BENDING TO SHEAR RATIO APPROACH FOR BEAM DESIGN

LAWRENCE A. SOLTIS
DOUGLAS R. RAMMER

ABSTRACT

Current design criteria for timber beams are often governed by calculating shear strength rather than bending strength. However, bending is often observed as the controlling factor in real structures and experimental studies. This report describes a design procedure based on predicting whether bending or shear will control the design and designing the member for that consideration. The prediction is based on comparing the ratio of maximum bending and shear stresses to experimentally determined bending and shear strengths. Stresses are based on load and support conditions and span/depth ratios. A lower limit bending-to-shear stress ratio of 10 is recommended for design on the basis of a lower bound fit to experimental strength values. For this stress ratio, typical designs for most load and support conditions will result in bending-controlled designs that do not require shear calculations. Shear will control the design for a simply supported span with a concentrated load near one support or for a span with a short overhang having a concentrated or uniform load. This approach is limited to unsplit and unnotched beams. Design considerations for notched beams and shear at connections are outside the scope of this report.

Both bending and shear strength must be considered in the design of timber beams. Design is often governed by shear calculations rather than bending calculations; however, bending rather than shear is often observed as the controlling factor in real structures and experimental studies. The difference between observed and calculated controlling factors is based on differences in derivation of allowable stresses for shear and bending.

This paper describes a design procedure based on lower bound theory of experimental shear and bending strength. The hypothesis is that if it is known whether shear or bending will control the design, only that design consideration needs to be checked during the design procedure. The controlling design consideration is defined by relating span/depth ratios that are analytically determined from shear and bending stresses for a variety of load and support conditions to experimentally determined beam shear and bending strength.

Shear and bending strength have been correlated in limited studies. Most research on shear strength has focused on shear block strength as defined by ASTM D 143 (2). A few studies have investigated beam shear in glued-laminated beams (8,9). Some studies have investigated bending strength, but few have yielded results on both bending and shear. Recent studies by Rammer and others (10, 11) are the only known studies of full-size beams in which correlated samples were tested in both bending and shear. Zakic (13) tested 10 laminated beams with a span/depth ratio of 8.5 and a concentrated midspan load. Two beams failed in shear and the remainder in bending. Buchanan and Madson (5) tested 2 by 4 and 2 by 8 Douglas-fir and spruce-pine-fir lumber and concluded that it is not necessary to design for shear for commercial lumber without end splits.

This paper summarizes shear and bending strength results for glued-laminated and solid-sawn timber beams and describes a modified or simplified design procedure for shear considerations. This approach is limited to unsplit and unnotched beams. Design considerations for notched beams and shear at connections are outside the scope of this study.

EXPERIMENTAL PROCEDURE

Approximately 500 glued-laminated beams and 280 solid-sawn beams were tested to determine shear strength and bending modulus of rupture (MOR). Detailed experimental procedures and complete test results were published previously for glued-laminated beams (10) and solid-sawn beams (11).

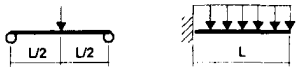
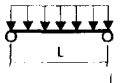
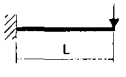
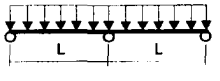
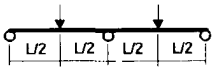
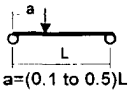
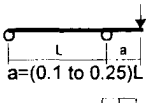
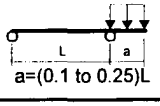
Southern pine and Douglas-fir glued-laminated beams ranged in size from 1-1/2 by 4 inches (38 by 89 mm) to 5 by 24 inches (127 by 610 mm). The larger southern pine beams conformed to a 24F-V5 configuration; the smaller beams were cut from the larger ones. Most laminations in the high shear

The authors are, respectively, Supervisory Research Engineer and General Research Engineer, USDA Forest Serv., Forest Products Lab. (FPL), One Gifford Pinchot Dr., Madison, WI 53705-2398. The FPL is maintained in cooperation with the Univ. of Wisconsin.
This paper was received for publication in January 1996. Reprint No. 8480.
©Forest Products Society.
Forest Prod. J. 47(l):104-108.

TABLE 1. — Matrix of specimens tested and matched.

Beam type and size[a]	Matching method	No. of specimens tested: Shear	No. of specimens tested: Bending
Southern pine glulam			
1-1/2 by 4	Side matched	22	22
2-1/2 by 5-1/2 LQ	Side matched	39	39
2-1/2 by 5-1/2 HQ	Side matched	38	38
3 by 11	End matched	20	20
5 by 22	End matched	20	20
Douglas-fir glulam			
1-1/2 by 5-1/2 LQ	Side matched	30	30
1-1/2 by 5-1/2 HQ	Side matched	26	26
2-1/2 by 6-1/2 LQ	Side matched	14	14
2-1/2 by 6-1/2 HQ	Side matched	37	37
3-1/8 by 12	End matched	20	20
4-3/8 by 11-1/2 LQ[b]	Side matched	16	16
4-3/8 by 11-1/2 HQ[b]	Side matched	17	17
5-1/8 by 24	End matched	20	20
Douglas-fir solid-sawn			
2 by 4	End matched	40	40
2 by 10	MOE matched	40	20
4 by 8	MOE matched	40	20
4 by 12	MOE matched	20	20
4 by 14	MOE matched	20	20

[a] LQ = No. 2 medium grain Southern Pine or L2 Douglas-fir inner laminations; HQ = specially graded tension laminations.
[b] Originally tested at Washington State University.

Case	Type of load and support conditions	C	L/d where shear failure governs:
1,2		2	L/d < 5
3		1	L/d < 10
4		4	L/d < 2.5
5		0.8	L/d < 12.5
6		1.09	L/d < 9.2
7[a]		0.4 to 2[b]	L/d < 25 to L/d < 5[b]
8		0.4 to 1	L/d < 25 to L/d < 10
9		0.2 to 0.5	L/d < 50 to L/d < 10

[a]Case 7 applicable to moving concentrated load.
[b]The first expressions, 0.4 and L/d < 25, correspond to a = 0.1L; the second expressions, 2 and L/d < 5, correspond to a = 0.5L. For 0.1L < a < 0.5L, use linear interpolation.

Figure 1. — Values of C for several commonly occurring load and support conditions.

stress zones were No. 2 medium-grain material. The larger Douglas-fir beams conformed to a 24F-V8 configuration; the smaller beams were cut from the larger ones. Most laminations in the high shear stress zones were L2 material. Douglas-fir No. 2 or Better solid-sawn beams ranged in size from 2 by 4 inches (38 by 89 mm) to 4 by 14 inches (89 by 337 mm).

Matched samples for bending and shear tests were obtained by either side matching, end matching, or modulus of elasticity (MOE). Side-matched samples were obtained by ripping a wide sample in half. End-matched samples were obtained by cutting a long member into two short members. The MOE-matched samples were obtained by separating a large group of specimens into two smaller groups based on the rank of each MOE. Beam size, lumber species, number of specimens, and matching techniques are shown in Table 1.

The glued-laminated bending and shear specimens were tested in a dry condition with moisture contents (MCS) of 10 to 11 percent. The solid-sawn specimens were tested in a green condition with MCS above 20 percent.

The bending tests were simple spans of lengths either 14 or 15 times the member depth loaded with two concentrated loads, each located 2.5 times the member depth from the centerline of the span (2). Shear tests were two span members with each span length 5 times the member depth loaded with a mid-span concentrated load. All specimens were loaded to failure. Members tested for shear sometimes failed in bending; only results of those members failing in shear and matched specimens failing in bending are reported here.

RESULTS

The results are presented as a comparison of the ratio of maximum bending and shear parallel-to-grain stresses, f_b and f_v, to the ratio of maximum bending and shear parallel-to-grain strengths, F_{ub} and F_{uv}.

Maximum bending moment *(M)* and shear force *(V)* are dependent on types of loads and support conditions. Maximum bending and shear stresses are determined from moments and forces by mechanics equations for rectangular members:

$$f_b = \frac{M}{S} \quad [1]$$

$$f_v = \frac{3V}{2A} \quad [2]$$

where:

S and A = section modulus and cross-sectional area in common units

Combining Equations [1] and [2] results in a bending to shear stress ratio:

$$\frac{f_b}{f_v} = C\frac{L}{d} \quad [3]$$

where:

C = a constant depending on types of loads and support conditions

L and d = span length and member depth in common units

Values of C for several commonly occurring load and support conditions are given in **Figure 1.**

Maximum bending and shear strengths, F_{ub} and F_{uv}, for each individually matched specimen were determined by test and are plotted in Figures **2,3,** and **4** for southern pine glued-laminated timber, Douglas-fir glued-laminated timber, and green Douglas-fir solid-sawn specimens, respectively. Superimposed on these figures is a line corresponding to a ratio of maximum bending to shear stresses equal to 10.

The bending and shear strength values plotted in **Figures 2** and **3** are data from tests on dry specimens, and the values in **Figure 4** are data from tests on green specimens. In **Figure 5,** the strength values of **Figure 4** are adjusted from a green to dry condition using the moisture adjustment procedure given in ASTM D 245 (3).

The test data plotted in **Figures 2** through **5** are ultimate strength values. Different factors of safety for bending and shear are used to derive allowable strength values; thus plotting allowable values would skew the results.

Discussion

No statistical correlation was found between bending and shear strength for either the glued-laminated timber (10) or solid-sawn timber (11). This was evidenced by low coefficients of determination of the statistical regressions of bending strengths versus shear strengths. Because no correlation was found, we plotted results from tests on individually matched bending and shear specimens in **Figures 2** through **5.** Green et al. (7) compared ASTM shear-block strengths to MOR values for machine-stress-rated lumber and found a positive trend but a low coefficient of determination.

The strength of beams is influenced by the member size for both bending (1) and shear (12). The bending and shear test results automatically included this size effect; thus, no correction was required in **Figures 2** through **5.**

Ultimate strength values for MOR and shear strength are plotted in **Figures 2** through **5.** The derivation of allowable bending stress values is based on dividing the fifth percentile value of the MOR by a factor of 2.1, which includes

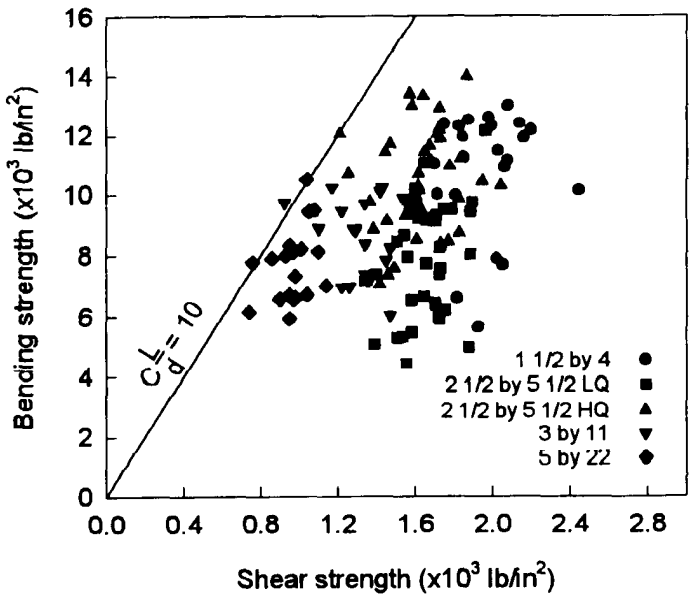


Figure 2. — Shear strength versus modulus of rupture for various sizes of dry (10% to 11% moisture content) glued-laminated southern pine beams.

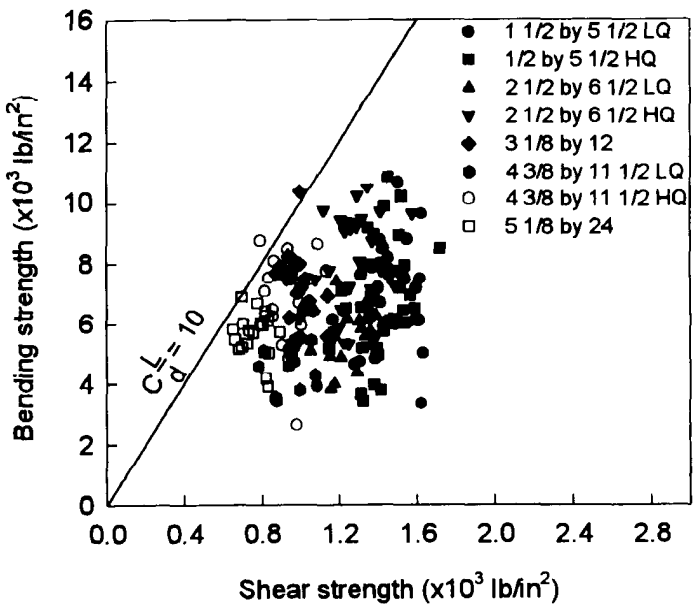


Figure 3. — Shear strength versus modulus of rupture for various sizes of dry glued-laminated Douglas-fir beams.

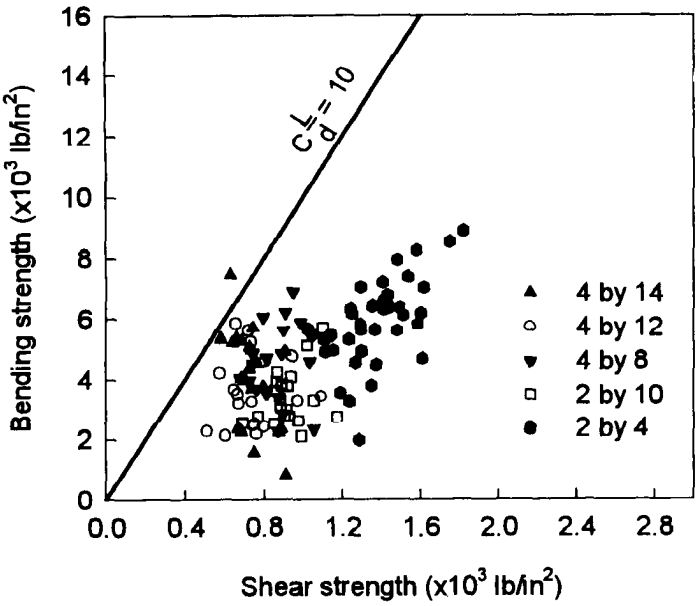


Figure 4. Shear strength versus modulus of rupture for various sizes of green solid-sawn Douglas-fir beams.

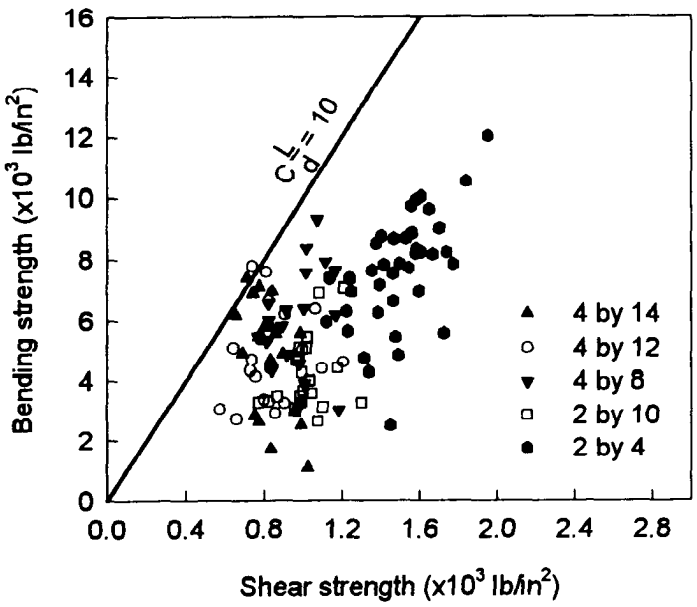


Figure 5. — Shear strength versus modulus of rupture for various sizes of solid-sawn Douglas-fir beams adjusted to 12 percent moisture content.

safety and load duration factors (3). The derivation of allowable shear stress values is based on dividing the fifth percentile values of the ASTM D 143 shear-block strength by a factor of 4.1, which includes safety, load duration, and stress concentration factors (3,4). Thus, comparing allowable bending and shear values derived with different factors was concluded to be inappropriate.

The principle of comparing maximum bending and shear stress to ultimate strength is shown in **Figure 6.** The data point illustrates the bending (F_{ub}) and shear (F_{uv}) strength of one beam based on matched specimens and represents the point where shear and bending failures would occur simultaneously. The ratio of maximum bending to shear stresses (Eq. **[3])** is linear, with a slope dependent on types of loads, support conditions, and span/depth ratios **(Fig. 1).** If the stress ratio line is to the left of the data point **(Fig. 6a),** this particular beam will reach its MOR strength before reaching its shear strength; thus, bending is the controlling factor, If the stress ratio line is to the right of the data point **(Fig. 6b),** this particular beam will reach its shear strength before reaching its MOR; thus, shear is the controlling factor.

Using the principle illustrated in **Figure 6,** a $C\frac{L}{d}$ value of 10 was selected for Figures 2 through 5. As a result of this value, bending will control design before shear for nearly all of the matched glued-laminated and solid-sawn beams tested; thus, this value is recommended as a design basis. This approach is based on the lower bound of individual data points because no correlation exists between bending and shear. Thus, a span/depth ratio can be determined for a beam with various load and support conditions such that the $C\frac{L}{d}$ ratio is 10. For example, a simply supported beam with concentrated midspan load or a uniformly loaded cantilever beam **(Fig. 1,** Cases 1 and 2) will be controlled by bending if the span/depth ratio is greater than 5. Conversely, these beams will only need to be checked for shear if the span/depth ratio is less than 5. A simply supported, uniformly loaded beam **(Fig. 1,** Case 3) need only be checked for shear if the span/depth ratio is less than 10. In practice, most simply supported beams have a span/depth ratio of 15 to 25, so no shear check is required unless there is a concentrated load near one support **(Fig. 1,** Case 7) or a short overhang with a concentrated or uniform load **(Fig. 1,** Cases 8 and 9).

The selection of a bending-to-shear stress ratio of 10 implies that shear strength is about 10 percent of bending strength. European standards (6) have long used this value as a basis for allowable shear strength.

The proposed design procedure is more conservative for glued-laminated timber than for solid-sawn lumber because both have equal shear parallel-to-grain design values but glued-laminated timber has higher bending design values.

Conclusion

Design criteria for beams are currently governed by shear rather than

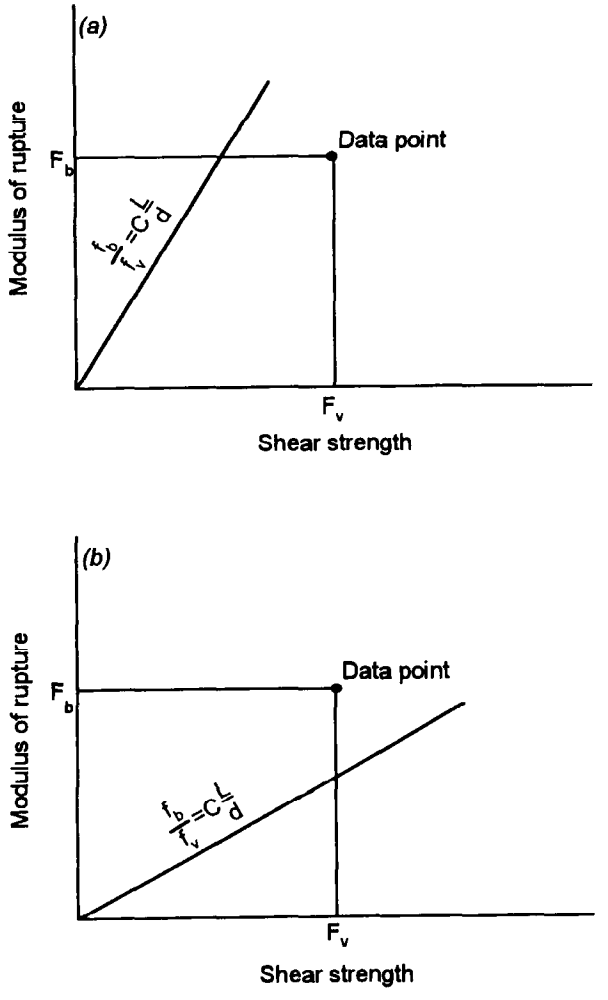


Figure 6. — The principle of comparing bending-to-shear stress ratio to modulus of rupture and shear strength: (a) bending failure occurs before shear failure; (b) shear failure occurs before bending failure.

bending because shear calculations have larger safety factors. However, in real structures and experimental studies, bending rather than shear is observed as the controlling factor. The proposed design procedure is based on predicting whether bending or shear will control the design and then designing the member for that design condition. A bending-to-shear stress ratio of 10 (lower limit) is recommended for design; this ratio is based on lower bound fit to experimental strength values. For this stress ratio, typical designs for most load and support conditions will result in bending-controlled designs; no shear calculations are necessary. Shear will control the design for a simply supported span with a concentrated load near one support or for a span with a short overhang with a concentrated or uniform load. This approach is limited to unsplit and unnotched beams and does not apply to shear considerations at a connection.

LITERATURE CITED

1. American Forest & Paper Association (formerly National Forest Products Assoc.). 1991. National design specification for wood construction. ANSI/NFPA NDS. AF&PA, Washington, D.C.
2. American Society for Testing and Materials. 1995. Standard methods of static tests of timber in structural sizes. ASTM D 198-84. ASTM, West Conshohocken, Pa.
3. _____. 1995. Establishing structural grades and related allowable properties for visually graded lumber. ASTM D 245-93. ASTM, West Conshohocken, Pa.
4. _____. 1995. Establishing allowable properties for visually-graded dimension lumber from in-grade tests of full-size specimens. ASTM D 1990-91. ASTM, West Conshohocken, Pa.
5. Buchanan, A.H. and B. Madson. 1984. Shear design for lumber. Dept. of Civil Engineering, Univ. of British Columbia, Vancouver, B.C., Canada.
6. Fewell, A.R. 1989. Stress classes for structural lumber. Document CEN/TC/WG2 N66E. British Standard Inst., London, U.K.
7. Green, D.W., D.E. Kretschmann, and J.A. Fantozzi. 1994. Shear and compression perpendicular to grain property estimation for 2400f MSR lumber. Forest Prod. J. 44(11/12):75-82.
8. Longworth, J. 1977. Longitudinal shear strength of timber beams. Forest Prod. J. 27(8):19-23.
9. Quaile, A.T. and F.J. Keenan. 1978. Shear strength of small composite wood beams. Wood Sci. 11(1):1-9.
10. Rammer, D. R. and L.A. Soltis. 1994. Experimental shear strength of glued-laminated beams. Res. Rept. FPL-RP-527. USDA Forest Serv., Forest Prod. Lab., Madison, Wis.
11. _____, _____, and P. Lebow. 199_. Experimental shear strength of unchecked solid-sawn Douglas-fir. FPL Res.Pap., USDA Forest Serv., Forest Prod. Lab., Madison, Wis. In preparation.
12. Soltis, L.A. and D.R. Rammer. 1994. Shear strength of unchecked glued-laminated beams. Forest Prod. J. 44(1):1-8.
13. Zakic, B.D. 1983. Shearing stress in plastic bending of wood beams. J. of Structural Engineering 109(4):1042-1060.

NOTATION

L = length of beam span; d = depth of member; A = cross-sectional area; S = section modulus; f_b = maximum bending stress parallel to grain; f_v = maximum shear stress parallel to grain; F_{ub} = ultimate bending strength parallel to grain; F_{uv} = ultimate shear strength parallel to grain; M = maximum bending moment; V = maximum shear force.