DYNAMIC CHARACTERISTICS OF WOOD AND GYPSUM DIAPHRAGMS

By Robert H. Falk and Rafik Y. Itani, Members, ASCE

ABSTRACT: Wood diaphragms are used in low-rise, wood-framed buildings to resist the lateral forces produced by wind and earthquakes. Since wood buildings are known to be efficient in absorbing the energy produced by these loadings, it is of importance to more fully understand the dynamic behavior of their components. Presented in this paper are the results from an experimental study performed to measure the dynamic characteristics of ten plywood and gypsum-board-sheathed diaphragms. Four walls, three floors, and three ceilings were tested to determine natural frequencies and damping ratios. Results indicate that natural frequencies for the diaphragms proportions tested range from 8–29 Hz and vary depending on diaphragm displacement level. Using results of regression analysis, natural frequencies for various diaphragms may be predicted. Damping ratios were calculated and were found to range from 0.09–0.34.

INTRODUCTION

Wood diaphragms are important structural components of wood buildings that resist the lateral forces produced by wind and earthquakes.

Considerable research effort has been directed toward the behavior of diaphragms; however, most of this has been limited to static load response. As a result, there is a lack of information on the dynamic characteristics of wood diaphragms.

This paper presents results from diaphragm tests performed to determine basic static and dynamic properties. Ten floor, ceiling, and wall diaphragms ranging in size from 8 × 24 ft to 16 × 28 ft were tested. The tests focused on the determination of natural frequencies, damping ratios, and nonlinear stiffness characteristics. Free vibration tests were performed to determine natural frequencies and damping ratios, while nonlinear stiffness characteristics were determined from ultimate static load tests. The effects of two typical sheathing materials, plywood and gypsum wallboard, were considered, as well as the effect of a stairwell opening in one floor diaphragm and door and window openings in two walls.

RELATED RESEARCH

Considered experimental research has been performed on wood diaphragms; however, little has been directed towards the determination of dynamic properties (Atherton 1981; Easley 1982; Ewing 1980; Gupta 1985; Itani 1984; Tuomi 1978).

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Note.—Discussion open until November 1, 1987. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on June 19, 1986. This paper is part of the Journal of Structural Engineering, Vol. 113, No. 6, June, 1987. ©ASCE, ISSN 0733-9445/87/0006-1357/$01.00. Paper No. 21603.
A comprehensive bibliography prepared by Carney (1976) and later updated by Peterson (1983) lists diaphragm research from as early as 1930. Most of the experimental tests listed were performed to determine the relative stiffness and strengths of various sheathing materials and fastener types.

Allowable shear load tables for horizontal diaphragm design currently used by code agencies originated from static floor tests performed by the Douglas Fir Plywood Association. Various types of plywood sheathing, fasteners, and framing were considered. These tables were later expanded as a result of additional tests performed by Tissell (1967; 1977).

More recently, experimental studies have been undertaken to investigate the effects of wall openings, wall length, addition of gypsumboard sheathing, and cycled static loading (Kallsner 1983; Kamiya 1981; Patton-Mallory, et al. 1984; Soltis, et al. 1981; Yasumura and Sugiyama 1983; 1984).

The energy absorption characteristics of wood shear walls was investigated by Young and Madearis in 1962. Damping ratios averaging 0.10 for walls with plywood on both sides and 0.07 for walls with plywood on one side were recommended.

In 1977, Freeman tested several building partitions constructed of wood or metal studs and sheathed with gypsumboard and plywood. Damping ratios were estimated to range from 0.07–0.20.

In 1975, Polensek experimentally tested 34 wood-joist floor diaphragms for damping capacity using horizontal vibrations. These diaphragms ranged in size from 6 × 20 ft to 16 × 24 ft and were sheathed with various materials. It was found that for low displacement levels, the average damping ratios for the floors ranged from 0.07–0.11.

A verification of the decrease in stiffness and load carrying capacity of floors due to cyclic loading was performed by GangaRao (1980). A 16 × 24-ft floor was subjected to sinusoidal loading. Impulsive loading was also applied to determine damping ratios. The results obtained indicated that the dynamic load carrying capacity was reduced to about half the static value. Maximum damping coefficients were found to be about 0.15.

**Experimental Program**

Test Specimens.—The diaphragms tested in this study included four walls, three ceilings, and three floors. These specimens were chosen to represent components that might be found in conventionally constructed wood-framed homes.

Four walls 8 × 24 ft in size were constructed as shown in Fig. 1 and Table 1. Walls W1 and W3 were sheathed with 1/2-in. CDX grade four-ply plywood, while W2 and W4 were sheathed with 1/2-in. gypsumboard. W3 and W4 included door and window openings. The door and window sizes are typical for wood-frame construction used in the United States.

Floor and ceiling construction configurations are shown in Fig. 2. F1, F2, and F3 represent floor diaphragms 16 × 16 ft to 16 × 28 ft in size and were sheathed with 1/2-in. plywood. F2 included an opening representing a stairwell.
To represent a first-story ceiling in a two-story building, diaphragms with similar framing to F1 and F3 were sheathed with 1/2-in. gypsumboard. These diaphragms are designated C1, C2, and C3.

Joints were not taped in the gypsumboard-sheathed specimens and no gaps were allowed between the plywood or gypsumboard sheathing. For the walls, 2 × 4-in. studs at 16-in. centers and single sill and sole plates were used. Framing for the floors and ceilings consisted of 2 × 10-in. joist lumber spaced 16-in. on center (o.c.). A single rim joist and typical frame nailing with 16d common nails were used.

All wood materials were purchased from a local lumber supplier. To maximize consistency in material properties, only material from a single lot was used. All material was allowed to equilibrate to laboratory conditions before testing (10% M.C.).

Test System.—A structural test system was constructed in the Structural Engineering Laboratory at Washington State University. The system can be used to perform free vibration, forced vibration, and static tests on wood diaphragms up to 16 × 28 ft in size. This system is essentially a one-dimensional shake table capable of testing specimens in the horizontal position (see Fig. 3). A steel beam 30 ft long and capable of low-friction longitudinal motion is connected to a MTS (Multiple Test System) 22-kip hydraulic actuator. The base of each diaphragm was bolted to the steel beam, which acted as a sliding base support.

Tests Performed.—To determine natural frequencies, damping ratios, and nonlinear load characteristics, three tests were performed. The first involved vibrating each diaphragm harmonically to determine its first mode resonant frequency. A constant amplitude sine wave signal was used to excite each specimen through a range of frequencies (typically 1–30 Hz). Vibrating the base of each specimen while measuring the lateral displacement of the top of the specimen at each frequency identified the first mode-resonant frequency (or natural frequency).

A second test involved the measurement of diaphragm free vibration. This test was performed at various displacement levels and provided measures of natural frequency and damping ratio. This test was performed by applying an initial displacement to the top of the diaphragm at a slow (static) rate. A mechanical release device allowed quick release of the force holding the diaphragm in its deflected position. Lateral dis-
<table>
<thead>
<tr>
<th>Dia-</th>
<th>Type</th>
<th>Size, ft (m)</th>
<th>Framing size, *</th>
<th>Sheathing type, *</th>
<th>Sheathing arrangement</th>
<th>Nail size</th>
<th>Nail Spacing</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>W1</td>
<td>Wall</td>
<td>8 x 24</td>
<td>2 x 4</td>
<td>1/2-in. ply</td>
<td>Vertical</td>
<td>6d</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>W2</td>
<td>Wall</td>
<td>8 x 24</td>
<td>2 x 4</td>
<td>1/2-in. gyp</td>
<td>Vertical</td>
<td>1/2 in.</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>W3</td>
<td>Wall</td>
<td>8 x 24</td>
<td>2 x 4</td>
<td>1/2-in. ply</td>
<td>Vertical</td>
<td>6d</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>W4</td>
<td>Wall</td>
<td>8 x 24</td>
<td>2 x 4</td>
<td>1/2-in. gyp</td>
<td>Vertical</td>
<td>1/2 in.</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Floor</td>
<td>16 x 16</td>
<td>2 x 10</td>
<td>1/2-in. ply</td>
<td>Staggered</td>
<td>6d</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Floor</td>
<td>16 x 16</td>
<td>2 x 10</td>
<td>1/2-in. ply</td>
<td>Stacked</td>
<td>6d</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>F3</td>
<td>Floor</td>
<td>16 x 28</td>
<td>2 x 10</td>
<td>1/2-in. ply</td>
<td>Staggered</td>
<td>6d</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>C1</td>
<td>Ceiling</td>
<td>16 x 16</td>
<td>2 x 10</td>
<td>1/2-in. gyp</td>
<td>Staggered</td>
<td>1/2 in.</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>C2</td>
<td>Ceiling</td>
<td>16 x 16</td>
<td>2 x 10</td>
<td>1/2-in. gyp</td>
<td>Stacked</td>
<td>1/2 in.</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>C3</td>
<td>Ceiling</td>
<td>16 x 28</td>
<td>2 x 10</td>
<td>1/2-in. gyp</td>
<td>Staggered</td>
<td>1/2 in.</td>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

1/2 in. = 12.70 mm.
6 in. = 15.2 cm; 12 in. = 30.5 cm.
6d common nails = 2 in. (50.1 mm) long; 0.113 in. (2.9 mm) diameter
1/2 in. gypsumboard nails = No. 13 gauge; 1-3/8 in. (35 mm) long; 19/64 in. (7.5 mm) head; 0.098 in. (2.5 mm) diameter.
2 in. x 4 in. framing construction grade (Douglas Fir), average s.g. = 0.55; 2 in. x 10 in. joists = grade No. 2 (Douglas Fir), average s.g. = 0.55.
placement was continuously recorded while the diaphragm equilibrated. A final test determined the ultimate load capacity of the diaphragm. A point load was applied to the top of each specimen while fixing the base. The load was increased in increments until the diaphragm would carry no higher load or the capacity of the hydraulic actuator was reached.

FIG. 2.—Sheathing and Corresponding Framing Configuration for 16 × 16 ft and 16 × 28 ft Floors and Ceilings

FIG. 3.—Diaphragm Test System
**TEST RESULTS**

**Forced Vibration (Sine Wave).**—The sliding steel beam built into the test system described earlier allowed forced vibration tests to be performed. Each diaphragm was vibrated using a sine wave signal of constant amplitude over varying frequencies. Measuring the resulting diaphragm displacements using a linear varying displacement transducer (LVDT) identified the natural frequency. The results of this test for floor F2 are shown in Fig. 4. (Note the peak amplitude at 10 Hz and refer to column 2 of Table 2.) With the exception of walls W1 and W2, each diaphragm showed a distinct peak in displacement identifying the fundamental frequency. Due to the nature of the supports on the sliding steel beam, some system vibration occurred during the testing of these two specimens that made it difficult to identify distinct natural frequencies.

Free Vibration.—All test specimens were subject to free vibration using a mechanical release device. Several displacement levels were in-

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**FIG. 4.—Response of Floor F2 Due to Forced Vibration (Sine Wave) Base Excitation**

<table>
<thead>
<tr>
<th>Diaphragm</th>
<th>Forced vibration sine wave</th>
<th>0.10 in. (2.5 mm)</th>
<th>0.20 in. (5.1 mm)</th>
<th>0.30 in. (7.6 mm)</th>
<th>0.40 in. (10.2 mm)</th>
<th>0.50 in. (12.7 mm)</th>
<th>0.60 in. (15.1 mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>W1</td>
<td>—</td>
<td>29</td>
<td>28</td>
<td>27</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>W2</td>
<td>—</td>
<td>25</td>
<td>23</td>
<td>20</td>
<td>19</td>
<td>16</td>
<td>—</td>
</tr>
<tr>
<td>W3</td>
<td>24</td>
<td>27</td>
<td>27</td>
<td>27</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>W4</td>
<td>18</td>
<td>23</td>
<td>21</td>
<td>16</td>
<td>15</td>
<td>12</td>
<td>—</td>
</tr>
<tr>
<td>F1</td>
<td>10</td>
<td>12</td>
<td>—</td>
<td>—</td>
<td>9</td>
<td>—</td>
<td>8</td>
</tr>
<tr>
<td>F2</td>
<td>10</td>
<td>10</td>
<td>—</td>
<td>10</td>
<td>—</td>
<td>8</td>
<td>—</td>
</tr>
<tr>
<td>F3</td>
<td>11</td>
<td>14</td>
<td>—</td>
<td>12</td>
<td>—</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>C1</td>
<td>9</td>
<td>12</td>
<td>—</td>
<td>10</td>
<td>—</td>
<td>8</td>
<td>—</td>
</tr>
<tr>
<td>C2</td>
<td>9</td>
<td>12</td>
<td>—</td>
<td>10</td>
<td>—</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>C3</td>
<td>9</td>
<td>12</td>
<td>—</td>
<td>10</td>
<td>—</td>
<td>8</td>
<td>—</td>
</tr>
</tbody>
</table>
FIG. 6.—Free Vibration of Floor F2

investigated. Typical experimental traces of the lateral displacement of specimens W1 and F2 are shown in Figs. 5 and 6, respectively. A continuous recording over a test duration of approximately two seconds captured the full response. Note the few cycles produced for wall W1. This is a result of the high stiffness and relatively low mass of the wall diaphragm.

Computation of natural frequency from these test results was made by computing power spectral estimates of the free vibration time series. A fast Fourier transform algorithm was used to transform this time series data into the frequency domain where the relative densities of each frequency could be determined. The dominant frequency corresponded to the natural frequency of the diaphragm. See Table 2 for natural frequency results for all diaphragms.

For the diaphragm proportions tested, wall natural frequencies were generally two to three times those of the floors and ceilings. In general, natural frequencies decreased with increased displacement. Wood dia-
Phragm stiffness is dominated by the distribution and stiffness of fasteners securing the sheathing to the framing. Since these fasteners possess nonlinear material properties, the stiffness of each fastener decreases with increased slip. Therefore, diaphragm stiffness decreases with increased diaphragm displacement reducing the natural frequency.

The natural frequency values for gypsumboard-sheathed diaphragms were generally lower at all displacement levels than similarly constructed diaphragms with plywood sheathing. This is expected since gypsumboard diaphragms are lower in stiffness.

The frequency content of various earthquake ground-motion signals have been determined using power spectral density functions (Penzien 1965). Results indicate a dominance of frequencies between about 0.5 and 8 Hz. Natural frequency levels for the floor and ceiling diaphragms tested in this study decrease to as low as 8 Hz at higher displacement levels. Therefore, the possibility of a diaphragm vibrating at its resonant frequency due to earthquake excitation is greater as its displacement level increases. This indicates the importance of determining the change in frequency properties of diaphragms as displacement changes.

A comparison of the natural frequency values resulting from the different tests indicates some variation. The natural frequency values produced from the sine wave tests were, in all cases, lower than those computed from the free vibration traces. Some test system vibration occurred during the sine wave tests, producing extraneous frequencies that shifted the dominant frequency to the values shown. In spite of this problem, natural frequencies determined from the sine wave tests were no more than 25% lower than the values determined from the free vibration tests.

A review of the natural frequency values generated from the free vibration tests indicated a tendency for natural frequency to increase as diaphragm stiffness increased. Therefore, a regression analysis was performed between diaphragm stiffness and natural frequency. Diaphragm stiffness was expressed by the following stiffness modulus:

\[ K = \frac{P}{\delta} \]  

(1)

where \( P \) = load on diaphragm, in kips; and \( \delta \) = deflection at load level \( P \), in in.

Performing an analysis which included all diaphragms, an equation of the following form provided the best fit:

\[ f = 10.89 \ln(K) - 15.43 \]  

(2)

and resulted in a correlation coefficient of 0.86 (see Fig. 7). A slightly higher correlation coefficient (0.89) was obtained by analyzing only the walls; however, a separate analysis of the floor and ceiling yielded a low correlation coefficient of 0.64.

In the lower frequency portion of Fig. 7 there is a significant amount of scatter in the data. Floor and ceiling data are clustered in this portion of the figure.

Since mass significantly influences the natural frequency of the diaphragm, the stiffness ratios presented in Fig. 7 were adjusted by dividing each value by the mass of the diaphragm. Again, a regression anal-
FIG. 7.—Regression Analysis between Stiffness Modulus and Natural Frequency

FIG. 8.—Regression Analysis between Stiffness Modulus/Mass and Natural Frequency

Analysis was performed. The results of this analysis are shown in Fig. 8. Note the more predictable trend of increasing frequency with the increasing stiffness modulus/mass ratio. From basic dynamics theory, the linear behavior of a single degree-of-freedom mass \( m \) is given by

\[
f = \frac{1}{2\pi} \sqrt{\frac{K}{m}} \quad \text{................................................................. (3)}
\]

where \( f \) = cyclic frequency (Hz); and \( K \) = stiffness of the system. A curve of the same form as this equation was fitted to the data in Fig. 8 and resulted in a correlation coefficient of 0.91. This indicates that the above
single-degree-of-freedom expression predicts quite well the relationship between diaphragm stiffness, mass, and frequency. A slightly higher correlation coefficient (0.96) was found when a logarithmic function was used:

\[ f = 6.24 \ln \left( \frac{K}{m} \right) - 38.34 \]  \hspace{1cm} (4)

This function is shown in Fig. 8.

Using this relationship, the natural frequency of diaphragms at various displacement levels can be estimated. This assumes that the stiffness and mass of the diaphragm are known. The stiffness at different displacement levels could be estimated from experimental load tests or from mathematical models capable of predicting diaphragm response.

Damping Ratio.—Damping ratios were calculated from results of the free vibration tests described earlier. If one mode of frequency is dominant and a structure can be approximated as a one degree-of-freedom system, damping ratios can be calculated from two successive amplitudes of the structure’s response using the following formula:

\[ \lambda = \frac{1}{2\pi} \ln \left( \frac{a_n}{a_{n+1}} \right) \]  \hspace{1cm} (5)

Though this equation was derived assuming the linear behavior of a single-degree-of-freedom mass, it has been used to estimate damping ratios for nonlinear wood diaphragms (Polensek 1975).

An alternative equation was also used to compute damping ratios. This equation requires the measurement of energy capacity, \( \Delta W \), and energy dissipation, \( W \), from static hysteresis loops and is accurate as long as the hysteresis loops indicate linear behavior:

\[ \lambda = \frac{1}{2\pi} \ln \left( \frac{\Delta W}{W} \right) \]  \hspace{1cm} (6)

Though not discussed previously, static hysteresis tests were performed at the same displacement levels as the free vibration tests. Computation of damping ratio using the above two equations indicated close comparison for lower levels of diaphragm displacements. For this reason, damping ratios are reported only for lower displacement levels.

Table 3 shows the results of computation of damping ratio using Eq. 5. Generally, there is a tendency for damping ratio to increase with increased diaphragm displacement. This increase is a result of the increased energy absorption of each fastener at higher displacements. Calculated values ranged from 9–34% of critical damping.

The presence of openings had an effect on the level of damping ratio. Specimens with openings (W3, W4, F2) generally produced lower damping ratios than similar diaphragms without openings (W1, W2, F1). It has been shown that the fasteners used in the construction of wood structures play a dominant role in dissipating energy (Yeh 1971). Since fewer nails were used to secure the sheathing to the framing in the specimens with openings, less energy was dissipated.

A regression analysis similar to that described for natural frequency was performed between diaphragm stiffness and damping ratio. When
TABLE 3.—Damping Ratios

<table>
<thead>
<tr>
<th>Diaphragm</th>
<th>FREE VIBRATION</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Displacement Levels</td>
</tr>
<tr>
<td></td>
<td>0.10 in. (2.5 mm)</td>
</tr>
<tr>
<td>W1</td>
<td>0.16</td>
</tr>
<tr>
<td>W2</td>
<td>0.17</td>
</tr>
<tr>
<td>W3</td>
<td>0.09</td>
</tr>
<tr>
<td>W4</td>
<td>0.18</td>
</tr>
<tr>
<td>F1</td>
<td>—</td>
</tr>
<tr>
<td>F2</td>
<td>—</td>
</tr>
<tr>
<td>F3</td>
<td>—</td>
</tr>
<tr>
<td>C1</td>
<td>—</td>
</tr>
<tr>
<td>C2</td>
<td>—</td>
</tr>
<tr>
<td>C3</td>
<td>—</td>
</tr>
</tbody>
</table>

damping ratio values for all diaphragms were lumped together and analyzed, a rather poor correlation between stiffness and damping is found (0.45). However, if the data were grouped according to diaphragm construction type, higher correlations result. See Fig. 9 for a typical regression analysis for walls with openings.

Ultimate Load Results.—Following the tests described above, ultimate lateral load tests were performed. A lateral point load was applied to the top of the diaphragm while fixing the base, until the diaphragm could no longer carry higher load or the capacity of the hydraulic actuator was reached. A continuous recording of lateral displacement and corresponding resisting load was made. The results of these tests for the walls are shown in Fig. 10.

Because of the relatively small nail sizes used (6d), the mode of failure for all plywood diaphragms was nail pullout. Generally the gypsumboard-sheathed diaphragms failed due to nail pullthrough. This was because of the relatively small nail sizes used. One exception was F1, which failed because of a premature corner joint failure. This occurred well below its ultimate load capacity.

A comparison of the relative stiffness of the wall diaphragms was made by comparing the load capacity at 0.50 in. At this displacement level, wall W1 was about 34% stiffer than W3, while W2 was 36% stiffer than W4. These results indicate that plywood-sheathed walls are consistently stiffer than gypsumboard-sheathed walls, regardless of the presence of openings. Conversely, walls without openings were consistently stiffer than walls with openings, regardless of the type of sheathing material used.

A similar comparison of stiffness for floors and ceilings was made at a 1.0-in. displacement level. The presence of the stairwell opening in F2 was shown to have little effect on stiffness. This is due to the location of the opening close to the diaphragm centroid. There was little effect on ceiling diaphragm stiffness due to a gypsumboard sheathing arrangement. A rather surprising result was the fact that floor F1 was only 15%
stiffer than ceiling C1 at a 1.0-in. displacement level. A comparison of floor F3 and ceiling C3, however, indicated F3 to be 45% stiffer. These results indicate the significant stiffness contributed by the gypsumboard sheathing.

**Conclusions**

Ten full-size wall, floor, and ceiling diaphragms ranging in size from 8 x 24 ft to 16 x 28 ft were experimentally tested to determine basic static and dynamic characteristics. These tests focused on the determination of natural frequencies, damping ratios, and nonlinear stiffness characteristics.

The following conclusions are based on the results of the tests reported in this paper:
1. Natural frequency generally decreased with increased diaphragm displacement. For the diaphragm proportions tested, Eq. 3 was found to predict quite well the relationship between diaphragm stiffness, mass, and natural frequency.

2. Damping ratios were found to range from 0.09–0.34 and generally increased with increased diaphragm displacement. Damping capacity is dependent on displacement level and the number of nails securing the sheathing to the framing.

3. The reduction in stiffness in a wall diaphragm due to the presence of openings is about equal to the proportion of the wall occupied by the openings.

ACKNOWLEDGMENTS

The research reported was sponsored by the National Science Foundation under contract CEE 830121. Findings and conclusions expressed in this paper are those of the writers and do not necessarily reflect the views of the National Science Foundation.

The assistance provided by Dr. Fumio Kamiya and the staff of the Wood Technology Laboratory at Washington State University is gratefully appreciated.

APPENDIX.—REFERENCES


