Mathematical Model to Describe
The Propagation of Smoldering
In Cellulosic Insulation

By

Susan LeVan
and
Erwin L. Schaffer

Forest Products Laboratory
Forest Service
U. S. Department of Agriculture
Madison, Wis.

MATHEMATICAL MODEL TO DESCRIBE THE PROPAGATION
OF SMOLDERING IN CELLULOSIC INSULATION

By

SUSAN LeVAN and ERWIN L. SCHAFFER

ABSTRACT

A mathematical model was developed which predicted the weight loss that would occur under smoldering conditions when the test was conducted according to Federal Specification HH-I-515D. Also a smoldering propagation parameter was identified that was sensitive to various treatment levels.

The mathematical model to predict the weight loss was derived from the volumetric shape of the residual char, char density, and wave-front velocity. The predicted weight loss values were in good agreement with the experimental values.

The smoldering wave-front velocity was determined for various treatment levels of borax:boric acid-treated newsprint insulation and was found to be a sensitive indicator of the effects of treatment levels. The propagating smoldering wave-front velocities were $17.65 \times 10^{-2}$ for untreated, $25.92 \times 10^{-2}$ for 10 percent treatment, $13.97 \times 10^{-2}$ for 18 percent, and 0 cm/min for 25 percent treatment. The increased smoldering velocity of the 10 percent treatment with respect to untreated is caused by the smolder-promoting effect of boras.

INTRODUCTION

The present energy crisis has created a great interest in cellulosic insulation for reducing home heating losses. Cellulosic fibers provide an economical method of insulating homes, and properly treated fibers significantly reduce any potential fire hazard. The reports of fire incidence involving cellulosic insulation indicate that the fires are initiated by smoldering combustion caused by the overheating of recessed light fixtures, junction boxes, or electrical wiring that have been improperly covered with insulation. Smoldering combustion is a self-sustaining, propagating process (fueled by exothermic reactions) that is a nonflaming mode of combustion characterized by thermal degradation, charring, and evolution of volatile gases. The rate of smoldering propagation is directly related to the smoldering tendency of the material--the higher the smoldering tendency, the faster the propagation.
To measure the ease of ignition and propensity to propagate smoldering combustion, a smoldering test \((6)^{1/}\) had been developed. However, this pass/fail test relays little information on the variables that influence smoldering propagation or on the relative effectiveness of various fire retardants added to the material to control smoldering.

The objective of this work was twofold. The first one involved developing a mathematical model to predict the weight loss which occurs in the above test. If the model successfully predicted the weight loss under several conditions, it would permit a better understanding of the smoldering propagation phenomenon and the variables that influence it. The second objective was to identify a measurable parameter that could quantify the effects of different treatments.

Literature Review of Smoldering Mechanism

The smoldering phenomenon is very complex and much work has been done to analyze this mode of combustion. Kinbara et al. \((7)^{1/}\) and Moussa et al. \((10)^{1/}\) developed mathematical models for determining the propagating velocity. Ohlemiller and Rogers et al. \((11-14)^{1/}\) studied the smoldering process in polyurethane foams and concluded that heat source and smoldering geometry, treatment of material, and thermophysical factors are the variables influencing smoldering initiation.

McCarter \((8,9)^{1/}\), Day et al. \((3)^{1/}\), and Issen \((6)^{1/}\) studied the effects of fire retardants on smoldering. McCarter reported on smolder-promoting compounds in cellulosics. Day et al. and Issen reported that boric acid reduced the smoldering tendency for loose fiber-fill cellulosics. They also found that the addition of borax reduced flaming combustion but also decreased the capability of boric acid to reduce smoldering.

Brenden and Schaffer \((2)^{1/}\) studied smoldering in fiberboard and concluded that the smoldering wave velocity could be used as a direct indicator of the smoldering tendency of a material.

EXPERIMENTAL PROCEDURES

The general test procedure of Federal Specification HH-I-515D was modified with implanted thermocouples. Thermocouples were positioned in the smoldering apparatus at 1/2-inch intervals to determine the distance of the smoldering wave-front from the cigarette ignition source. Chromel alumel 30-gage wire was used for the thermocouples. The wires were inserted into 3- x 130-mm pyrex tubing to eliminate any displacement of the thermocouples when the cellulosic insulation was packed into the smoldering apparatus. The tips of the thermocouples were perpendicular to the thermocouple wire. Hammer-milled newsprint, treated with borax:boric acid \((1:1)^{1/}\) at treatment levels of 10, 18, and 25 percent based upon original weight of the materials, were packed at a bulk density

\(^{1/}\) Italicized numbers in parentheses refer to literature cited at the end of this report.
of 0.0908 ± 0.0002 g/cm$^3$ (5.67 ± 0.01 lb/ft$^3$). Smoldering propagation could not be achieved at any density for the 25 percent level.

The time from ignition and the temperature of each thermocouple were recorded with an automatic multipoint recorder. Smoldering was allowed to continue until the smoldering front has passed each thermocouple and the temperature at the last position was observed to decrease. (A temperature of 288°C (550°F) was selected as evidence that the smoldering wave-front had reached a particular thermocouple. This temperature corresponds to Beall's (1) Zone C where active pyrolysis takes place under exothermic conditions.) The smoldering apparatus was positioned on an automatic top loading balance and the weight was recorded initially and then every 5 minutes. The weight difference between time point and the initial weight yielded the weight loss due to smoldering combustion.

After the smoldering test was completed the density of the char was determined. For the three treated materials, the charred material was removed and weighed, and the radius of the space occupied by the removed char measured in the longitudinal and horizontal directions. For the untreated material, the char density was estimated.

The treated material, shredded newsprint that had received a spray application of a 1:1 mixture of 5 mole borax and boric acid, was obtained from Wegner et al. (15).

Weight Loss Model

The model developed is a material balance performed around the smoldering apparatus. The model assumes that, from a volumetric analysis of the char and the determination of the smoldering wave velocity, the weight loss to the atmosphere can be calculated. Assumptions made for simplification of the model include the following:

1. The radial and longitudinal rates of propagation are the same.
2. The unburned material has constant density throughout the apparatus.
3. The propagation of the smoldering proceeds as either an increasing cylindrical or hemispherical volume.
4. The rate of smoldering is assumed to be symmetrical around the cigarette.
5. Steady propagation stage is linear.

With these assumptions one can write a mass balance around the contents of the smolder test container:

$$\text{original weight} = \text{unburned material} + \text{charred material} + \text{weight loss to atmosphere} \quad (1)$$
where
\[ \rho_o (V_o - V_c) = \text{unburned material} \]
\[ \rho_c V_c = \text{charred material} \]
\[ \omega = \text{weight loss to atmosphere} \]
\[ \rho_o V_o = \text{original weight} \]

By substituting these expressions into equation (1) one gets

\[ \rho_o V_o = \rho_o (V_o - V_c) + \rho_c V_c + \omega \]  \hspace{1cm} (2)

or upon rearrangement

\[ \omega = (\rho_o - \rho_c) V_c \]  \hspace{1cm} (3)

However, from assumption 3, one can write for a cylindrical shape model of the char

\[ V_c = \pi r^2 L \]  \hspace{1cm} (4)

or for a hemispherical shape model

\[ V_c = \frac{2}{3} \pi r^3 \]  \hspace{1cm} (5)

Since the rates of propagation are assumed to be equal in the radial and longitudinal direction for the cylindrical model, equation (4) can be rewritten as

\[ V_c = \pi r^3 \]  \hspace{1cm} (6)

If one assumes that the rate of movement of the propagation front is linear, this can be expressed as

\[ r = v(t - \tau_D) \]  \hspace{1cm} (7)

where \( \tau_D \) is a correction factor to eliminate the time delay due to the initiation period. Equation (3) can now be written for a cylindrical model as

\[ \omega = (\rho_o - \rho_c) \pi v^3 (t - \tau_D)^3 \]  \hspace{1cm} (8)
Figure 1.--Plot of the time it took for each thermocouple to reach the set point temperature of 288° C (550° F). The range of the values is shown in brackets: (A) untreated, (B) 10 percent, (C) 18 percent.

Figure 2.--Weight loss, w, for both experimental and theoretical results was determined at various Δt's. This particular graph is for 18 percent treatment level.

Figure 3.--Rate of weight loss, w, for both experimental and theoretical results is determined for various Δt's. Particular graph is for 18 percent treatment level.
and for a hemispherical model as

\[
\dot{w} = \frac{2}{3}(\rho_o - \rho_c)\pi v^3(t - \tau_D)^3 \tag{9}
\]

The first derivative of equations (8) and (9) is the rate of weight loss which will occur at any point in time; therefore, the rate of weight loss can then be expressed for their respective shapes as

\[
\dot{w} = 3(\rho_o - \rho_c)\pi v^3(t - \tau_D)^2 \quad \text{and} \tag{10}
\]

\[
\dot{w} = 2(\rho_o - \rho_c)\pi v^3(t - \tau_D)^2 \tag{11}
\]

RESULTS AND DISCUSSION

Stages of Propagation

The progression of the smoldering wave can be experimentally broken down into three stages: Initiation, steady propagation, and extinction. The initiation stage involves ignition of the material and the buildup of heat. In figure 1, this initiation stage corresponds to the initial nonlinear section of the curve. The steady propagation stage is then the linear portion of the same curve. The smoldering wave-front proceeds in a radial and longitudinal direction from the ignition source and is evidenced by the growth of charred material. Propagation persists until the amount of fuel is limiting or the heat losses become significant. At this point the extinction stage begins and the rate of smoldering decreases.

Weight Loss--Experimental vs. Predicted

The weight loss and rate of weight loss for the experimental results and the two calculated theoretical models are plotted against \(\Delta t\) of 10, 15, 20, 25, and 30 minutes in figures 2 and 3. These particular graphs represent the 18 percent treatment level; however all concentrations gave similar responses. These particular time intervals were selected for the theoretical calculation since they were within the experimental \(\Delta t\)’s listed in table 1. The theoretical weight loss and rate of weight loss were calculated from equations (8) and (10) for the cylindrical shape and equations (9) and (11) for the hemispherical model using these time intervals. The standard error in figures 2 and 3 was calculated at a 95 percent confidence level. The experimental values fell between the two assumed shapes, indicating a volumetric shape for the charred material that is a combination of the two--such as a teardrop shape.

The discrepancies between the experimental and theoretical values of the models can be attributed to two factors--the inaccuracy of positioning the thermocouples and invalid assumptions in the model. The cellulosic material was not completely homogeneous in density throughout as assumed but had clusters of
very dense material. Also the rate of smoldering was not always symmetrical around the cigarette. The longitudinal and radial rate of smoldering assumption might also be inaccurate for the cylindrical model. The assumptions only approximate experimental observation and one would expect differences; however, the similarities in the results between experimental and theoretical values suggest that the proposed model adequately describes smoldering propagation.

Initiation Stage Time Delay

Table 1 and figure 1 give the distance of the thermocouple from the cigarette and the time needed for the smoldering front to reach that position for three of the treatment levels evaluated. The 25 percent treatment level is not listed because it would not smolder and thus no information could be obtained. Also listed in table 1 is a time delay correction factor, $\tau_D$, which eliminates the nonlinear initiation period by assuming that it is the time delay of the system to the response of ignition. It is determined for each experimental run by extrapolating the linear steady propagation stage of the curve back to the time axis. The term $\Delta t$ refers to the time interval between some time $t$ and $\tau_D$. For the 0, 10, and 18 percent treatment levels in table 1 the coefficient of variation for $\tau_D$ for each level of fire retardant is 37.3, 18.6, and 32.4 percent, respectively. This indicates that $\tau_D$ has the greatest variability between each set of tests. This can be due to differences in how the cigarette is positioned, deviations in packing density localized around the cigarette, or other uncontrollable factors, all of which affect the response of the system to ignition. The mean time delay is determined for each treatment level and plotted in figure 4—increasing the level of borax:boric acid increases the time it takes for steady smoldering propagation to occur.

Smoldering Wave Velocity

The slope of the linear steady propagation stage is termed the smoldering wave velocity (table 1). For the 0 and 10 percent treatment level the slope of the line connecting all four thermocouple positions has a correlation factor $(r^2)$ of around 0.99. For the 18 percent treatment level (fig. 3C) the linear steady propagation stage occurs after thermocouple position No. 1; the $r^2$ value for this treatment is 0.99 if the last three thermocouple positions are used. With higher loading levels of borax:boric acid, the ignition stage occurs for longer times and to distances farther from the cigarette. A similar phenomenon was observed for the 25 percent treatment level. Thermocouple position No. 1 ($r = 1.27$ cm) reached an average maximum temperature of around $304^\circ$ C ($580^\circ$ F). The temperature then dropped and no significant temperature rise was observed at the last three thermocouple positions.

The variability in the smoldering wave-front velocity between the six tests of a particular treatment level is small. The coefficients of variation for the 0, 10, and 18 percent fire retardant levels are 9.6, 13.9, and 9.8 percent, respectively. Standard error calculated at a 95 percent confidence level is also included in table 1.
Figure 4.—Time delay of the response as a function of the treatment level.

Figure 5.—SVU is shown as a function of different treatment levels.
Table 1.--Smoldering wave data

<table>
<thead>
<tr>
<th>Experiment No.</th>
<th>$\tau_D$</th>
<th>$\Delta t$ at distance from ignition source, $r =$</th>
<th>Smoldering wave velocity factor $v/\tau^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1.27 cm</td>
<td>2.54 cm</td>
</tr>
<tr>
<td>1</td>
<td>6.61</td>
<td>7.64</td>
<td>16.14</td>
</tr>
<tr>
<td>2</td>
<td>6.25</td>
<td>8.00</td>
<td>15.75</td>
</tr>
<tr>
<td>3</td>
<td>9.68</td>
<td>6.82</td>
<td>11.82</td>
</tr>
<tr>
<td>4</td>
<td>7.15</td>
<td>7.10</td>
<td>15.85</td>
</tr>
<tr>
<td>5</td>
<td>15.34</td>
<td>7.16</td>
<td>13.16</td>
</tr>
<tr>
<td>6</td>
<td>10.85</td>
<td>6.15</td>
<td>15.65</td>
</tr>
<tr>
<td>Mean</td>
<td>9.31</td>
<td>7.15</td>
<td>14.73</td>
</tr>
<tr>
<td>Coefficient of variation (Pct)</td>
<td>37.3</td>
<td>9.0</td>
<td>12.2</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>3.47</td>
<td>.644</td>
<td>1.79</td>
</tr>
<tr>
<td>Standard error</td>
<td>±3.64</td>
<td>±.68</td>
<td>±1.88</td>
</tr>
</tbody>
</table>

10 PERCENT TREATED

<table>
<thead>
<tr>
<th>Experiment No.</th>
<th>$\tau_D$</th>
<th>$\Delta t$ at distance from ignition source, $r =$</th>
<th>Smoldering wave velocity factor $v/\tau^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1.27 cm</td>
<td>2.54 cm</td>
</tr>
<tr>
<td>1</td>
<td>15.47</td>
<td>5.28</td>
<td>10.03</td>
</tr>
<tr>
<td>2</td>
<td>8.87</td>
<td>7.51</td>
<td>9.88</td>
</tr>
<tr>
<td>3</td>
<td>12.08</td>
<td>5.30</td>
<td>11.30</td>
</tr>
<tr>
<td>4</td>
<td>13.37</td>
<td>5.01</td>
<td>9.88</td>
</tr>
<tr>
<td>5</td>
<td>14.71</td>
<td>3.92</td>
<td>9.42</td>
</tr>
<tr>
<td>6</td>
<td>14.84</td>
<td>4.26</td>
<td>8.46</td>
</tr>
<tr>
<td>Mean</td>
<td>13.22</td>
<td>5.21</td>
<td>9.83</td>
</tr>
<tr>
<td>Coefficient of variation (Pct)</td>
<td>18.6</td>
<td>24.1</td>
<td>9.4</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>2.46</td>
<td>1.26</td>
<td>.922</td>
</tr>
<tr>
<td>Standard error</td>
<td>±2.58</td>
<td>±1.32</td>
<td>±.97</td>
</tr>
</tbody>
</table>
Table 1:--Smoldering wave data--continued

<table>
<thead>
<tr>
<th>Experiment No.</th>
<th>$T_D$</th>
<th>1.27 cm</th>
<th>2.54 cm</th>
<th>3.81 cm</th>
<th>5.08 cm</th>
<th>$v = \frac{\text{Smoldering wave latency}}{r^2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>27.45</td>
<td>19.65</td>
<td>30.95</td>
<td>39.55</td>
<td>0.1267</td>
<td>0.997</td>
</tr>
<tr>
<td>2</td>
<td>16.74</td>
<td>18.26</td>
<td>28.64</td>
<td>36.76</td>
<td>0.1367</td>
<td>0.998</td>
</tr>
<tr>
<td>3</td>
<td>23.77</td>
<td>19.11</td>
<td>30.76</td>
<td>38.48</td>
<td>0.1293</td>
<td>0.993</td>
</tr>
<tr>
<td>4</td>
<td>27.12</td>
<td>16.43</td>
<td>26.43</td>
<td>33.08</td>
<td>0.1506</td>
<td>0.993</td>
</tr>
<tr>
<td>5</td>
<td>44.29</td>
<td>18.94</td>
<td>29.06</td>
<td>38.04</td>
<td>0.1328</td>
<td>0.999</td>
</tr>
<tr>
<td>6</td>
<td>28.66</td>
<td>15.54</td>
<td>24.02</td>
<td>31.24</td>
<td>0.1615</td>
<td>0.999</td>
</tr>
<tr>
<td>Mean</td>
<td>28.01</td>
<td>17.99</td>
<td>28.31</td>
<td>36.19</td>
<td>0.1397</td>
<td>--</td>
</tr>
<tr>
<td>Coefficient of variation (Pct)</td>
<td>32.4</td>
<td>9.1</td>
<td>9.4</td>
<td>9.1</td>
<td>9.8</td>
<td>--</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>9.07</td>
<td>1.64</td>
<td>2.67</td>
<td>3.30</td>
<td>0.0127</td>
<td>--</td>
</tr>
<tr>
<td>Standard error</td>
<td>±9.53</td>
<td>±1.72</td>
<td>±2.8</td>
<td>±3.46</td>
<td>±0.015</td>
<td>--</td>
</tr>
</tbody>
</table>

18 PERCENT TREATED
Brenden and Schaffer (2) have defined a smoldering velocity unit (SVU) to be 1.0 x 10^-2 in./min. (The conversion of this unit in metric is 2.54 x 10^-4 m/min.) Friedman (5) reported SVU rates of various materials, ranging from 3.9 SVU for coal dust with a size less than 104 microns to an SVU of 425 for cotton string 0.045 cm in diameter. The SVU value for untreated newsprint in this experiment is 6.95, fairly close to the SVU of 7.9 reported for fiberboard. The mean SVU's for the 10 and 18 percent treatment levels reported here are 11.65 and 5.50, respectively. These values are plotted in figure 5, where the peak SVU occurs at the 10 percent treatment level. The explanation for this phenomenon is found in the works of Day (3), McCarter (8, 9), and Issen (6); increased amounts of boric acid must be added to the formulation to compensate for the smoldering promoted by the borax. This result confirms their findings and also indicates the sensitivity of the smoldering wave-front velocity to distinguish treatment effects.

Summary of Findings

1) The weight loss due to smoldering combustion can be predicted by a mathematical model which relies on the volumetric shape of the char, density, and the smoldering wave-front velocity.

2) The smoldering wave-front velocity effectively measures the smoldering tendencies of a given material and is a sensitive parameter that can quantify the effects of fire retardants. At the 10 percent treatment level, the smoldering velocity was increased over that of untreated material. However at a treatment level of 18 percent the smoldering velocity decreased over that of untreated material. At the 25 percent treatment level, smoldering extinguished.

3) The variability between smoldering tests, according to HH-I-515D, is due to the initiation stage and can be eliminated by treating this stage as a time delay of the response of the system to ignition.

4) Higher concentrations of boric acid must be added when used in conjunction with borax.


NOMENCLATURE

$\rho$ = material density [g/cm$^3$]

$V$ = volume [cm$^3$]

$\omega$ = absolute weight loss [g]

$\dot{\omega}$ = weight loss rate [g/min]

$t$ = time [min]

$\tau_D$ = time delay due to initiation stage [min]

$v$ = smoldering wave velocity [cm/min]

$r$ = radius [cm]

$L$ = cylindrical length [cm]

$\Delta t = t - \tau_D$ [min]

SVU = smoldering velocity unit [1 x $10^{-2}$ in./min = 1 SVU] = 2.54 x $10^{-4}$ m/min

Subscripts

$o$ = original

c = char