^3 m^2 if used in a B- or C-flute corrugated fiberboard. If the same linerboard is used in an A-flute corrugated fiberboard, the expected strength is reduced 9.5% to a B_u of 4.37 kN/m (point G, Fig. 8).

The analysis of a 210-g/m^2 linerboard is also plotted in Figure 8. Point H is at the same strength level as point C but at a lower cost C_t of \$26.99/10^3 m^2. For a required strength of 4.82 kN/m and for our included assumptions, the 210-g/m^2 linerboard is 19% less expensive than the 185-g/m^2 linerboard. The pulp represented by point H is refined to a level of 663 mL CSF compared to 485 mL CSF for the pulp at point C. In this example, it is less expensive to add fiber than to increase refining.

The 210-g/m^2 linerboard (Fig. 8) has another inherent advantage. The Performance predicted by point H is the same for B-, C-, and A-flute structures. By contrast, the 185-g/m^2 linerboard becomes weaker in the A-flute structure, which could generate additional costs. Additional analysis at our assumed conditions would show that the 210-g/m^2 linerboard is more economical than the 185-g/m^2 linerboard at any required strength level.

The analytical results could change if market conditions change. Figure 9 predicts the performance at the same two BW levels assuming a higher fiber cost C_F of \$140/t and a lower energy cost C_E of 4.41 cents/kW·h. For these market conditions and for a required strength of 4.82 kN/m, linerboards at both BW levels cost \$33.25/10^3 m^2. Points C and H merge in Figure 9. However, if a lower strength, such as 4.37 kN/m, were required in a B- or C-flute structure, it would cost \$29.92/10^3 m^2 (point J, Fig. 9) to use 185-g/m^2 linerboard compared with \$31.72/10^3 m^2 (point I, Fig. 9) to use 210-g/m^2 linerboard.

The pulp represented by point J (Fig: 9) was refined to a level of 597 mL CSF compared to 706 mL CSF for the pulp at point I. In this example, when energy costs are low, it is less expensive to expend energy and increase refining than to add fiber. In contrast with the scenario in Figure 8, the 185-g/m^2 linerboard is 5.7% less expensive than the 210-g/m^2 linerboard. This cost advantage is limited to the flute size considered. If the linerboard is required to retain the same strength level in an A-flute structure, it must be refined to 485 mL CSF at a combined cost of \$33.25/10^3 m^2 (point G, Figure 9).

CONCLUSIONS

The mechanical properties ofpaper at a single basis weight and a single targeted refining freeness level have traditionally been used to evaluate the feasibility of using small-diameter trees, waste wood, and unconventional wood species for papermaking. While the conclusions drawn may have been valid, we see that a more global characterization of the variation of mechanical properties and refining energy consumption with freeness provides a more rational economic justification of refining levels. At recent price levels, the cost of refining energy to increase paper performance is of the same magnitude as the cost of adding fiber to obtain an equal increase in performance. The costs of energy and fiber vary dramatically with geography and market conditions. We provide scenarios for optimizing the performance of the linerboard component in corrugated fiberboard based on real world data, showing how the lowest cost linerboard depends on market conditions. For the single recycled pulp investigated, an economic justification for increased refining occurs only with a combination of relatively inexpensive energy and expensive fiber. Otherwise. it is more economical to use low refined pulp and heavier hasis weight papers.

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