

United States  
Department of  
Agriculture

Forest Service

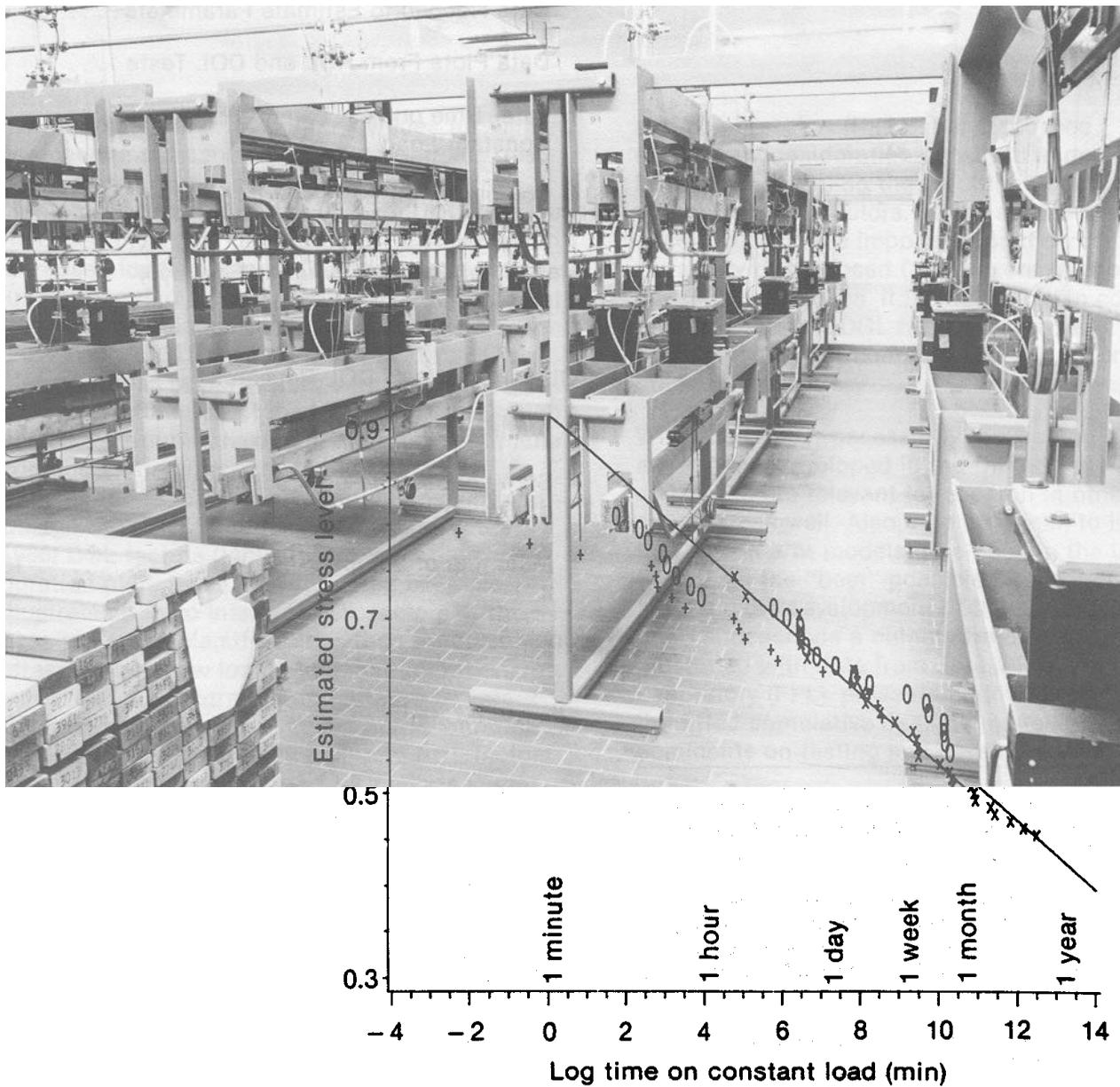
Forest  
Products  
Laboratory

Research  
Paper  
FPL-RP-487



# Statistical Considerations in Duration of Load Research

Carol L. Link



## Abstract

Duration of load factors are the most significant factors that reduce allowable design stresses for lumber. This paper discusses statistical considerations for the design of duration of load tests and for analysis of the resulting data. Duration of load factors along with their associated confidence intervals are estimated for one model using Douglas-fir data from tests at Forest Products Laboratory.

Keywords: Duration of load, duration of load factors, cumulative damage models.

July 1988

---

Link, Carol L. 1988. Statistical considerations in duration of load research. Res. Pap. FPL-RP-487. Madison, WI: U.S. Department of Agriculture, Forest Service, Forest Products Laboratory. 20 p.

A limited number of free copies of this publication are available to the public from the Forest Products Laboratory, One Gifford Pinchot Drive, Madison, WI 53705-2398. Laboratory publications are sent to over 1,000 libraries in the United States and elsewhere.

The Laboratory is maintained in cooperation with the University of Wisconsin.

## Contents

	<i>Page</i>
Introduction . . . . .	1
Calculation of Duration of Load Factors. . . . .	2
Data Variability. . . . .	2
Estimation of Stress Levels. . . . .	2
Cumulative Damage Models . . . . .	7
Data Needed to Estimate Parameters. . . . .	7
Data Plots From ROL and DOL Tests . . . . .	7
Total Time on Test Versus Time on Constant Load . . . . .	9
Calculation of DOL Factors. . . . .	10
Results of Tests on Douglas-Fir 2 by 4 Structural Lumber by Grades . . . . .	11
Other Aspects of DOL Research. . . . .	15
Additional Modeling. . . . .	15
Step Loads and Required Percentage of Failures . . . . .	15
Matching of Lumber Samples . . . . .	15
Conclusions . . . . .	18
Literature Cited . . . . .	18
Appendix . . . . .	20

# Statistical Considerations in Duration of Load Research

Carol L. Link, Mathematical Statistician  
Forest Products Laboratory, Madison, WI

## Introduction

In determining allowable stress values for lumber, engineers must consider duration of load. This phenomenon is demonstrated in both rate of loading (ROL) and duration of load (DOL) tests. In ROL tests, the strength of lumber increases with increased loading rate. In DOL tests, a piece of lumber carrying a given load may be unable to carry that load indefinitely. Building codes account for DOL by using multiplicative DOL factors. These factors change the allowable stress as a function of load duration time, which equals time under maximum load for a particular condition. The current National Design Specifications for Wood Construction (National Design Specifications for Wood Construction 1986) DOL factors (table 1) assume a baseline duration of 10 years. The following example illustrates how to interpret DOL factors. If the baseline allowable stress is  $x$ , then the allowable stress under snow load (of 2-month duration) would be  $1.15x$ . Duration of load researchers often use a baseline of 5 minutes (the duration of a standard static-strength test) and DOL factors that reduce allowable stress for periods longer than 5 minutes. These factors are given in table 1.

The National Design Specifications DOL factors (table 1) are based on Wood's (1951) bending tests of small, clear wood specimens. Current DOL research in the United States, Canada, and Europe includes testing of structural lumber and development of models. The ultimate goal of this research is to derive new DOL factors based on tests of structural lumber in bending, tension,

and compression. This paper discusses statistical considerations involved in designing DOL tests, modeling the resulting data, and computing DOL factors. These statistical considerations are important for knowing the variability of proposed DOL factors. Without variability estimates, it is impossible to compare new estimates of DOL factors to those currently in use or compare estimates from other experiments.

While this paper necessarily focuses on the data and models developed in the United States, the comments are relevant to research in other countries as well. Also, I do not mean to imply that the data or models developed in the United States are the "best" possible. The paper recounts the development of the current DOL factors, describes a cumulative damage model illustrated with data from Forest Products Laboratory (FPL), presents additional data with the fitted cumulative damage model, and finally comments on testing and development of models.

Table 1 - Current National Design Specifications (1996) for wood construction duration of load (DOL) factors

Type of load	Load duration	DOL factors baseline	
		10 years	5 minutes
Static strength Wind	5 minutes	1.00	
	1 day	1.33	.82
	1 week	1.25	.77
Snow Live Permanent	2 months	1.15	.71
	10 years	1.00	.62
	50 years	.90	.56

# Calculation of Duration of Load Factors

Current DOL factors were derived from research on small, clear wood specimens (Wood 1951). Both ROL and DOL tests of primarily Douglas-fir specimens were combined in a stress level versus log time-to-failure plot. The stress level was estimated; it was calculated from matched specimens. Wood fit a hyperbolic curve to these data (fig. 1) using the equation

$$SL(X) = \frac{(108.4)}{(60X)^{0.04635}} + 18.3$$

where  $X$  is time in minutes and  $SL$  is stress level in percent. Calculation of DOL factors from this equation is possible given the baseline and the desired load duration times. For example, the DOL factor for 2 months given a baseline of 10 years is then the ratio of  $SL(2 \text{ months})$  to  $SL(10 \text{ years})$  which is equal to 1.15.

$$\begin{aligned} SL(2 \text{ months}) &= SL(60 \times 24 \times 61 \text{ minutes}) \\ &= SL(87,840 \text{ minutes}) = 71.2 \end{aligned}$$

(2)

$$\begin{aligned} SL(10 \text{ years}) &= SL(60 \times 24 \times 365 \times 10) \\ &= SL(5,256,000 \text{ minutes}) = 62.1 \end{aligned}$$

DOL factors for other time periods can be computed in the same manner.

## Data Variability

Real data are inherently variable. While Wood's 1951 curve may appear to give a precise definition of the DOL phenomenon, one should be aware of the data variability behind this curve. The variability of the small clear data in ROL (fig. 2) and DOL (fig. 3) tests is not trivial. The strength of wood specimens is extremely variable, even for small, clear pieces. This underlying variability is not expressed in the point estimate of a DOL factor. The underlying variability can only be expressed in a confidence interval for this factor. Confidence intervals are a measure of how precisely one knows or can estimate a given quantity. Point estimates without confidence intervals ignore the underlying variability and give the misguided illusion of precision. Without confidence intervals, one cannot determine if DOL factors obtained from various experiments are actually similar or different from each other or from currently used DOL factors.

Unfortunately, confidence intervals for DOL factors using Wood's equation are not straightforward because there are no variability estimates for its parameters. Such estimates would be difficult to obtain because the stress levels are only approximate and some of the data are censored. Censored data arise when the exact failure time is unknown. For specimens that have not failed under constant load at the conclusion of the constant load tests, the precise time to failure is unknown, although it must exceed the duration of that constant load test. While a curve can be fit to the stress level versus log time-to-failure data, this curve may vary with the exclusion or inclusion (and plotting position) of the censored data points. Confidence intervals for parameters of this fitted curve cannot be determined in the usual fashion due to the estimated stress levels and censoring. Therefore, some type of model must be used to obtain confidence intervals.

## Estimation of Stress Levels

Results from DOL tests are commonly shown in a plot of stress level versus log time to failure. Theoretically, a stress level is the ratio of the load at which a specimen failed in a constant load test to the load at which it would fail in a short-term static strength test. Stress levels are always approximate because strength of any specimen is unknown under a loading scheme different from the one in which it failed. Wood determined a stress level by using the failure load from a matched specimen that had failed in a short-term static strength test. Underlying the failure load of a matched specimen is some standard rate of loading (since the strength of a specimen varies with the rate of loading). Stress level will vary as the standard rate of loading is varied.

Since matched specimens are only practical for tests of small, clear wood specimens (and even these are not perfectly matched), another procedure must be used to estimate the stress level for tests of structural lumber. At least three methods are found in the literature: (1) matching specimens by order of failure, (2) estimating an underlying failure distribution, and (3) using a cumulative damage model. All of these methods

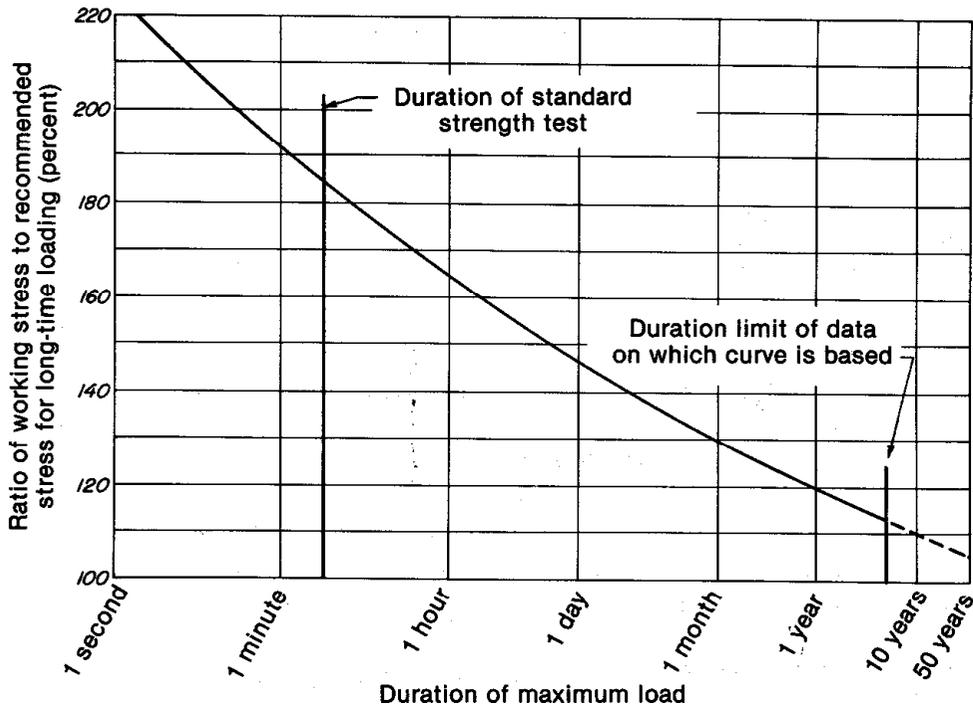


Figure 1 - Relation of working stress to duration of load (Fig. 4 of Wood 1951). (M 89 380)

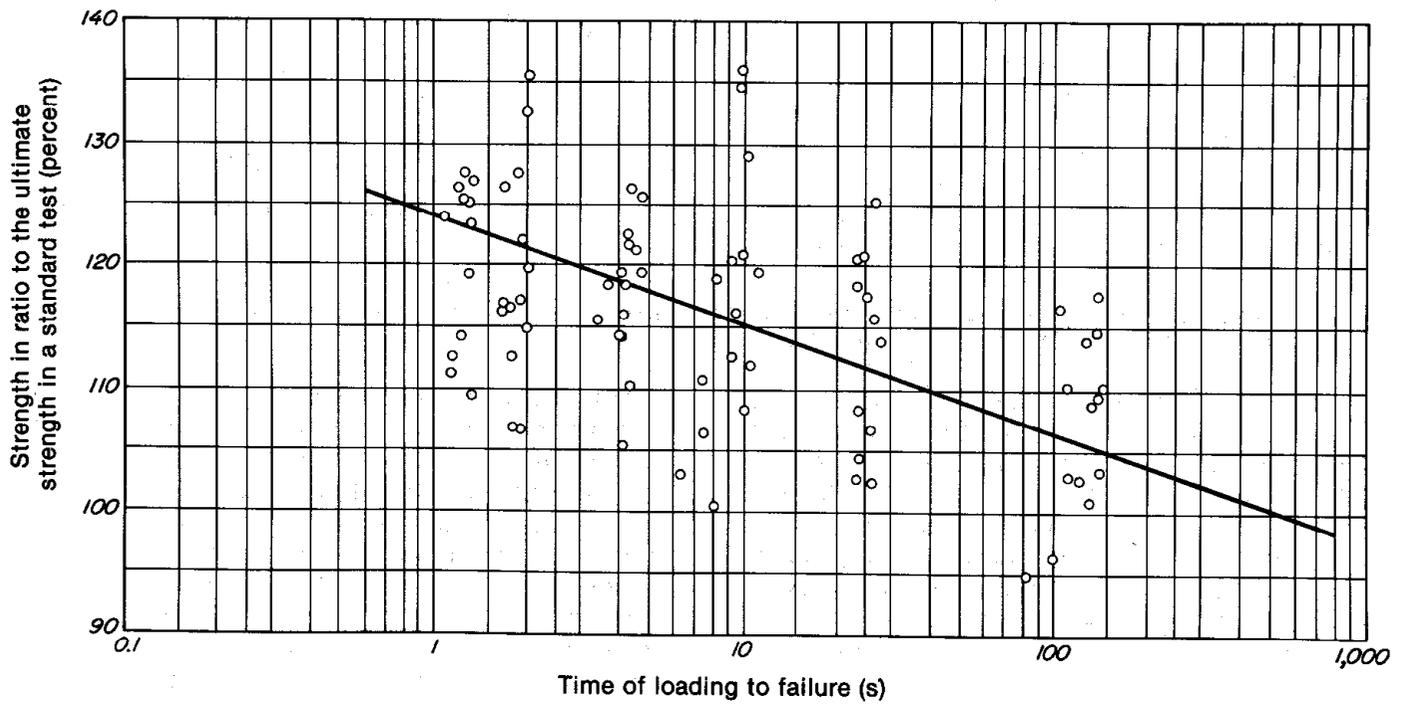


Figure 2 - Relation of strength to time of loading in rapid-loading bending tests of small, clear Douglas-fir specimens (Fig. 2 of Wood 1951). (M 89 381)

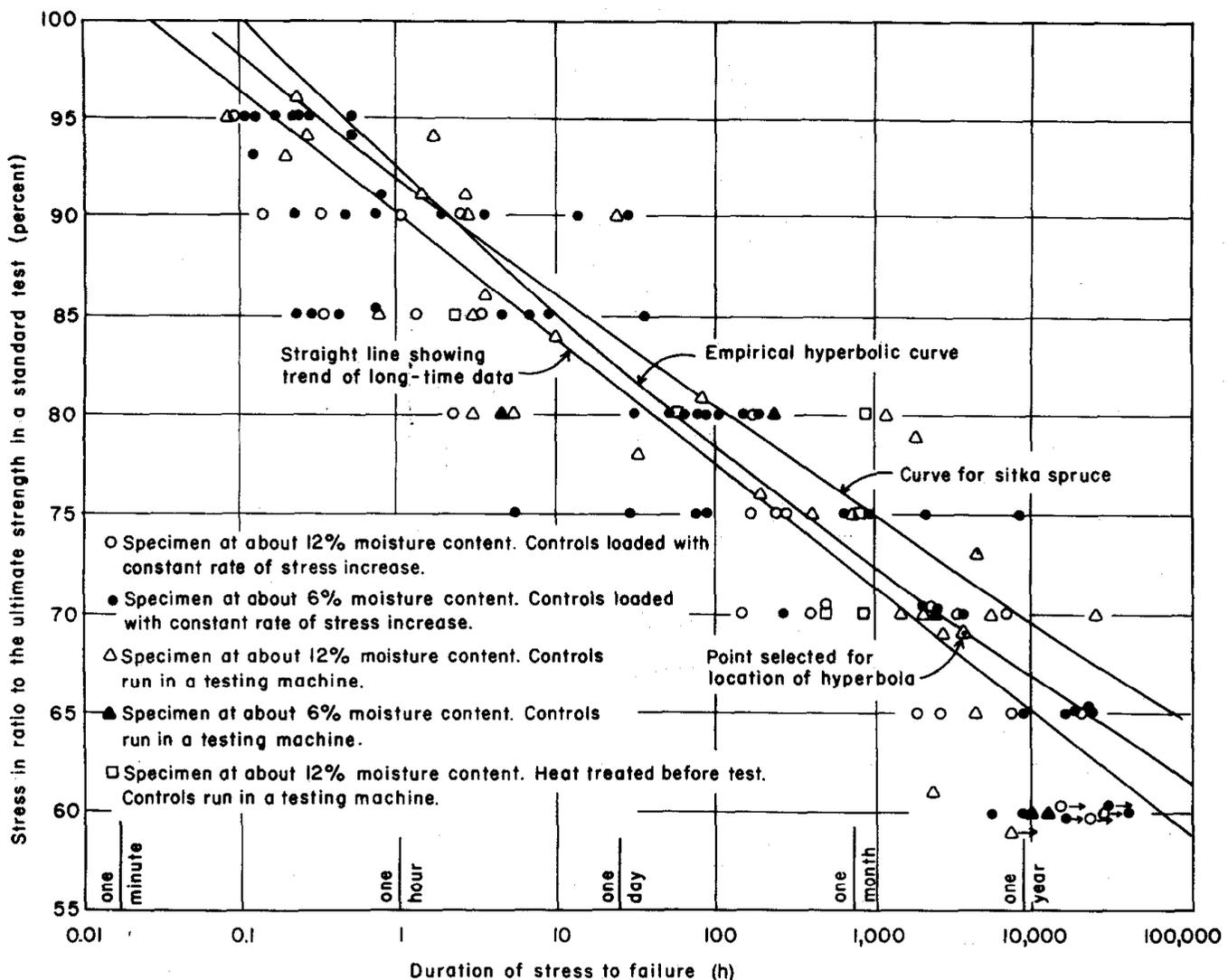


Figure 3 - Relation of duration of constant stress to level of stress in long-time loading bending tests of Douglas-fir specimens (Fig. 1 of Wood 1951). (ML87 5570)

require specifying a standard rate of loading. The first two methods require data from a group of specimens that have failed under the standard rate of loading. This group of specimens is usually matched with specimens tested under constant load, using non-destructive measures such as strength ratio and modulus of elasticity. All three methods require an equal rank assumption. That is, the order of failure in any group of specimens is the same under ROL and DOL tests.

These methods can be illustrated by applying them to a set of data from the "edge knot" study of Gerhards and Link (1987). The data consisted of 294 pieces of Douglas-fir 2 by 4 lumber chosen such that each piece contained an edge knot of a specified size as the strength-controlling defect. Using modulus of elasticity, the specimens were split into six matched groups. Three of these groups were tested under ramp loading, and the other three groups were tested under constant load.

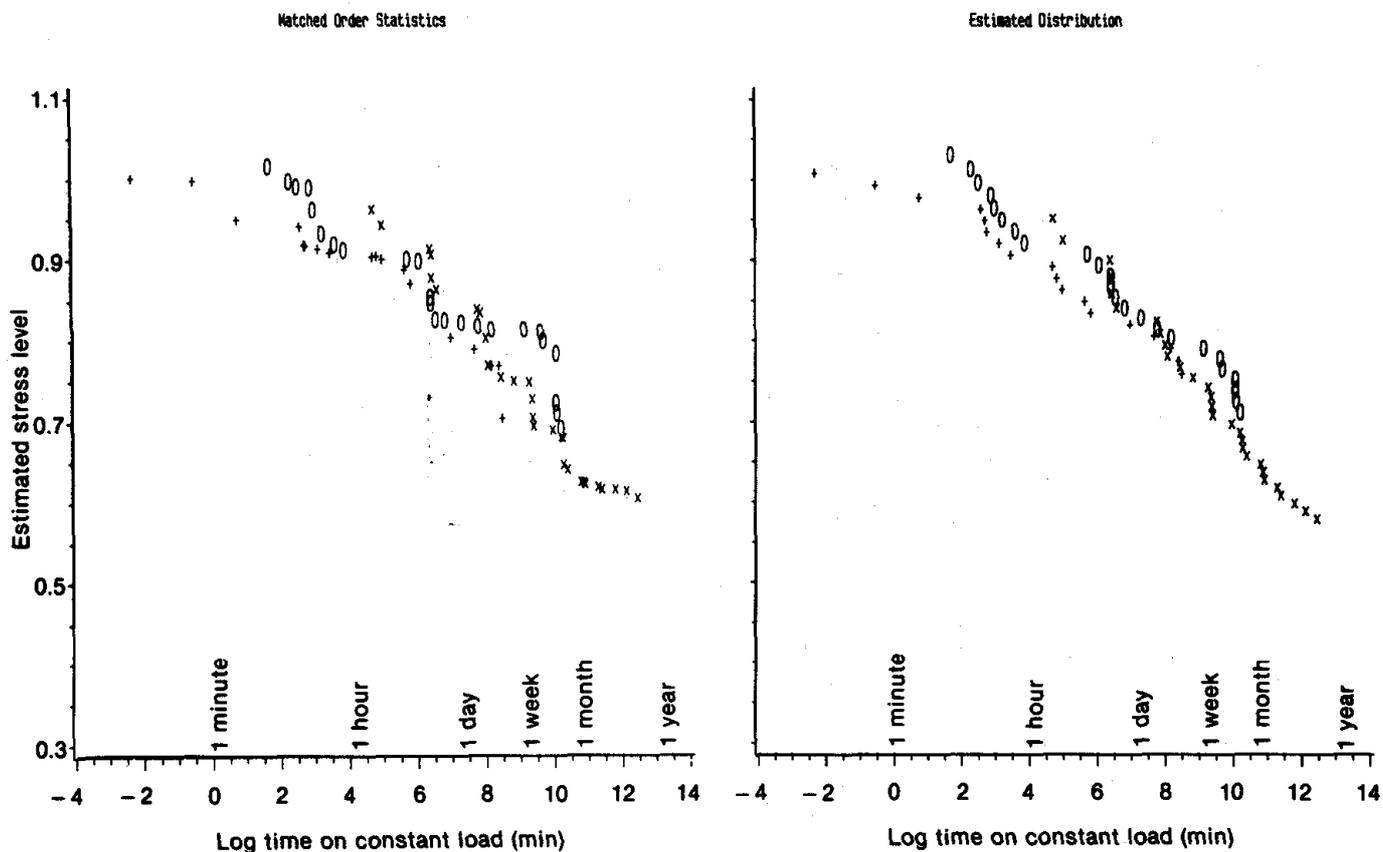


Figure 4 - Estimated stress levels for edge knot study using matched order statistics and estimated failure distribution (+, high constant load; 0, medium constant load; x, low constant load). y axis: stress level (proportion), x axis: natural logarithm of time-to-failure in minutes. (ML88 5412)

The first method for estimating stress level-matching specimens by order of failure (Johns and Madsen 1982)-can be most easily illustrated if the sample size of the group that failed under ramp loading is the same as the sample size of the group tested under constant load. The pieces in the ramp failure group are ordered by actual static strength (which is also the order of failure time if each piece has the same ramp rate), and the pieces of the constant load group (including initial ramp failures) are ordered by time of failure. The stress level of the  $i$ th failure of the constant load test is then the load at failure

divided by the load of the  $i$ th failure under ramp (short-term static strength test) load. Using the edge knot study as an example, let the middle ramp rate (approximately 6 pounds per minute) be chosen as the standard rate. For failures under constant load, the resulting stress level versus log time-to-failure plot can be seen in figure 4 (preliminary ramp failures not shown). If the sample sizes of the constant load group and the standard ramp group are different, one may use this method by interpolating estimated static strength between adjacent failure times.

In the second method for estimating stress level (Gerhards 1986, 1988), a failure distribution is specified for the standard rate of loading, and the parameters of this distribution are then estimated. The stress level of the  $i$ th failure (out of  $n$  specimens) is then the  $i \times 100/n$  percentile of the fitted distribution. Using the edge knot data, if the standard ROL data are assumed to have a lognormal distribution, the estimated parameters are the usual mean and standard deviation of the logarithms of the static strength. Gerhards (1988) fit a regression line to a plot of normal scores and failure loads (similar to fig. 5). For one ROL test, this method should give comparable results to the usual mean and standard deviation. However, Gerhards added initial ramp failures from the constant load tests to presumably improve the estimation procedure. Since added failures in one region of the data may bias the regression, initial failures should not be added to the data. Using an estimated underlying failure distribution, the resulting stress level plot for the edge knot data is very similar to the plot obtained through matched order statistics (fig. 4). Unless the ramp data deviate greatly from the fitted failure distribution, methods 1 and 2 should give similar stress level plots.

The final method, used by Gerhards and Link (1987), obtains stress levels through the use of a cumulative damage model. The numerator is still the constant load at failure, but the denominator is found by using the estimated parameters from the cumulative damage model. The resulting plot is almost identical to that shown in figure 4; a slightly different scale on the y axis is due to another choice of the standard ROL.

DOL factors may be calculated from stress level plots as a ratio of the stress level at the desired load duration and the stress level at the baseline. A fitted equation can be used, but the fitted line changes with the addition of different failure times for the censored data. If sufficient data exist, a representative point can be chosen at the baseline as well as the time for the desired load duration, and a ratio can be derived from these

times. This is usually impractical for load durations of greater than a few months because long-term (years) data do not exist. Also, due to the problems of censored data and estimated stress levels (versus true stress levels), confidence intervals on the ratio estimates are difficult to obtain.

Gerhards (1988) fit a regression line to the stress level versus log time to failure, using the parameter and variance estimates from the fitted regression to give confidence intervals for the estimated DOL factors. While the ratio estimates may be reasonable, the reported confidence intervals were derived from incorrect methodology and are also much too small.

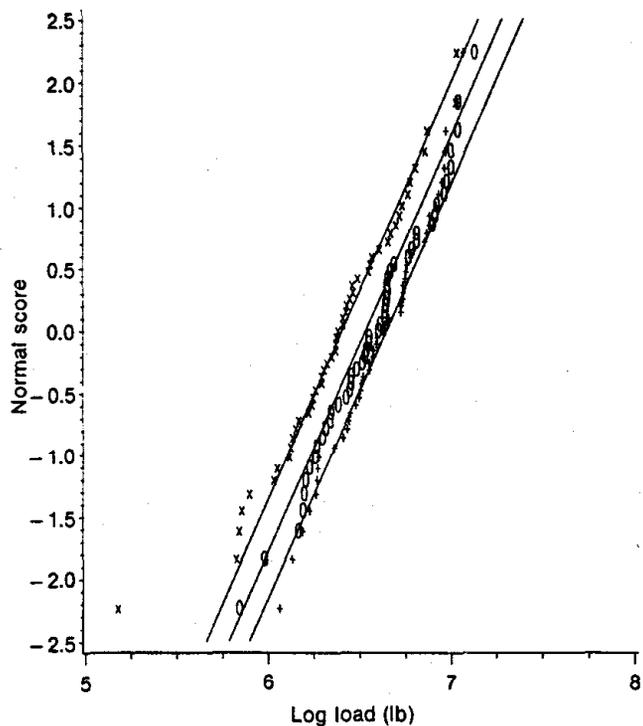


Figure 5 - Lognormal probability plot for edge knot rate of loading (ROL) tests (+, high ROL; 0, medium ROL; x, low ROL). y axis: normal score from a standard normal distribution with mean 0 and standard deviation 1 (- 1.645 is 5th percentile, 0 is 50th percentile, etc.), x axis: natural logarithm of load in pounds. Lines indicate Gerhards' fitted exponential cumulative damage model. (ML88 5413)

## Cumulative Damage Models

Several cumulative damage models have been developed for the DOL phenomenon. Nielson (1986a, 1986b) proposed a model based on visco-elastic fracture mechanics. Schaffer's model (1973, 1982) included compensation for temperature of exposure. Barrett and Foschi (1978) and Foschi and Barrett (1982) proposed a cumulative damage model with stress- and damage-dependent factors. Gerhards (1979) proposed an exponential cumulative damage model with a stress-dependent factor based on Miner's rule (1945). Caulfield (1985) obtained the same model as Gerhards by a chemical kinetics approach.

Use of these cumulative damage models to obtain reliable ratio estimates and confidence intervals requires methodology for estimating the parameters and associated variability. I am unaware of any variability estimates for the Nielson or Barrett and Foschi models. For the Gerhards model, details of parameter estimation and associated variability are given in Link, Gerhards, and Murphy (1988).

A brief review of the Gerhards model follows. This cumulative damage model describes the damage,  $\alpha$ , as a function of time,  $t$ , and load history,  $\sigma(t)$ . Damage ranges from 0 (undamaged specimen) to 1 (failed specimen) using the following equation:

$$\frac{d\alpha}{dt} = \exp \left[ -\frac{b}{c} + \frac{b\sigma(t)}{X^w} \right] \quad (3)$$

where  $b$ ,  $c$ , and  $w$  are model parameters and  $X$  is a random variable used to model the underlying variability of the strength of lumber. The underlying distribution of the strength of lumber is assumed to be either a lognormal or a Weibull distribution. In this discussion, the lognormal distribution is used. This damage model gives the following mean trends ( $X = 1$ ) for ROL and DOL tests (assuming instantaneous application of load):

$$\text{ROL load} = (1/c) + (1/b) \ln(bk) \quad (4)$$

$$\begin{aligned} \text{DOL } \ln(T) &= (b/c) - b(\text{load}) \text{ or} \\ \text{load} &= (1/c) - (1/b) \ln(T) \end{aligned} \quad (5)$$

where  $k$  is the ROL and  $T$  is the time to failure.

### Data Needed to Estimate Parameters

ROL and DOL tests are conducted using groups of matched specimens. A group usually contains 50 to 200 specimens. Matching is based on estimated failure strength using nondestructive measurements of modulus of elasticity and strength ratio.

For Gerhards' exponential cumulative damage model, the variability parameter,  $w$ , can be estimated as long as there are multiple specimens within a test group because it is a measure of variability within a test group. Estimation of the other two parameters,  $b$  and  $c$ , requires at least two groups of data from specimens that failed under two different ROL or at two different load levels. It is preferable to have more than two groups of data to judge the lack of fit of the data to the hypothesized model.

The Barrett and Foschi model (1978) contains six parameters; two are variability parameters. The model, as estimated by Foschi and Barrett (1982), is used with only two groups of data (using a larger sample size). It is not clear whether the two variability parameters or the other four parameters can be uniquely estimated with only two groups of data. Thus, with a fixed total sample size, it would be preferable to test more load levels with fewer specimens than to test only two load levels.

### Data Plots From ROL and DOL Tests

Data plots are useful for checking the distributional assumptions and the fit of the damage models. One of the most useful plots for checking distributional assumptions is a probability plot, which plots the failure data by the cumulative probability distribution function. A cumulative probability distribution function ranges from 0 to 1 (0 to 100 pct). If the failure data come from the hypothesized distribution, then the probability plot should look like a straight line. Therefore, the scale on the cumulative probability distribution axis depends on the distributional assumption. For example, if one assumes that the data come from a normal distribution, the failure data are plotted by a normal score. A normal score is a function of the cumulative probability of a standard normal

distribution: - 1.645 is the 5th percentile, 0 is the median, and 1.645 is the 95th percentile, etc. If one assumes that the data come from a lognormal distribution, then the logarithm of the failure data is plotted versus the normal score. A good discussion of probability-plots can be found in Nelson (1982).

ROL tests provide information about failure load, failure time, ROL, and order of failure. For ROL tests, an appropriate plot of the data would be a function of failure load versus some cumulative probability distribution. If one assumes that the underlying strength distribution is lognormal, then the appropriate plot would be of the log load versus a normal score. The ramp test data from the edge knot study can be used to illustrate this kind of plot (fig. 5). The plot used by many researchers of log time to failure versus logarithm of loading rate (fig. 6) is inappropriate for ROL data; there is no way to gauge the fit of the model because the difference in failure times overwhelms anything else. Foschi and Barrett (1982) used log time to failure versus logarithm of loading rate to demonstrate that the data are linear. But a plot of the median failure load versus logarithm of ROL (which should also be linear by their model) shows that the data are not linear.

A DOL test is made up of three segments: (1) initial ramp loading of the specimen to the desired constant load; (2) period during which the specimen is under constant load, given that it has survived the initial ramp loading; (3) second ramp loading (from 0 to failure) which determines the residual strength of a specimen that survives the constant load segment. In general, some specimens fail in each segment. DOL tests provide information about failure load, failure time, order of failure, and the segment in which the specimen failed.

For DOL tests, an appropriate plot of the data is a function of the logarithm of the time on test versus a cumulative probability distribution. The calculation of time on test may vary from one segment to another. I distinguish between total time on test (from initial ramp loading to failure) and time on test in a particular segment of the test.

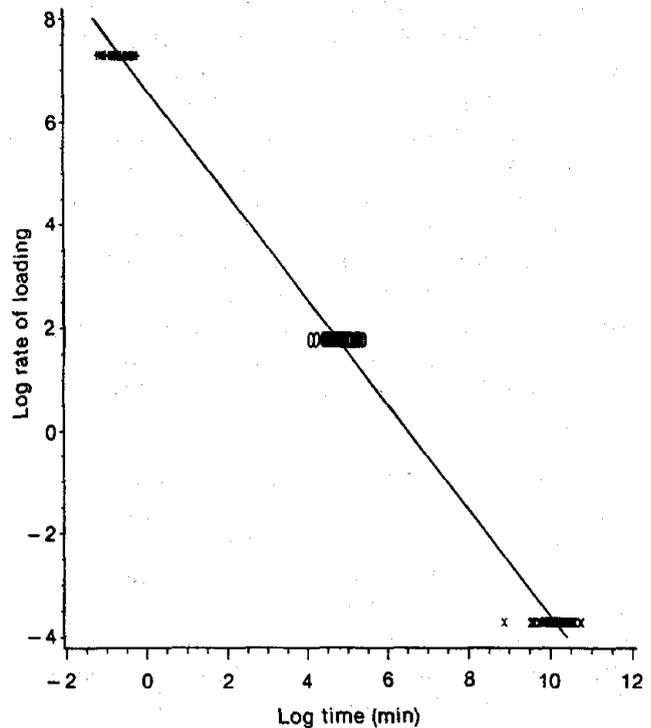


Figure 6 - Log rate of loading (ROL) versus log failure time for the edge knot ROL test (+, high ROL; 0, medium ROL; x, low ROL). y axis: natural logarithm of ROL in pounds per minute, x axis: natural logarithm of time to failure in minutes. (ML88 5414)

For specimens that fail during the first segment, that time is the total time on test. For specimens that fail during the second load segment, total time on test is either initial ramp time plus constant load time or the time on constant load only. If the latter is designated as total time on test, the time clock is reset to zero, creating a discontinuity in the plots between the initial ramp and constant load failures. The choice of time to failure under constant load affects the parameter estimation. If time on constant load is chosen, the first few failures under constant load (within seconds of uploading) may force the fitted model to be closer to these early failures than later failures; a very early failure may have an arbitrary short failure time that is subject only to the accuracy of the test equipment. The logarithm of this time on test is then an arbitrary large negative number. This influences the convergence criterion of the estimation procedure, which is to minimize the sum of squares of the difference

between actual and predicted logarithms of time. This problem does not occur if the time on test includes the ramp loading time, which is in minutes not seconds.

For specimens that fail in the third load segment, the time used is the failure load divided by the ROL during the third segment. This would be equivalent to the failure time obtained by setting the clock to zero at the beginning of the second ramp loading. This will appear as a continuation of the curve defined by the initial ramp loading failures, assuming there is no constant load segment and that rates of loading are the same for the first and third segments. There is no way to avoid the discontinuity of the constant load failures and the residual strength times, as total time on test would essentially appear as a straight vertical line. The plot for the edge knot study along with Gerhards' fitted exponential cumulative damage model (fig. 7) uses total time on test for the constant load failures.

### Total Time on Test Versus Time on Constant Load

The edge knot study had quite a few early failures in the constant load phase of the test (some failures occurred within seconds of uploading). The difference between the log total time on test and the log time on constant load was trivial after some time had elapsed (minutes or hours), but this difference was substantial for early constant load failures. This is seen in figure 8, which plots only the failures under constant load. The solid lines designate the same model shown in figure 7, and the dashed lines are the estimated model when the dependent variable is time on constant load. Note that the dashed lines fit the early failures quite well but are less suited to later failures, whereas the reverse is true for the solid lines. This discrepancy occurs only when there are a large number of early failures. The use of a different model will not change this situation, as the analysis must use log time to failure rather than the actual time. Of the data sets currently available at FPL, the total time on test versus the time on constant load matters in only the data set from the edge knot study because of the early failures.

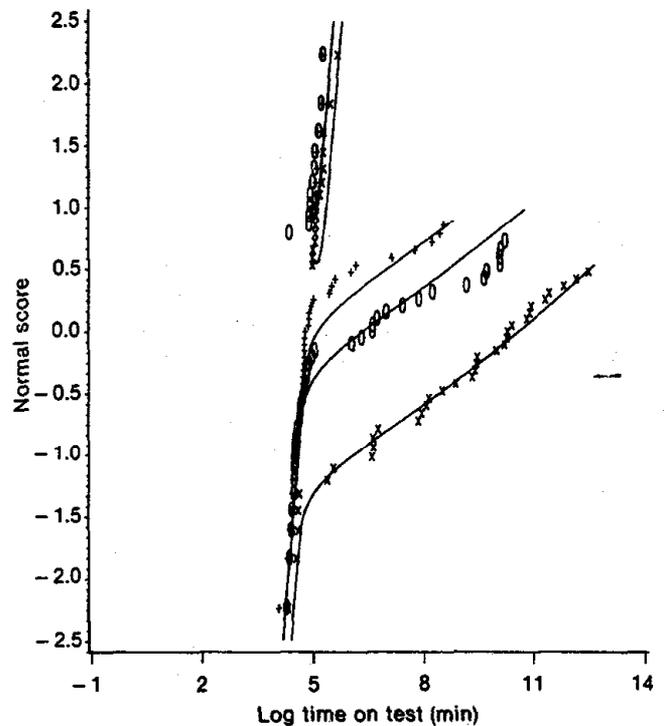


Figure 7 - Results from edge knot duration of load (DOL) study using total time on test (+, high constant load; 0, medium constant load; x, low constant load). y axis: normal score from a standard normal distribution with mean 0 and standard deviation 1 (-1.645 is 5th percentile, 0 is 50th percentile, etc.), x axis: natural logarithm of time to failure in minutes. Lines indicate Gerhards' fitted exponential cumulative damage model. (ML88 5415)

The early failures in the edge knot study occurred because the specimens sustained more damage during ramp uploading than predicted in Gerhards' model or any other model currently under consideration. DOL factors are needed for periods ranging from minutes to years. Using total time on constant load forces the fitted models to early failures (seconds, minutes, and hours), which flattens the estimated model (fig. 8). DOL factors computed with a flatter line (using total time on constant load) will be larger (e.g., DOL has a smaller effect) than those calculated with the steeper line (using total time on test). This is unlikely to be a conservative approach toward estimating DOL factors. Results of DOL research at FPL have often been reported using time on constant load. Because I prefer to fit

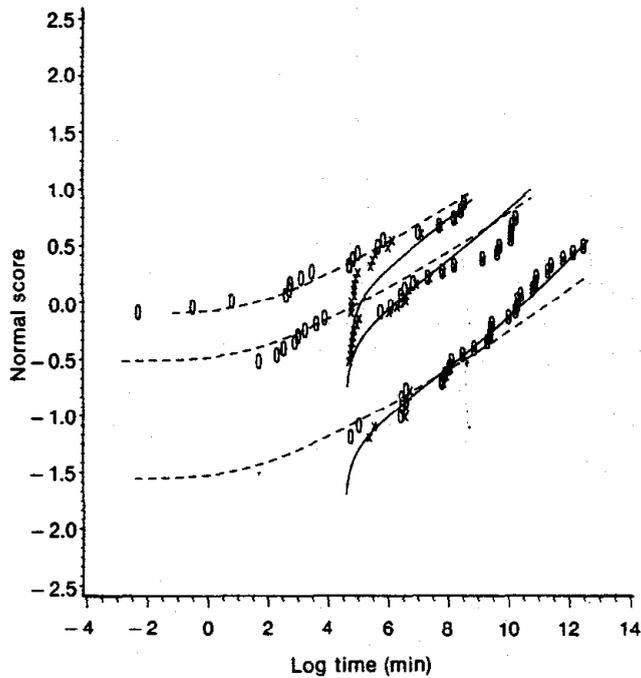


Figure 8 - Results from edge knot duration of load (DOL) study: x and solid line designate total time on test; o and dashed line designate time on constant load. y axis: normal score from a standard normal distribution with mean 0 and standard deviation 1 (-1.645 is 5th percentile, 0 is 50th percentile, etc.), x axis: natural logarithm of time to failure in minutes. (ML88 5416)

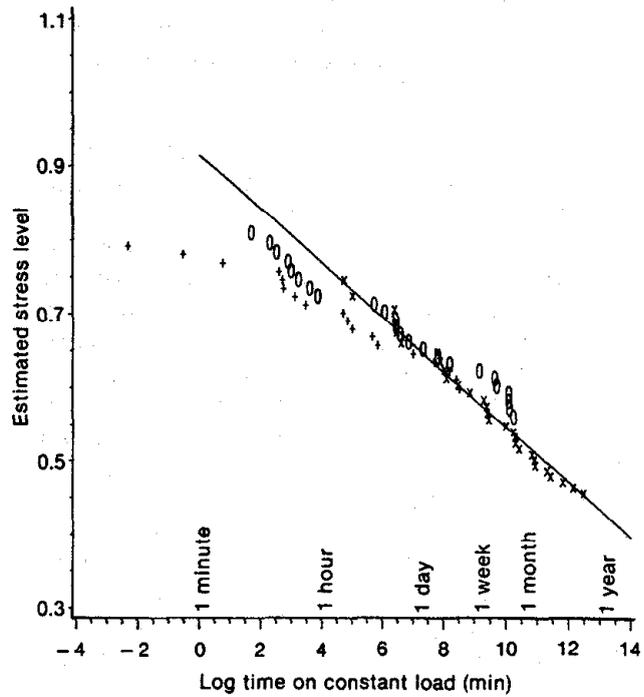


Figure 9 - Estimated stress levels for edge knot study using a cumulative damage model, with estimated parameters from Gerhards' model (+, high constant load; o, medium constant load; x, low constant load). y axis: stress level (proportion), x axis: natural logarithm of time to failure in minutes. (ML88 5417)

later failures rather than early failures, this paper presents only plots and fitted models that use total time on test.

### Calculation of DOL Factors

Estimated parameters and their variance estimates can be used to draw a line to the stress level versus log time-to-failure plots (fig. 9). Note that this curve is not fit to the data shown

but is the result of the parameter estimation. The appendix to this paper contains technical details of how to estimate DOL factors and their confidence intervals. As explained in the appendix, the stress level versus log time-to-failure plots are a function of a standard ROL, but the DOL factors are not.

## Results of Tests on Douglas-Fir 2 by 4 Structural Lumber by Grades

The FPL performed additional ROL (Gerhards and Link 1986) and DOL (Gerhards 1988) tests on Douglas-fir 2 by 4's. The lumber was graded, using strength ratio, into Select Structural, No. 1, No. 2, and No. 3 samples. No. 1 lumber was not tested. There were up to three ROL tests (ROL1, ROL2, and ROL3) and up to three DOL tests (DOL1, DOL2, and DOL3) for each grade. The sample sizes and test conditions are given in table 2. The rates of loading were 300, 3, and 0.03 pounds per minute for the ramp tests. The constant load tests contained several load levels in each test. These tests are known as step constant load tests and are used to get additional constant load failures. The specimens were loaded to the first constant load level (300 pounds per minute), held on that constant load level for a predetermined time, loaded to the second load level, etc. The steps used in the constant load tests were estimated percentiles of the underlying failure distribution. The constant load tests used step constant loads with a combination of 40th and 70th (DOL1), 5th, 15th, and 40th (DOL2), and 5th and 15th (DOL3) percentile loads.

**Table 2 - Sample sizes and test conditions of Douglas-fir 2 by 4's**

Grade	Sample size					
	Rate of loading tests <sup>a</sup>			Duration of load tests <sup>b</sup>		
	1	2	3	1	2	3
Select Structural	100	50	100	50	50	50
No. 2	100	50	100	50	50	50
No. 3	100	—	—	50	50	—

<sup>a</sup> Test conditions for rate of loading (ROL) tests were: ROL1, 300 lb/min; ROL2, 3 lb/min; and ROL3, 0.03 lb/min.

<sup>b</sup> Step constant loads for duration of load (DOL) tests were: DOL1, steps at 40th and 70th percentiles; DOL2, steps at 5th, 15th, and 40th percentiles; and DOL3, steps at 5th and 15th percentiles.

The fitted model for ROL data uses only failures from ROL1, ROL2, and ROL3 tests. Likewise, the fitted model for DOL data uses only constant load failures from DOL1, DOL2, and DOL3 constant load tests. For the FPL DOL tests, the parameter estimates do not change substantially if one uses the constant load failures from all the constant load phases or just the first set of failures, nor do they change substantially if one

uses total time on test or time on constant load. Therefore, the plotted fit is that of total time on test using only the constant load failures in the first "step." Plots of the fitted damage model for the ROL data (fig. 10) and the DOL data (fig. 11) give a visual impression of the model fit to the data. No model was fit to the No. 3 ROL test as data exist for only one ROL which is insufficient to estimate the model parameters. Confidence intervals can be plotted for these fitted curves (Link, Gerhards, and Murphy 1988). The overall impression is that the model generally fits the data, but there appear to be systematic departures from the model which indicate additional modeling might be useful.

The stress level versus log time-to-failure plots (fig. 12) show the fitted DOL model and the first step constant load failures. The corresponding DOL factors, along with 95 percent confidence intervals, for these data as well as edge knot data are given in table 3. The DOL and ROL curves are plotted along with Wood's equation (fig. 13). Wood's equation does not match up well with the experimental data because of the definition of the standard ROL. The relative vertical position

**Table 3 - Duration of load factors for Forest Products Laboratory Douglas-fir 2 by 4's point estimates<sup>a,b</sup>**

Source	Type	Duration of load				
		1 Day	1 Week	2 Months	10 Years	50 Years
NDS		.82	.77	.71	.62	.56
Edge knot	ROL <sup>c</sup>	.87 (.83,.92)	.83 (.77,.89)	.78 (.70,.86)	.69 (.58,.81)	.66 (.53,.78)
	DOL <sup>d</sup>	.75 (.67,.83)	.67 (.56,.78)	.58 (.44,.71)	.40 (.20,.59)	.33 (.11,.55)
SS	ROL	.88 (.83,.92)	.83 (.78,.89)	.79 (.71,.86)	.70 (.59,.80)	.66 (.55,.78)
	DOL	.80 (.71,.89)	.73 (.61,.86)	.65 (.49,.82)	.51 (.28,.74)	.45 (.20,.71)
No. 2	ROL	.91 (.87,.95)	.88 (.82,.94)	.85 (.77,.92)	.78 (.68,.88)	.76 (.64,.87)
	DOL	.78 (.69,.87)	.70 (.58,.82)	.61 (.46,.77)	.45 (.23,.67)	.39 (.14,.63)
No. 3	DOL	.72 (.61,.83)	.62 (.47,.77)	.51 (.32,.70)	.31 (.03,.58)	.23 (.00,.53)

<sup>a</sup> Baseline = 5 minutes.

<sup>b</sup> Ninety-five percent confidence intervals in parentheses.

<sup>c</sup> ROL = rate of loading.

<sup>d</sup> DOL = duration of load.

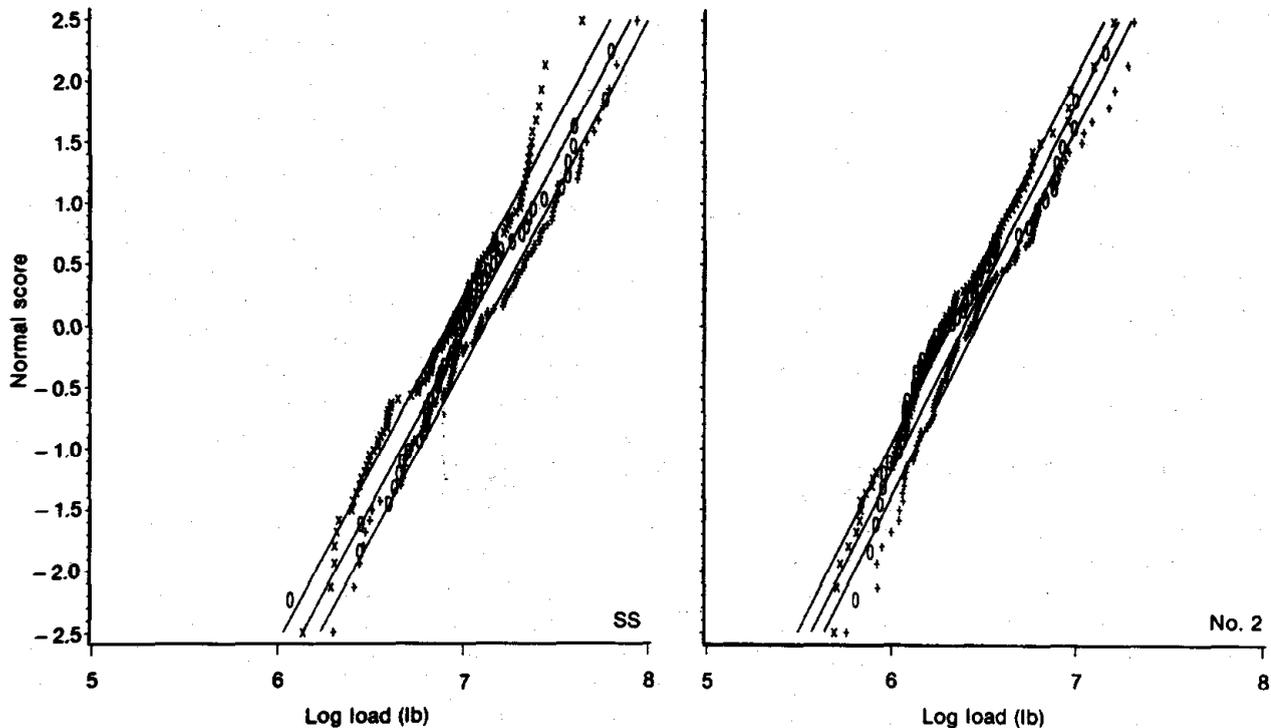


Figure 10 - Lognormal probability plots for rate of loading (ROL) tests of Douglas-fir Select Structural and No. 2 lumber (+, high ROL; 0, medium ROL; x, low ROL). y axis: normal score from a standard normal distribution with mean 0 and standard deviation 1 (-1.645 is 5th percentile, 0 is 50th percentile, etc.), x axis: natural logarithm of load in pounds. (ML88 5418)

is not important; it is the ratio of two values which matters.

Several results using Gerhards' exponential cumulative damage model are immediately apparent from table 3 and figure 13:

(1) The parameter estimates and DOL factors are not the same for DOL and ROL tests. Although, theoretically (by Gerhards' (1979) model or any of the other models currently under consideration for modeling the effect of DOL) they should give the same result; in practice they do not. Therefore one cannot substitute ROL tests, which can be done much more quickly, for DOL tests. ROL tests predict a much smaller DOL effect. If both the ROL and DOL data sets are used to estimate the cumulative damage parameters, the ROL tests will determine the parameter estimates because the variability of log time to failure in a ROL test is at least 20 times smaller than the variability of log time to failure under DOL tests.

(2) The DOL factors for lumber DOL tests appear to be smaller than those derived by Wood (1951).

(3) Stress level curves using DOL tests were steeper as the quality of the material decreases, and those using ROL tests had the reverse trend. However, these trends are not statistically significant.

(4) Confidence intervals increase as the load duration increases. This is to be expected as there is less information available at longer load times.

(5) Confidence intervals for ROL tests were smaller than the corresponding intervals for DOL tests because (a) sample size was larger in the ROL tests and (b) variability of the parameter estimates was larger for DOL tests compared to ROL tests.

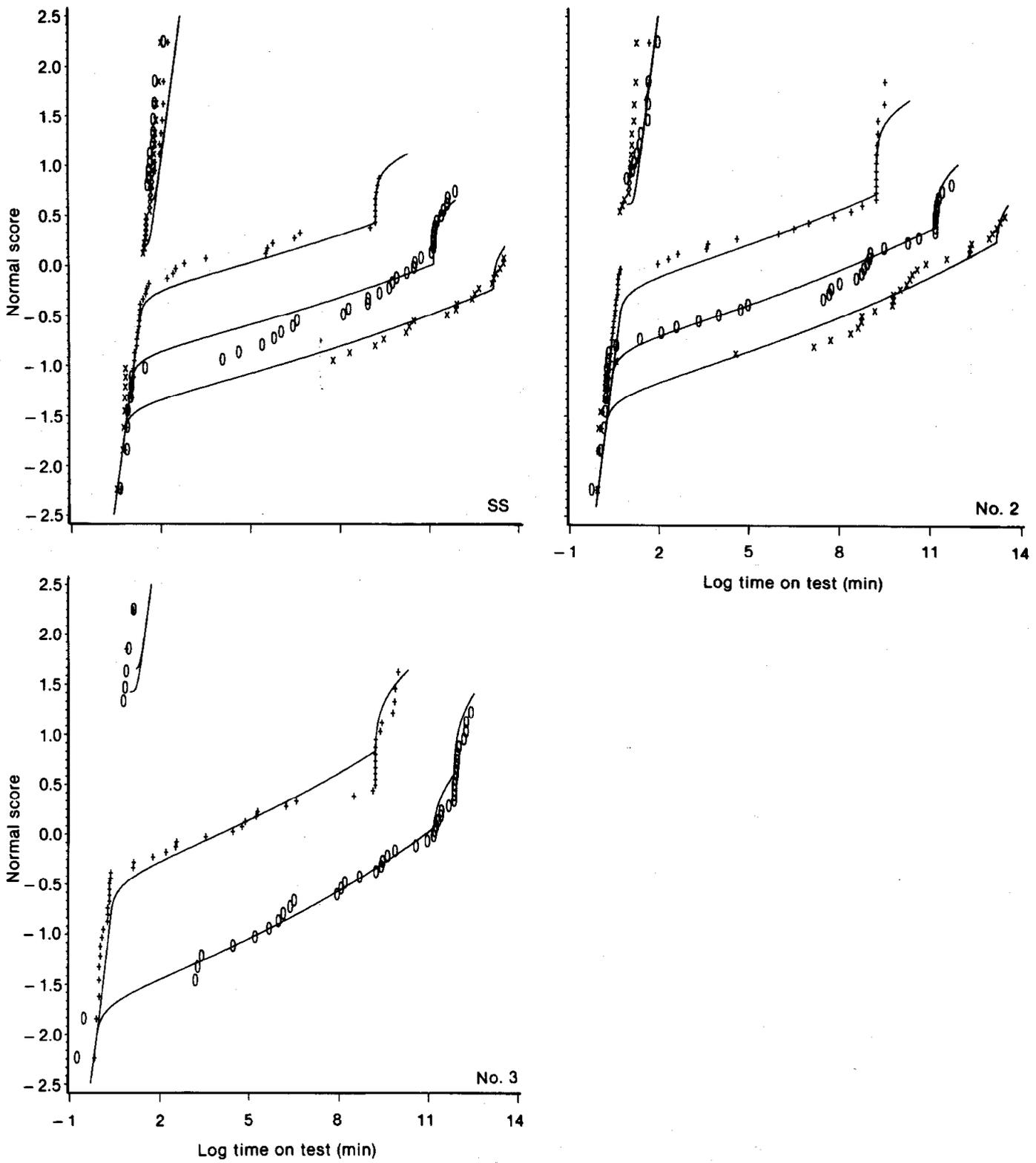


Figure 11 - Results from the Douglas-fir constant load study of Select Structural, No. 2, and No. 3 lumber using total time on test (+, high constant load; 0, medium constant load; x, low constant load). y axis: normal score from a standard normal distribution with mean 0 and standard deviation 1 (- 1.645 is 5th percentile, 0 is 50th percentile, etc.), x axis: natural logarithm of time to failure in minutes. (ML88 5419)

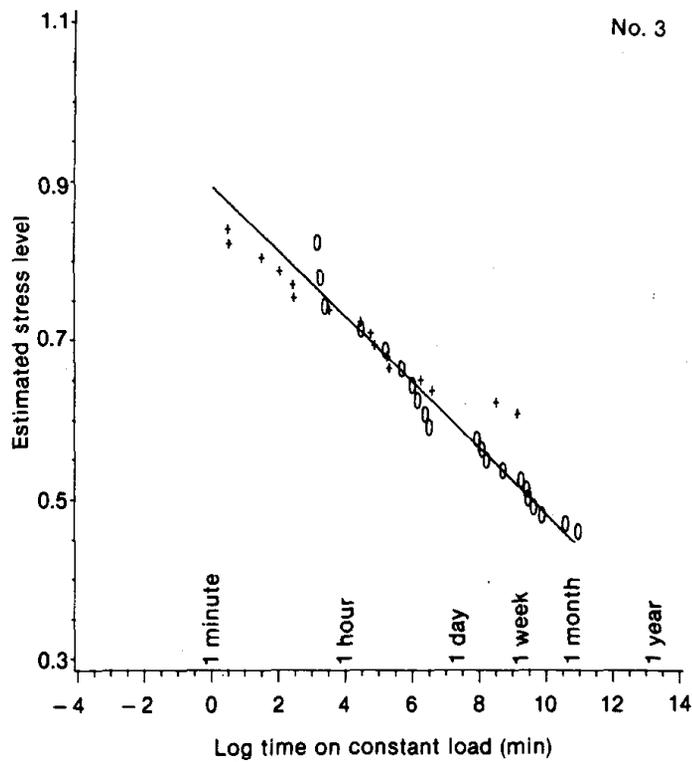
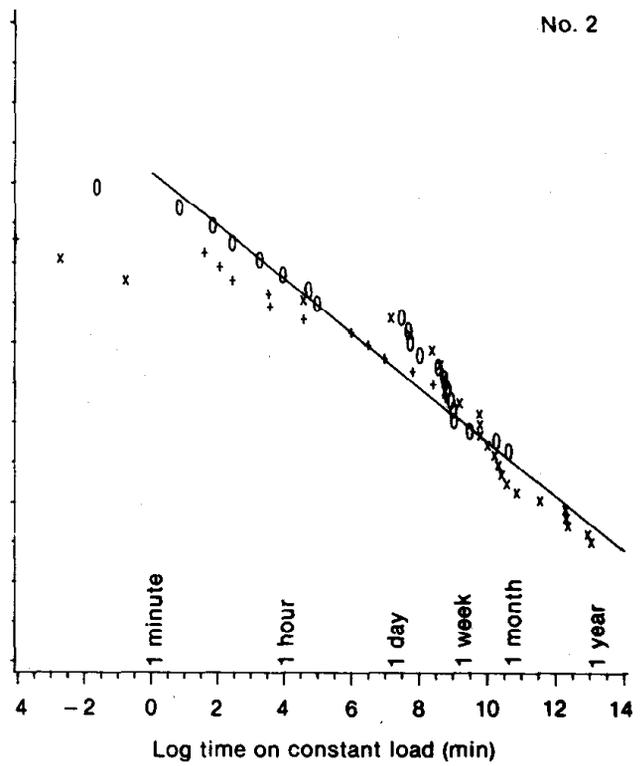
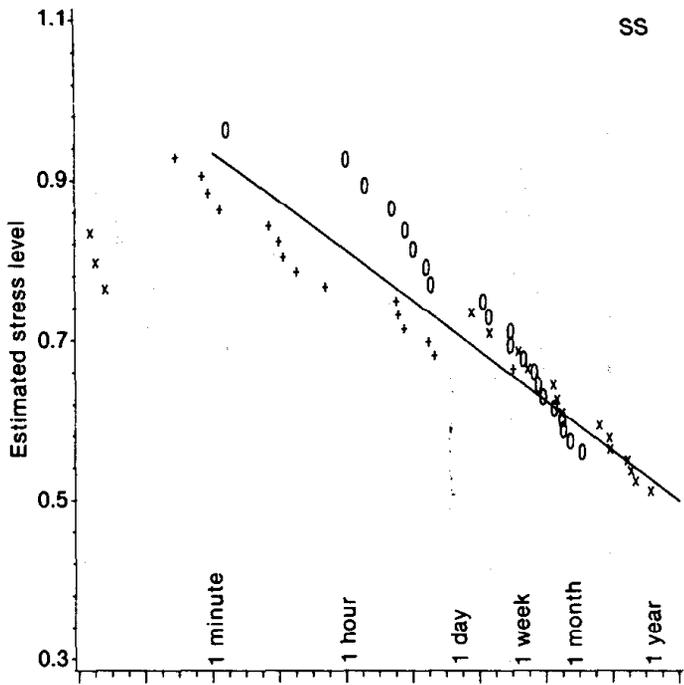


Figure 12 - Estimated stress levels using cumulative damage model, with estimated parameters from Gerhards' model, for Douglas-fir constant load study of Select Structural, No. 2, and No. 3 lumber; failures from first constant load level only (+, high constant load; 0, medium constant load; and x, low constant load). y axis: stress level (proportion), x axis: natural logarithm of time to failure in minutes. (ML88 5420)

## Other Aspects of DOL Research

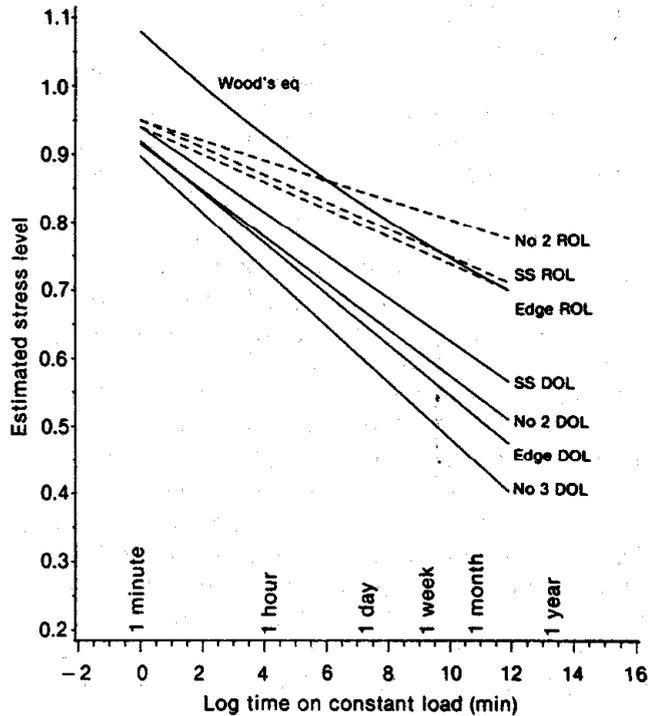


Figure 13 - Estimated stress level equations for Forest Products Laboratory Douglas-fir 2 by 4 studies and Wood's duration of load curve (solid lines: constant load studies, dashed lines: rate of loading studies). y axis: stress level (proportion), x axis: natural logarithm of time to failure in minutes. (ML88 5421)

Three other aspects of damage modeling and design of DOL tests deserve comment: (1) the need for additional modeling, (2) usefulness of step loads and required percentage of failures, and (3) matching of lumber samples.

### Additional Modeling

Systematic departures of the data from Gerhards' exponential cumulative damage model suggest the need for continued effort in modeling the effect of DOL. All of the current models use an exponential function for damage accumulation which implies that the damage accumulates only near the failure time or load. It has been recognized that more damage accumulates at lower stress levels than the models predict. Two suggested modifications to Gerhards' model, which model damage as a power function rather than an exponential function, have not solved the problem. The first modification is

$$\frac{d\alpha}{dt} = nt^{n-1} \left[ \exp \left( -a + \frac{b\sigma(t)}{X^w} \right) \right]^n$$

It is impossible to estimate  $n$  using ROL tests. Although an estimate of  $n$  was obtained for DOL tests, it did not change the plot of the fitted model.

The second alternative modification is

$$\frac{d\alpha}{dt} = \frac{[\sigma(t)]^b}{X^w \exp(c)}$$

One can estimate the parameters of this model for ROL and DOL tests. However, the fitted model for ROL tests does not change. For DOL tests, the underprediction of failure times was worse using the power model (dashed line) than using the exponential model (solid line) (fig. 14).

### Step Loads and Required Percentage of Failures

DOL tests using step loads (progressively higher stresses) are common in the testing of electronic parts (Yurkowsky, Schafer, and Finkelstein 1967). In an effort to obtain more information from additional failures and to accelerate the tests, Gerhards (1986) used a three-step constant load to compare the DOL effect for southern pine

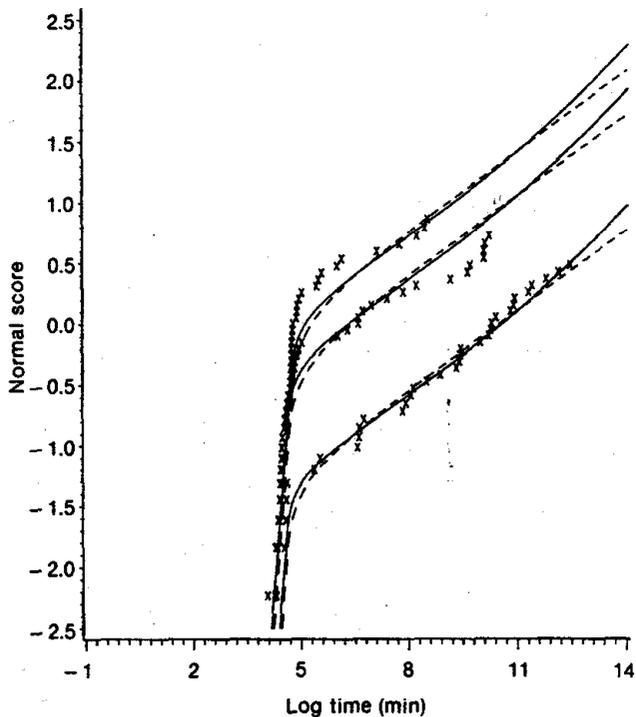


Figure 14 - Gerhards' exponential (solid line) and power models (dashed lines) fitted to the edge knot study constant load tests. y axis: normal score from a standard normal distribution with mean 0 and standard deviation 1 (- 1.645 is 5th percentile, 0 is 50th percentile, etc.) x axis: natural logarithm of time to failure in minutes. (ML88 5422)

lumber dried conventionally and at high temperatures. Only one ramp test and one three-step constant load test were used. Because there were very few failures on either of the two lower steps, the experiment was essentially reduced to one ramp and one constant load test. The resultant data were insufficient for estimating the parameters of Gerhards' exponential cumulative damage model. Gerhards combined these data into one data set and plotted a stress level versus log time to failure using a hypothesized distribution of static strength. He concluded that the data exhibited a much smaller DOL effect than that predicted using Wood's equation. However, different results are obtained if the conventional and high-temperature data sets are kept separate (fig. 15). The different symbols denote drying temperature and load level. There are no consistent patterns.

Gerhards' (1988) study of the effect of grade on the DOL behavior of Douglas-fir 2 by 4's also featured two or three constant load levels within a single test. Although these tests contained a reasonable number of failures at each constant load level, the estimated parameters and their standard errors do not change much if all the constant load failures or just those at the lowest step are used. The stress level versus log time-to-failure plots for this study (fig. 12) show constant load failures from only the lowest step. Because the damage model predicts little damage, it is tempting to include constant load failures from the higher load levels as well, but this is likely to increase the variability of the data. Figure 16 shows estimated stress levels for the Douglas-fir Select Structural specimens using failures from all constant load levels. Note that some damage occurred in the lowest step; the estimated stress levels for the second step failures were lower than those for the first step failures during the first week of constant load.

A sufficient number of failures are needed in each constant load test. As mentioned earlier, early and late failures do not follow the same pattern. When the high constant load is terminated, later failures cannot be observed. In the edge knot study (Gerhards and Link 1987), the high constant load was held for 3 to 4 days. After the initial 2 hours of constant load, there were only 10 (out of 49) failures at the high constant load level versus 16 at the middle constant load level and 29 at the low constant load level. Therefore, the lower constant load levels had a greater influence in estimating the parameters. In the later Douglas-fir 2 by 4 test, the high constant load level was maintained for a longer period. The estimation procedure requires that 30 to 50 percent of the specimens fail in the constant load segment of the test.

### Matching of Lumber Samples

Matching the lumber samples for each test is important. Lumber contains a large amount of variability, and samples should therefore be matched in as large a group as practical. Small groups of samples, such as the matched groups of 25 specimens used in the Douglas-fir study (Gerhards 1988), have a high degree of variability.

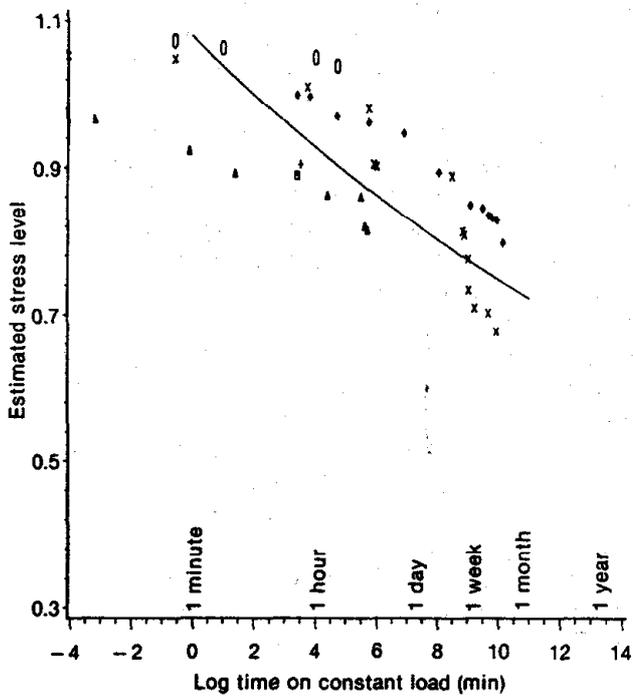


Figure 15 - Estimated stress levels using a cumulative damage model with estimated parameters from Gerhards' model for the high-temperature drying constant load study, using failures from all constant load levels. (Conventional drying: +, step 1; 0, step 2; x, step 3. High-temperature drying: Δ, step 2; ◇, step 3). y axis: stress level (proportion), x axis: natural logarithm of time to failure in minutes. (ML88 5423)

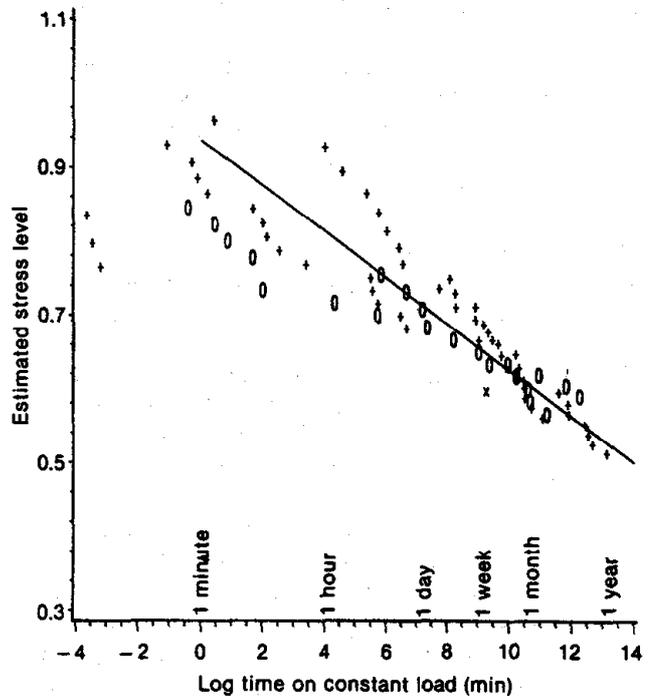


Figure 16 - Estimated stress levels using a cumulative damage model with estimated parameters from Gerhards' model for the Douglas-fir Select Structural constant load study, using failures from all constant load levels (+, first constant load; 0, second constant load). y axis: stress level (proportion), x axis: natural logarithm of time to failure in minutes. (ML88 5424)

A larger matching group will decrease some of the variability. That is, if the sample size is 100, the lumber should be matched in groups of 100. The sample size in each group will depend for the

most part on the resources at hand. The results of the Douglas-fir lumber tests indicate that a minimum of 50 samples should be used.

## Conclusions

The results of current FPL tests on DOL suggest

- Inclusion of confidence intervals increases the usefulness of point estimates of DOL factors by allowing comparison of results from different experiments and the currently used values.
- Fitting regressions to stress level versus log time-to-failure plots does not give statistically correct confidence intervals for DOL factors.
- Using some cumulative damage model that includes estimates of both standard errors and parameters provides confidence intervals for DOL factors.
- The exponential cumulative damage model provides reasonable starting estimates of DOL factors. Additional modeling might be useful, since the exponential cumulative damage model apparently does not account for damage at a lower stress level, nor does the model fit both late and early constant load failures.
- Parameter estimates from ROL tests are apparently not useful for predicting the DOL results in structural lumber.
- Step loads within a DOL test do not appear to add information about the model parameters.
- DOL tests should be designed to allow for at least 30 to 50 percent failures under constant load. Also, one should run at least one more test than the number of tests needed to estimate the model parameters. This allows at least a visual assessment of lack of fit to the model.
- Matching of lumber samples is important; a minimum of 50 specimens should be used for each test sample.

## Literature Cited

- Barrett, J. D.; Foschi, R. O.** 1978. Duration of load and probability of failure in wood. *Canadian Journal of Civil Engineering*. 5(4): 505-532.
- Caulfield, D. F.** 1985. A chemical kinetics approach to the duration-of-load problem in wood. *Wood and Fiber Science*. 17(4): 504-521.
- Foschi, Richardo O.; Barrett, J. David.** 1982. Load-duration effects in western hemlock lumber. *Journal of the Structural Division, American Society of Civil Engineers*. 108(ST7): 1494-1510.
- Gerhards, Charles C.** 1979. Time-related effects of loading on wood strength: A linear cumulative damage theory. *Wood Science*. 11(3):139-144.
- Gerhards, C. C.** 1986. High-temperature drying of southern pine 2 by 4's: Effects on strength and load duration in bending. *Wood Science Technology*. 20(4): 349-360.
- Gerhards, C. C.** 1988. Effect of grade on load duration of Douglas-fir lumber in bending. *Wood and Fiber Science*. 20(1): 146-161.
- Gerhards, C. C.; Link, C. L.** 1986. Effect of loading rate on bending strength of Douglas-fir 2 by 4's. *Forest Products Journal*. 36(2): 63-66.
- Gerhards, C. C.; Link, C. L.** 1987. A cumulative damage model to predict load duration characteristics of lumber. *Wood and Fiber Science*. 19(2): 147-164.
- Johns, Kenneth; Madsen, Borg.** 1982. Duration of load effects in lumber. Part I: A fracture mechanics approach. *Canadian Journal of Civil Engineering*. 9(3): 502-525.
- Link, Carol L.; Gerhards, Charles C.; Murphy, Joseph F.** 1988. Estimation and confidence intervals for parameters of a cumulative damage model. Res. Pap. FPL 484. Madison, WI: U.S. Department of Agriculture, Forest Service, Forest Products Laboratory.
- Miner, M. A.** 1945. Cumulative damage in fatigue. *Journal of Applied Mechanics, Transactions of American Society of Mechanical Engineers*. 12(3). A159-A164.
- National Design Specifications for Wood Construction.** 1986. National Forest Products Association, Washington, DC.
- Nelson, Wayne.** 1982. Applied life data analysis. John Wiley and Sons.

- Nielson, Lauge Fuglsang.** 1986a. Wood as a cracked viscoelastic material. Part I: Theory and applications. In: Proceedings of international workshop on duration of load in lumber and wood products; 1985 September 12-14; Richmond, BC, Canada. Special Publication SP-27. Forintek Canada Corporation, pp 67-78.
- Nielson, Lauge Fuglsang.** 1986b. Wood as a cracked viscoelastic material. Part II: Sensitivity and justification of a theory. In: Proceedings of international workshop on DOL in lumber and wood products; 1985 September 12-13; Richmond, BC, Canada. Special Publication SP-27. Forintek Canada Corporation, pp 79-89.
- Schaffer, E. L.** 1973. Effect of pyrolytic temperatures on the longitudinal strength of dry Douglas-fir. *Journal of Testing and Evaluation*. 1(4): 319-329.
- Schaffer, Erwin L.** 1982. Influence of heat on the longitudinal creep of dry Douglas-fir. In: Robert W. Meyer and Robert M. Kellogg, eds. Proceedings of structural use of wood in adverse environments; 1978 May 15-18; Vancouver, BC, Canada. New York, NY: Van Nostrand Reinhold Company, pp 20-32.
- Wood, Lyman W.** 1951. Relation of strength of wood to duration of load. Rep. No. 1916. Madison, WI: U.S. Department of Agriculture, Forest Service, Forest Products Laboratory.
- Yurkowsky, W.; Schafer, R. E.; Finkelstein, J. M.** 1967. Accelerated testing technology. Vol. 1 and 2. Tech. Rep. RADC-TR-420. New York: Rome Air Development Center, Air Force Systems Command, Griffiss Air Force Base.

## Appendix Estimation of DOL Factors and Confidence Intervals Using Gerhards' Exponential Cumulative Damage Model

Stress level is the ratio of constant load to a standard static strength. It is also the ratio of load under ramp loading to the load under constant load at any given time:

$$SL(t) = \text{load under constant}/\text{load under ramp}$$

$$= \frac{b - c \ln(t)}{b + c \ln(bk)}$$

using equations (4) and (5) of the main text. While this equation assumes mean trends and instantaneous constant load, it can be modified for the ramp time. Note that the stress level is still a function of the ROL,  $k$ . This equation defines the line on the stress level versus log time-to-failure plot (fig. 10). The constant load data can also be shown on the same plot. The equation used is similar but uses the variability parameter,  $w$ , to estimate the ramp failure load.

DOL factors may be calculated from the stress level equation. Again, the DOL factor for time,  $t_2$ , given a baseline time,  $t_1$ , is the ratio of  $SL(t_2)/SL(t_1)$ :

$$\text{DOL factor} = \frac{b - c \ln(t_2)}{b - c \ln(t_1)}$$

Note that this DOL factor is not a function of the ROL,  $k$ . By taking the ratio of two stress levels, one effectively eliminates the problem of having

to specify a standard ROL. In that the DOL factor is a function of the baseline and target times as well as the estimated parameters, confidence intervals can be obtained for the DOL factor using the estimated variability of the parameters  $b$  and  $c$ .

$$\begin{aligned} \text{Standard error} \\ (\text{DOL factor}) &= \text{sedol} \\ &= [(\text{sec} \times \text{dc})^2 + (\text{seb} \times \text{db})^2 \\ &\quad + 2(\text{rbc} \times \text{sec} \times \text{dc} \times \text{seb} \\ &\quad \times \text{db})]^{1/2} \end{aligned}$$

$$\begin{aligned} \text{where: } \text{sec} &= \text{standard error of } c \\ \text{seb} &= \text{standard error of } b \\ \text{rbc} &= \text{correlation of } b \text{ and } c \\ \text{dc} &= b[\ln(t_2) - \ln(t_1)]/[b - c \ln(t_2)]^2 \\ \text{db} &= c[\ln(t_1) - \ln(t_2)]/[b - c \ln(t_2)]^2 \end{aligned}$$

Parameter estimates, standard errors, and correlations of the parameter estimates may be found using the computer program in Link, Gerhards, and Murphy (1988). Confidence intervals for the DOL factor are then: DOL factor  $\pm z_\alpha$  (sedol), where  $z_\alpha$  is the appropriate standard normal coefficient, e.g., 1.96 for a 95 percent confidence interval.