The Compressive Load-Strain Curve of paperboard: Rate of Load and Humidity Effects

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This paper reports the effect of load rate and humidity on edgewise compression properties of paperboard. One linerboard and one corrugating medium were tested at three load rates; 263 N/m/s, 2.63 N/m/s, and 0.0263 N/m/s, under constant 50% and 90% relative humidity (rh). Both paperboards exhibited lower compressive strength, smaller initial stiffness, and larger failure strains as load rate was reduced. Compressive strength and initial stiffness were found to vary with the logarithm of load rate. The results also show that the paperboards in this study were more affected by load rate at 90% rh than at 50% rh. The hyperbolic tangent model is extended to determine load as a function of strain and load rate. In this model, comparison of the machine direction (MD) and the cross-machine direction (CD) response shows that the CD is more sensitive to load rate at 50% rh, but that the MD is more sensitive at 90% rh.

INTRODUCTION

Knowledge of the basic engineering properties of paperboard is essential to optimum use. The papermaking and packaging industries have gained much needed information from the recent development of methods for measuring the edgewise compressive strength of linerboard and corrugating medium [1,2]. We anticipate that in the near future this industry will accelerate research investigating the reduction in compressive load-carrying ability resulting from change in the service environment and extended time of load.

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Caulfield [3] has shown that duration of load can be predicted from the strength data obtained at various load rates. Rance [4], Andersson and Sjöberg [5], and Steenberg [6] reported a rate-of-load effect while testing paperboard in tension. Moody and Koning [7] reported a reduction of compressive strength of corrugated board at reduced strain rates. There are no data in the literature concerning the rate-of-load effect on the compressive strength of paperboard, per se.

This paper reports the effect of load rate on the compression properties of two paperboards. The samples were tested at three load rates, in the machine and cross-machine directions, and at constant 50% and 90% relative humidity (rh). We also present an empirical model, which accurately represents the compressive load-strain behaviour at different load rates.

EXPERIMENTAL PROCEDURE

All tests were performed in an apparatus, developed at the Forest Products Laboratory [8,9], that consists of a load frame and a vacuum restraint system enclosed in a chamber. The load frame is servo driven, so that it achieves the computer-specified load values regardless of specimen strain magnitude. During conditioning of the specimen, the servo compensates for changes in specimen length due to sorption. Humid and dry air are proportioned by a flow control valve and drawn into the specimen chamber. The humidity-controlled air flows through and around the specimen. With such a configuration, dimensional stability can be reached in 5 min.

Specimens are preconditioned at 50% rh, 23°C for several days. A test is begun by placing the specimen in the vacuum restraint system, engaging the load frame, and setting the bell jar to enclose the system. Initially, dry air (< 50% rh) is drawn through the specimen. Using the flow control valve, the rh in the chamber is slowly increased to the rh desired for the test. The specimen is held at the test rh for 30 min before the test begins. A computer controls the application of load and also records the load-strain data.

We performed tests on two paperboards; a softwood kraft, material A[1], formed on the FPL pilot machine, and a

[1] 100% Lake States softwood, unbleached kraft; grammage, 205 g/m^2; density, 670 kg/m^3.

TABLE I
COMPRESSIVE STRENGTH, FAILURE STRAIN, AND EXPERIMENTALLY DETERMINED VALUES FOR MODELING THE COMPRESSION CURVE OF LINERBOARD A AND CORRUGATING MEDIUM B AT SELECTED RATES OF LOAD: 50% AND 90% RH [SEE EQUATION (1) IN TEXT]. VALUES IN PARENTHESES REPRESENT ONE STANDARD DEVIATION, SIX TESTS PER DATA POINT.

Material	Direction*	Relative humidity	LR (N/m/s)	C_1 (kN/m)**	C_2 (kN/m)**	P (kN/m) +	ϵ (10^{-6}) +	%C_2	%P
A	MD	50	263.0	8.65 (0.48)	2,131 (75)	5.30 (0.27)	2,899 (212)	—	—
			2.63	7.38 (0.51)	2,116 (50)	5.02 (0.33)	2,916 (312)	—	—
			0.0263	5.35 (0.48)	1,870 (245)	4.45 (0.31)	3,303 (385)	—	—
		90	263	4.32 (0.25)	1,662 (79)	3.65 (0.25)	3,356 (273)	78	69
			2.63	3.05 (0.44)	1,373 (143)	2.84 (0.31)	3,726 (297)	65	57
			0.0263	2.90 (0.29)	964 (64)	2.41 (0.13)	3,802 (295)	52	54
	CD	50	263.0	4.58 (0.25)	1,022 (71)	3.64 (0.21)	4,849 (468)	—	—
			2.63	3.86 (0.24)	864 (62)	3.03 (0.25)	4,781 (392)	—	—
			0.0263	3.19 (0.14)	857 (74)	2.90 (0.11)	5,024 (648)	—	—
		90	263.0	2.59 (0.13)	694 (40)	2.40 (0.09)	6,019 (350)	68	66
			2.63	2.02 (0.14)	465 (34)	1.85 (0.16)	6,526 (608)	54	61
			0.0263	1.91 (0.19)	415 (45)	1.67 (0.18)	6,458 (919)	48	58
B	MD	50	263.0	5.11 (0.39)	1,138 (69)	3.29 (0.23)	3,475 (275)	—	—
			2.63	4.00 (0.13)	1,105 (59)	2.88 (0.17)	3,309 (205)	—	—
			0.0263	2.81 (0.11)	887 (26)	2.31 (0.08)	3,737 (225)	—	—
		90	263.0	1.82 (0.09)	726 (37)	1.67 (0.08)	4,017 (357)	64	51
			2.63	1.31 (0.14)	548 (61)	1.26 (0.14)	4,366 (437)	50	44
			0.0263	1.15 (0.13)	381 (32)	1.05 (0.09)	4,488 (665)	43	45
	CD	50	263.0	2.13 (0.07)	390 (30)	1.65 (0.11)	5,656 (280)	—	—
			2.63	1.80 (0.03)	311 (14)	1.41 (0.04)	6,050 (191)	—	—
			0.0263	1.69 (0.22)	262 (16)	1.28 (0.14)	6,614 (577)	—	—
		90	263.0	1.19 (0.06)	255 (10)	1.05 (0.05)	6,297 (737)	65	64
			2.63	0.97 (0.04)	191 (7)	0.86 (0.05)	7,255 (575)	61	61
			0.0263	0.79 (0.04)	149 (14)	0.73 (0.05)	8,137 (491)	57	57

* MD = machine direction; CD = cross machine direction.
** Composite values for six curves.
\+ Average values for six curves.

commercial NSSC corrugating medium, material B[2]. Tests were run in both the machine direction and cross-machine direction, at 50% and 90% rh, and at three load rates: 263 N/m/s, 2.63 N/m/s, and 0.0263 N/m/s. Six tests were performed at each condition, for a total of 72 tests per paperboard. For this study, the shortest tests took 6 s while the longest tests took 50 h.

RESULTS AND DISCUSSION

We used a model and program suggested by Urbanik [10] to characterize the load-strain data for each compression test. The compressive load-strain model is

$$p = C_1 \tanh(C_2 \epsilon / C_1) \qquad (1)$$

where

p = load/width (kN/m)

ϵ = strain

and C_1, C_2 are constants with units of kN/m.

The constant C_2 represents the initial stiffness, and the constant C_1 represents the asymptote of the hyperbolic tangent curve. After each test curve was fit individually with Equation (1), we fit the six curves at each condition simultaneously with Equation (1) to get composite values for C_1 and C_2, as suggested by Suhling [11]. The termination point for each is the average failure strain for that data set. The composite values for C_1, C_2 and average values for load and strain at failure are shown in Table I. The energy absorbed can be calculated by intergrating Equation (1). Standard deviations, for the individual tests, are shown in parenthesis.

The Rate of Load Effect

Figure 1 is a set of load-strain curves for test materials A and B based on the composite C_1 and C_2 values at the specified load rates. These curves show a clear trend to lower initial stiffness, smaller failing loads, and larger failing strains as the load rate is reduced. As is typical of tensile and compressive tests, the coefficients of variation are generally larger for strain at failure than for strength. Consequently, the trend for strain is more variable than for strength.

Moody and Koning [7] have used a linear equation to relate failing load to strain rate for corrugated containerboard. Using load rate instead of strain rate, the equation is

$$P = b + a \log L \qquad (2)$$

[2]NSSC commercial corrugating medium; grammage, 127 g/m^2; density, 530 kg/m^3.

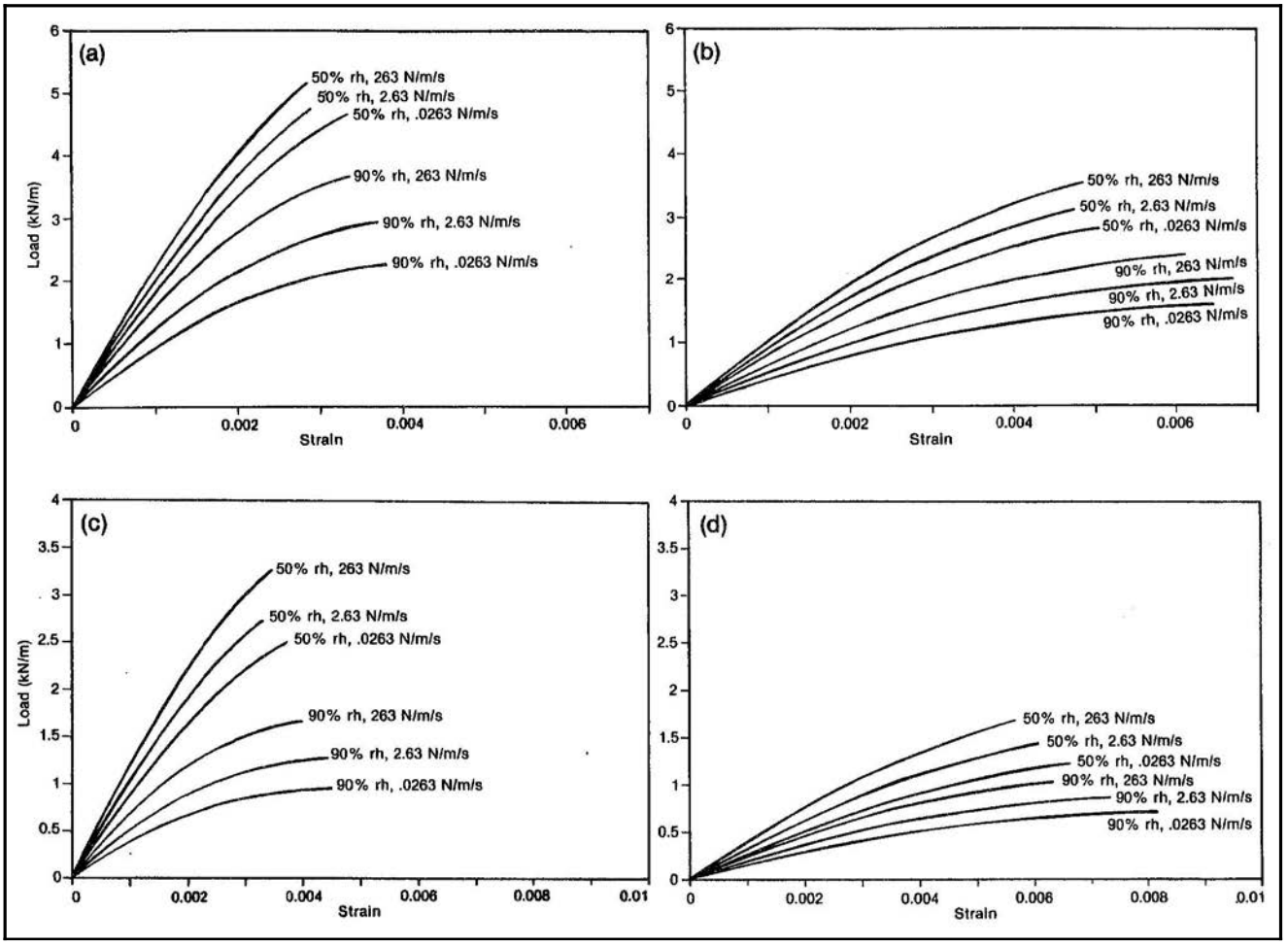

Fig. 1. Composite load-strain curves for linerboard A and corrugating medium B. (a) Material A, MD; (b) Material A, CD; (c) Material B, MD; (d) Material B, CD. (ML87 5409).

TABLE II
EXPERIMENTALLY DETERMINED VALUES FOR MODELING COMPRESSIVE STRENGTH AND INITIAL STIFFNESS AS A FUNCTION OF LOAD RATE FOR LINERBOARD A AND CORRUGATING MEDIUM B [SEE EQUATIONS (2) AND (3) IN TEXT]; r(2) AND r(3) ARE CORRELATION COEFFICIENTS. EACH LINE IS THE REGRESSION OF SIX TESTS AT EACH OF THREE LOAD RATES.

Material	Direction*	Relative humidity	a+	b+	r(2)	c+	d+	r(3)
A	MD	50	0.214	5.48	0.77	72.97	2,220	0.63
		90	0.311	3.78	0.91	178.60	1,787	0.95
	CD	50	0.187	3.67	0.82	35.63	997	0.65
		90	0.185	2.46	0.88	66.95	702	0.90
B	MD	50	0.245	3.46	0.93	58.63	1,190	0.82
		90	0.155	1.73	0.92	86.63	776	0.96
	CD	50	0.092	1.69	0.83	28.81	399	0.91
		90	0.082	1.09	0.95	26.95	268	0.98

* MD = machine direction; CD = cross-machine direction.
\+ Units = kN/m.

where

P = failing load (kN/m)
L = load rate normalized with respect to 1 kN/m/s
a, b = constants.

Fitting our experimental data with Equation (2), using the least squares method, produces the regression coefficients, a and b, shown in Table II. Column r(2) lists the correlation coefficients for the data fit to Equation (2). Each value of a and b represents the fit to a total of 18 data points, 6 data points at three rates of load.

From Fig. 1, it also seems reasonable to consider an equation of the same form to relate initial stiffness to load rate. Using

$$S = d + c \log L \qquad (3)$$

where

S = initial stiffness (kN/m)
L = load rate normalized with respect to 1 kN/m/s
c,d = constants

we obtain regression coefficients, c and d, shown in Table II; column r(3) shows correlation coefficients for the data fit to Equation (3).

The generally high values for columns r(2) and r(3) in Table II suggest that both compressive strength and initial stiffness vary in proportion to the logarithm of load rate. There are no comparable compression data in the literature with which these results can be compared. Anderson and Sjöberg [5] conducted tensile tests at variable rates of strain covering three orders of magnitude. They report that the initial modulus is unaffected by strain rate. Our compressive tests of materials A and B show a clear reduction in initial modulus at reduced load rates. The tensile case should be reexamined to resolve this apparent conflict. Moody and Koning [7] did not report on the relationship between stiffness and strain rate for corrugated containerboard.

To examine rate-of-load behaviour, it is helpful to define a base value for stiffness and failing load in order to calculate a percentage reduction in compressive strength and stiffness per decade of decrease in load rate. We selected the y intercept value (1 kN/m/s), column b for strength and column (d) for stiffness, as the base. Columns a and c in Table II represent the rate of decline in failing load and initial stiffness, respectively, per decade decrease in load rate. For material A, machine direction and 50% rh, we calculate that the percentage reduction in compressive strength per decade of decrease in load rate, from columns a and b of Table II, is 0.214/5.48 × 100, or 3.9%. Similarly, for material B, cross-machine direction and 90% rh, the reduction is 0.082/1.09 × 100, or 7.5%. Stiffness reductions can be computed in the same way. The percentage reductions in strength and stiffness per decade of decrease in rate of load are listed in Table III in the P% and the S% columns, respectively.

The strength reductions, shown in the P% column of Table III, span the 7.5% loss of strength per decade reduction in strain rate for corrugated fiberboard reported by Moody and Koning [7].

Humidity

Both strength and stiffness decrease as rh is raised from 50% to 90%. Strength and stiffness values are shown in Table I at columns C_2 and P. The percentages of strength and stiffness retained at 90% rh (based on 50% rh values) are also shown in columns $\%C_2$ and %P. These strength retention values at fixed rate of load are comparable to those reported by Back [12] for similar materials. But strength retention depends on load rate. Looking at Table III, we observe that the loss in strength and stiffness per decade reduction in load rate is greater at 90% rh than at 50% rh.

A Load Rate Model

Objecting to the lack of physical significance of Equations (2) and (3), Andersson and Sjöberg [5] modeled tensile strength data with an equation of the form

$$P = \alpha s^{\beta} \tag{4}$$

where

P = failing stress
s = strain rate
α, β = constants.

The constant β is called the rate coefficient.

We used a combination of Equations (1) and (4) to predict the shape of the load-strain curve as a function of load rate. This yields

$$p = C_3 L^{\beta} \tanh(C_4 \epsilon / C_3) \tag{5}$$

where

p = load/width (kN/m)
ϵ = strain
L = load rate normalized with respect to 1 kN/m/s
C_3, C_4, β = constants.

Table IV shows the results of fitting Equation (5) with our data. When the load rate model, Equation (5), is compared with our composite fits of Equation (1), we find that the difference in load is consistently less than 10% over the full strain range.

The rate coefficient indicates the effect of load rate on the load-strain curve. In Table IV, we see that for both materials A and B the rate coefficient is larger at 90% rh than at 50% rh, which is consistent with the analysis in Table III. Based

TABLE III
STRENGTH AND STIFFNESS REDUCTIONS AS A FUNCTION OF LOAD RATE BASED ON THE LOGARITHM MODEL. P% IS PERCENTAGE LOSS OF COMPRESSIVE STRENGTH PER DECADE DECREASE OF LOAD RATE. S% IS PERCENTAGE LOSS OF INITIAL STIFFNESS PER DECADE DECREASE OF LOAD RATE.

Material	Direction*	Relative humidity	P%	S%
A	MD	50	3.9	3.3
		90	8.2	10.0
	CD	50	5.1	3.6
		90	7.5	9.5
B	MD	50	7.1	4.9
		90	9.0	11.2
	CD	50	5.5	7.2
		90	7.5	10.1

* MD = machine direction; CD = cross-machine direction.

TABLE IV
EXPERIMENTALLY DETERMINED PARAMETERS FOR MODELING THE LOAD-STRAIN CURVE AS A FUNCTION OF LOAD RATE [SEE EQUATION (5) IN TEXT]. VALUES IN PARENTHESES ARE +/− 95% CONFIDENCE INTERVALS.

Material	Direction*	Relative humidity	β	C_3	C_4
A	MD	50	0.0207 (0.0008)	7.17 (0.161)	2,268 (25.90)
		90	0.0560 (0.0008)	4.18 (0.054)	1,640 (18.52)
	CD	50	0.0265 (0.0005)	4.29 (0.050)	1,025 (8.85)
		90	0.0452 (0.0007)	2.61 (0.028)	633 (6.90)
B	MD	50	0.0337 (0.0007)	4.32 (0.087)	1,211 (13.26)
		90	0.0624 (0.0009)	1.78 (0.020)	700 (8.71)
	CD	50	0.0434 (0.0004)	2.16 (0.022)	388 (2.54)
		90	0.0475 (0.0005)	1.18 (0.008)	240 (1.56)

* MD = machine direction; CD = cross-machine direction.

on comparison of ß values, we observe that at 50% rh the cross-machine direction is more affected by load rate, while at 90% rh the machine direction is more affected by load rate. This trend is also apparent in Table III.

CONCLUSIONS

We investigated the effect of load rate on two paperboards, in the machine direction and cross-machine direction, and at constant 50% and 90% rh. Our results indicate that the paperboards in this study were more adversely affected by reduced load rate at 90% rh than at 50% rh. We found that both compressive strength and initial stiffness vary in proportion to the logarithm of load rate. A model is presented which accurately fits the load-strain curves. The model shows that, for the paperboards tested, the machine direction is less sensitive than the cross-machine direction when tested at 50% rh, but is more sensitive when tested at 90% rh.

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ABSTRACT: This paper reports the effect of load rate and humidity on edgewise compression properties of paperboard. One linerboard and one corrugating medium were tested at three load rates; 263 N/m/s, 2.63 N/m/s, and 0.0263 N/m/s, under constant 50% and 90% relative humidity (rh). Both paperboards exhibited lower compressive strength, smaller initial stiffness, and larger failure strains as load rate was reduced. Compressive strength and initial stiffness were found to vary with the logarithm of load rate. The results also show that the paperboards in this study were more affected by load rate at 90% rh than at 50% rh. The hyperbolic tangent model is extended to determine load as a function of strain and load rate. In this model, comparison of the machine direction (MD) and the cross-machine direction (CD) response shows that the CD is more sensitive to load rate at 50% rh, but that the MD is more sensitive at 90% rh.

RÉSUMÉ: Notre article traite des effets des taux de charge et d'humidité sur les caractéristiques de compression sur rive du carton. Nous avons mis à l'essai du carton doublure et du papier à canneler en faisant appel à trois taux de charge de 263 N/m/s, 2.63 N/m/s et 0.0263 N/m/s à des taux constants d'humidité relative (hr) de 50% et 90%. Ces deux échantillons de carton ont laissé voir une résistance à la compression plus faible, une rigidité initiale moins élevée et une défaillance de contrainte supérieure en fonction de la réduction du taux de charge. Nous avons observé que la résistance à la compression et la rigidité initiale variaient avec la logarithme du taux de charge. Les résultats de nos essais montrent que les échantillons de carton étaient davantage affectés par le taux de charge à un taux d'humidité relative de 90% qu'à un taux de 50%. Nous avons par ailleurs étendu le modèle de la tangente hyperbolique en vue de déterminer la charge comme une fonction de la contrainte et du taux de charge. Dans notre modèle, la comparaison entre les effets dans le sens machine (SM) et dans le sens travers (ST) montrent que le ST est plus sensible à un taux d'humidité relative de 50% et que le SM est lui plus sensible à un taux d'humidité relative de 90%.

KEYWORDS: PAPER BOARD TESTS, HUMIDITY, EDGE CRUSH TESTS, LINER BOARDS, CORRUGATING MEDIUM, COMPRESSION STRENGTH, STIFFNESS, LOADS, DYNAMIC TESTS, MATHEMATICAL MODELS.