A PARAMETRIC STUDY OF HEAT AND MASS TRANSFER IN DRYING OF CAPILLARY-POREUS MEDIA

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ABSTRACT
This paper evaluates the relative importance of the dimensionless numbers in the Luikov system of heat and mass transfer equations in predicting the temperature and moisture distributions in capillary-porous media during drying. The six dimensionless numbers considered are the Lukomskii number Lu, Biot heat transfer number Bi_q, Biot mass transfer number Bi_m, phase transformation number E, Posnov number Pn, and Kossovich number Ko. The thermophysical properties of spruce were used to derive the reference set of input data for the dimensionless numbers. By varying each of the six numbers twice while keeping the others constant, 13 sets of input data were generated. The Fourier number, also defined as dimensionless time, covered the range of interest. The solution technique applied is somewhat simpler than those reported by others in the literature.

INTRODUCTION
This paper evaluates the relative importance of the dimensionless numbers in the Luikov (1966) system of heat and mass transfer equations in predicting the temperature and moisture distributions in capillary-porous media during drying. Luikov equations with boundary conditions of the third kind were solved by Luikov and Mikhailov (1965) using the Laplace transform technique for the cases of a plate, a cylinder, and a sphere. These same problems were also treated by Mikhailov and OziSik (1984) using the finite integral transform technique. In the case of drying of capillary-porous media such as lumber, the boundary conditions must relate the transfer potentials at the surfaces of the body to the corresponding potentials of the surroundings. Therefore, the boundary conditions of the third kind (Luikov and Mikhailov, 1965b) should be applied, which makes the process of obtaining solutions more complex.

In our study we solved the Luikov equations in dimensionless form using the same analytical technique developed by Liu and Cheng (1991). The method of solution was somewhat simpler than that of Luikov and Mikhailov (1965a), and the complex eigenvalues, when existent, were taken into account. The thermophysical properties of spruce (Liu and Cheng, 1991) were used to form the reference set of input data for the six dimensionless numbers in the equations, namely, the Lukomskii number Lu, Biot heat transfer number Bi_q, Biot mass transfer number Bi_m, phase transformation number E, Posnov number Pn, and Kossovich number Ko. By varying each of the six numbers twice while keeping the other numbers the same as in the reference set, we obtained 13 sets of data. The Fourier number, also defined as dimensionless time, covered the range of interest. From the solutions of the 13 cases, the effects of the six numbers and their relative importance on heat and mass transfer in the drying of capillary-porous media similar to spruce were evaluated after the approach of Robbins and Ozisik (1988).

HEAT AND MASS TRANSFER EQUATIONS
For the one-dimensional case shown in Figure 1, heat and mass move along the x axis only. Under constant pressure,
the governing equations (Luikov and Mikhailov, 1965a) can be written as

\[ \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} - \epsilon \frac{\partial \Theta}{\partial t} \quad (-1 < x < 1; \quad t > 0) \]  
\[ \frac{\partial \Theta}{\partial t} = \frac{\partial^2 \Theta}{\partial x^2} - \frac{\partial^2 T}{\partial x^2} \quad (-1 < x < 1; \quad t > 0) \]  

where \( T \) is dimensionless temperature, \( \Theta \) dimensionless mass transfer potential, \( \epsilon \) Fourier number, \( x \) dimensionless coordinate, \( \xi \) phase transformation number, \( \kappa \) Koosovitch number, \( \alpha \) Lukomskii number, and \( P \) Posnov number (see Nomenclature). (According to Luikov and Mikhailov (1965a), mass transfer potential of the bound matter in the liquid state need be considered only because at atmospheric pressure the specific mass of vapor is negligibly small compared with that of the liquid in the pores of the body.)

At the surfaces of the specimen in Figure 1, \( x = \pm 1 \), the boundary conditions of the third kind (Luikov and Mikhailov, 1965b) apply. They are

\[ \frac{\partial T}{\partial x} = B_i (1 - T) - (1 - \epsilon) \alpha K B_i x (1 - \Theta) \quad (X = \pm 1; \quad t > 0) \]  
\[ \frac{\partial \Theta}{\partial x} = \alpha K \frac{\partial T}{\partial x} + B_i (1 - \Theta) \quad (X = \pm 1; \quad t > 0) \]  

where \( B_i \) and \( B_i \) are the Biot heat and mass transfer numbers, respectively.

Because of symmetry, at \( x = 0 \) we should have

\[ \frac{\partial T}{\partial x} = 0 \quad (X = 0; \quad t > 0) \]  
\[ \frac{\partial \Theta}{\partial x} = 0 \quad (X = 0; \quad t > 0) \]  

The initial conditions are assumed to be constant and are represented by

\[ T = 0 \quad (-1 \leq x \leq 1; \quad t = 0) \]  
\[ \Theta = 0 \quad (-1 \leq x \leq 1; \quad t = 0) \]  

**METHOD OF SOLUTION**

The boundary conditions (3) and (4) are non-homogeneous. One way to make them homogeneous is to introduce two functions \( T_1 \) and \( \Theta_1 \) of \( x \) and \( t \) such that

\[ T = T_1 + 1 \]  
\[ \Theta = \Theta_1 + 1 \]  

and substitute for \( T \) and \( \Theta \) in all the preceding equations. We then obtain the following:

1. **Basic equations**

\[ \frac{\partial T_1}{\partial x} = \frac{\partial^2 T_1}{\partial x^2} - \epsilon \frac{\partial \Theta_1}{\partial x} \quad (-1 < x < 1; \quad t > 0) \]  
\[ \frac{\partial \Theta_1}{\partial x} = \frac{\partial^2 \Theta_1}{\partial x^2} - \frac{\partial^2 T_1}{\partial x^2} \quad (-1 < x < 1; \quad t > 0) \]  

2. **Boundary conditions**

\[ \frac{\partial T_1}{\partial x} = -B_i T_1 (1 - \epsilon) \alpha K B_i x \Theta_1 \quad (X = \pm 1; \quad t > 0) \]  
\[ \frac{\partial \Theta_1}{\partial x} = \alpha K \frac{\partial T_1}{\partial x} - B_i x \Theta_1 \quad (X = \pm 1; \quad t > 0) \]  
\[ \frac{\partial T_1}{\partial x} = 0 \quad (X = 0; \quad t > 0) \]  
\[ \frac{\partial \Theta_1}{\partial x} = 0 \quad (X = 0; \quad t > 0) \]  

3. **Initial conditions**

\[ T_1 = -1 \quad (-1 \leq x \leq 1; \quad t = 0) \]  
\[ \Theta_1 = -1 \quad (-1 \leq x \leq 1; \quad t = 0) \]  

Introducing a potential function \( \phi(x, \quad t) \) such that

\[ T_1 = \left( \frac{\partial}{\partial t} - \frac{\partial^2 \phi}{\partial x^2} \right) \phi \]  
\[ \Theta_1 = -\frac{\partial^2 \phi}{\partial x^2} \]  

we find that Equation (12) is automatically satisfied and Equation (11) becomes

\[ \left[ \frac{\partial^4 \phi}{\partial x^4} - \left( 1 + \frac{1}{\alpha K} + \epsilon \frac{\partial^2 \phi}{\partial x^2} \right) \frac{\partial \phi}{\partial x^2} \right] = 0 \]  
\[ \text{or} \]  
\[ \left[ \frac{\partial^2 \phi}{\partial x^2} - \nu^2 \frac{\partial^2 \phi}{\partial x^2} \right] \phi = 0 \]  

with

\[ \nu^2 = \frac{1}{2} \left( 1 + \frac{1}{\alpha K} + \epsilon \frac{\partial^2 \phi}{\partial x^2} \right) \]  
\[ - \frac{1}{4} \left( 1 + \frac{1}{\alpha K} + \epsilon \frac{\partial^2 \phi}{\partial x^2} \right)^2 \frac{1}{\alpha K} \]  
\[ \nu^2 = \frac{1}{2} \left( 1 + \frac{1}{\alpha K} + \epsilon \frac{\partial^2 \phi}{\partial x^2} \right) \]  
\[ + \frac{1}{4} \left( 1 + \frac{1}{\alpha K} + \epsilon \frac{\partial^2 \phi}{\partial x^2} \right)^2 \frac{1}{\alpha K} \]
Equation (22) can be broken into two equations as follows:

\[
\begin{align*}
\frac{\partial^2 \phi_1}{\partial X^2} &= \nu_1^2 \frac{\partial \phi_1}{\partial \text{Fo}} \\
\frac{\partial^2 \phi_2}{\partial X^2} &= \nu_2^2 \frac{\partial \phi_2}{\partial \text{Fo}}
\end{align*}
\]

(25)  
(26)

which are of the diffusion type. The general solution of Equation (22) is the sum of the solutions of Equations (25) and (26) and can therefore be written as

\[
\phi = \phi_1(X, \text{Fo}) + \phi_2(X, \text{Fo})
\]

in which, using the method of separation of variables (Crank, 1975).

\[
\phi_1(X, \text{Fo}) = A e^{-\nu_1^2 \text{Fo} \cos \nu_1 \mu X}
\]

\[
\phi_2(X, \text{Fo}) = B e^{-\nu_2^2 \text{Fo} \cos \nu_2 \mu X}
\]

Hence

\[
\phi = e^{-\nu_1^2 \text{Fo}}(A \cos \nu_1 \mu X + B \cos \nu_2 \mu X)
\]

(27)

where A and B are arbitrary constant coefficients. (Sine functions do not appear because of the symmetry conditions represented by Equations (15) and (16).)

Substituting Equation (27) into Equations (19) and (20) yields

\[
T_1 = \mu_2^2 e^{-\nu_1^2 \text{Fo}}(a_1 A \cos \nu_1 \mu X + a_2 B \cos \nu_2 \mu X)
\]

(28)

\[
\Theta_1 = \mu_2^2 e^{-\nu_1^2 \text{Fo}}(b_1 A \cos \nu_1 \mu X + b_2 B \cos \nu_2 \mu X)
\]

(29)

where

\[
\begin{align*}
a_1 &= L u_1^2 - 1 \\
a_2 &= L u_2^2 - 1 \\
b_1 &= L u P n_1^2 \\
b_2 &= L u P n_2^2
\end{align*}
\]

(30)

Substituting Equations (28) and (29) into the boundary conditions (13) and (14), we obtain

\[
\begin{align*}
(c_1 \mu_1 \sin \nu_1 \mu + c_2 \cos \nu_1 \mu) A \\
+ (c_3 \mu_1 \sin \nu_2 \mu + c_4 \cos \nu_2 \mu) B &= 0 \\
(d_1 \mu \sin \nu_1 \mu + d_2 \cos \nu_1 \mu) A \\
+ (d_3 \mu \sin \nu_2 \mu + d_4 \cos \nu_2 \mu) B &= 0
\end{align*}
\]

(31)  
(32)

where

\[
\begin{align*}
c_1 &= \nu_1 - L u_1^3 \\
c_2 &= -B_{1q} + B_{1q} L u_2^2 - (1 - \epsilon) L u_2^2 K o B_{1m} P n_1^2 \\
c_3 &= \nu_2 - L u_2^3 \\
c_4 &= -B_{1q} + B_{1q} L u_2^2 - (1 - \epsilon) L u_2^2 K o B_{1m} P n_2^2 \\
d_1 &= \nu_1 \\
d_2 &= -B_{1m} \nu^2 \\
d_3 &= \nu_2 \\
d_4 &= -B_{1m} \nu_2^2 \\
\end{align*}
\]

For nontrivial solutions of Equations (31) and (32) to exist, the determinant of the coefficients of A and B must vanish, giving the characteristic equation

\[
(\mu_1 \tan \nu_1 \mu + \psi_1)(\mu_2 \tan \nu_2 \mu + \psi_2) = \psi_3
\]

(33)

in which

\[
\psi_1 = [B_{1q} - B_{1q} L u_2^2 + (1 - \epsilon) L u_2^2 K o B_{1m} P n_1^2 \\
- B_{1m} \nu(1 - L u_2^2)]/L u(\nu_1^2 - \nu_2^2)
\]

\[
\psi_2 = [B_{1q} - B_{1q} L u_2^2 + (1 - \epsilon) L u_2^2 K o B_{1m} P n_2^2 \\
- B_{1m} L u_2(1 - \nu_2^2)]/L u(\nu_2^2 - \nu_1^2)
\]

\[
\psi_3 = -B_{1m} B_{1q} + \psi_1 \psi_2
\]

From Equation (32) we can also derive

\[
\frac{B}{A} = -\frac{\mu_1 \sin \nu_1 \mu + \mu_2 L u_2^2 \cos \nu_1 \mu}{\mu_2 \sin \nu_2 \mu - \mu_1 L u_2^2 \cos \nu_2 \mu} = g(\mu)
\]

(34)

where the ratio $B/A \equiv g(\mu)$ so that B can be expressed in terms of A by means of g(\mu).

In Equation (33) the constant parameter $\mu$ can take an infinite number of real values and may also take some complex values. These values are called eigenvalues of $\mu$. For each eigenvalue, corresponding values for A and B should exist. Therefore, Equations (28) and (39) can be put in the following series form:

\[
T_1 = \sum_{n=1}^{\infty} \mu_n^2 e^{-\nu_1^2 \text{Fo}} A_n [a_1 \cos \nu_1 \mu_n X \\
+ a_2 g(\mu_n) \cos \nu_2 \mu_n X]
\]

(35)

\[
\Theta_1 = \sum_{n=1}^{\infty} \mu_n^2 e^{-\nu_1^2 \text{Fo}} A_n [b_1 \cos \nu_1 \mu_n X \\
+ b_2 g(\mu_n) \cos \nu_2 \mu_n X]
\]

(36)

where the function $g(\mu_n)$ from Equation (34) is used to eliminate $B_n$. Since $\mu_n$ can be either positive or negative in Equations (35) and (36) without changing the results, we need to take only the positive real values and complex values with positive real components in the numerical calculations.

Now we evaluate the coefficients A, in Equations (35) and (36). These coefficients are independent of time. By setting Fo = 0 in Equations (35) and (36) and making use of the initial conditions (17) and (18), these coefficients can be evaluated using a least-squares technique (Cheng and Angsirikul, 1977; Hildebrand, 1974). First, we set up the following integral:

\[
\Omega = \int_{0}^{1} \left[ 1 + \sum_{n=1}^{\infty} \mu_n^2 A_n [a_1 \cos \nu_1 \mu_n X \\
+ a_2 g(\mu_n) \cos \nu_2 \mu_n X] \\
\times \left[ 1 + \sum_{n=1}^{\infty} \mu_n^2 A_n [b_1 \cos \nu_1 \mu_n X \\
+ b_2 g(\mu_n) \cos \nu_2 \mu_n X] \right] \\
\times \left[ 1 + \sum_{n=1}^{\infty} \mu_n^2 A_n [b_1 \cos \nu_1 \mu_n X \\
+ b_2 g(\mu_n) \cos \nu_2 \mu_n X] \right] \right] dX
\]

(37)
which must be a minimum. The parameters \( \bar{\mu}_n \) and \( \bar{A}_n \) in Equation (37) are complex conjugates of \( \mu_n \) and \( A_n \), respectively.

The condition that \( \Omega \) be a minimum requires that its partial derivatives with respect to \( A_m \) or \( \bar{A}_m \) shall be zero. We therefore have

\[
\frac{\partial \Omega}{\partial A_m} = \int_0^1 \left\{ \left[ \mu_n^2 A_n (a_1 \cos \nu_1 \mu_n X + a_2 g(\mu_n) \cos \nu_2 \mu_n X) + b_2 g(\mu_n) \cos \nu_2 \mu_n X \right] \times \frac{\partial \mu_n}{\partial A_m} (a_1 \cos \nu_1 \bar{\mu}_m X + a_2 g(\bar{\mu}_m) \cos \nu_2 \bar{\mu}_m X) \\
+ \left[ \mu_n^2 A_n (b_1 \cos \nu_1 \mu_n X + b_2 g(\mu_n) \cos \nu_2 \mu_n X) \right. \\
+ \mu_n^2 (b_1 \cos \nu_1 \bar{\mu}_m X + b_2 g(\bar{\mu}_m) \cos \nu_2 \bar{\mu}_m X) \right] dX
\]

\[
= 0 \quad (m = 1, 2, 3, \ldots)
\]  

(38)

Note that the same results are obtained if we set \( \frac{\partial \Omega}{\partial A_m} = 0 \). In matrix form, Equation (38) generates a Hermitian matrix as

\[
[C_{mn}] \{A_n\} = \{R_m\}
\]  

(39)

in which

\[
C_{mn} = \int_0^1 \left\{ \mu_n^2 (a_1 \cos \nu_1 \mu_n X + a_2 g(\mu_n) \cos \nu_2 \mu_n X) \\
\times \frac{\partial \mu_n}{\partial A_m} (a_1 \cos \nu_1 \bar{\mu}_m X + a_2 g(\bar{\mu}_m) \cos \nu_2 \bar{\mu}_m X) \\
+ \mu_n^2 (b_1 \cos \nu_1 \mu_n X + b_2 g(\mu_n) \cos \nu_2 \mu_n X) \times \frac{\partial \mu_n}{\partial A_m} (b_1 \cos \nu_1 \bar{\mu}_m X + b_2 g(\bar{\mu}_m) \cos \nu_2 \bar{\mu}_m X) \right\} dX
\]

\[
R_m = -\int_0^1 \left\{ \mu_n^2 (a_1 + b_1) \cos \nu_1 \bar{\mu}_m X \\
+ (a_2 + b_2) g(\bar{\mu}_m) \cos \nu_2 \bar{\mu}_m X \right\} dX
\]

(40)

The coefficients \( A_n \) can be determined from the system of linear equations (39). We can then calculate \( T \) and \( \Theta \) from Equations (35) and (36), and \( T \) and \( \Theta \) from Equations (9) and (10). The expressions for \( T \) and \( \Theta \), their averages across \(-1 \leq X \leq 1\), \( \bar{T} \) and \( \bar{\Theta} \), and the derivatives of all these functions with respect to \( F_0 \) (that is, the heating rates and the drying rates) are summarized as follows:

\[
T = 1 + \sum_{n=1}^{\infty} \mu_n^2 e^{-\mu_n^2 F_0} A_n (a_1 \cos \nu_1 \mu_n X \\
+ a_2 g(\mu_n) \cos \nu_2 \mu_n X)
\]

\[
\bar{T} = 1 + \sum_{n=1}^{\infty} \mu_n^2 e^{-\mu_n^2 F_0} A_n \left( \frac{a_1}{\nu_1} \sin \nu_1 \mu_n \\
+ \frac{a_2 g(\mu_n)}{\nu_2} \sin \nu_2 \mu_n \right)
\]

\[
\bar{T} = 1 + \sum_{n=1}^{\infty} \mu_n^2 e^{-\mu_n^2 F_0} A_n \left( \frac{b_1}{\nu_1} \sin \nu_1 \mu_n \\
+ \frac{b_2 g(\mu_n)}{\nu_2} \sin \nu_2 \mu_n \right)
\]

\[
\frac{dT}{dF_0} = -\sum_{n=1}^{\infty} \mu_n^2 e^{-\mu_n^2 F_0} A_n (a_1 \cos \nu_1 \mu_n X \\
+ a_2 g(\mu_n) \cos \nu_2 \mu_n X)
\]

\[
\frac{d\Theta}{dF_0} = -\sum_{n=1}^{\infty} \mu_n^2 e^{-\mu_n^2 F_0} A_n \left( \frac{a_1}{\nu_1} \sin \nu_1 \mu_n \\
+ \frac{a_2 g(\mu_n)}{\nu_2} \sin \nu_2 \mu_n \right)
\]

\[
\frac{d\bar{T}}{dF_0} = -\sum_{n=1}^{\infty} \mu_n^2 e^{-\mu_n^2 F_0} A_n \left( \frac{b_1}{\nu_1} \sin \nu_1 \mu_n \\
+ \frac{b_2 g(\mu_n)}{\nu_2} \sin \nu_2 \mu_n \right)
\]

NUMERICAL RESULTS AND DISCUSSION

Thirteen sets of input data for the six dimensionless numbers, \( Lu, Bi_1, Bi_2, E, P_n, \) and \( Ko \), are shown in Table 1. The reference set (case 1 in Table 1), was derived from the values at the bottom of the table based on the thermophysical properties of spruce (Liu and Cheng, 1991). Each of the six dimensionless numbers was varied twice with respect to the reference set, making a total of 13 sets of data.

The real eigenvalues in Equation (33) were obtained using a bisection procedure. For the complex eigenvalues, we used a method by Müller (1956), which has been included in IMSL (1957). Of the 13 cases, all except cases 2 and 6 contain a pair of complex eigenvalues (Table 1).

Equations (7) and (8) specify that initially the heat and mass potentials are uniformly distributed in the specimen; however, as drying progresses, their distributions are known to be approximately parabolic. For heat potential or temperature distribution, the maximum value is at the surfaces \( X = \pm 1 \) and the minimum value at the center \( X = 0 \); for mass or moisture potential distribution, the opposite is true. It is therefore convenient to plot the temperature and mass potential against time for the center and the surface \( X = 1 \)
only. This will also contain implicit information on heating rate and drying rate.

Figure 2 shows the variations of temperature with time for $Lu = 0.008(1), 0.02(2),$ and $0.04(3)$ at the center and surface of the specimen in Figure 1. (Numbers in parentheses are case numbers in Table 1.) The variations of mass transfer potential with time for the same cases are plotted in Figure 3. The Lu number has the most essential influence on the heat and mass transfer processes and can be called the coupling parameter for heat and mass transfer. It characterizes the inertial properties of the mass transfer potential field compared with the inertial properties of the temperature field. As Lu increases, both temperature and moisture decrease. Large values of Lu will therefore intensify the heating and drying process and decrease the undesirable effects of high-temperature drying.

The Biot heat transfer number $Bi_t$ characterizes the rate of external heat transfer with respect to the rate of internal heat transfer. The variations of temperature and mass transfer potential against time for $Bi_t = 0.1(4), 0.4(1),$ and $1.0(5)$ are displayed in Figures 4 and 5, respectively. $Bi_t$ has a strong influence on $\Theta$; its influence on $\Theta$ is small and becomes negligible for $Fo > 5$. In Figure 4 for small times, the gradients of curves 1 and 4 increase with time, but the gradient of curve 5 decreases with time. This phenomenon was observed in comparing test data and numerical prediction by Thomas et al. (1980). By varying the value of $Bi_t$, the test data, and the numerical prediction could more closely coincide.

The Biot mass transfer number $Bi_m$ characterizes the rate of external mass transfer compared with that of internal mass transfer. Figures 6 and 7 present the variations of temperature and mass transfer potential with time for $Bi_m = 0.7(6), 1.4(1),$ and $2.8(7)$. $Bi_m$ has a strong influence on $\Phi$ except at the center, where the three curves coincide until $F = 10$; its influence on $\Phi$ is not so pronounced.

The role of the phase transformation number $\epsilon$ in heat and mass transfer in spruce is very small. $4$ value of $\epsilon = 0$ implies that within the body mass, transfer occurs as liquid only and evaporation takes place at the surfaces. A value of $\epsilon = 1$ implies that evaporation takes place inside the body and mass transfer occurs as vapor only (Luikov, 1966). For $\epsilon = 0.15(8), 0.3(1),$ and $0.6(9)$, the temperature and mass transfer potential variations with time are essentially the same as shown in Figures 2 and 3.

The Posnov number $Pn$ characterizes only the internal mass transfer as described in Equation (2) and has no direct effect on heat transfer (Luikov, 1966). For $Pn = 1.2(10), 2.4(1),$ and $4.8(11)$, the temperature variations with time described by curve 1 of Figure 2 apply; the variations of mass transfer potential with time shown in Figure 8 are close together.

The Kossovich number $Ko$ characterizes only the internal heat transfer, as seen in Equation (1), and has no direct effect on mass transfer (Luikov, 1966). For $Ko = 4(12), 8(1),$ and $16(13)$, the temperature variations with time are shown in Figure 9; the mass transfer potential variations with time are essentially the same as in curve 1 of Figure 3.

Local variations of temperature and moisture in a medium during drying must be determined for drying stress analyses. However, determining local variations is usually very complex. Knowledge of the variations of average temperature and moisture with time can be applied to obtain information on the drying environment, the quality of the products, and the drying cost estimate. Figures 10 and 11 plot the average temperature and the average mass transfer potential, respectively, against time for all 13 cases in Table 1.

For the product $\epsilon Ko Pn$ to be constant, $v_1$ and $v_2$ vary only with Lu, as seen in Equations (23) and (24). Also, for the product $Ko Pn$ to be constant while the other dimensionless numbers remain the same, the eigenvalues in Equation (33) will not change. This explains why the complex eigenvalues for cases 10 and 12 and those for cases 11 and 13 are the same (Table 1). However, the heat and mass transfer potentials are not the same, as seen from the expressions for $b_1$ and $b_2$ in Equation (30), where $Pn$ does not appear together with Ko.

<table>
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<th>Case</th>
<th>$Lu$</th>
<th>$Bi_t$</th>
<th>$Bi_m$</th>
<th>$\epsilon$</th>
<th>$Pn$</th>
<th>$Ko$</th>
<th>$a \times 10^2$</th>
<th>$b \times 10^4$</th>
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Figure 2. Variation of dimensionless temperature $T$ with time $Fo$ for cases 1–3 and 8–11 (Table 1).

Figure 3. Variation of dimensionless mass transfer potential $\Theta$ with time $Fo$ for cases 1–3, 8, 9, 12, and 13 (Table 1).

Figure 4. Variation of dimensionless temperature $T$ with time $Fo$ for cases 1, 4, and 5 (Table 1).

Figure 5. Variation of dimensionless mass transfer potential $\Theta$ with time $Fo$ for cases 1, 4, and 5 (Table 1).

Figure 6. Variation of dimensionless temperature $T$ with time $Fo$ for cases 1, 6, and 7 (Table 1).

Figure 7. Variation of dimensionless mass transfer potential $Q$ with time $Fo$ for cases 1, 6, and 7 (Table 1).
the thermophysical properties of spruce, 11 sets yielded complex eigenvalues.

Of the six dimensionless numbers concerned, the Lukomskii number $Lu$ has strong effects on both heat and mass transfer; the Biot heat transfer number $Bi_q$ has significant effects on temperature distribution only; the Biot mass transfer number $Bi_m$ has substantial effects on moisture distribution only; the effects of the Posnov number $Pn$ and the Kossovich number $Ko$ on temperature and moisture distributions are moderate; and the effects of the phase transformation number $E$ are negligible in the cases considered. For any capillary-porous media, the numbers $Lu$, $Bi_q$, and $Bi_m$ must therefore be more carefully evaluated than the other numbers.

REFERENCES


NOMENCLATURE

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Subscripts

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