

PREDICTING THE EFFECT OF MOISTURE CONTENT ON THE FLEXURAL PROPERTIES OF DOUGLAS-FIR DIMENSION LUMBER

David W. Green

Research Engineer
Forest Products Laboratory,¹ One Gifford Pinchot Dr.
Madison, WI 53705-2398

James W. Evans

Mathematical Statistician
Forest Products Laboratory, One Gifford Pinchot Dr.
Madison, WI 53705-2398

*J. David Barrett*²

Manager, Wood Engineering
Forintek Canada Corp., Western Laboratory
Vancouver, BC, Canada

and

*E. Nicholas Aplin*³

Research Scientist
Forintek Canada Corp., Western Laboratory
Vancouver, BC, Canada

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ABSTRACT

Current procedures for adjusting estimates of the mechanical properties of lumber for changes in moisture content are based on trends in the observed means. The present study was initiated to develop analytical procedures for adjusting estimates of the flexural properties of 2-inch-thick Douglas-fir dimension lumber that would be applicable to all levels of the flexural properties. Equations are derived for adjusting modulus of rupture (MOR), modulus of elasticity (MOE), moment capacity ($RS = MOR \times$ section modulus), and flexural stiffness ($EI = MOE \times$ moment of inertia) for changes in moisture content. The best of these equations are found to be significantly more accurate than current procedures for adjusting estimates of strength properties such as MOR and RS. Because MOE and EI are less affected by changes in moisture content, most of the equations work well for these properties.

Keywords: Flexural properties, modulus of rupture, modulus of elasticity, flexural stiffness, moment capacity, moisture content, mathematical models.

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²Now Professor and Department Head, Department of Harvesting and Wood Science, Faculty of Forestry, University of British Columbia, Vancouver, BC.

³Now with Aplin Engineering, Ottawa, ON.

INTRODUCTION

The objective of the present study is to develop and evaluate analytical models that can be used to adjust estimates of the flexural properties of Douglas-fir dimension lumber from one moisture content (MC) to another. Nine models are presented, belonging to the six general types of models considered relevant. The models are evaluated and their predictions compared with each other and with the data. Applicability of the results is uncertain beyond the range of the available data.

BACKGROUND

In the United States and Canada, major research programs have been conducted to evaluate the mechanical properties of full-size structural lumber, visually graded according to current practice (Green 1983; Forintek 1983). The results of these studies provide a basis for the establishment of new values of allowable design stress for visually graded structural lumber. It was not economically feasible in these studies to evaluate the effect on mechanical properties of varying levels of MC. Most of the lumber was tested at the MC at which it was produced, not conditioned to a single MC. Thus, the In-Grade data produce insufficient information on the relationships between MC and lumber properties to adjust mechanical properties of lumber to specific end use MC levels or to interpret the variations of these properties within and between mills.

Current procedures for adjusting estimates of the mechanical properties of dimension lumber for changes in MC are based on adjustment factors presented in American Society for Testing and Materials Standard D 245-81 (ASTM 1986). The adjustment factors are assumed to be independent of lumber quality and applicable to all levels of the cumulative frequency distribution of the lumber property.

When dried from green to a maximum MC of 19 % or of 15 % , these adjustment factors are:

<i>Property</i>	<i>Maximum MC</i>	
	<i>19%</i>	<i>15%</i>
Bending	25	35
Modulus of elasticity	14	20
Tension parallel to grain	25	35
Compression parallel to grain	50	75
Horizontal shear	8	13
Compression perpendicular to grain	50	50

Older versions of D 245, however, cite procedures for adjusting the estimates for changes in MC that are dependent upon lumber quality (Green 1982).

$$SR_{\text{dry}} = SR_{\text{green}} + \frac{1}{2}(SR_{\text{green}} - 50) \quad (1)$$

where

SR = strength ratio, percent—the ratio of the strength of a member containing a defect to the strength of an equivalent defect-free member.

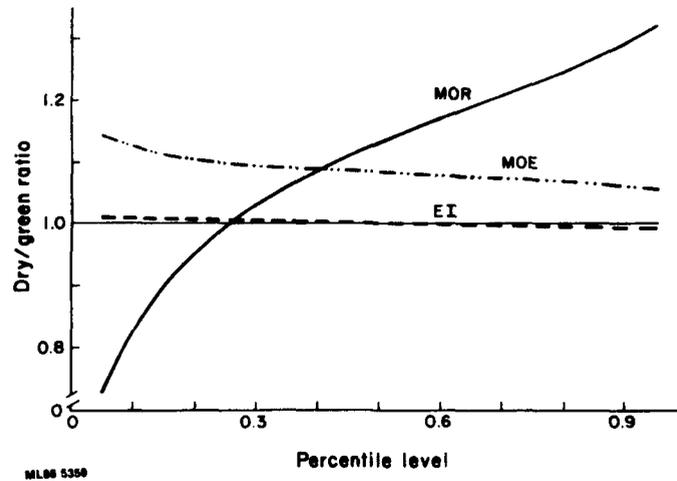


FIG. 1. Effect of MC on the dry:green ratio for three flexural properties of 2×6 , No. 2 and Better, Douglas-fir dried to an MC of 15% (adapted from Weibull parameter given in Madsen et al. 1980). (ML86 5359)

Thus the adjustment factor ranged from 25 % for green lumber having an SR of 100 % to no increase for green lumber having an SR of 50 %.

Earlier research supports the concept of a quality-dependent adjustment factor. Gerhards (1968, 1970) used matched pairs of 4×8 southern pine beams to investigate the effect of seasoning on flexural properties. He concluded that the MC adjustment factor, F , for modulus of rupture (MOR) when green wood was dried to an MC of 12 % could be expressed as a function of SR.

$$F = 0.994 + 0.00503 (SR) + 0.0104 (DSR - GSR) \quad (2)$$

where

F = the ratio of MOR at 12 % MC to MOR green, and
 $DSR - GSR$ = the within-pair difference in the SR of the matched specimens tested dry (DSR) and green (GSR).

The adjustment factor for modulus of elasticity (MOE) was found to be independent of SR. When dried by a mild schedule, the MOE increased about 23 % in drying from green to an MC of 12 %.

Studies by Madsen (1975) and Madsen et al. (1980) on the flexural properties of Douglas-fir, Hem-Fir, and Spruce-Pine-Fir dimension lumber indicated that the magnitude of the adjustment for MOR or MOE was dependent upon location in the cumulative density function for MOR or MOE. Using Douglas-fir as a typical example (Fig. 1), it was noted that MOR was much more sensitive than MOE to position in the distribution.

PROCEDURES

In this section we review the experimental procedures briefly before introducing and deriving the analytical models. Finally, we discuss the intersection MC, M_p , which is the MC above which properties are assumed to be independent of MC.

Experimental procedures

Green Douglas-fir lumber of three grades (Select Structural, No. 2, and No. 3) and three sizes (2×4 , 2×6 , and 2×8) was sampled from one sawmill near Vancouver, B.C. The lumber of each size and grade was divided into four equivalent populations in terms of estimated stiffness in the green condition. Each population or group contained approximately 100 pieces. Three of the groups were then equilibrated to 10, 15, and 20 % MC. The fourth group was maintained in the green condition prior to testing, by use of a water spray. All pieces were tested to failure on edge in third-point bending using a span-to-depth ratio of 17:1. Flexural properties were calculated using the actual dimensions of the piece at the time of test. The MOE values were not corrected for deflection caused by shear stresses. Additional details concerning the experimental procedures and the data analysis we employed are given in Aplin et al. (1986).

Analytical models

We considered six types of models that may be used to make MC adjustments of the four flexural properties, MOR, MOE, RS (moment capacity), and EI (flexural stiffness).⁴ Several variations of each type of model may be produced by making different assumptions concerning the form of the analytical expression or by using various subsets of the data. Coefficients of the individual models are given in the appendix.

Zero adjustment model. – The simplest model is one in which no adjustment of estimated properties is made for changes in MC; the MC adjustment factor, F, is unity. This model is useful primarily as a baseline against which to compare the performance of the more complicated models.

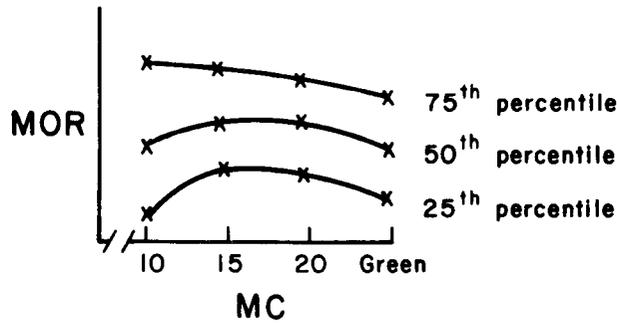
Constant percentage adjustment models. – These models adjust estimates of a given property by a constant percentage when the MC is changed from one level to another. All current standard adjustment procedures for dimension lumber and for clear wood are of this type. Several variations are possible with this type of model (Green et al. 1986). We present only a linear adjustment formula (models 1, 2, and 3 in Table 1) by which, when adjusting from a property value of P_1 at an MC of M_1 percent, the value of the property P_2 at an MC of M_2 percent is given by

$$P_2 = P_1 * \left[\frac{a + bM_2}{a + bM_1} \right] \quad (3)$$

where b represents a change in property per percent change in MC. In obtaining a value for b, MC values greater than the intersection MC, M_p , were replaced by the assumed value of M_p .

Surface model. – A group of surface models may be obtained by plotting non-parametric estimates of the percentiles of a property against MC. In this study, percentile levels of 2, 5, 10, 15, 20, . . . , 90, 95, 98, were plotted for MCs of 10,

⁴Moment capacity (RS) is the product of modulus of rupture and section modulus, S, where $S = 1/6 \times \text{thickness} \times (\text{width})^2$. Flexural stiffness (EI) is the product of the modulus of elasticity and the moment of inertia, I, where $I = 1/12 \times \text{thickness} \times (\text{width})^3$.



ML86 5358

FIG. 2. Contours of percentiles for MOR such as would be used to create a surface model (actual model used 21 percentiles: 2, 5, 10, 15, . . . , 90, 95, 98). (ML86 5358)

15, 20, and green for each grade and size of lumber. Thus, for any grade, size, and percentile level, four values of the property appear on the plot, one for each MC (Fig. 2). These contour lines define a surface that is followed when adjusting from one MC level to another. Contour surfaces were approximated by fitting to each contour line a quadratic curve of the form

$$P = a + bM + cM^2 \tag{4}$$

where a, b, and c are unique for each contour. The contour, in turn, depends upon the value of the property, P_1 , and the MC, M_1 . The values of a, b, and c can then be estimated using P_1 and M_1 . Then

$$P_2 = P_1 + b(M_2 - M_1) + c(M_2^2 - M_1^2). \tag{5}$$

We considered several variations of this type of model. One variation (model 4, Table 1) uses the linear form of Eq. (5).

In a second variation, b and c were modeled using a polynomial function of P and MC. Given values of P_1 and M_1 , the polynomial predicted b and c so that Eq. (5) could be used. This procedure produced what we call a “nonfixed surface.” It is nonfixed in the sense that, when predictions of P_2 at M_2 made from P_1 and M_1 are used to predict a property value at M_1 , the value predicted may not be exactly P_1 . This is because the contour estimate is slightly different when b and c are predicted from P_2 and M_2 instead of from P_1 and M_1 . Model 4 is a linear version and model 5 a quadratic version of Eq. (5) treated in this way. The nonfixed surface models are easier to compute than the fixed surface model described next. However, care must be taken always to use real data as the starting values P_1 and M_1 .

It is possible to fix the contours so that the original values of P_1 and M_1 can be obtained from the predicted value of P_2 at M_2 . This is done by fixing a reference MC that is used to estimate b and c. For this paper a reference MC of 15 % was chosen, and the P at 15 % was found that, when adjusted to an M_1 value, gave P_1 . This P_{15} and $M = 15$ were then used to predict b and c for use in Eq. (5). This step, added to insure that the contours are fixed, can result in some difficult calculations. In the most useful fixed surface model, a quadratic curve of the form

TABLE 1. Moisture content adjustment models for flexural properties of Douglas-fir at various moisture contents.

Model number	Model type	Variation	Dependence on grade and size	Number of coefficients	Average maximum absolute difference ¹								
					Mean for MC		5th Percentile for MC						
					10%	15%	20%	Green	10%	15%	20%	Green	
Modulus of rupture, MOR													
0	Zero adjustment	—	Independent	0	2,277	2,277	2,277	2,277	2,277	727	727	727	727
1	Constant percentage	Linear	By grade and size	18	641	577	513	461	715	636	557	493	493
2			By grade	6	761	686	612	553	713	637	562	501	501
3			Independent	2	930	833	737	659	654	586	518	464	464
4	Surface	Linear, regression	Function	10	788	690	592	516	485	427	388	359	359
5		Quadratic, regression	Function	30	675	523	423	382	472	509	468	357	357
6		Quadratic, regression (fixed)	Function	8	681	546	434	360	503	516	457	357	357
7	Normal	Regression	Function	25	461	409	314	260	816	724	555	460	460
8	Weibull	3-P, regression	Function	35	462	416	309	264	506	479	424	324	324
9		2-P, regression	Function	22	464	411	309	264	589	555	482	403	403
10	Strength ratio	Linear, 0% cutoff	Yes	2	808	761	715	692	647	532	424	354	354
Modulus of elasticity, MOE													
0	Zero adjustment	—	Independent	0	0.268	0.268	0.268	0.268	0.268	0.162	0.162	0.162	0.162
1	Constant percentage	Linear	By grade and size	18	0.047	0.044	0.041	0.039	0.057	0.053	0.050	0.047	0.047
2			By grade	6	0.053	0.050	0.046	0.043	0.061	0.057	0.053	0.050	0.050
3			Independent	2	0.053	0.050	0.046	0.043	0.063	0.059	0.055	0.051	0.051
4	Surface	Linear, regression	Independent	10	0.047	0.045	0.042	0.040	0.065	0.060	0.056	0.052	0.052
5		Quadratic, regression	Function	30	0.045	0.042	0.039	0.039	0.071	0.064	0.058	0.057	0.057
6		Quadratic, regression (fixed)	Function	8	0.044	0.041	0.038	0.036	0.067	0.063	0.059	0.055	0.055
7	Normal	Regression	Function	27	0.038	0.034	0.031	0.029	0.062	0.056	0.051	0.047	0.047
9	Weibull	2-P, regression	Function	23	0.040	0.037	0.033	0.031	0.062	0.057	0.051	0.048	0.048
10	Strength ratio	Linear, 0% cutoff	Yes	2	0.106	0.104	0.103	0.102	0.078	0.096	0.078	0.080	0.080

TABLE 1. Continued.

Model number	Model type	Variation	Dependence on grade and size	Number of coefficients	Average maximum absolute difference ¹								
					Mean for MC		5th Percentile for MC						
					10%	15%	20%	Green	20%	15%	10%	Green	
..... in · lb													
Moment capacity, RS													
0	Zero adjustment	—	Independent	0	12,819	12,819	12,819	12,819	12,819	4,044	4,044	4,044	4,044
5	Surface	Quadratic, regression	Function	30	5,006	4,444	4,048	3,835	4,822	4,587	4,268	3,969	3,969
6	Surface	Quadratic, regression (fixed)	Function	8	4,836	4,066	3,409	2,944	3,566	3,829	3,530	2,884	2,884
7	Normal	Regression	Function	24	3,804	3,694	2,796	2,524	5,078	5,009	3,740	3,405	3,405
9	Weibull	2-P, regression	Function	24	4,274	4,087	3,088	2,767	3,816	3,851	3,387	3,001	3,001
..... 10 ⁶ lb · in ²													
Flexural stiffness, EI													
0	Zero adjustment	—	Independent	0	2,326	2,326	2,326	2,326	2,326	1,846	1,846	1,846	1,846
5	Surface	Quadratic, regression	Function	30	0,753	0,730	0,712	0,714	1,642	1,597	1,770	1,796	1,796
6	Surface	Quadratic, regression (fixed)	Function	8	0,802	0,812	0,819	0,822	1,513	1,463	1,425	1,403	1,403
7	Normal	Regression	Function	28	0,502	0,501	0,499	0,498	1,489	1,467	1,445	1,427	1,427
9	Weibull	2-P, regression	Function	25	0,557	0,543	0,540	0,545	1,478	1,420	1,391	1,387	1,387

¹ Based on the average of each of the 9 grade and size combinations at the indicated percentile (or mean level).

of Eq. (4) was fitted to each surface. Then b and c were estimated at 15 % , giving b_{15} and c_{15} where

$$b_{15} = D_0 + D_1 P_{15} + D_2 P_{15}^2 + D_3 P_{15}^3 \quad (6a)$$

and

$$c_{15} = E_0 + E_1 P_{15} + E_2 P_{15}^2 + E_3 P_{15}^3 \quad (6b)$$

The parameters $D_0, D_1, D_2, D_3,$ and E_0, E_1, E_2, E_3 in Table A-5 were estimated from the data. The problem was to find P_{15} . We know that going to M_1 from P_{15} at M_{15} must give PI. The change in property value is

$$P_{15} - P_1 = b_{15}(15 - M_1) + c_{15}(15^2 - M_1^2). \quad (7)$$

Inserting Eqs. (6a) and (6b) in Eq. (7) gives

$$0 = [P_1 + D_0(15 - M_1) + E_0(15^2 - M_1^2)] + [D_1(15 - M_1) + E_1(15^2 - M_1^2) - 1]P_{15} \\ + [D_2(15 - M_1) + E_2(15^2 - M_1^2)]P_{15}^2 + [D_3(15 - M_1) \\ + E_3(15^2 - M_1^2)]P_{15}^3$$

Because everything except P_{15} is known, we can use a root-finding procedure to find P_{15} , then use this value to predict b_{15} and c_{15} in Eqs. (6a) and (6b). Finally, the predicted values of b_{15} and c_{15} are used in Eq. (5) to go from P_1 at M_1 to any P_2 at M_2 . The resulting contours are fixed. Several versions of this model type were tried, the best of which is the model just described (model 6 in Table 1), the only model of this type reported here.

Normal model. – A fourth type of model is obtained by fitting a normal distribution to the data for each combination of grade, size, and MC (Aplin et al. 1986). Given a mean property value X and standard deviation S at M_1 and at M_2 , we assumed that if the strength of a particular piece of lumber was the p th percentile at one MC, it would be the p th percentile at any MC. By this assumption, the property P_2 at M_2 was related to the property P_1 at M_1 by

$$P_2 = S_2 \frac{P_1 - \bar{X}_1}{S_1} + \bar{X}_2 \quad (8)$$

To generalize this model to grades, sizes, and MC levels not tested in this study, the mean and standard deviation were related to lumber width, grade, and MC through a regression equation. In this study the minimum SR assigned to the grade and the standard dressed dry width were used as the grade and width parameters.⁵ The generalized normal model is model 7 in this report.

Weibull model. – Another model may be derived in a fashion similar to that used for the normal model except that a Weibull distribution is first fitted to the data. Again, we equated percentile levels in any pair of strength distributions for two specified MC levels. In this instance the relationship between the properties at the two specified MC levels (M_1 and M_2) is

$$P_2 = \omega_2 \left(\frac{P_1 - \ell_1}{\omega_1} \right)^{m_1/m_2} + \ell_2 \quad (9)$$

⁵Strength ratios (USDA 1974) are: Select Structural = 65 % , No. 2 = 45 % , No. 3 = 26 % . Standard dressed "dry" widths (U.S. Department of Commerce 1986) are: 4 inches = 3.5 inches, 6 inches = 5.5 inches, 8 inches = 7.25 inches.

where

- m_i = the Weibull shape parameter at M_i , $i = 1, 2$,
- ω_i = the Weibull scale parameter at M_i , $i = 1, 2$, and
- ξ_i = the Weibull location parameter at M_i , $i = 1, 2$.

The Weibull parameters at each of the four MC levels tested in this study are given in Aplin et al. (1986).

Equation (9) may be used to convert properties from one to another of the four MC levels for a given grade and size combination. To generalize the procedure to MC levels not tested experimentally, a relationship is required that relates each of the Weibull parameters to MC for each grade and size combination. In a previous paper on modeling the effect of MC on the flexural properties of southern pine (Green et al. 1986), a different quadratic function of MC was used for each individual grade and size combination. Although this procedure worked well, it was difficult to generalize the model to sizes and especially to grades not tested.

Alternatively, a regression approach such as was used with the normal model may be used to relate the Weibull parameters given in Eq. (9) to MC, grade, and size. We derived both two-parameter and three-parameter forms of the Weibull regression model (model 8 and 9 of Table 1). The shape and scale parameters for both take the form

$$\text{Parameter} = a_0 + a_1(W) + a_2(SR) + a_3(M) + \dots + a_{20}(W)^2(M)^2 \quad (10)$$

The location parameter used for the three-parameter form was determined by observing trends in the minimum MOR values for each combination of grade, size, and MC. The variables W (specimen width), SR (strength ratio), and M (moisture content) used to derive Eq. (10) are defined in Table A-3 of the appendix.

Strength ratio model. – Another type of model assumes that the MC adjustment factor depends upon the SR of the individual piece of lumber. The 25 % rule (Eq. 1) and Gerhard's model (Eq. 2) are historical models of this type.

To determine the form of the relationship between property and SR, the data were stratified and the results plotted in several subsets of SR. These plots indicated that change in MOR with change in MC increased erratically with increasing SR. Similar plots indicated that change in MOE was virtually independent of change in SR. Because there was no clear-cut trend in the SR relationships, only a linear SR model was investigated.

In this model, the change in property with a given change in MC was assumed equal to unity for SRs between zero and some cutoff value, y_0 . A linear relationship was used between y_0 and F^* , where F^* is the change in property between MCs for green lumber and lumber with SR = 100. Four values of y_0 were assumed: 0, 26, 45, and 50 %. Then

$$\begin{aligned} P_2 - P_1 &= 0 & 0 \leq SR \leq y_0 \\ P_2 - P_1 &= [100 - F^*y_0 + (F^* - 1)SR]/(100 - y_0) & y_0 < SR \leq 100 \end{aligned} \quad (11)$$

The value of F^* was obtained using a linear regression between F^* and MC for the 1,083 pieces that had an estimated SR of 100 %.

TABLE 2. Estimated intersection moisture content M_p for Douglas-fir dimension lumber.

Lumber grade and width		Moisture content adjustment model			
		MOR		MOE	
		Linear	Quadratic	Linear	Quadratic
%					
Surface model ¹					
Select structural	4	25.3	39.5	22.7	20.9
	6	22.0	56.7	22.1	23.1
	8	24.0	22.0	23.8	22.7
No. 2	4	30.3	29.5	22.0	21.1
	6	34.2	22.6	21.9	21.6
	8	34.3	20.5	22.5	31.0
No. 3	4	34.9	23.5	23.4	22.6
	6	29.9	34.0	21.2	22.6
	8	24.1	19.3	22.1	25.1
All	All	26.0	21.3	22.5	21.6
Constant percentage model ²					
All	All	31.8	62.7	23.7	23.2

¹ Linear surface model by regression (model 4 of Table 1), quadratic surface model by regression (model 5).

² Linear constant percentage model (model 3). Quadratic model not shown in text.

Intersection moisture content

It is generally assumed that the mechanical properties of small, clear wood specimens decrease with increasing MC up to some MC level. Past this level, properties are assumed to be independent of MC (U.S. Department of Agriculture 1974). The MC above which properties are independent of MC is called the intersection MC, M_p (Wilson 1932).

Traditionally, the value of M_p is chosen as the intersection of two lines on a plot of the logarithm of the property versus MC. The first line describes the linear relationship between log property and MC over a range of MCs for "dry" wood, and the second is a horizontal line representing the property value for green wood. For clear specimens of Douglas-fir, the M_p value is 24 % (Wilson 1932).

Estimates of M_p evaluated in this study vary with the property and the MC adjustment model used (Table 2).⁶ Because of this variation, there is no solid empirical evidence to reject the clear wood value of $M_p = 24$. This value is used in all modeling.

EVALUATION AND COMPARISON OF MODELS

In this section the MC adjustment models are compared for each grade and size by adjusting the property estimates at the four MCs to a common MC level. If the model were a perfect predictor of the effect of MC on property change, all four property estimates would be identical. The maximum absolute difference between the estimates is, therefore, an indication of the performance of the model. The maximum absolute difference is the maximum property estimate minus the minimum property estimate after each of the four values has been adjusted to

⁶ Procedures for estimating M_p for the various types of models are given in Green et al. (1986).

the common MC level. Because no trends were observed in the variation of maximum absolute difference with grade and size, the nine values of maximum absolute difference for individual grade and size combinations were averaged and are reported in Tables 1 and 3.

In general, models in which the coefficients are fitted using all the data are preferred to models in which the coefficients are fitted to each grade and size separately. Although the latter tend to fit the experimental data better, the former are more likely to be adequate for grades and sizes not tested. Also, given the limited number of samples tested for any given combination of grade, size, and MC, a model obtained using all the data is likely to be more stable and less likely to overfit the peculiarities of the given data set. Coefficients of each model discussed in this section are given in Tables A-1 to A-6 in the appendix.

Modulus of rupture

With the exception of the zero adjustment model, all the models are more accurate when adjusting to green than to dry MC levels (Table 2). However, models that perform well at one MC level also perform well at other MC levels.

As was also observed for southern pine, neither the constant percentage adjustment models (1, 2, and 3) nor the SR model (10) provided a very accurate adjustment of MOR (Table 3). The results are not surprising since the coefficient of determination, R^2 , between MOR and SR was only 0.19 using all the data.

Of the 11 models evaluated in this paper, the normal model (7) and the two Weibull models (8 and 9) consistently produced the smallest maximum absolute difference at the mean with only minor differences between the three models (Table 1). The fixed and nonfixed surface models (4, 5, and 6) were usually best at the 5th percentile.

Either the two- or three-parameter Weibull model is generally best for adjusting the entire distribution of MOR (Table 3). Although there is little difference between the normal and the Weibull models at the mean, the Weibull models are generally better throughout the rest of the distribution and especially at the 5th percentile (Tables 1 and 3). Except at the 5th percentile, the fixed and unfixed surface models generally produce larger average maximum absolute differences than the Weibull models.

Comparison of the two- and three-parameter Weibull models indicates little difference in performance except at the 5th and 95th percentiles (Tables 1 and 3). Because the location parameter to be used with the three-parameter Weibull MC adjustment model must be selected in a somewhat arbitrary manner, this model may be overfitted to the peculiarities of this particular data set. Until more experience is gained with fitting this model to other data sets, the two-parameter model (9) is recommended for the adjustment of MOR distributions.

A direct comparison between the performance of the two-parameter Weibull model and the performance of the other models is shown in Table 4. The numbers in the body of the table indicate the difference between the average maximum absolute value for the two-parameter Weibull and each of the other models. Positive values indicate that the two-parameter Weibull provides the best estimate; negative values otherwise.

Although the maximum absolute difference generally provides the better esti-

TABLE 3. Comparison of moisture content adjustment models for the properties of Douglas-fir adjusted to a moisture content of 15%.

Model number	Model type	Variation	Dependence on grade and size	Number of coefficients	Average maximum absolute difference at indicated point in cumulative frequency distribution ^a					
					Mean	5	25	50	75	95
psi										
Modulus of rupture, MOR										
0	Zero adjustment		Independent	0	2,277	727	1,662	2,201	3,182	4,075
1	Constant percentage	Linear	By grade and size	18	577	636	627	800	1,087	1,379
2			By grade	6	686	637	650	896	1,137	1,563
3	Surface	Linear, regression	Independent	2	833	586	743	1,107	1,323	1,512
4			Function	10	690	427	649	889	985	1,159
5			Quadratic, regression	30	523	509	607	667	770	915
6			Quadratic, regression (fixed)	8	546	516	605	701	793	989
7	Normal	Regression	Function	25	409	724	557	643	682	1,007
8	Weibull	3-P, regression	Function	35	416	479	543	645	693	934
9			2-P, regression	22	411	555	551	635	720	874
10	Strength ratio	Linear, 0% cutoff	Independent		761	533	727	932	1,250	1,506
10 ⁶ psi										
Modulus of elasticity, MOE										
0	Zero adjustment		Independent	0	0.268	0.162	0.206	0.262	0.303	0.432
1	Constant percentage	Linear	By grade and size	18	0.044	0.053	0.056	0.050	0.048	0.097
2			By grade	6	0.050	0.057	0.063	0.058	0.061	0.094
3	Surface	Linear, regression	Independent	2	0.050	0.059	0.065	0.057	0.065	0.097
4			Function	10	0.045	0.060	0.065	0.055	0.064	0.083
5			Quadratic, regression	30	0.042	0.064	0.063	0.051	0.062	0.082
6			Quadratic, regression (fixed)	8	0.041	0.063	0.062	0.051	0.068	0.083
7	Normal	Regression	Function	27	0.034	0.056	0.052	0.050	0.053	0.074
9	Weibull	2-P, regression	Function	23	0.037	0.057	0.053	0.052	0.060	0.077

TABLE 3. Continued.

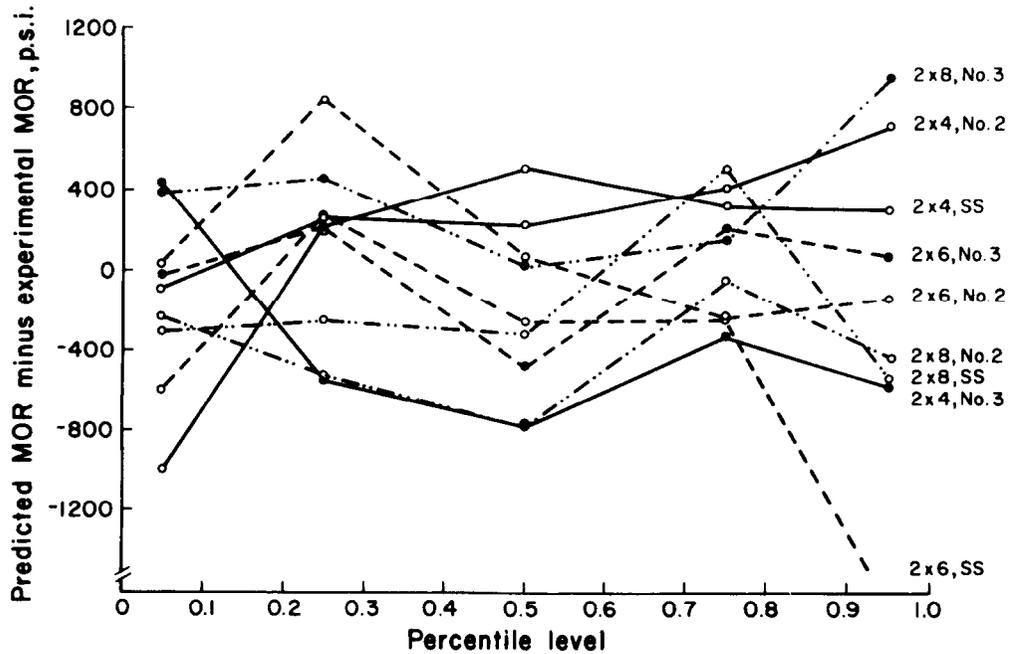
Model number	Model type	Variation	Dependence on grade and size	Number of coefficients	Average maximum absolute difference at indicated point in cumulative frequency distribution ¹					
					Mean	5	25	50	75	95
in · lb										
Moment capacity, RS										
0	Zero adjustment		Independent	0	12,819	4,044	9,192	12,225	19,938	22,607
5	Surface	Quadratic, regression	Function	30	4,444	4,587	5,100	6,595	7,082	8,733
6		Quadratic, regression (fixed)	Function	8	4,066	3,829	4,513	4,986	5,891	7,750
7	Normal	Regression	Function	24	3,694	5,009	3,977	5,568	6,342	7,933
9	Weibull	2-P, regression	Function	24	4,087	3,851	3,713	5,832	6,517	7,842
..... 10 ⁶ lb · in										
Flexural stiffness, EI										
0	Zero adjustment		Independent	0	2.326	1.846	2.497	2.432	2.766	2.919
5	Surface	Quadratic, regression	Function	30	0.750	1.597	1.379	0.787	1.335	1.676
6		Quadratic, regression (fixed)	Function	8	0.812	1.463	1.299	0.902	1.336	1.754
7	Normal	Regression	Function	28	0.501	1.467	1.178	0.826	0.979	1.094
9	Weibull	2-P, regression	Function	25	0.543	1.420	1.216	0.837	0.903	1.165

¹ Based on the average of each of the 9 grade and size combinations at the indicated percentile (or mean) level.

TABLE 4. Performance of moisture content adjustment models compared to the two-parameter Weibull model (model 9) for modulus of rupture adjusted to a moisture content of 15%.

Model number	Model type	Variation	Dependence on grade and size	Num-ber of coef-ficients	Average maximum absolute difference (model X _i - model 9) at indicated level of cumulative frequency distribution, ¹ psi					Average of 5 through 95	
					Mean	5	25	50	75		95
Modulus of rupture, MOR											
0	Zero adjustment		Independent	0	1,866	172	1,111	1,566	2,462	3,201	1,702
1	Constant percentage	Linear	By grade and size	18	166	81	76	165	367	505	239
2			By grade	6	275	82	99	261	417	689	310
3			Independent	2	422	31	192	472	603	638	387
4	Surface	Linear, regression	Independent	10	279	-128	98	254	265	285	155
5		Quadratic, regression	Independent	30	112	-47	56	32	50	41	26
6		Quadratic, regression (fixed)	Independent	8	135	-39	54	66	73	115	54
7	Normal	Regression	Function	25	-2	243	184	240	269	272	144
8	Weibull	3-P, regression	Function	35	5	-76	-8	10	-27	60	-8
10	Strength ratio	Linear, 0% cutoff	Independent	15	350	-22	176	297	530	632	323
Modulus of elasticity, MOE											
0	Zero adjustment		Independent	0	0.231	0.105	0.153	0.210	0.243	0.355	0.213
1	Constant percentage	Linear	By grade and size	18	0.007	-0.004	0.003	-0.002	-0.012	0.020	0.001
2			By grade	6	0.012	0.000	0.010	0.006	0.001	0.017	0.007
3			Independent	2	0.013	0.002	0.012	0.005	0.005	0.020	0.009
4	Surface	Linear, regression	Function	10	0.008	0.003	0.012	0.003	0.004	0.006	0.006
5		Quadratic, regression	Function	30	0.005	0.007	0.010	-0.001	0.002	0.005	0.005
6		Quadratic, regression (fixed)	Function	8	0.004	0.006	0.009	-0.001	0.008	0.006	0.006
7	Normal	Regression	Function	27	-0.003	-0.001	-0.001	-0.002	-0.007	-0.003	-0.003
10	Strength ratio	Linear, 0% cutoff	Independent	15	0.067	0.018	0.043	0.052	0.051	0.104	0.054

¹ Based on the average of each of the 9 grade and size combinations at the indicated percentile (or mean) level. Positive values indicate that the two-parameter Weibull provides the best estimate; negative values otherwise.



ML86 5357

FIG. 3. Differences at various percentile levels between MOR tested at 10% MC and MOR predicted by adjusting green MORs to 10% MC. (ML86 5357)

mate of performance, a comparison between experimental and predicted MOR values for a given change in MC is also of interest. Such a comparison is shown in Fig. 3 where the MOR'S measured in the green condition were adjusted to an MC of 10 % using the two-parameter Weibull model (9). The adjusted values, when compared to the MOR'S measured at 10 % , indicate that the difference is not a function of the position in the MOR cumulative frequency distribution.

In a previous paper, Madsen (1982) suggested that for different MC levels "changes in strength are minor for design purposes and that the same bending stresses should be used for dry and wet service conditions." Figure 4 compares the dry: green ratio predicted by the two-parameter Weibull model (9) for No. 2, 2 x 6, to those obtained by Madsen for Douglas-fir, No. 2 and Better, 2 x 6.⁶ From these plots it is obvious that the changes in strength are significant.

Modulus of elasticity

Modulus of elasticity tends to be normally distributed (Aplin et al. 1986; McLain et al. 1984). Also, the effect of MC is not very dependent upon lumber quality (or position in the MOE cumulative frequency distribution) (Green 1982; Green et al. 1986). For these reasons, several of the models gave a satisfactory fit to the data (Tables 1 and 3). The normal model (7) provides the best overall fit, the two-parameter Weibull model (9) being essentially as good (Table 4). Even the simplistic constant percentage adjustment models (1, 2, and 3) provide a reasonable fit to the data.

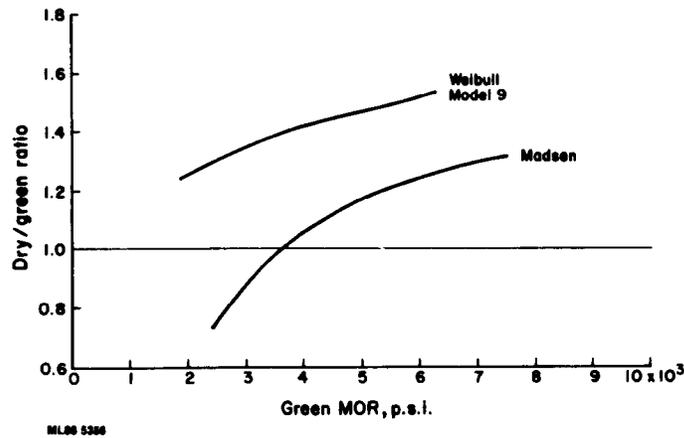


FIG. 4. Dry : green ratio for 2 × 6 Douglas-fir dimension lumber dried to an MC of 15%. (ML86 5356)

Moment capacity and stiffness

As noted with southern pine (Green et al. 1986), the type of model that best fits the MOR/MOE data generally provides the best fit for RS/EI. For this reason, only the two quadratic surface models (5 and 6), the normal model (7), the two-parameter Weibull model (9), and the zero adjustment model (for comparison) are shown in Tables 1 and 3. In general, the normal model tends to give a slightly better fit in the middle of the cumulative frequency distribution, while the two-parameter Weibull fits better in the tails.⁷

MISUSE OF THE MODELS

The analytical models were developed to predict the effect of changes in MC on the strength of Douglas-fir dimension lumber and are unsuitable for predicting absolute values of properties. A fundamental assumption of the models developed is that changes in properties with change in MC are relatively insensitive to the particular geographic location from which the lumber was sampled. The absolute magnitude of the properties at a given MC may vary considerably from sample to sample.

Care should be exercised when applying the equations to lumber properties or MC levels outside the range of data used to establish the coefficients. It is our experience that a failure to place limits on the use of these equations may lead to unrealistic or even illogical results. We do not recommend that these equations be used for MC less than 8 %.

The MOR and MOE limits were established by comparing trends predicted using the models with actual trends observed near the extremes of the data. These limits are given in Table A-7. Applicable limits for EI and RS may be determined by appropriate scaling of the MOR and MOE limits.

⁷Because of the effect of specimen width on the magnitude of EI and RS, the average values of the maximum absolute difference will be dominated by the values for the wider widths. Therefore, the values of the average maximum absolute error given in Tables 2 and 3 should only be used to compare models.

CONCLUSIONS

Of the models evaluated in this study we conclude:

1. Neither of the model types used in the past (constant percentage adjustment and SR) provides an acceptable adjustment for the effect of MC on MOR.
2. The analytical model based on the Weibull distribution with parameters determined by a regression procedure (models 8 and 9) provides the best overall fit to the MOR and RS data. Although the three-parameter version (model 8) is slightly more accurate than the two-parameter version (model 9), we lack sufficient experience with similar data sets for other species to justify the use of the location parameters used in this study. Therefore, the two-parameter version is recommended for adjusting lumber strength distributions.
3. The model obtained by fitting a normal distribution to the data and predicting distribution parameters through regression (model 7) provides the best fit to the MOE and EI data. However, all models except the SR model give reasonable adjustments for MOE and EI.

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APPENDIX

Tables A-1 through A-6 list the coefficients to be used with most of the models presented in this paper. For all the models given in this paper, except the Weibull and normal models given in Tables A-3 and A-4, MOR and RS are expressed in units of 1 psi, MOE and EI as 10^6 psi. The Weibull and normal models are expressed in 10^3 psi. Moisture content (M , M_p) is expressed as a percent. In models 7, 8, and 9 (Tables A-3 and A-4), the specimen width (W) is assumed to be the standard dressed dry width (3.5, 5.5, 7.25, etc.), and the strength ratio (SR) is assumed to be the minimum SR for the structural light-framing grades (26, 45, 55, 65). (The SR for Select Structural 2×4 s is actually 67 but 65 is assumed in this report.)

TABLE A-1. *Coefficients of the linear constant percentage adjustment model.*¹

Model number	Grade	Nominal width	MOR		MOE	
			a	b	a	b
1	Select	4	14,450.3	-316.33	1.9175	-0.02074
1	structural	6	12,810.2	-241.09	2.1818	-0.02122
1		8	10,813.8	-208.81	2.0386	-0.02410
1		No. 2	4	9,404.5	-179.02	1.6421
1	No. 2	6	7,596.1	-138.55	1.6020	-0.01771
1		8	6,303.4	-85.22	1.5339	-0.01691
1		No. 3	4	6,312.1	-75.59	1.4363
1	No. 3	6	5,503.1	-72.63	1.3673	-0.01462
1		8	5,392.6	-81.49	1.3712	-0.01639
2		Select	All	12,590.38	-250.4762	2.056623
2	structural	All	7,691.10	-130.0149	1.589974	-0.018285
2	No. 2	All	5,595.99	-68.1797	1.390843	-0.016645
2	No. 3	All	5,595.99	-68.1797	1.390843	-0.016645
3	All	All	8,601.64	-148.02	1.67839	-0.01916

¹ Equation (3).

TABLE A-2. Regression coefficients of (nonfixed) quadratic surface models (model 5).¹

Variable	MOR ²			MOE ³		
	b	c	c	b	c	c
Constant	-0.7697463E+04	0.1672728E+03		0.7563579E+00		-0.2390278E-01
M ₁	0.1937799E+04	-0.3922224E+02		-0.1138130E+00		0.4117220E-02
P ₁	0.7071369E+00	-0.2952671E-01		-0.8844447E+00		0.2106855E-01
M ₁ ²	-0.1808354E+03	0.3448722E+01		0.905915E-02		-0.34826E-03
P ₁ ²	-0.3206511E-04	0.14691064E-05		0.9270093E+00		-0.2376689E-01
M ₁ P ₁	-0.9667900E-01	0.417799E-02		0.1041533E-01		0.5917250E-05
M ₁ ³	0.7098496E+01	-0.1273483E+00		-0.35143E-03		0.1378824E-04
P ₁ ³	0.1636167E-08	-0.5855939E-10		-0.3623098E+00		0.9371550E-02
M ₁ ² P ₁	0.700956E-02	-0.26101E-03		0.1226970E-02		-0.4624055E-04
P ₁ ² M ₁	-0.1005114E-05	-0.4967575E-07		-0.2322769E-01		0.56063E-03
M ₁ ⁴	-0.1001912E+00	0.168830E-02		0.5088906E-05		-0.2017113E-06
P ₁ ⁴	-0.5885839E-13	0.1534856E-14		0.4914017E-01		-0.1281570E-02
M ₁ P ₁ ³	0.9710008E-10	-0.9744899E-12		0.5164390E-02		-0.12762E-03
M ₁ ² P ₁ ²	-0.5035050E-07	0.2815968E-08		0.7217547E-04		-0.1265619E-05
M ₁ ³ P ₁	-0.14130E-03	0.4870561E-05		-0.2843562E-04		0.991323E-06
Constant	-0.2339686E+02	0.3193140E+00		0.4613937E+05		-0.1098555E+04
M ₁	0.5973324E+01	-0.8235865E-01		-0.1204662E+05		0.2923694E+03
P ₁	-0.6011880E-02	0.67865E-03		0.3690677E+00		-0.1413041E-01
M ₁ ²	-0.5425985E+00	0.7438140E-02		0.1157441E+04		-0.2822527E+02
P ₁ ²	0.1193750E-02	-0.3411746E-04		-0.2882066E-05		0.1394021E-06
M ₁ P ₁	-0.1050260E-01	0.17790E-03		-0.5580503E-01		0.1839680E-02
M ₁ ³	0.2110891E-01	-0.28691E-03		-0.4752492E+02		0.1158631E+01
P ₁ ³	-0.1141844E-04	0.3525323E-06		0.7590174E-11		-0.7075964E-12
M ₁ ² P ₁	0.80272E-03	-0.1410402E-04		0.2609380E-02		-0.8345374E-04
P ₁ ² M ₁	-0.5836668E-04	0.1239208E-05		0.3229176E-06		-0.1095333E-07
M ₁ ⁴	-0.298730E-03	0.40192248E-05		0.7065035E+00		-0.1719949E-01
P ₁ ⁴	0.5946199E-07	-0.1830510E-08		-0.1212617E-16		0.1675731E-17
M ₁ P ₁ ³	0.2747526E-06	-0.6371371E-08		-0.5211465E-12		0.2212559E-13
M ₁ ² P ₁ ²	0.6522413E-06	-0.1404858E-07		-0.7733330E-08		0.2266034E-09
M ₁ ³ P ₁	-0.1690938E-04	0.30244268E-06		-0.3856020E-04		0.1239378E-05

¹ Equation (5) of text.² In psi.³ In 10⁶ psi.

TABLE A-3. Regression coefficients to be used with normal and Weibull models for MOR and MOE.

Variable ¹	Two-parameter Weibull (model 9)			Three-parameter Weibull (model 8) ²		
	MOR ³	Scale, ω	Shape, M	MOE ⁴	Scale, ω	Shape, M
Constant	0.95990E+01	0.54270E+01	-0.11947E+02	-0.11947E+02	-0.45279E+00	-0.54112E+01
W	-0.45489E-01	0.38492E+01	0.62989E+01	0.62989E+01	0.58613E+00	0.50809E+01
SR	-0.11587E+00	0.13688E+00	0.84498E+00	0.84498E+00	0.93137E-01	0.72533E+00
M ₁	-0.11224E+01	-0.10376E+01	0	0	0.63903E-01	-0.10456E+01
(W) ²	0	-0.36469E+00	-0.57900E+00	-0.57900E+00	-0.49959E-01	-0.46670E+00
(SR) ²	0.18427E-02	0	-0.92431E-02	-0.92431E-02	-0.12810E-02	-0.86848E-02
M ₁ ²	0.75053E-01	0.27434E-01	0	0	-0.52950E-02	0.69495E-01
W(SR)	0	-0.30331E+00	-0.32874E+00	-0.32874E+00	-0.35680E-01	-0.29549E+00
W(SR) ²	0	0.39487E-02	0.36436E-02	0.36436E-02	0.56389E-03	0.36732E-02
(W) ² (SR)	0	0.26499E-01	0.29959E-01	0.29959E-01	0.30200E-02	0.26272E-01
(W) ² (SR) ²	0	-0.36134E-03	-0.32636E-03	-0.32636E-03	0.48760E-04	-0.32546E-03
W(M ₁) ²	0	0	0.13638E-03	0.13638E-03	0	0
(W)(M ₁)	0	0	0	0	0	0.45144E-02
(SR)(M ₁)	0	0.70987E-01	0	0	-0.13887E-03	-0.80463E-03
(M ₁) ³	-0.15075E-02	0	0	0	0.10986E-03	-0.14128E-02
(SR)(M ₁) ²	0	-0.20777E-02	0	0	0	0
W(SR)(M ₁)	0	0.43511E-03	0	0	0	0
(SR) ² (M ₁)	0	-0.94275E-03	0	0	0	0
(W) ² (M ₁)	0	0	0	0	0	0
(SR) ² (M ₁) ²	0	0.25252E-04	0	0	0	0
(W) ² (M ₁) ²	0	0	0	0	0	0

TABLE A-3. Continued.

Variable ¹	Three-parameter Weibull (model 8) ²				Normal (model 7)			
	MOR ³		MOR ³		MOR ³		MOE ³	
	Scale, ω	Mean, \bar{x}	Standard deviation, S	Mean, \bar{x}	Standard deviation, S	Mean, \bar{x}	Standard deviation, S	
Constant	-0.46134E+01	0.41601E+01	0.20322E+01	-0.70262E+00	-0.87410E-01	-0.70262E+00	-0.87410E-01	
W	0.67475E+01	0.35300E+01	-0.10199E+01	0.66508E+00	0.20859E+00	0.66508E+00	0.20859E+00	
SR	0.63672E+00	0.13700E+00	0	0.10276E+00	-0.81839E-02	0.10276E+00	-0.81839E-02	
M ₁	-0.10768E-01	-0.86352E+00	0.47008E+00	0.49413E-01	0.76781E-01	0.49413E-01	0.76781E-01	
(W) ²	-0.57317E+00	-0.33381E+00	0.88150E-01	-0.58769E-01	-0.19769E-01	-0.58769E-01	-0.19769E-01	
(SR) ²	-0.68442E-02	0	-0.3803E-03	-0.31081E-01	0.38585E-04	-0.3803E-03	0.38585E-04	
M ₁ ²	0.31177E-01	0.22914E-01	-0.31081E-01	0.39874E-01	-0.23567E-02	-0.31081E-01	-0.23567E-02	
W(SR)	-0.46079E+00	-0.27410E+00	0.19693E-01	0.60391E-03	0.27204E-02	-0.46079E+00	0.27204E-02	
W(SR) ²	0.59880E-02	0.35053E-02	0	0	0	0.59880E-02	0	
(W) ² (SR)	0.38465E-01	0.23919E-01	-0.18626E-02	0.34526E-02	-0.27128E-03	0.38465E-01	-0.27128E-03	
(W) ² (SR) ²	-0.52078E-03	-0.32072E-03	0	-0.52417E-04	0	-0.52078E-03	0	
W(M ₁) ²	0	0	0	0	0	0	0	
(W)(M ₁)	-0.16351E-01	0	0	0.96801E-03	0.10108E-02	-0.16351E-01	0.10108E-02	
(SR)(M ₁)	0.75538E-01	0.61020E-01	-0.58373E-03	0	-0.35165E-01	0.75538E-01	-0.35165E-01	
(M ₁) ³	0	0	0.58521E-03	0.95732E-04	0	0	0	
(SR)(M ₁) ²	-0.22569E-02	-0.17886E-02	0	0	0	-0.22569E-02	0	
W(SR)(M ₁)	0.76219E-03	0.39894E-03	0	-0.26215E-04	0	0.76219E-03	0	
(SR) ² (M ₁)	-0.10008E-02	-0.80874E-03	0	0	0	-0.10008E-02	0	
(W) ² (M ₁)	0	0	0	0	0	0	0	
(SR) ² (M ₁) ²	0.26900E-04	0.21579E-04	0	0	0.33380E-02	0.26900E-04	0.33380E-02	
(W) ² (M ₁) ²	0	0	0	0	0	0	0	
(W) ² (M ₁) ³	0	0	0	0	-0.96428E-04	0	-0.96428E-04	

¹ Equation (10) with location parameters (β_1 and β_2) = 0. Shape (M) and scale (ω) parameters expressed as parameter = $a_0 + a_1(W) + a_2(SR) + \dots + a_{10}(W)(SR)^2$. M = moisture content, %, W = standard dressed dry widths, 3.5, 5.5, 7.25, etc. SR = assumed minimum strength ratio for the grade, select structural = 65, No. 1 = 55, No. 2 = 45, No. 3 = 26, etc.
² Location parameters, β_1 are 2 x 4 SS = 3,200, 2 x 6 and 2 x 8 SS = 2,200, all No. 2 = 1,000 ksi and all No. 3 = 700 ksi.
³ In 10³ psi.
⁴ In 10⁶ psi.

TABLE A-4. *Coefficients to be used with normal and Weibull models for RS and EI.*

Variable	Two-parameter Weibull (model 9) ¹			
	RS ²		EI ³	
	Shape, M	Scale, ω	Shape, M	Scale, ω
Constant	0.74936E+01	0.85539E+02	-0.13768E+02	0.26771E+02
W	0.106475E+01	-0.38568E+01	0.71750E+01	-0.16962E+02
SR	-0.11813E+00	-0.62195E+01	0.95927E+00	0.63778E+00
M	-0.99738E+00	-0.69665E+01	0	-0.37136E+00
(W) ²	-0.11052E+00	0.18675E+01	-0.66776E+00	0.33342E+01
(SR) ²	0.18847E-02	0.59482E-01	-0.10198E-01	-0.10703E-01
(M) ²	0.77591E-01	0	0	-0.14143E-01
W(SR)	0	0	-0.37551E+00	-0.20645E+00
W(SR) ²	0	0.55665E-02	0.41275E-02	0.31682E-02
(W) ² (SR)	0	-0.18383E-01	0.34478E-01	0.75496E-02
(W) ² (SR) ²	0	0	-0.37318E-03	0
W(M) ²	0	0	0	0.50787E-02
(W)(M)	-0.17642E-01	0	0.50338E+02	0.19963E+00
(SR)(M)	0	0.78505E+00	-0.87914E-03	0.10702E-02
(M) ³	-0.15523E-02	0.65916E-02	0	0
(SR)(M) ²	0	-0.21163E-01	0	0
(W)(SR)(M)	0	-0.56632E-02	0	0
(SR) ² (M)	0	-0.88122E-02	0	0
(W) ² (M)	0.74768E-02	0	0	-0.43735E-01
(SR) ² (M) ²	0	0.23945E-03	0	0
(W) ² (M) ²	0	0	0	0

TABLE A-4. Extended.

Normal (model 7)			
RS ²		EP ³	
Mean, \bar{x}	Standard deviation, S	Mean, \bar{x}	Standard deviation, S
0.43104E+02	-0.42227E+02	0.12420E+02	0.10051E+02
0.90883E+01	0	-0.14711E+02	-0.55373E+01
0.10166E+01	0	0.11472E+01	0
-0.87577E+01	0.76124E+01	0.10501E+01	-0.23393E+00
0	0.64246E-00	0.29847E+01	0.90306E+00
0	0	-0.15794E-01	-0.16236E-02
0.23300E+00	-0.45185E+00	-0.52453E-01	0
-0.10770E+01	0.54937E-01	-0.21260E+00	0.18904E-01
0.18546E-01	-0.57103E-03	0.30015E-02	0.43875E-03
0.85488E-01	0	0.92176E-02	-0.52588E-02
-0.12605E-02	0	0	0
-0.16149E-01	0	0.55919E-02	0
0.57810E+00	0	0.14621E+00	0.97899E-01
0.44581E+00	0	-0.638045E-01	0
0	0.85523E-02	0	0
-0.11519E-01	0	0.18606E-02	0
-0.52989E-02	0	0	0.21489E-03
-0.51822E-02	0	0.72559E-03	0
0	-0.17048E-01	-0.39671E-01	-0.11658E-01
0.13623E-03	0	-0.20954E-04	0
0	0	0	0

¹ Equation (10) with location parameters (ℓ_1 and ℓ_2) = 0. Shape (M) and scale (ω) parameters expressed as parameter = $a_0 + a_1(W) + a_2(SR) + \dots + a_{10}(W)^2(SR)^2$. M = moisture content, %. W = standard dressed dry widths, 3.5, 5.5, 7.25, etc. SR = assumed minimum strength ratio for the grade, select structural = 65, No. 1 = 55, No. 2 = 45, No. 3 = 26, etc.

² In 10³ psi.

³ In 10⁶ psi.

TABLE A-5. Regression coefficients of fixed quadratic surface models (model 6).

Coefficient	MOR ¹	MOE ²	EI ²	RS ¹
D ₀	-0.36161121E+03	-0.7400681E-01	-0.46983780E-01	-0.37405354E+03
D ₁	0.29592626E+00	0.14654033E+00	0.10134717E-01	0.30897147E+00
D ₂	-0.39783127E-04	-0.12649923E+00	-0.9818598E-01	-0.40693328E-04
D ₃	0.12503366E-08	0.3051976E-01	0.2621110E-01	0.13410251E-08
E ₀	0.994437082E+01	0.195770E-02	0.150273E-02	0.10187048E+02
E ₁	-0.815532E-02	-0.414948E-02	-0.342433E-02	-0.82443200E-02
E ₂	0.95704266E-06	0.324488E-02	0.303597E-02	0.96345528E-06
E ₃	-0.29208423E-10	-0.78286E-03	-0.76892E-03	-0.30486442E-10

¹ In psi.² In 10⁶ psi.

TABLE A-6. Coefficients of linear strength ratio model (model 10).

Size	Grade	Coefficient symbol	Coefficient for indicated property ¹	
			MOR ²	MOE ³
----- psi -----				
All	All	a	11,486.116530	1.8667432
		b	-228.150890	-0.0217115

¹ $F^* = (a + bM_2)/(a + bM_1)$ for lumber with SR = 100.
² In psi.
³ In 10⁶ psi.

TABLE A-7. Recommended property limits for the models presented in this report.¹

Property	Size	Grade	Applicable range at MC = 15%	
			----- psi -----	
Modulus of rupture	All	Select structural	1,000	12,000
		No. 2	1,000	10,000
		No. 3	1,000	8,000
Modulus of elasticity	All	Select structural	200,000	2,300,000
		No. 2	200,000	1,900,000
		No. 3	200,000	1,700,000

¹ The recommended limits on moisture content are 8% ≤ MC ≤ 24%.