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Deckboard Bending Theory for Three-Stringer Wood Pallets in Drive-In Racks


ABSTRACT: Wood pallets are increasingly being used in drive-in racks and are often positioned for easy access with supports parallel to the outer stringers. The performance of these pallets is governed by the bending behavior of the deckboards as affected by the distribution of the load and the rigidity of the nailed joints. This report gives design formulas for calculating the short-term constant environmental condition bending stiffness and bending strength of three-stringer pallets used in racks. The formulas treat an arbitrary rigidity of the nailed joints and an arbitrary area of the top deck covered by a uniformly distributed, symmetric, and centrally applied load.

KEYWORDS: pallets, racks, warehouses, fork trucks, materials handling, design, wood structure

Nomenclature

- $a$: Position on beam
- $B_a$, $B_b$: Cumulative width of deckboards
- $c$: Fraction of beam length covered by load
- $C_d$: Deflection factor calculated from $c$ and $J$
- $C_p$: Capacity factor calculated from $c$ and $J$
- $d$: Centermost deflection of pallet
- $E$: Modulus of elasticity
- $G$: Geometry factor calculated from $B_a$, $B_b$, $L$, and $S$
- $I_a$, $I_b$: Moment of inertia
- $J$, $J_t$, $J_b$: Joint modulus ratio, $J$ is determined from $J = \frac{K_a(EL/right)}{K_b(EL/left)} = \frac{K_a(EL)}{S}$
- $K_a$, $K_b$: Joint rotation modulus
- $l$: Beam length
- $L$: Distance between inside edges of outer stringers
- $m$: Moment at end of beam
- $m_e$, $m_b$: Deckboard end moment
- $M$, $M_1$, $M_2$: Moment at arbitrary point along beam
- $M_a$, $M_b$: Maximum absolute bending moment
- $p$: Load on pallet
- $R$, $R_e$: Load at end of beam
- $R_f$: Force on outermost stringer
- $S$: Distance between support edges
- $t$: Deckboard thickness

$T$: Ratio of bending stress in top deckboards to bending stress in bottom deckboards

$T_1$, $T_2$: Term for determining location of maximum bending stress

$w$: Distributed load on beam

$x$, $x_a$, $x_b$, $x_0$: Distance along beam

$y_0$: Deflection of end of beam

$\theta_0$, $\theta_{eb}$: Rotation of end of beam

$\theta_{eb}$: Rotation of outermost end of deckboards

$\sigma$: Maximum stress

Subscripts

- $b$: Bottom deckboards
- $t$: Top deckboards

Introduction

Regional warehouses supply a number of manufacturers and therefore store diversified goods. Usually their operations involve palletized products stored temporarily on steel racks and transported to and from loading areas by forklift trucks. This report is concerned with one increasingly popular storage method, the "drive-in" rack, and in particular the performance of wood pallets in this system. This report was written to guide material handling engineers, packaging engineers, pallet manufacturers, and others who need to evaluate the performance capability of pallets for use in drive-in racks. Design formulas for calculating the short-term constant environmental condition stiffness and strength of a pallet are given.

In the design of pallets for a rack facility, the products intended for storage and how the pallet deflects need to be considered. Sometimes the bending stiffness of a pallet is of primary interest. Unit loads of corrugated containers, for instance, if stacked on a bowing pallet, will "give" with the deflected shape and undergo sideways compression. With flowable loads, such as bagged food products, pallet stiffness is usually less critical whereas its load-carrying capacity is more important.

Ordinarily, the forks of the lifting equipment are inserted parallel to the pallet stringers. The loaded pallet is then supported along the outermost stringers, and the strength of the pallet involves primarily the bending strength of top and bottom deckboards acting like a truss.

Pallets are also stored in some racks in which the notched stringers of the pallets are located perpendicular to the forks of the forklift truck. This often occurs because of handling in tight quarters, such
as inside the carrier vehicle or at the ends of loading docks. Therefore, handling the loaded pallet through the notches sometimes produces a condition wherein they are supported only by the ends of notched stringers. Because of these two possibilities of loading, both mechanisms need to be considered in selecting wood pallets for use in racks.

The scope of this report is limited to situations where the top and bottom deckboards provide the load-carrying ability. The situation where pallets are used with the load supported only by the stringers requires a special treatment and is beyond the scope of this report.

Background

Whereas it might be of interest to provide a general theory for arbitrary pallets and arbitrary loading conditions, prior investigators have devised simple theories for specific applications. Heebink [1] developed a deckboard load capacity calculator that relates the performance of pallet deckboards to the design parameters of species, deckboard thickness, stringer length, and span between stringers. He assumed the most conservative situation for fastener behavior (effectively no fasteners at all) and characterized the load on the pallet as a compromise between a fully distributed load and a concentrated center load. This previous research dealt only with the support configuration of pallets placed on the floor or other continuous surface, and thus the calculator cannot accurately evaluate pallets placed in racks and supported along the outside edges.

To evaluate pallets used in racks, Wallin et al. [2] and Wallin [3] proposed some pallet bending formulas for calculating deflection and load capacity based on species, pallet member dimensions, support conditions, and fastener and load characteristics. The formulas treat either fully restrained or freely hinged deckboard-stringer joint conditions and either a uniformly distributed or concentrated center load condition. These formulas are thus limited to a few combinations of ideal fastener and load characteristics.

Kyokong [4] developed new models and wrote a computer program for evaluating pallets based also on bending characteristics. The program treats arbitrary rigidities in the nailed joints and either concentrated or uniformly distributed loads. Kyokong’s theory of pallets in racks treats more general joint conditions than does Wallin’s, but its application is more complex.

Objective and Scope

The objective of this report is to propose a theory that can be used as an alternative to short-term laboratory tests to investigate primarily the contribution of deckboards and fasteners in evaluating three-stringer pallets supported in drive-in racks. The theory emphasizes the derivation of closed form equations that are as easy to use as Wallin’s. The theory treats arbitrary joint conditions as does Kyokong’s and expands the loading conditions beyond both theories. Simplified design formulas are given for calculating both pallet stiffness, as determined by its load-deflection ratio, and strength, as governed by its load-carrying capacity.

Severe service conditions may, of course, override the need for deckboard bending stiffness and strength. Rough handling and forklift impacts necessitate diagonal rigidity and shock absorbing ability that preserve pallet life. The effects of creep and duration of load accompanying long-term loading always lower a pallet’s predicted short-term performance ability.

The theory is developed around the linear and elastic behavior of a two-dimensional model of a pallet section. A complete pallet sometimes comprises a mixture of species varying in quality. A uniform load tends to distribute itself to stiffer members stressing them to higher levels and premature failure. Extending the two-dimensional model to a complete pallet assumes that upper deckboards share the load in proportion to their widths and therefore necessitates that material properties represent working values with allowances for material variation and defects. Within the scope of this report, the two-dimensional model represents a step toward more accurate design methodologies, which encompass the effects of variability.

Nail, type, pattern, number, wood density, and environmental conditions all affect the deckboard end restraint, which may doubtlessly exhibit considerable variability. The joint restraint enters the theory as a dimensionless parameter derived relative to the deckboard bending resistance. Compromising the effects of upper and lower joints greatly simplifies the theory. In light of the expected joint variability, the compromise seems reasonable if the nails per unit board width and the effective deckboard lengths do not differ significantly between top and bottom decks.

The assumed loading condition fits a large class of products, which distribute their weight uniformly and symmetrically about the pallet. As long as the pallet is supported near its edges, the internal moment caused by the outer stringer reaction times the pallet overhang, is insignificant and is thus ignored. Concentrating loads on single deckboards or positioning the supports too far inward generates stresses that may exceed those within the scope considered here.

The primary loading condition is depicted in Fig. 1. The uniform load is assumed to cover a fraction, between zero and one, of the top deckboards. When the fractional coverage approaches zero, the loading condition becomes a concentrated center load.

As noted by the two extreme deflected shapes, the rigidity of the nailed joints significantly improves pallet performance. A primary contribution of this report beyond the earlier theories is the introduction of a joint modulus ratio. Provided that values of the joint rigidity can be determined experimentally for specific joint designs, the joint modulus ratio characterizes the combined effects of the joint condition, the deckboard stiffness, and the loading condition. By greatly simplifying the analysis.

While it is of interest to quantify the stiffness of various pallet joints, doing that is beyond the scope of this report. Experiments were made to check the theory proposed here, and the results will be given in subsequent reports. Representative values of the joint modulus ratios for two popular pallet types were determined in that work and are given later in this report.

![FIG. 1 —Comparison between the pallet deflections that result from different deckboard end conditions. In Condition A the joints are pinned and in Condition B they are rigid.](image-url)
Short-Term Bending Theory for Wood Pallet Deckboards

Formulas are first developed here to calculate the end rotation and end deflection of a cantilever beam that are later used to analyze a symmetrically loaded pallet. Viscoelastic effects accompanying creep caused by long-term loading do not enter the theory. Figure 2 shows a cantilever beam of length $l$ with loads $w$ and $R$ and moment $m$. The load $w$ is uniformly distributed from $x = a$ to $x = l$, where $x$ is an arbitrary distance along the beam measured from the free end. $R$ is a concentrated load at $x_0$ and $m$ is a moment at $x_0$. The rotation $\theta_0$ and vertical deflection $y_0$ occurring at $x_0$ result from the internal bending moment $M$ in the beam at arbitrary $x$ where

$$M = M_1 = Rx - m \quad 0 < x \leq a$$  \hspace{1cm} (1a)$$

$$M = M_2 = Rx - m - \frac{1}{2}w(x - a)^2 \quad a < x \leq l$$  \hspace{1cm} (1b)$$

End Rotation

According to elementary strength of materials theory [5], and ignoring shear deformation

$$\theta_0 = \int_0^l \frac{M}{EI} dx = \frac{1}{EI} \left[ \int_0^a M_1 dx + \int_a^l M_2 dx \right]$$  \hspace{1cm} (2)$$

where $E$ is the modulus of elasticity of the material and $I$ is the cross-sectional moment of inertia for the beam. Entering Eqs la and lb into Eq 2 and performing the integrations gives

$$\theta_0 = \frac{1}{EI} \left[ \frac{M_1 (a^2 - l^2)}{2} - \frac{ml}{6} - \frac{(wa)}{6} \right]$$  \hspace{1cm} (3)$$

End Deflection

According to Ref 5

$$y_0 = \int_0^l \frac{M}{EI} dx = \frac{1}{EI} \left[ \int_0^a M_1 dx + \int_a^l M_2 dx \right]$$  \hspace{1cm} (4)$$

Entering Eqs la and lb into Eq 4 and performing the integration gives

$$y_0 = \frac{1}{EI} \left[ \frac{M_1 (a^3)}{3} - \frac{(m l^2)}{2} - \frac{1}{24} (l - a)^2 (l + 2a) \right]$$  \hspace{1cm} (5)$$

Short-Term Deflection Analysis

To evaluate the deflection of a pallet symmetrically loaded for a short duration and supported along its outermost stringers, the previously developed beam formulas were used and the behaviors of top and bottom decks were analyzed. The moment $m$, at the end of the top deck is given by $m = \theta_0 K_t$, where $\theta_0$ is the end rotation of the top deck and $K_t$ is the joint rotation modulus caused by the bending resistance of the fastened joints. The joint modulus ratio $J_t$ is defined as the ratio between $K_t$ and the bending resistance of the top deck

$$J_t = \frac{K_t}{(EI_t/L)}$$  \hspace{1cm} (6)$$

where $l$ is the moment of inertia for the top deck and $L$ is the distance between the inside edges of the outermost stringers (Fig. 3). A similar expression for the bottom deck is given by

$$J_b = \frac{K_b}{(EI_b/S)}$$  \hspace{1cm} (7)$$

where the span $S$ is the distance between the support edges (Fig. 3).

A complete analysis would require simultaneously solving a set of three equations to get $d$, $\theta_t$, and $\theta_b$, where $d$ is the centermost deflection of the pallet and $\theta_b$ is the end rotation of the bottom deck. But when the top and bottom nailing patterns are identical, top and bottom deckboards are of equal thickness and have the same $E$, $I_t$, and $I_b$, and $S = L$. then

$$J_t = J_b = J$$  \hspace{1cm} (8)$$

Assuming the top and bottom nailing patterns are similar and $S = L$, approximate formulas can be given for $m_t$ and $m_b$, which are later used to calculate without having to calculate $\theta_t$ and $\theta_b$. Applying Eq 8 gives

$$m_t = \frac{\theta_t J E l_t}{L}$$  \hspace{1cm} (9a)$$

$$m_b = \frac{\theta_b J E l_b}{S}$$  \hspace{1cm} (9b)$$

based on the statistically average $J$ obtained from data on $K_t$, $K_b$, $E$, $l_t$, $l_b$, $L$, and $S$. It was found that typical values of $J$ are given by $J = 1.8$ for stringer pallets and $J = 4.9$ for block pallets. These represent average values determined for 1.22- by 1.02-m Douglas fir and red oak pallets tested under a variety of loading conditions.

Analysis of Top Deck

The total load on the pallet is $P$. It is centered and distributed over the distance $cL$ where $c$ is the fraction of $L$ covered by $P$. The vertical reaction caused by the outermost stringer is $R$. The end moment is given by $m_t$. To use the proposed theory, treat a deck supported by three stringers as a cantilevered deck of half the length with loads $P/2$, $R$, and $m$. It will later be shown that the magnitude of $R$ automatically accounts for the outer stringer reaction, which needs not

![FIG. 2 - Cantilever beam model showing the assumed loading conditions equivalent to those acting on half of a symmetrically loaded pallet deck. Conditions in the bottom deck yield $w = 0$.](image)

![FIG. 3 - Pallet model showing the loading condition assumed to act on half of a symmetrically loaded pallet.](image)
be calculated. Let \( l = L/2 \), \( w = P/(cL) \), \( R = R_s \), \( a = (1 - c) L/2 \), \( m = m_2 \), \( \theta_0 = \theta_1 \), and enter Eq 9a into Eq 3 to calculate

\[
\theta_t = \left[ \frac{L^2}{4EI} \right] (2 + J) \mid R_s - \left( \frac{Pc^2}{6} \right) \tag{10}
\]

Then enter Eq 10 into Eq 9a to get

\[
m_t = \left( R_s L/4 \right) [J/(2 + J)] - \left( \frac{Pc^2}{24} \right) J/(2 + J) \tag{11}
\]

Tocalculate the centermost deflection of the top deck \( d_t \). let \( y_0 = d_t \), and enter Eq 11 into Eq 5 to get

\[
d_t = \frac{L^3}{96EI} \left( \left[ R_s (8 + J) \right] \right) + \left( \frac{Pc^2}{4} \right) \left[ \frac{8 + J}{(2 + J)} \right] \tag{12}
\]

**Analysis of the Bottom Deck**

For the bottom deck \( w = 0 \). An end load caused by the support reaction and \( R_s \) is given by \( R = P/2 - R_s \). To proceed, use \( w \), \( R \), \( m = m_2 \), \( l = S/2 \), \( \theta_0 = \theta_2 \), and Eq 3 and 9b to compute

\[
\theta_b = \left[ \frac{S^2}{4EI_b} \right] (2 + J) \mid (P/2) - R_s \tag{13}
\]

Enter Eq 13 into Eq 9b to get

\[
m_b = \left( PS/8 \right) [J/(2 + J)] - \left( R_s S/4 \right) J/(2 + J) \tag{14}
\]

Then to calculate the centermost deflection of the bottom deck \( d_b \), let \( y_0 = d_b \), and enter Eq 14 into Eq 5 to get

\[
d_b = \frac{(S^3/192EI_b) \left[ (8 + J)/(2 + J) \right]}{(P - 2R_s)} \tag{15}
\]

**Pallet Stiffness Formula**

Equating Eqs 12 and 15 gives the statically indeterminant force \( R \), in the outer stringer

\[
R_s = \left( \frac{PS^3}{2} \right) \left[ \frac{Pc^2}{4} \right] \left( \frac{c(2 + J) - (8 + 2J)}{8 + J} \right) \tag{16}
\]

The pallet deflection \( d \) is determined from the assumption \( d = d_t = d_b \). Enter Eq 16 into Eq 15 to get

\[
d = d_0 = \left( P/192EI \right) \frac{S^3L^3}{I_b L^3 + I_s S^3} \left( \frac{8 + J}{2 + J} \right) \times \left[ 1 + \left( \frac{c^2}{2} \right) \frac{c(2 + J) - (8 + 2J)}{8 + J} \right] \tag{17}
\]

Given the deckboard thickness \( t \), cumulative widths of top deckboards \( B_t \), cumulative widths of bottom deckboards \( B_b \), and rewriting the \( Is \) according to \( I = B/12 \), Eq 17 becomes

\[
d = \frac{PC_d}{16 GEt^3} \tag{18}
\]

where the geometry factor \( G \) is given by

\[
G = \left( B_t/L^3 \right) + \left( B_b/S^3 \right) \tag{19}
\]

and the deflection factor \( C_d \) is given by

\[
C_d = \frac{8 + J}{2} \left[ 1 + \left( \frac{c^2}{2} \right) \left( \frac{c(2 + J) - (8 + 2J)}{8 + J} \right) \right] \tag{20}
\]

Then the pallet stiffness \( K_p \) is given by the ratio between \( P \) and \( d \)

\[
K_p = P/d = 16 GEt^3/C_d \tag{21}
\]

To aid the calculations, values of \( 1/C_d \) are plotted in Fig. 4 for various combinations of \( c \) and \( J \).

**Short-Term Strength Analysis**

To determine the strength of a pallet loaded for a short term and supported according to the previously discussed methods, the maximum stresses \( \sigma_t \) and \( \sigma_b \) occurring in the top and bottom decks are considered first. A strength formula using \( \sigma_b \) as a maximum design stress is then developed. and finally, a correction factor is introduced for use whenever \( \sigma_t \) is greater than \( \sigma_b \).

**Maximum Stress in the Bottom Deck**

To evaluate the maximum stress caused by the maximum absolute bending moment \( M_b \) in the bottom deck, assume \( w = 0 \) and an arbitrary joint condition. Then \( 0 \leq m \leq Rl/2 \) in the cantilever beam model of Fig. 2 and \( M_b \) occurs at \( x \). To compute \( M_b \), use Eqs 1b, 14, 16. and let \( M_t = M_b \), \( R = P/2 - R_s \), \( x = S/2 \), \( m = m_1 \), and \( w = 0 \). Then rearrange terms to get

\[
8M_b/PS = \left[ 1 - 2R_s/P \right] \left[ 2 - J/(2 + J) \right] \tag{22}
\]

From elementary theory [5]

\[
\sigma_b = M_b t/2I_b \tag{23}
\]

Determine \( R/P \) from Eq 16. and enter this into Eq 22. Then determine \( M_t \) from Eq 23 and enter this into Eq 22. Finally, again rewrite

![FIG. 4 - Reciprocal of the deflection factor versus the joint modulus ratio for various coverage factors.](image-url)
the Is in terms of Bs and ts, enter these into the result, and rearrange terms to get

\[ (24) \]

where the capacity factor \( C_p \) is given by

\[ C_p = (3/4)[(4 + J)/(2 + J)] \]

\[ \left( 1 + (c^2/2) \left[ \frac{c(2 + J) - (8 + 2J)}{8 + J} \right] \right)^{1/2} \]

Values for \( 1/C_p \) are plotted in Fig. 5 for various combinations of \( c \) and \( J \).

**Maximum Stress in the Top Deck**

The maximum stress in the top deck is determined from its maximum absolute bending moment \( M_t \).

\[ \sigma_t = \frac{M_t}{2I_t} \]

(26)

The location of \( M_t \) and thus its magnitude, in the beam model of Fig. 2 depends on the loading condition. However, by using Eqs 1, 11, and 16 to search for values of \( x \) that maximize \( M_t \), it can be shown that typical values of \( J \) will position \( M_t \) at \( x = L/2 \). Thus, to compute \( M_t \) use Eqs 1b, 11, 16, 22 and let \( M_t = M_s, R = R_s, x = L/2, m = m_s \), \( w = P/(cL) \), and \( (s - a) = cL/2 \). Then rearrange terms to get an equation having a term equal to the right side of Eq 22.

\[ (-8M_t/PL) + (c^2/3)[J/(2 + J)] - c + 2 \]

\[ - (J/(2 + J)) = 8M_b/PS \]

(27)

To determine \( \sigma_t \) in terms of \( \sigma_b \), first evaluate \( M_t \) and \( M_s \) from Eqs 23 and 26. Then rewrite \( I_t \) and \( I_b \) in terms of \( B_t, B_b, \) and \( t \). Finally, determine \( \sigma_t \) from Eq 24, and enter this into the result to get

\[ \sigma_t/\sigma_b = (1/4C_p)[(c^2 - 3)J/(2 + J) + 3(2 - c)] \]

\[ \times [(S^2/L^2) + (B_bL/B_tS) - (B_tL/B_bS)] \]

(28)

**Pallet Short-Term Strength Formula**

To formulate a strength formula based on the bottom deck, the magnitude of \( \sigma_t/\sigma_b \) given by Eq 28 must be less than one. To check this, let the right side of Eq 28 equal one, and define the following terms

\[ T_1 = \left[ (c^2 - 3) \frac{J}{2 + J} + 3(2 - c) \right]/4C_p \]

(29a)

**FIG. 5** — Reciprocal of the capacity factor versus the joint modulus ratio for various coverage factors.

**FIG. 6** — (A) Contours of constant \( T_1 \) for various combinations of \( J \) and \( c \). (B) Contours of constant \( T_1 \) for various combinations of \( B/b \) and \( L/S \). When \( T_1 < T_0 \), the maximum bending stress in the pallet is in the bottom deckboards. When \( T_1 > T_0 \), the maximum bending stress is in the top deckboards.
\[ T_2 = \frac{[(B_b/L_b)S + 1]/[(B_b/L_b)S + (S^2/L^2)]}{(29b)} \]

It thus follows that whenever \( T_1 < T_2 \), then \( \alpha_1/\alpha_b < 1 \).

To determine if \( T_1 \) is always less than \( T_2 \), values of \( T_i \) for various combinations of \( J \) and \( c \) are plotted in Fig. 6a, and values of \( T_i \) for various combinations of \( B/B_s \) and \( L/S \) are plotted in Fig. 6b. Comparing values of \( T_1 \) and \( T_2 \) for specific values of \( J \), \( c \), \( B/B_s \), and \( L/S \) shows that some combinations of low \( L/S \) and low \( c \) values yield \( T_1 > T_2 \), and thus position the maximum stress in the top deck.

To continue with the development of a strength formula, a correction factor based on the maximum design stress allowed in the bottom deck is introduced and Eq 24 is used to compute

\[ P = \sigma GS^2t^2/C_pT \]  

In this formula \( P \) becomes the short-term load-carrying capacity for the pallet and \( \sigma \) becomes the maximum design stress for the material. The correction factor \( T \) corrects the formula whenever \( \alpha_1 > \alpha_b \). Thus when \( T_1 \leq T_2 \), let \( T = 1 \). When \( T_1 > T_2 \), use Eq 28, and let \( T = \alpha_1/\alpha_b \).

Summary

Design formulas are given for calculating the short-term, constant environmental condition bending strengths and bending stiffnesses of pallets used in drive-in racks. Pallets in drive-in racks are often supported along the edges coinciding with the outermost stringers, and the bending performance of the deckboards is of primary interest.

These formulas are valid whenever the loading condition is uniformly distributed, symmetric, and centrally applied, and the load is of a short term. The fraction of the top deck covered by the load can be varied. The effect of the joints is characterized by a ratio between the rotation modulus of the joints and the bending resistance of the deckboards. Using this ratio simplifies the pallet deflection formula and avoids the need to calculate rotation of the joints to get a deflection value. Strength and stiffness formulas are further simplified with proportionality factors that quantify the interaction between the loading condition and the joint condition.

References