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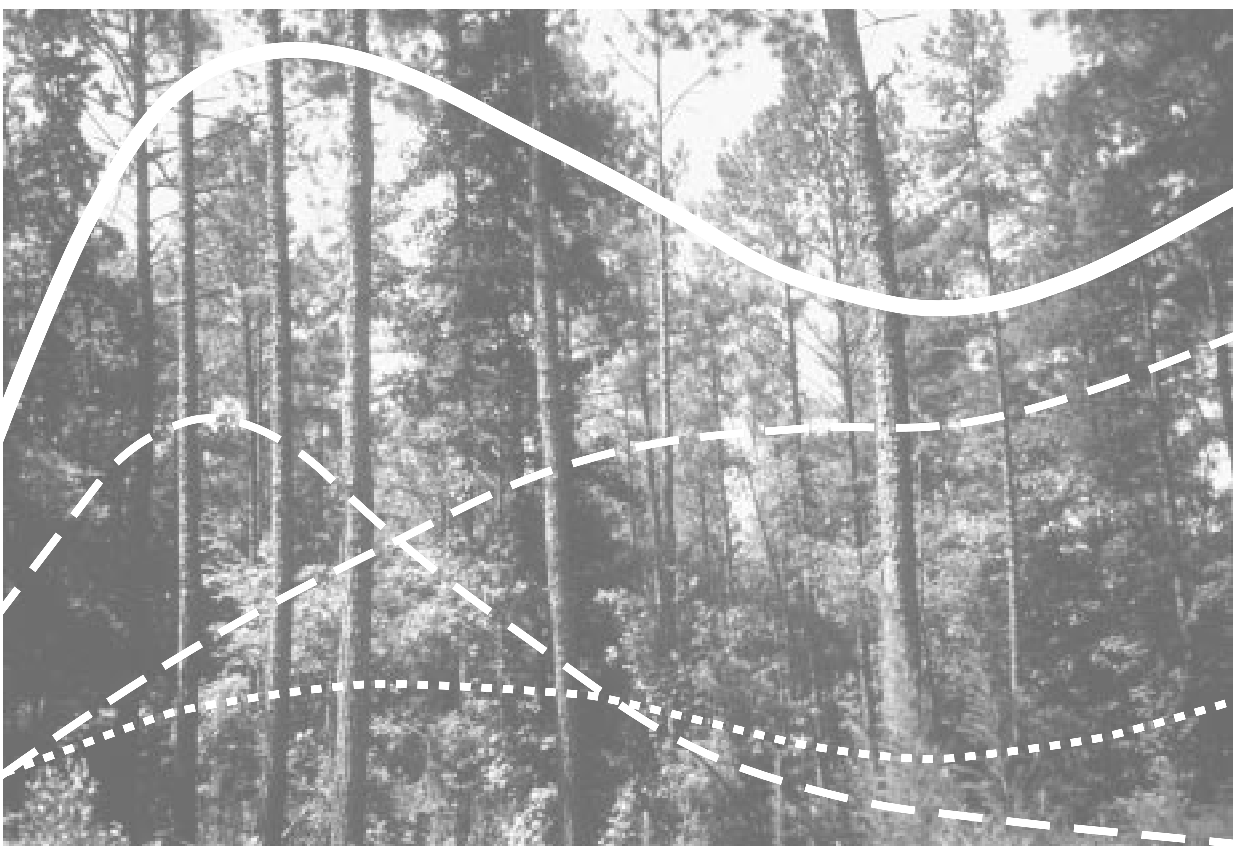
Growth Model for Uneven-Aged Loblolly Pine Stands

Simulations and Management Implications

Ching-Rong Lin
Joseph Buongiorno
Jeff Prestemon
Kenneth Skog



National Project on *Wood Utilization for Ecosystem Management*

Abstract

A density-dependent matrix growth model of uneven-aged loblolly pine stands was developed with data from 991 permanent plots in the southern United States. The model predicts the number of pine, soft hardwood, and hard hardwood trees in 13 diameter classes, based on equations for ingrowth, upgrowth, and mortality. Projections of 6 to 10 years agreed with the growth of stands between the last two inventories. In 300-year simulations of undisturbed growth, softwood species were replaced by hardwoods, in accord with previous knowledge. Soft hardwood species became dominant on good sites and hard hardwoods on poor sites. Basal area oscillated over time, converging slowly towards a steady state. Changes in tree size diversity were correlated positively with basal area. Without disturbance, species diversity would decrease. For economic analysis, equations were developed to predict total tree height, sawlog length and volume, pulpwood volume, and volume of top sawtimber, as functions of tree diameter and stand basal area. Simulations of three cutting regimes showed that management would lead to a steady state faster than would natural growth. Management aimed at maintaining the current average distribution would result in size and species diversity similar to that of an unmanaged stand. From a financial point of view, the *q*-factor guide and a 13-in.- (330-mm-) diameter-limit cut would be superior to the average current management regime. The diameter-limit regime would have the greatest effect on lowering tree size diversity and an effect on species diversity similar to that of the *q*-factor guide. A computer program, SOUTHPRO, was developed to simulate the effects of other management alternatives.

Keywords: Loblolly pine, growth model, uneven-aged, SOUTHPRO

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Contents

Page

Introduction........1

Structure of Growth Model........2

Data for Model........2

Estimation of Model Parameters........3

  Ingrowth Equations........3

  Upgrowth Equations........4

  Mortality Equations........4

  Projection Accuracy........4

Long-Term Growth of Unmanaged Stands........5

  Initial Stand State and Diversity Indices........6

  Expected Stand Growth, Without Management........6

Tree Diversity and Revenues in Managed Stands........8

  Tree Volumes and Values........8

  Long-Term Effects of Management........9

Conclusion........11

References........12

Appendix—Metric Conversion Factors........13

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Ching–Rong Lin,
Joseph Buongiorno,
Department of Forest Ecology and Management, University of Wisconsin, Madison, Wisconsin

Jeff Prestemon,
Forest Science Laboratory, Research Triangle Park, North Carolina

Kenneth Skog, Supervisory Research Forester
Forest Products Laboratory, Madison, Wisconsin

Introduction

Forestry has a significant impact on the economy of the southeastern United States. This region contains more than 211×10^6 acres (85×10^6 ha) of forest land, about 90% of which is privately owned. A quarter of private holdings are held by forest industry (Powell and others 1993). Federal holdings managed by the USDA Forest Service constitute a small share, about 12×10^6 acres (5×10^6 ha). Loblolly pine (*Pinus taeda L.*) and shortleaf pine (*P. echinata Mill.*) occupy the largest share of forest land acreage (50×10^6 acres (20×10^6 ha)) and constitute more than two-thirds of the region's standing merchantable volume (Powell and others 1993).

Loblolly pine, the primary commercial species in the southern pine region, is generally managed with even-aged silvicultural techniques (clear cut and plant, clear cut and regenerate naturally, shelterwood, or seed tree). Managed loblolly pine provides profitable returns for industry and individual forest landowners. Hence, commercial forests of the region are dominated by even-aged stands (Powell and others 1993), presumably at the expense of ecological diversity at the stand level, although even-aged stands can contribute to the diversity of a forest landscape.

Growing awareness of the importance of maintaining an ecologically diverse mix of species and size classes has heightened interest in alternatives to even-aged, single-species regimes. One major objective of current research is to quantify the consequences of uneven-aged management in terms of its effects on tree diversity at the stand level; for example, the diversity of tree sizes and species. Although these are simple measures of diversity, they appear to be fundamental components of overall biological diversity (Franzeb 1978, MacArthur and MacArthur 1961, Rice and others 1984, Willson 1974).

Past studies of uneven-aged management of southern pines addressed the problem of determining effective stand structures (number of trees in each size class) and cutting cycles (interval between harvests) for high timber production. Usually, the management criteria have been the present value of returns or the volume of timber produced per unit time (for examples, see Farrar 1981, Hotvedt and others 1989, Williston 1978). Murphy and Shelton (1994) initiated modeling of growth of uneven-aged stands of loblolly pine, and Guldin and Baker (1988) compared yields of even- and uneven-aged loblolly–shortleaf pine stands.

More research is available on the uneven-aged management of other forest types. Several papers have reported on the ecological and economic implications of uneven-aged management of mixed northern hardwood forests (Adams and Ek 1974, Bare and Opalach 1987, Buongiorno and Michie 1980), and a few have dealt with some ecological implications (Buongiorno and Lu 1990, Buongiorno and others 1994, Lu and Buongiorno 1993). Other research to model the interrelationships among trees of different species in uneven-aged stands includes that of Michie and McCandless (1986), on oak–hickory forest types in Pennsylvania; Bowling and others (1989), on Appalachian hardwoods; Vanclay (1989), on rain forests of Queensland, Australia; and Mengel and Roise (1990), on models for southeastern bottomland hardwoods.

In this research paper, we present a site- and density-dependent, multi-species matrix model for predicting the development of loblolly pine forests in the mid-South.

The recognition of tree species and size is critical in determining the implications of management regimes with ecological and economic objectives.

The paper is organized as follows. First, we present a general model, in which both tree species and tree size are recognized and the probability of trees moving between size classes depends on site productivity and stand density. Next, we report on model estimation and validation with data from permanent sample plots. Sawlog length, tree height, and volume equations leading to individual tree values are also reported. The growth and value models are then used to predict the long-term evolution of loblolly pine stands on good and poor sites, without harvest or with different cutting regimes.

Structure of Growth Model

The growth model is an extension of earlier matrix models (Buongiorno and Michie 1980, Lu and Buongiorno 1993). The elements of the growth matrix vary with residual stand basal area (Solomon and others 1986, Buongiorno and others 1995) and site productivity. The model has the following form:

$$\mathbf{y}_{t+1} = \mathbf{G}_t(\mathbf{y}_t - \mathbf{h}_t) + \mathbf{I}_t \qquad (1)$$

where $\mathbf{y}_t = [y_{ijt}]$ is a column vector containing the number of live trees per unit area of size j ($j = 1, 2, \ldots, n$) and species i ($i = 1, 2, ..., m$) at time t before harvest. Similarly, the column vector $\mathbf{h}_t = [h_{ijt}]$ represents the harvest at t. The upgrowth and ingrowth matrices $\mathbf{G}_t$ and $\mathbf{I}_t$ are defined as

$$\mathbf{G}_t = \begin{bmatrix} \mathbf{G}_{1t} & & & \\ & \mathbf{G}_{2t} & & \\ & & \ddots & \\ & & & \mathbf{G}_{mt} \end{bmatrix}$$

$$\mathbf{G}_{it} = \begin{bmatrix} a_{i1t} & & & & \\ b_{i1t} & a_{i2t} & & & \\ & \ddots & \ddots & & \\ & & b_{i(n-2)t} & a_{i(n-1)t} & \\ & & & 0 & a_{int} \end{bmatrix} \qquad (2)$$

$$\mathbf{I}_t = \begin{bmatrix} \mathbf{I}_{1t} \\ \mathbf{I}_{2t} \\ \vdots \\ \mathbf{I}_{mt} \end{bmatrix}$$

where

$$I_{it} = I_{it}(B_t, N_{it}, S) \qquad (3)$$

is the number of trees of species i that enter the smallest size class from t to $t + 1$. This ingrowth was assumed to be a function of residual stand basal area B_t, number of trees of species i after harvest, N_{it}, and site productivity S (Buongiorno and others 1995).

$$b_{ijt} = b_{ijt}(B_t, D_j, S) \qquad (4)$$

is the probability that a tree of species i and size j at t is alive and of size $j + 1$ at $t + 1$. The hypothesis was that this probability is a function of B_t, tree diameter D_j, and site productivity (Buongiorno and others 1995, Mengel and Roise 1990, Solomon and others 1986).

$$a_{ijt} = 1 - b_{ijt} - m_{ijt} \qquad (5)$$

is the probability that a tree of species i and size j is alive and of size j at $t + 1$, where

$$m_{ijt} = m_{ijt}(B_t, D_j, S) \qquad (6)$$

is the probability that a tree of species i and size j dies from t to $t + 1$, hypothesized to be a function of B_t, D_j, and site productivity (Lin and others 1996).

Data for Model

Forest Inventory and Analysis (FIA) data of the Forest Service were used to estimate the ingrowth equation (Eq. (3)) and the upgrowth and mortality equations (Eqs. (4) and (6)). Data were taken from stands in seven southern states (Table 1). (See Appendix for SI conversion factors.) The complete FIA data base consists of approximately 18,000 permanent-point plots, at the intersections of lines on a 3-mile (4.8-km) orthogonal grid. On each plot, stand volume, site, and individual tree characteristics were recorded by survey crews. Stand-level and site characteristics include a productivity class and forest type. Plots with more than one age class in the major or dominant species were classified as "mixed age." Site productivity on each plot was estimated on a scale of 1 to 7. The ratings indicate highest potential

Table 1—Distribution of sample plots by State

State	Survey year[a] Current	Survey year[a] Previous	Number of plots
Alabama	1990	1982	292
Arkansas	1988	1978	53
Louisiana	1991	1984	243
Mississippi	1994	1987	270
Oklahoma	1993	1986	4
Tennessee	1989	1980	2
Texas	1992	1986	127
Total	—	—	991

[a]State surveys may last more than 1 year.

mean growth in fully stocked stands for various stand sizes per year.

Productivity rating	Volume growth (ft^3/acre/year)[a]
Site 1	≥225
Site 2	165 to 224
Site 3	120 to 164
Site 4	85 to 119
Site 5	50 to 84
Site 6	20 to 49
Site 7	<20

[a] 1 ft^3/acre = 0.07 m^3/ha.

Measurements of individual trees on each plot were made by first systematically establishing 5 to 10 satellite points, or subplots, around the plot center, covering a sampling area of approximately 1 acre (0.405 ha). At each satellite point, trees of at least 5-in. (127-mm) diameter at breast height (dbh) were recorded in terms of species, diameter, total tree height, height to a merchantable top, percentage cull volume, crown class, tree and grade. Mortality was gauged by individual tree histories: as plots were remeasured, field crews recorded whether a tree that was previously sampled as "live" had been cut, had died, or was still alive.

Trees smaller than 5 in. (127 mm) dbh were sampled in three fixed-radius, 1/275-acre (1.5×10^{-3} ha) subplots. Ingrowth into the smallest size class was computed from the data in these fixed plots.

A subset of the 18,000 permanent plots constituted the data base for this study. This subset consisted of all the plots that conformed to the following criteria:

1. remeasured at least once,
2. classified within loblolly pine forest type in previous inventory,
3. classified as "mixed age" in previous inventory, and
4. consisting of natural stands (that is, no evidence of artificial regeneration).

This subdivision led to 991 plots that had been measured twice between 1978 and 1994 at intervals of 6 to 11 years, averaging 7.3 years between measurements. Most plots were in Alabama, Mississippi, Louisiana, and Texas (Table 1). While loblolly pine was the dominant species on all the plots, lesser amounts of shortleaf pine and hardwood species were sometimes present.

Estimation of Model Parameters

To estimate the parameters of the model, the trees in the data base were divided into three species groups, following the FIA classification:

1. Pines and other softwoods, mostly loblolly, shortleaf, and longleaf pines (*Pinus taeda*, *P. echinata*, *P. palustris*)
2. Soft hardwoods, dominated by sweetgum (*Liquidambar styraciflua*), blackgum (*Nyssa sylvatica*), and yellow-poplar (*Lirodendron tulipifera*)
3. Hard hardwoods, mostly southern red, water, and post oaks (*Quercus falcata* var. *falca*, *Q. nigra*, *Q. stellata*, and *Q. alba*).

Within each species group, trees were classified into thirteen 2-in.- (51-mm-) diameter classes ranging from 2 to 26+ in. (51 to 660+ mm). Each class was denoted by its midpoint diameter. For example, trees in the smallest class (2 in. (51 mm)) had 1 to <3 in. (25.4 to <76 mm) dbh, and those in the largest class (26+ in. (660+ mm)) had ≥25 in. (≥635 mm) dbh. Ingrowth was defined as the number of trees per sampling area that grew more than 1 in. (25.4 mm) between two forest inventories.

Each plot provided one observation on ingrowth, transition probabilities of upgrowth and mortality, and independent variables that affected tree growth. The transition probabilities and ingrowth were converted to a 1-year interval by linear interpolation. The parameters of the equations that predicted ingrowth (Eq. (3)), upgrowth (Eq. (4)), and mortality (Eq. (6)) were obtained by multiple regression, across all plots. In choosing the empirical form of the equations, the following criteria were followed: consistency with expectations based on prior knowledge, simplicity of form and parsimony of parameters, and statistical significance of parameters.

Ingrowth Equations

The empirical versions of the ingrowth equation (Eq. (3)) are in Table 2. For a particular species, ingrowth per year was a linear function of stand basal area and number of trees of that species. The coefficient of stand basal area was negative, so that ingrowth was lower at higher stand densities. Ingrowth of pines and other softwoods was hampered the most by stand density. Other things being equal, ingrowth of a species was affected positively by the number of trees of the same species in the stand.

The results also suggest that ingrowth of pines and soft hardwoods was independent of the site. Ingrowth of hard hardwoods was a quadratic function of site, such that ingrowth was higher on a poor site (site 6) than on a good site (site 2), for stands with equal basal area and number of hardwood trees.

Table 2—Ingrowth equations for various woods[a]

Statistic[b]	Basal area (ft^2/acre)	Trees/ acre	Site	$(Site)^2$	Con- stant
	Independent variable				
Pines and other softwoods					
Coefficient	–0.15	0.008	—	—	17.4
SE	0.02	0.003	—	—	1.7
	***	***	—	—	***
r^2	0.05				
dF	988				
Soft hardwoods					
Coefficient	–0.05	0.016	—	—	10.5
SE	0.01	0.003	—	—	1.1
	***	***	—	—	***
r^2	0.04				
dF	988				
Hard hardwoods					
Coefficient	–0.05	0.013	–6.9	1.1	19.8
SE	0.01	0.002	3.7	0.5	6.6
	***	***	*	**	***
r^2	0.04				
dF	988				

[a]Equations (trees/acre/year) for 2-in. (51-mm) dbh class. Asterisks denote level of significance: * , 0.10; **, 0.05; and ***, 0.01.
[b]SE is standard error; r^2, coefficient of determination; and dF , degrees of freedom.

Table 3—Equations for transition probability of trees between size classes in 1 year[a]

Statistic	Basal area (×10 ft^2/ acre)	dbh (in.)	dbh^2 (×10 in^2)	Site·dbh	Constant
	Independent variable				
Pines and other softwoods					
Coefficient	–0.0015	0.0119	–0.0036	–0.00020	0.029
SE	0.0002	0.0006	0.0002	0.00007	0.003
	***	***	***	***	***
r^2	0.12				
dF	5,107				
Soft hardwoods					
Coefficient	–0.0013	0.0064	–0.0022	–0.0006	0.033
SE	0.0003	0.0009	0.0004	0.0002	0.006
	***	***	***	***	***
r^2	0.05				
dF	2,015				
Hard hardwoods					
Coefficient	–0.0023	0.0112	–0.0025	–0.0007	0.029
SE	0.0003	0.0008	0.0003	0.0001	0.004
	***	***	*	**	***
r^2	0.15				
dF	2,589				

[a]Asterisks denote level of significance: * , 0.10; ** , 0.05; and ***, 0.01. dbh is diameter at breast height.

Upgrowth Equations

The estimates of the parameters of the upgrowth equation (Eq. (4)) are in Table 3. As expected, for a given diameter and site, the rate of upgrowth was lower for stands of higher basal area. For a given stand basal area, the upgrowth rate increased with tree diameter, reached a maximum, and then decreased, in agreement with the sigmoid curve of tree diameter over time. For the three species groups, the upgrowth rate was higher on land of higher productivity (smaller site number). This is consistent with the definition of site productivity—trees grew faster on the better sites.

Mortality Equations

Table 4 summarizes the results of estimates for the mortality equation (Eq. (6)). For the three species groups, the mortality rates were independent of site productivity. Pines and other softwoods had a higher mortality rate in stands of higher basal area, whereas the other two species groups were unaffected by basal area.

For all species, mortality per year was a quadratic function of tree diameter. The negative coefficient of tree diameter and the positive coefficient of the diameter squared confirm the presumed convex relationship between tree mortality and tree diameter. Mortality was high for the smallest trees, lowest for intermediate trees, and increased again for very large trees.

For all equations in Tables 2, 3, and 4, the coefficient of determination r^2 was small. The selected variables explained only a small part of the variation in annual ingrowth, mortality, and upgrowth rates. This does not necessarily mean that the models are inadequate, however, since the standard errors of the constant terms are small. Thus, little might have been lost by modeling ingrowth, upgrowth, and mortality rates as constant. Nevertheless, we decided to maintain the variables that had effects consistent with prior knowledge and that were statistically significant.

Projection Accuracy

To test the accuracy of the growth model for projections as long as the interval between two FIA inventories (6 to 10 years), the model was estimated for 80% of the plots selected randomly from the 991 available plots. The model was then used to predict the state of the remaining 195 plots at the time of their second measurement, given their state at the first inventory. The harvest, if any, was assumed to have occurred right after the first measurement.

Table 4—Equations for probability of mortality in 1 year[a]

Statistic	Independent variable: (Basal area)^2 ($\times 10^3$ ft^4/acre)	dbh (in.)	(dbh)^2 ($\times 10$ in^2)	Constant
Pines and other softwoods				
Coefficient	0.00020	–0.0064	0.0019	0.052
SE	0.00006	0.0003	0.0001	0.002
	***	***	***	***
r^2	0.12			
dF	5,180			
Soft hardwoods				
Coefficient	—	–0.0041	0.0016	0.030
SE	—	0.0004	0.0002	0.002
	—	***	***	***
r^2	0.04			
dF	2,071			
Hard hardwoods				
Coefficient	—	–0.0045	0.0015	0.038
SE	—	0.0004	0.0002	0.002
	—	***	***	***
r^2	0.05			
dF	2,650			

[a]Asterisks denote level of significance: * , 0.10; ** , 0.05; and ***, 0.01.

To predict the stand state at the time of the second measurement, the growth model (Eq. (1)) was applied iteratively, each iteration simulating 1 year's growth. For each plot, the transition matrix $\mathbf{G}_t$ changed as a function of site, and at each iteration, $\mathbf{G}_t$ changed as a function of stand basal area.

Figures 1 to 3 show how the predicted stand states compared with the actual states. In general, for the 195 plots, the average of the predicted number of trees of each species and diameter was within the 95% confidence interval of average observed number of trees. A series of *t* tests confirmed that the predicted mean number of trees in each size class was not significantly different from the observed mean, at the 5% significance level.

Long-Term Growth of Unmanaged Stands

Accuracy of a growth model over 6 to 10 years is obviously not sufficient to warrant applying the model to management. The model predictions should also be consistent with general knowledge of stand growth over at least one century. For example, basal area should remain within plausible limits, as must the number of trees and their distribution by size, and the species composition must change according to a plausible succession pattern.

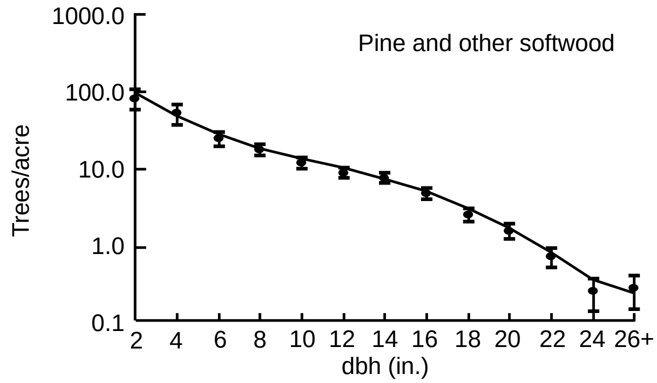

Figure 1—Average observed (dots, with 95% confidence intervals) and predicted (line) distribution of softwood trees on 195 post-sample plots, after average 8-year growth.

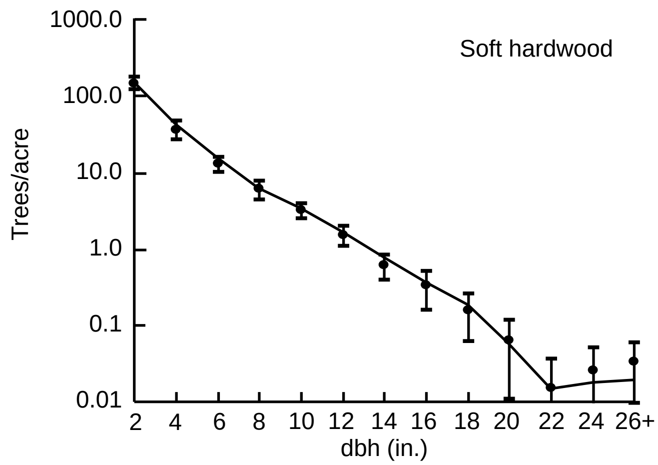

Figure 2—Average observed (dots, with 95% confidence intervals) and predicted (line) distribution of soft hardwood trees on 195 post-sample plots, after average 8-year growth.

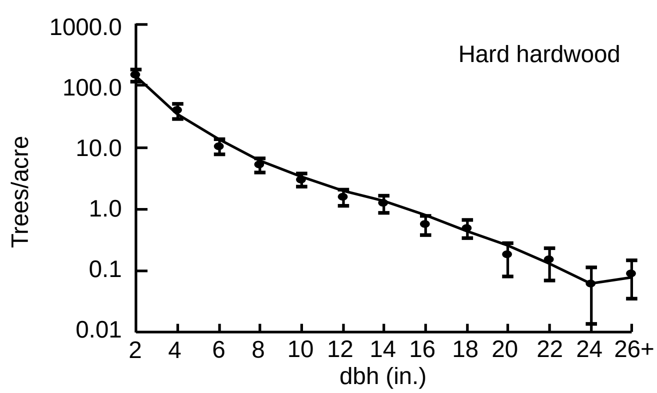

Figure 3—Average observed (dots, with 95% confidence intervals) and predicted (line) distribution of hard hardwood trees on 195 post-sample plots, after average 8-year growth.

Table 5—Average condition of 991 plots at second inventory

Species	dbh (in.)	Trees/acre All sites	Site 2	Site 6
Softwood	2	94.5	19.7	168.1
	4	47.9	27.2	107.0
	6	25.0	16.0	22.4
	8	18.2	14.2	11.9
	10	12.1	14.7	2.1
	12	9.5	12.0	3.4
	14	7.0	10	1.9
	16	4.4	7.4	0
	18	2.6	4.1	0
	20	1.4	2.4	0
	22	0.7	1.4	0.2
	24	0.3	0.6	0
	26+	0.2	0.5	0
Soft hardwood	2	144.5	158.9	45.8
	4	35.2	44.8	45.8
	6	14.3	20.3	8.9
	8	6.0	9.1	1.5
	10	2.8	4.3	0
	12	1.3	1.8	0
	14	0.6	1.2	0
	16	0.3	0.7	0
	18	0.2	0.3	0
	20	0.1	0.1	0
	22	0	0.1	0
	24	0	0	0
	26+	0	0	0
Hard hardwood	2	148.4	141.9	290.3
	4	32.8	33.3	45.8
	6	11.6	11.7	9.8
	8	5.9	6.2	6
	10	3.3	3.4	1.4
	12	1.7	1.9	1.6
	14	1.2	1.5	1.2
	16	0.8	0.8	0
	18	0.4	0.6	0.7
	20	0.3	0.4	0.3
	22	0.2	0.2	0.2
	24	0.1	0.1	0.2
	26+	0.1	0.1	0.1
Basal area (ft^2/acre)		90.2	110.5	56.3
Species diversity[a]		0.94	0.92	0.98
Tree size diversity[a]		2.45	2.46	1.98

[a]Diversity was measured by Shannon's index.

Initial Stand State and Diversity Indices

To test the long-term behavior of the model, the model was applied to predict the growth of a stand, without harvest, over several centuries. For the two simulations shown here, the initial condition was the average state of the plots on either site 2 or site 6 at the time of the second measurement. Table 5 shows the average number of trees by species and size class. The data within each species group reveal the reverse J-shaped distribution of the number of trees by size, typical of uneven-aged stands.

Shannon's index (Hunter 1990, Magurran 1988, Pielou 1977) measures stand diversity of tree species:

$$H_{\text{species}} = -\sum_{i=1}^{m} \frac{y_i}{y} \ln(\frac{y_i}{y}) \qquad (7)$$

where y_i is basal area of trees of species i and y is basal area of trees of all species. Tree size diversity is

$$H_{\text{size}} = -\sum_{j=1}^{m} \frac{y_j}{y} \ln(\frac{y_j}{y}) \qquad (8)$$

where y_j is basal area of trees of size j. Basal area was used instead of number of trees to assign more weight to larger trees.

Diversity is greatest when the basal area of trees is evenly distributed among species and sizes; then, max H_{species} = ln(3) = 1.10, and max H_{size}= ln(13) = 2.56. Thus, the average stand state at the time of the second inventory (Table 5) had 85% of the highest possible species diversity and 96% of the highest possible size diversity. However, the theoretical maximum of tree size diversity is usually not sustainable in natural uneven-aged stands (Buongiorno and others 1994). Table 5 shows that stands on the poorest site (site 6) had slightly higher species diversity, on average, but much lower size diversity than did stands on the best site (site 2), because of the absence of large softwood and soft hardwood trees.

Expected Stand Growth, Without Management

The growth model (Eq. (1)) with no harvest ($\mathbf{h}_t = 0$) was applied to forecast the expected growth of a stand initially in the average state shown in Table 5. Two simulations were done: initial condition of average of stands on a good site (site 2) and initial condition of average of stands on a poor site (site 6).

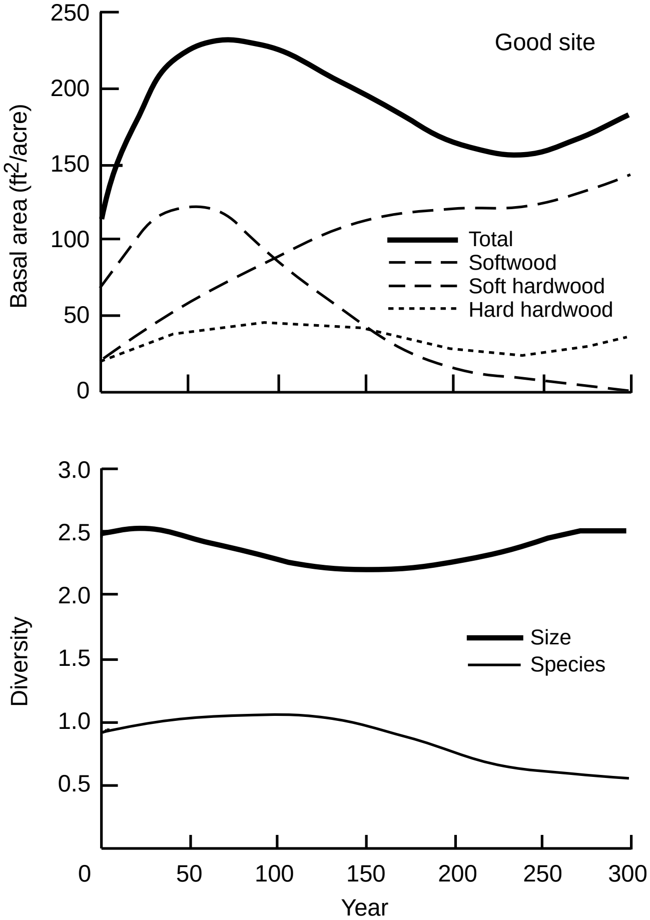

Figure 4—Basal area and diversity (Shannon's index) of undisturbed loblolly pine stand on good site (site 2).

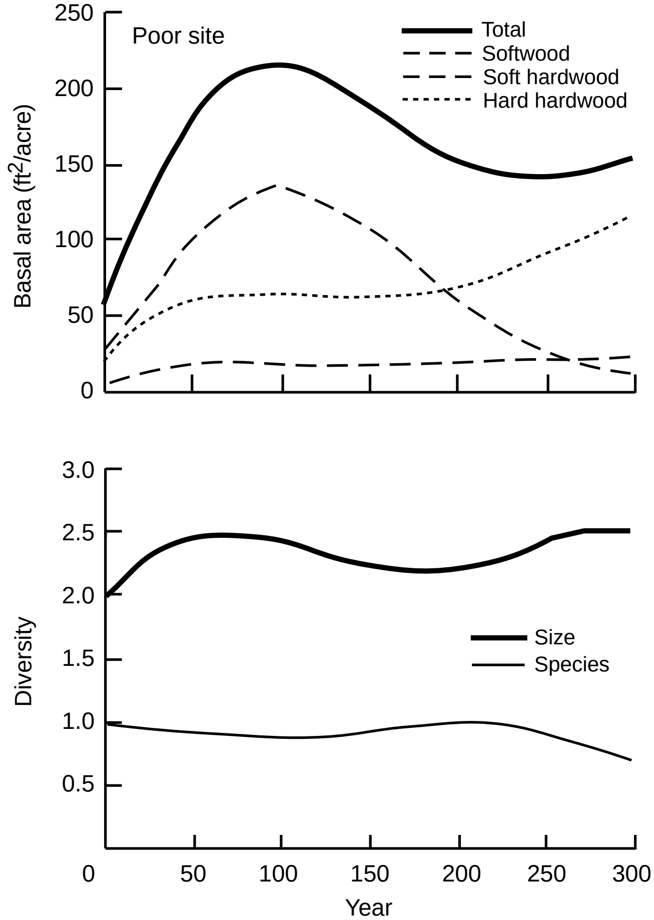

Figure 5—Basal area and diversity (Shannon's index) of undisturbed loblolly pine stand on poor site (site 6).

Figure 4 shows the succession of expected stand states on a good site, in terms of basal area and tree diversity. The expected total basal area fluctuated with decreasing amplitude over time. Even after three centuries, a steady state had not been reached. This phenomenon can be explained by the repeated cycle of the regeneration of numerous individuals (each with relatively small living biomass), the accumulation of living biomass as the trees grow, and eventually the death of large dominant trees at about the same age. Pines and other softwoods lost their dominance, in terms of basal area, after about 100 years, and they were totally replaced by hardwoods after about 300 years. This is consistent with prior knowledge of successional trends in this ecotype (Quarterman and Keever 1962, Switzer and others 1979). Fire and other catastrophic events could lead to trajectories very different from the smooth development shown in Figure 4. The figure shows the mean trajectory, taking into account the disturbances recorded in the FIA plots and embedded in the growth parameters.

The expected tree size diversity on a good site followed a path similar to that of basal area, though with less amplitude. At the peak of each cycle, the increase in the proportion of trees in the largest size classes led to a higher diversity of tree size. The expected tree species diversity increased slightly for the first century, and then tended to decline as a result of the dominance of hardwoods. However, species diversity, as defined here, refers to only the distribution between softwoods, hard hardwoods, and soft hardwoods. It does not take into account possible changes in the relative abundance of individual species within each group.

Development on a poor site was similar to that on a poor site for total basal area, but quite different for species composition (Fig. 5). Pine and other softwoods on a poor site followed the same trend as those on a good site, and they were eventually replaced by the hardwoods. But, hard hardwoods, instead of soft hardwoods, became the species that dominated the climax stand on a good site. The expected tree size diversity tended to increase slightly over 300 years, while species diversity tended to decrease towards half the initial level.

Tree Diversity and Revenues in Managed Stands

The model was also used to predict the effects of three cutting policies (involving both cutting cycle and harvest intensity) on the diversity of loblolly pine stands and the income they would produce, over 120 years. The diversity criteria were species and size diversity, as described in the previous text. The economic criterion was the present value of gross revenues from timber, per sampling area over a long time horizon. Gross rather than net revenues were used because costs were not known. The effect of these costs was assessed by sensitivity analysis.

Tree Volumes and Values

To calculate economic returns, the value of a tree in each species–size class was equal to its volume times the price per unit of volume. For saplings (trees <5 in. (<127 mm) dbh), tree values were set to zero. For trees ≥5 in. (≥127 mm) dbh, individual tree volumes of sawtimber and pulpwood depended on tree and stand conditions.

Total tree height of the average tree in each dbh class of a particular species was a function of stand basal area, site, and dbh. Table 6 shows the empirical tree height equations, obtained from more than 18,000 trees on the 991 plots used to develop the growth model. For a given diameter, trees were taller in stands with more basal area and on more productive sites. Sawlog length was a function of dbh and total tree height. The parameters, based on sawlog length of about 12,000 trees from the same plots, are in Table 7.

Sawlog volume (cubic feet or meters) was a function of dbh, total tree height, and sawlog length. Empirical equations, fitted to the data in the stem volume tables of Clark and Souter (1994), are shown in Table 8.

Pulpwood was derived from two sources: pulpwood (small-diameter) trees and the top of sawtimber trees. The volume of pulpwood trees was a function of dbh and total tree height (Table 9). The volume of top sawtimber was a function of dbh, total tree height and sawlog length (Table 10). Parameters in Tables 9 and 10 were fitted to the stem volume tables of Clark and Souter (1994).

Cubic-foot volumes of sawtimber were converted to board-foot volumes with Koch's conversion table (Koch 1972).

The prices per thousand board feet and cord were stumpage values in southeastern states, published in Timber Mart–South (1995). The sum of the products of the single-tree volumes of sawtimber and pulpwood by their stumpage prices gave the desired single-tree value by species and diameter class.

Table 6—Equations for total tree height (in feet)[a]

	Independent variable			
Statistic	Basal area (ft^2/acre)	dbh (in.)	Site	Constant
Pines and other softwoods				
Coefficient	0.060	35.3	–4.6	–5.2
	***	***	***	***
SE	0.002	0.3	0.1	0.9
r^2	0.69			
dF	13,127			
Soft hardwoods				
Coefficient	0.070	33.0	–2.7	–11.5
SE	0.006	0.6	0.3	1.9
	***	***	***	***
r^2	0.58			
dF	2,330			
Hard hardwoods				
Coefficient	0.070	30.5	–2.9	–9.2
SE	0.006	0.5	0.3	1.7
	***	***	***	***
r^2	0.061			
dF	3,155			

[a]Asterisks denote level of significance: * , 0.10; ** , 0.05; and ***, 0.01.

Table 7—Equations for sawlog length (in feet)[a]

	Independent variable			
Statistic	dbh (in.)	$(dbh)^2$ (in^2)	Total tree height (ft)	Constant
Pines and other softwoods				
Coefficient	2.8	–0.090	0.81	–39.2
SE	0.1	0.003	0.01	1.0
	***	***	***	***
r^2	0.68			
dF	10,212			
Soft hardwoods				
Coefficient	3.3	–0.07	0.47	–39.3
SE	0.4	0.01	0.03	41
	***	***	***	***
r^2	0.44			
dF	708			
Hard hardwoods				
Coefficient	0.5	–0.010	0.41	–9.3
SE	0.2	0.004	0.02	1.8
	***	***	***	***
r^2	0.33			
dF	1,584			

[a]Asterisks denote level of significance: * , 0.10; ** , 0.05; and ***, 0.01.

Table 8—Equations for sawlog volume per tree (in cubic feet)[a]

Statistic	(dbh)^2 (in^2)	Total tree height (ft)	Sawlog length (ft)	Constant
	Independent variable			
Pines and other softwoods				
Coefficient	0.120	0.08	0.93	–37.3
SE	0.003	0.02	0.02	1.8
	***	***	***	***
r^2	0.95			
dF	281			
Soft hardwoods				
Coefficient	0.100	0.12	1.07	–41.3
SE	0.002	0.02	0.03	1.9
	***	***	***	***
r^2	0.96			
dF	228			
Hard hardwoods				
Coefficient	0.100	0.07	1.08	–36.5
SE	0.002	0.02	0.03	1.7
	***	***	***	***
r^2	0.97			
dF	224			

[a]Asterisks denote level of significance: * , 0.10; ** , 0.05; and *** , 0.01.

Table 9—Equations for pulpwood volume per tree (in cubic feet)[a]

Statistic	(dbh)^2 (in^2)	Total tree height (ft)	Constant
	Independent variable		
Pines and other softwoods			
Coefficient	0.110	0.140	–7.9
SE	0.005	0.006	0.4
	***	***	***
r^2	0.97		
dF	47		
Soft hardwoods			
Coefficient	0.100	0.180	–10.0
SE	0.004	0.006	0.4
	***	***	***
r^2	0.97		
dF	80		
Hard hardwoods			
Coefficient	0.110	0.150	–8.4
SE	0.003	0.006	0.4
	***	***	***
r^2	0.97		
dF	80		

[a]Asterisks denote level of significance: * , 0.10; ** , 0.05; and *** , 0.01.

Table 10—Equations for top sawtimber volume per tree (in cubic feet)[a]

Statistic	(dbh)^2 (in^2)	Total tree height (ft)	Sawlog length (ft)	Constant
	Independent variable			
Pines and other softwoods				
Coefficient	0.061	0.60	–0.93	–18.5
SE	0.002	0.02	0.02	1.7
	***	***	***	***
r^2	0.89			
dF	281			
Soft hardwoods				
Coefficient	0.053	0.62	–1.06	–22.3
SE	0.001	0.02	0.03	1.8
	***	***	***	***
r^2	0.91			
dF	228			
Hard hardwoods				
Coefficient	0.056	0.55	–1.07	–17.1
SE	0.002	0.02	0.03	1.6
	***	***	***	***
r^2	0.91			
dF	224			—

[a] Asterisks denote level of significance: * , 0.10; ** , 0.05; and ***, 0.01.

Long-Term Effects of Management

To illustrate an application, the growth model (Eq. (1)) was used to simulate the effects of three management regimes (guides) on stand characteristics and revenues over 120 years. The simulations were done for a good site (site 2) and a poor one (site 6). The initial condition was the average species–size distribution of the plots on site 2 or site 6 at the second inventory (Table 5).

The three management guides were defined by cutting cycle and target state or distribution. The harvesting rule was that trees in excess of the target number in any species–size class would be cut. The first harvest would occur immediately.

The three management guides were as follows:

1. Current guide—cutting cycle of 8 years. The target state was the average stand state at the first inventory minus the cut between the two inventories (Table 11).
2. *q*-factor guide—cutting cycle of 6 years, as suggested by Williston (1978). The residual basal area was 53 ft^2/acre (12.2 m^2/ha) for softwood trees >5 in. (>127 mm), the *q* ratio was 1.44 for 2-in. (51-mm) dbh classes, and the largest dbh class was 16 in. (406 mm). All hardwoods (sawlogs and pulpwood) were used, and intensive hardwood control was applied. See target distribution in Table 11.

Table 11—Stand target states after cut and ending states of three harvest regimes, over 120 years (trees/acre)

		Current guide			*q*-factor guide			Diameter-limit cut		
	dbh class	Target	Final state		Target	Final state		Target	Final state	
Species	(in.)	state	Site 2	Site 6	state	Site 2	Site 6	state	Site 2	Site 6
Softwood	2	103.4	79.2	84.9	*	118.8	122.0	*	128.6	135.2
	4	49.6	35.2	37.3	*	54.0	55.1	*	58.7	61.2
	6	28.0	21.2	22.2	36.8	33.1	33.5	*	36.2	36.8
	8	17.6	15.3	16.0	25.6	24.4	24.6	*	26.9	26.6
	10	12.3	12.5	12.8	17.8	19.0	19.0	*	22.7	21.9
	12	8.2	9.9	9.9	12.3	14.4	14.3	*	20.9	20
	14	5.3	7.3	7.2	8.6	10.6	10.5	0	10	8.8
	16	3.2	5.0	4.8	5.9	7.7	7.5	0	2.7	2.1
	18	1.7	3.1	3.0	0	3.4	3.0	0	0.4	0.3
	20	0.9	1.8	1.7	0	0.8	0.6	0	0	0
	22	0.4	0.9	0.8	0	0.1	0.1	0	0	0
	24	0.2	0.4	0.4	0	0	0	0	0	0
	26+	0.1	0.3	0.2	0	0	0	0	0	0
Soft hardwood	2	116.7	134.2	152.7	0	37.2	39.1	0	38.8	41.3
	4	28.1	43.5	31.5	0	3.1	1.3	0	3.3	1.5
	6	12.2	17.1	13.2	0	0.2	0	0	0.2	0.1
	8	4.8	7.4	6.0	0	0.1	0	0	0	0
	10	2.2	3.5	2.9	0	0	0	0	0	0
	12	1.0	1.6	1.4	0	0	0	0	0	0
	14	0.5	0.8	0.7	0	0	0	0	0	0
	16	0.3	0.4	0.4	0	0	0	0	0	0
	18	0.1	0.2	0.2	0	0	0	0	0	0
	20	0.1	0.1	0.1	0	0	0	0	0	0
	22	0	0	0.2	0	0	0	0	0	0
	24	0	0	0	0	0	0	0	0	0
	26+	0	0	0	0	0	0	0	0	0
Hard hardwood	2	128.7	138.3	201.4	0	38.5	83.8	0	39.8	85.3
	4	26.9	39.5	41.9	0	3.0	5.3	0	3.2	5.8
	6	10.5	14.9	13.9	0	0.2	0.3	0	0.2	0.3
	8	5.3	7.7	6.5	0	0	0	0	0	0
	10	2.7	3.9	3.5	0	0	0	0	0	0
	12	1.6	2.3	2.0	0	0	0	0	0	0
	14	1.1	1.5	1.3	0	0	0	0	0	0
	16	0.6	0.9	0.8	0	0	0	0	0	0
	18	0.3	0.6	0.5	0	0	0	0	0	0
	20	0.2	0.8	0.3	0	0	0	0	0	0
	22	0.1	0.2	0.1	0	0	0	0	0	0
	24	0	0.1	0	0	0	0	0	0	0
	26+	0.1	0.1	0	0	0	0	0	0	0
Length of cutting cycle (years)				8			6			6

*No target.

3. Diameter-limit cut— cutting cycle of 6 years, removal of all trees each harvest, except softwood trees <13 in. (<330 mm) dbh. See target distribution in Table 11.

In addition to the target stand states of the three management guides on good and poor sites, Table 11 shows the ending stand states at year 120, before harvest. For a given regime, the diameter distributions after 120 years were similar on the two sites, except that the poor site had a greater number of the smallest hard hardwood trees.

Table 12 shows the present value of gross income and timber productivity by site and management guide, at 1995 prices and with a real rate of interest of 3% per year. Income is gross of management costs, but net of harvest costs reflected in the stumpage prices. Regardless of regime, present value was \$3,000 to \$4,000 more on good sites than on poor ones, largely as a result of the much higher quantity of pine and softwood sawtimber trees on the good site initially. On both sites, the diameter-limit cut yielded about \$200/acre (\$81/ha) more than did the *q*-factor guide and about \$700 more than the present value obtained by continuing the current cutting guide. A sensitivity analysis showed that the current regime was less affected by fixed costs per sampling area (acre (0.405 ha)) than the other two regimes, as a result of the longer cutting cycle. Present value per sampling area would decrease by \$4.6 for each \$1 increase in fixed cost for the current management guide, and by \$6.0/sampling area for the other two guides.

All three guides cut 3.0 to 3.2 ft^2 (0.28 to 0.30 m^2) per sampling area per year on site 2, and 2.3 to 2.7 ft^2 (0.21 to 0.25 m^2) on site 6. The composition of the harvest was quite different by guide; a much higher proportion of hardwoods was obtained by following the current cutting guide.

Figures 6 and 7 show the development of basal area and tree diversity on a good stand (site 2) managed according to the three management regimes. Regardless of the cutting guide, a steady state (sustainable yield and stock) was reached after about 60 years. Basal area was much larger on a stand managed according to the current guide, fluctuating between 97 ft^2 (9 m^2) (before cut) and 75 ft^2 (7 m^2) (after cut) per sampling area. Compared with the *q*-factor guide, the diameter-limit guide led to a lower basal area during the first 60 years. However, after that time, both cutting guides obtained similar basal areas before and after harvest.

The largest long-run average diversity of tree size was obtained by the current guide, followed by the *q*-factor and then the diameter-limit guide (Fig. 7). The current guide also produced much higher species diversity by always maintaining more hardwood trees than did the other guides. Species and size diversity were almost constant under the current management guide, in contrast to the strong initial drop and subsequent larger fluctuations under the other guides.

Conclusion

The model of uneven-aged loblolly pine stands presented in this report is based on extensive data from the southern United States. The model predicts the number of pine, soft hardwood, and hard hardwood trees in thirteen classes based on tree diameter at breast height. Post-sample forecasts showed that predictions of 6 to 10 years were accurate.

Simulations of growth without harvest over three centuries showed different stand dynamics on poor and good sites. A strong similarity between sites was the disappearance of softwoods in favor of hardwoods after about 300 years, and thus a decline in species diversity on both sites under undisturbed growth. The main difference was the long-run dominance of soft hardwood species on good sites and of hard hardwoods on poor sites. Without management, starting from the current average stand state, basal area increased as the number of large trees increased, with an attendant rise in tree size diversity. Then, basal area declined as a result of the death of old large trees, and size diversity consequently

Table 12—Income and timber productivity of stands managed under different harvest regimes, over 120 years

Income or productivity	Current guide		*q*-factor guide		Diameter-limit guide	
	Site 2	Site 6	Site 2	Site 6	Site 2	Site 6
Present value of gross income (\$/acre)	4,777	1,069	5,373	1,533	5,617	1,735
Basal area cut (ft^2/acre)	3.2	2.3	3.0	2.6	3.0	2.7
Annual productivity (ft^3/acre)						
Softwood sawtimber	59.5	27.0	80.1	39.7	82.1	44.3
Softwood pulpwood	4.0	3.6	4.0	4.9	0.9	1.0
Soft hardwood sawtimber	3.1	0.6	0.7	0.0	0.7	0.0
Soft hardwood pulpwood	7.1	1.6	1.7	0.1	1.8	0.1
Hard hardwood sawtimber	5.3	2.0	1.0	0.5	1.0	0.5
Hard hardwood pulpwood	7.1	3.2	1.5	0.7	1.5	0.7
Interest rate	3%					
Stumpage prices						
Softwood sawtimber	284	\$/$\times 10^6$ fbm, Scribner				
Softwood pulpwood	24	\$/cord				
Soft hardwood sawtimber	151	\$/fbm, Doyle				
Soft hardwood pulpwood	14	\$/cord				
Hard hardwood sawtimber	232	\$/fbm, Doyle				
Hard hardwood pulpwood	14	\$/cord				

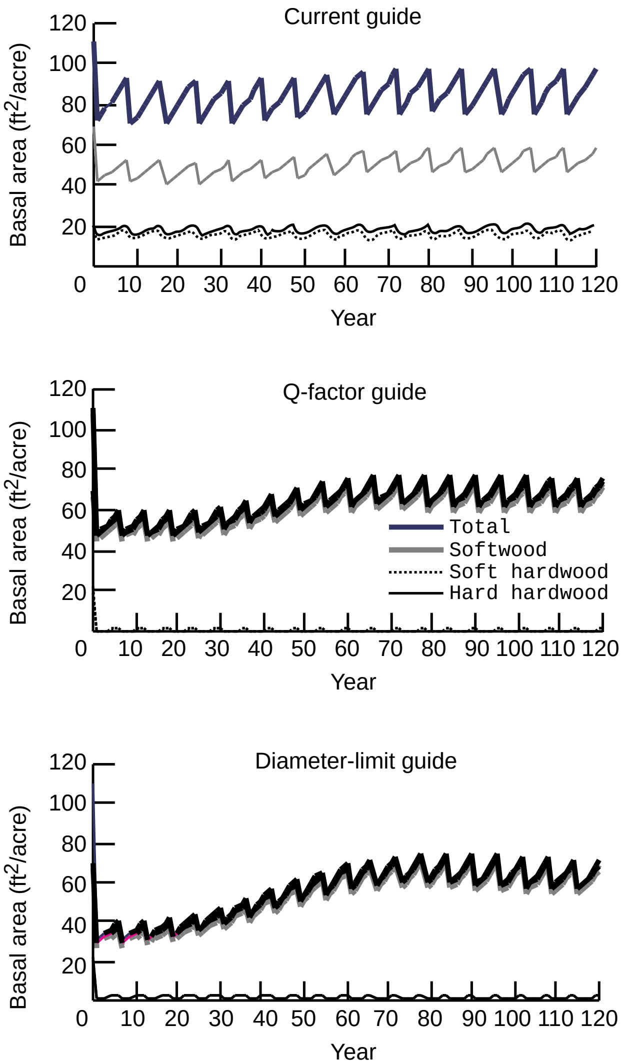

Figure 6—Basal area of loblolly pine stand by management regime (site 2).

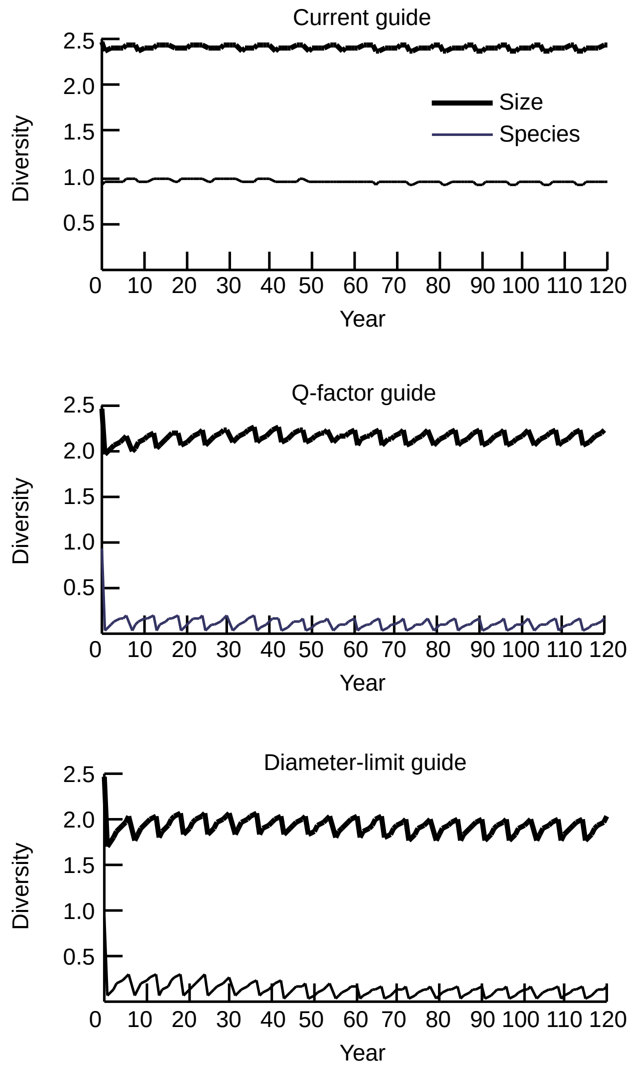

Figure 7—Diversity of loblolly pine stand by management regime (site 2).

declined, oscillating towards an equilibrium. Meanwhile, species diversity declined slowly as a result of the dominance of hardwoods.

Simulations of three cutting regimes showed that over 120 years, management would lead to a steady state (sustainable regime) faster than would natural growth. Management aimed at maintaining the average diameter and species distribution currently observed throughout the plots (current regime) would result in species and size diversity similar to that of an unmanaged stand. From a financial point of view, the *q*-factor guide would be superior to the current management regime; the *q*-factor guide would be similar to a 13-in.- (330-mm-) diameter-limit cut in terms of financial criteria and diversity.

The growth model could be improved by increasing the number of species groups. Still, as the model stands, it seems to be detailed enough for a wide range of applications in forest management. In particular, the model should be useful for investigating other management scenarios, with full consideration of the complete cost of each alternative. The computer program SOUTHPRO has been developed for such simulations (Schulte and others, in preparation). SOUTHPRO predicts the growth, volume, income, and diversity indices discussed in this paper; it allows the user to choose the initial stand state, cutting cycle, target distribution, and price and cost data.

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Appendix—Metric Conversion Factors

English unit	Conversion factor	Metric unit
inch (in.)	25.4	millimeters (mm)
foot (ft)	0.3048	meter (m)
square foot (ft^2)	0.093	square meter (m^2)
ft^2/acre	0.230	m^2/hectare
cubic foot (ft^3)	0.028	cubic meter (m^3)
ft^3/acre	0.070	m^3/hectare
board foot (fbm)	0.00236	cubic meter (m^3)