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Estimation and Confidence Intervals for Parameters of a Cumulative Damage Model

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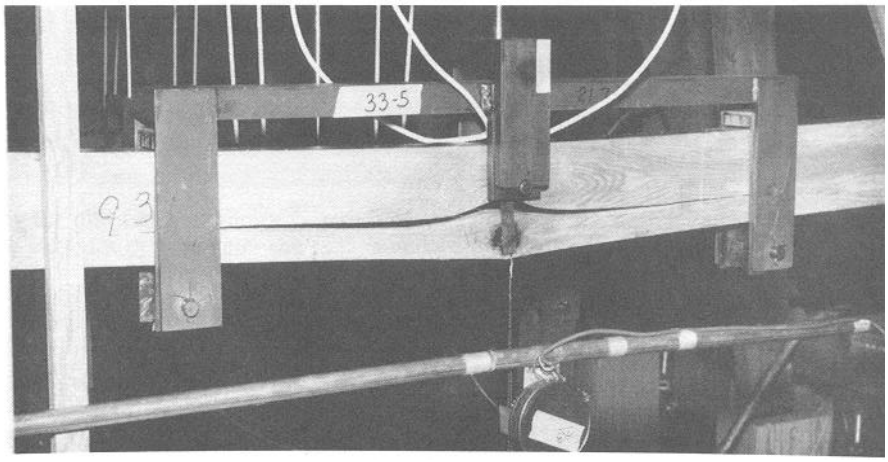

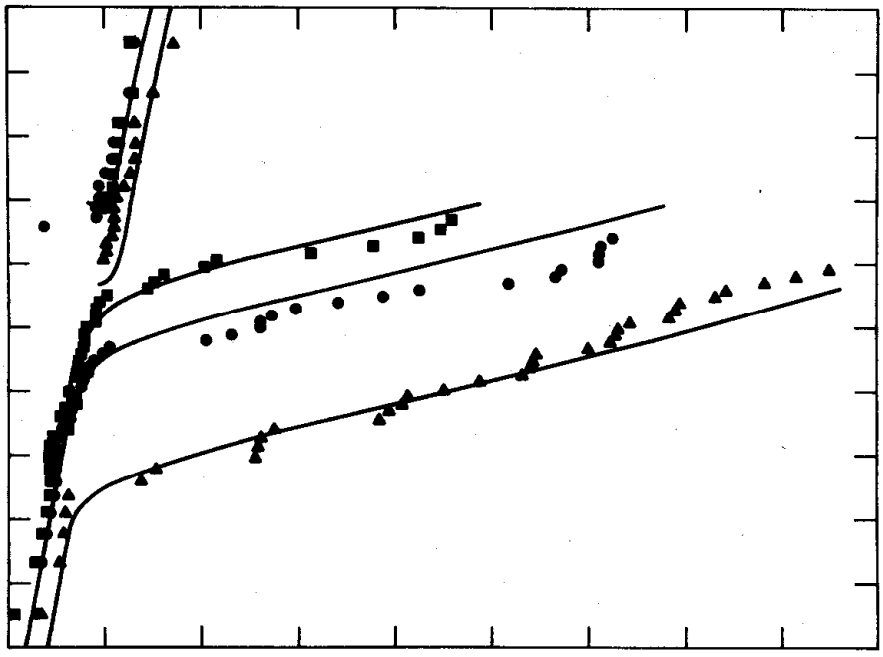

Abstract

An exponential cumulative damage model, $d\alpha/dt = \exp(-a + b'\sigma(t)/\sigma_o)$, has been proposed for the duration of load phenomenon in lumber. The amount of damage α (0 = no damage, 1 = failure) depends upon the load history, $s(t)$; the underlying inherent strength, σ_o ; and parameters a and b´. Experimental load histories include ramp loading, $\sigma(t) = kt$, and constant loading, $\sigma(t)$ = load, after a period of ramp loading. We model the underlying inherent strength by a lognormal or Weibull distribution. The parameters are then estimated using iterative reweighted nonlinear least squares. A heuristic argument is used to derive variances which are shown to be approximate through simulation. A computer program used to estimate the parameters is available from the National Technical Information Service (Link 1987).

Keywords: Iterative reweighted least squares, simulation, duration of load.

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Estimation and Confidence Intervals for Parameters of a Cumulative Damage Model

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Introduction

Lumber is known to be affected by the phenomenon known as duration of load. The following behavior demonstrates this phenomenon:

- the "strength" of a piece of lumber depends upon the rate with which it is loaded to failure;
- a piece of lumber may be unable to carry a given load indefinitely. Failure means that the piece of lumber is no longer able to carry a given load.

Gerhards (1979) proposed an exponential cumulative damage model based on Miner's (1945) rule to model this duration of load phenomenon. Damage, α, to a piece of lumber ranges from zero, indicating no damage, to 1, indicating failure. Damage accumulates over time, t, by the following equation:

$$d\alpha/dt = \exp(- a + b'\sigma(t)/\sigma_o) \qquad (1)$$

where a and b´ are parameters, $\sigma(t)$ is the load history over time, and s_o is the underlying inherent strength of the piece of lumber. Typical experimental load histories are

ramp loading: $\sigma(t) = kt$, where k is the rate of loading

constant loading: $\sigma(t)$ = load (usually preceded by a period of ramp loading to the constant load level).

We model the underlying strength of a piece of lumber by either a two-parameter lognormal or Weibull distribution; $\sigma_o = b'' X^w$. X is a random variable equal to

lognormal $X = \exp(R)$ where R is a standard normal (0, l) random variable.

Weibull $X = - \ln(1 - R)$ where R is a uniform (0, l) random variable.

The parameter b″ is a scale parameter, and w is the standard deviation of ln(strength) for the lognormal distribution and the inverse of the shape parameter for the Weibull distribution. Since b´ and b″ appear only as a ratio, b = b´/b″ will be substituted. Therefore, one can rewrite the cumulative damage model, equation (1), as

$$d\alpha/dt = \exp(- a + b\sigma(t)/X^w) \qquad (2)$$

where a, b, and w are the parameters to be estimated.

Although one can integrate the cumulative damage model for any load history, $\sigma(t)$, it is useful to determine the time to failure for two simple load histories, ramp and constant loading. For ramp loading, $\sigma(t) = kt$, and the time to failure, T, is

$$T = [X^w/(bk)] \ln [(\exp(a)bk/X^w) + 1]$$

$$\text{or load} = kT = [X^w/b] \ln [(\exp(a)bk/X^w) + 1]$$

$$\approx [X^w/b] [a + \ln(bk) - w \ln(X)] \qquad (3)$$

$$\text{or } \ln(T) \approx w \ln(X) - \ln(bk) + \ln[a + \ln(bk) - w \ln(X)] \qquad (4)$$

To see the effect of the parameters a and b on load or ln(T), eliminate the effect of w by setting X equal to 1. If the random variable X equals 1, the median load for the lognormal distribution and the 63rd percentile for the Weibull distribution is given by

$$\text{load} \approx a/b + (\ln(bk))/b \qquad (5)$$

Thus, the (median or 63rd percentile) load that a piece of lumber is able to carry before failure (its "strength") is linearly related to the natural logarithm of the rate of loading. Equation (5) is the mathematical model of the duration of load behavior under ramp loading.

Under a constant load (assuming that the load was instantaneously applied at time t = 0), the time to failure, T, is

$$T = \exp(a - b(\text{load})/X^w)$$

or

$$\ln(T) = a - b(\text{load})/X^w \qquad (6)$$

Thus, even if the load is zero, this theory implies that a piece of lumber will fail eventually, although for practical purposes this time is infinite. The natural logarithm of the time to failure of the median or 63rd percentile piece is linearly related to the constant load level. Equation (6) is the mathematical model of the duration of load behavior under constant loading.

Estimation of Parameters

To estimate the parameters, the dependent and independent variables must be specified. For failure under ramp loading, a logical dependent variable is the natural logarithm of the time to failure, ln(T). Actual time to failure turns out to be an inappropriate choice as the longest failure times (those from the slowest ramp rate) dominate the estimation procedure due to their larger variability. For failures under constant load, natural logarithm of time to failure, In(T), assuming instantaneous uploading would be an appropriate choice for the dependent variable. In practice, instantaneous uploading to a given constant load level is impossible, and specimens are usually subjected to a ramp loading, from time 0 to t_1, up to the desired constant load level. Therefore, two logical choices for the dependent variable are

1. The natural logarithm of the total time on test, In(T), or
2. The natural logarithm of the time on constant load, $\ln(T - t_1)$.

If one chooses the latter, the parameter estimation may be more heavily influenced by the early failures under constant load, if there are several failures immediately after uploading such that $T - t_1$ is less than one. In this case $\ln(T - t_1)$ may be a large negative number and will unduly affect the estimation process. Since it is impractical to carry out an experimental test until all the specimens have failed under constant load, surviving specimens are unloaded after a period of time, t_2. The survivors are then subjected to ramp loading until failure occurs. The dependent variable chosen for the survivors of constant load is the natural logarithm of the time on the second ramp loading, $\ln(T - t_2)$. To use the natural logarithm of the total time on test would not differentiate these failures as $T - t_2$ is quite small compared to t_2.

The independent variable is the underlying random variable X. Since this variable is unknown, some value must be specified in order to estimate the parameters. To do this, specimens are generally tested in groups of n. One test of n specimens consists of a single rate of loading, or one constant load level. (A test might start specimens at one load level and then increase the load to other levels, but this type of test is not considered here.) The order of failure, i of n, is known. A logical choice of R (the independent variable, X, is a function of R) is the expected value of the appropriate order statistic from either the standard normal (0, 1), or uniform (0, 1) distribution. Exact values are difficult to calculate, but Blom (1958) has shown that $R = \Phi^{-1}((i - 0.375)/(n + 0.25))$ is an adequate approximation for the standard normal; Φ^{-1} is the inverse of the standard normal cumulative distribution function. Likewise, $R = (i - 0.375)/(n + 0.25)$ will suffice for the uniform distribution.

If one assumes the lognormal model (similar results will work for the Weibull model), the dependent variable for ramp load failures is (equation (4), X = exp(R), and R is a standard normal (0, 1) random variable)

$$\ln(T) \approx wR - \ln(bk) + \ln(a - wR + \ln(bk))$$

The variance of ln(T) is approximately w^2 (the influence of the second wR is negligible). For failures under constant load (assuming, for the moment, instantaneous uploading), the dependent variable is (equation (6))

$$\ln(T) = a - b(\text{load})\exp(-wR)$$
$$\approx a - b(\text{load})(1 - wR)$$

The variance of ln(T) is approximately $(b(\text{load})w)^2$. The linearization also provides starting values for the nonlinear regression. A nonlinear least squares procedure assumes equal error variances. The residuals from failures under constant load must be weighted by (1/(b(load))), if the ramp and constant load failures are to be used together to estimate the parameters.

In practice, b and w which are needed to weight the residuals are the unknown parameters. Therefore, we use an iterative reweighted nonlinear least squares procedure to estimate the unknown parameters:

1. Obtain unweighted estimates of a, b, and w.
2. Reestimate a, b, and w using the current estimate of b to weight any residuals from constant load failures.
3. Continue to iterate until successive parameter estimates converge.

Given the specified dependent and independent variables, the parameters a, b, and w can be estimated using iterative nonlinear least squares if at least two different rates of loading (i.e. k values), or two constant load levels are used. Otherwise a and b are not both estimable. The variability parameter w is estimable only if there are at least two observations in a test.

Variability of the Parameter Estimates

To discuss an estimator of the variability of these parameter estimates, additional notation is needed. Let $\underset{\sim}{y}$ be the vector of the dependent variables, $\underset{\sim}{x}$ the vector of the independent variables, $\underset{\sim}{\theta}$ the vector of parameters to be estimated, and $\underset{\sim}{e}$ the vector of residuals. The model is then $\underset{\sim}{y} = f(\underset{\sim}{x}, \underset{\sim}{\theta}) + \underset{\sim}{e}$, where f is a nonlinear function of $\underset{\sim}{x}$ and $\underset{\sim}{\theta}$. Let the dimensionality of $\underset{\sim}{y}$, $\underset{\sim}{x}$ and $\underset{\sim}{e}$ be ng (g groups of n specimens each), and that of $\underset{\sim}{\theta}$, be p. If the errors are independent and identically distributed normal random variables, then the nonlinear least squares estimator, $\hat{\underset{\sim}{\theta}}$, of $\underset{\sim}{\theta}$ is known to asymptotically have a normal distribution with mean $\underset{\sim}{\theta}$ and variance $\sigma^2(\underset{\sim}{F}'\underset{\sim}{F})$ where $\underset{\sim}{F}$ is the ng x p matrix of first partial derivatives of $f(\underset{\sim}{x}, \underset{\sim}{\theta})$ with respect to $\underset{\sim}{\theta}$ (Gallant 1975). At first glance, it is apparenly sufficient to estimate σ^2 by $(\underset{\sim}{y} - f(\underset{\sim}{x}, \hat{\underset{\sim}{\theta}}))'(\underset{\sim}{y} - f(\underset{\sim}{x}, \hat{\underset{\sim}{\theta}}))/(ng - p)$. However, this is incorrect in this case since the variability term w is in the regression, as the following example shows.

Gupta (1952) gave an estimator of the standard deviation, σ^*, of a normal distribution which is a linear combination of the expected values of order statistics from a standard normal distribution. Let $x_{(i)}$ be the ith order statistic and u_i the expected value of the ith order statistic. Then $\sigma^* = \Sigma u_i x_{(i)}/\Sigma u_i^2$ (all summations are over i = 1, . . . ,n for sample size n). Ali and Chan (1964) showed that asymptotically σ^* is distributed as a normal random variable with mean σ and variance $\sigma^2/[2n(\Sigma u_i^2/n)]$. They also showed that the limit (as n goes to infinity) of $\Sigma u_i^2/n$ is one, so the asymptotic variance of σ^* is $\sigma^2/2n$. This is the Cramer-Rao lower bound for the variance of an unbiased estimator of σ. The estimator, σ^*, can also be obtained as the slope of the regression of $x_{(i)}$ on u_i: $x_{(i)} = \Sigma x_{(i)}/n + \sigma^* u_i + e_i$. The usual estimator of the variance of the slope from a linear regression is $(\Sigma e_i^2/(n - 2))/\Sigma u_i^2$. However, the sum of squares of the residuals divided by n asymptotically goes to zero, not to a constant σ^2:

$$\begin{aligned}
(1/n)\Sigma e_i^2 &= (1/n)\Sigma(x_{(i)} - \bar{x})^2 - 2\sigma^*\Sigma(x_{(i)} - \bar{x})u_i/n \\
&\quad + \sigma^{*2}\,\Sigma u_i^2/n \qquad (\bar{x} = \Sigma x_{(i)}/n) \\
&= (1/n)\Sigma(x_{(i)} - \bar{x})^2 - 2\sigma^*\Sigma x_{(i)}u_i/n \\
&\quad + \sigma^{*2}\,\Sigma u_i^2/n \qquad (\Sigma u_i = 0) \\
&= (1/n)\Sigma(x_{(i)} - \bar{x})^2 - 2(\Sigma x_{(i)}u_i/\Sigma u_i^2)\,\Sigma x_{(i)}u_i/n \\
&\quad + (\Sigma x_{(i)}u_i/\Sigma u_i^2)^2\Sigma u_i^2/n \\
&= (1/n)\Sigma(x_{(i)} - \bar{x})^2 - (\Sigma x_{(i)}u_i)^2/(n\Sigma u_i^2) \\
&= (1/n)\Sigma(x_{(i)} - \bar{x})^2 - \sigma^{*2}\Sigma u_i^2/n
\end{aligned}$$

Since the limit (as n goes to infinity) of $\Sigma(x_{(i)} - \bar{x})^2/n$ is σ^2, since σ^* is an unbiased estimator of σ, and since $\Sigma u_i^2/n$ is one, the limit of $(1/n)\Sigma e_i^2$ is zero, when it should be $\sigma^2/2$.

In our problem, we are also using expected values of order statistics as independent variables in a regression problem to estimate several parameters including the variability parameter w. Just as one can regress x values on expected values of order statistics from a normal distribution to get an estimate of the standard deviation for a normal distribution, one can regress x values on expected values of order statistics from a Weibull distribution to get an estimate of the inverse shape parameter (White 1969). The mean square error (sum of squares of residuals divided by the sample size) will be essentially zero. A heuristic solution, in the case of an assumed lognormal distribution, would be to add $w^2/2$ to the mean square error. This addition to the mean square error will give the appropriate variance at least for Gupta's estimate of the standard deviation and may prove adequate for our much more complicated problem. Menon (1963) showed that the Cramer-Rao lower bound for an unbiased estimator of the inverse shape parameter, d, is $0.61\, d^2/n$. Therefore we will add $0.61\, w^2$ to the mean square error when using a Weibull distribution as a model for the underlying inherent strength.

Simulation Results and Discussion

Simulations were carried out to determine the accuracy of the parameter and variability estimates. The form of the simulations was based on practical limits on experimentation. For ramp loading, an "optimal" design (if one believed the model) would be to use an equal number of observations at two extreme loading rates. In practice, three loading rates equally spaced in ln (rate of loading) have usually been chosen. Ideally these would be spaced as far apart as possible, given the limitations of the test equipment. For this simulation the three rates of loading which were chosen were 0.03, 3, and 300 pounds per minute as these were close to levels used in practice. In preliminary simulations (using only ramp tests) the parameter estimates of a and b were highly correlated (estimated correlation above 0.98), and to a lesser extent each was correlated with w. As an attempt to reduce this correlation, a new parameter, c, was introduced and the parameter a was replaced by b/c. This choice of reparameterization also gives a separate estimate of slope and intercept in equation (5). This reparameterization reduced the correlation problem considerably (estimated correlation between b and c of 0.8 or less). For constant load tests, an "optimal" design would again be to use two extreme load levels. In practice, the load levels chosen must be low enough so that too many specimens do not break on uploading to the constant load level, and high enough so that enough specimens break in a reasonable time. Again, three load levels have usually been chosen.

In a simulation, actual values of the random variable X generate the failure times. However, since the actual values are unknown in practice, when estimating parameters, these values are replaced by the approximate expected values of order statistics involving (i - 0.375)/(n + 0.25). Better parameter estimates should be expected if these "order statistics" are close to the true random variable X. One way to do this is to "match" errors in each of the test groups. If g groups of n observations each are to be generated, generate ng random variables X and sort them from lowest to highest. Assign the lowest g randomly to each of the g test groups, assign the next g, etc. Each test group then has equal distributions of the random variable X and the values should be close enough to their true value.

Unfortunately for lumber, it is impossible to predict the exact "strength" for a piece of lumber before breaking it. Since one would like to have equal distributions of "strength" for each test, some alternative measurement must be used. Differing distributions of strength will give a bias to the estimation of the parameters. A nondestructive measurement of stiffness is known to correlate with lumber "strength," with a correlation coefficient within the range of 0.5 to 0.8. To study the effect of correlation in preliminary simulations, three values of the correlation coefficient are assumed:

1. No matching (none r = 0).
2. Part matching (part r = 0.67).
3. Full matching (full r = 1).

Therefore, in the simulation, pairs of random variables are generated from bivariate normal distributions. The pairs are sorted by the first random variable and assigned to the g groups. The second random variable is then used to generate the failure times using b = 0.03, c = 0.001, and w = 0.3 as the "true" parameter values.

The effect of this "matching" of errors can be seen in table 1. Three rates of ramp loading (g = 3) are used in the simulation with 50 pieces (n = 50) at each rate. Summary results include the average and standard deviation of the parameter estimates for 500 replications. Also included is the average of the standard error of the parameter estimate. Asymptotic confidence intervals were constructed using the estimated standard errors and assuming asymptotic normality of the parameter estimates. The actual coverages of 50, 90, 95, and 99 percent confidence intervals are also given. Only simulations involving the lognormal distribution are given in table 1, as results from the Weibull are similar. Matching does little to the average standard error but does affect the variability of the parameter estimates of c and b. As the "matching" increases, the variability decreases. Coverage probabilities of the nominal 50, 90, 95, and 99 percent confidence intervals are likewise affected. As the sample size n increases, the effect of matching disappears. Coverage probabilities should be close to their nominal value if the parameter estimates are unbiased and the average standard error is roughly equal to the standard deviation of the parameter estimates. Asymptotically, this effect of "matching" errors will disappear. Simulations showed that this was the case for n = 200. Using part "matching" appears to give adequate coverage probabilities.

Summary results for simulations using three ramp tests with sample sizes of 50 and 200 in each group (additional simuiations using 100 and 400 in each group were also done) and part matched (r = 0.67) errors are given in table 2. Results from both the lognormal and Weibull distributions are given. The average of the parameter estimates are close to their true values. The heuristic variances appear to work well, as the average standard error is approximately equal to the standard deviation, and the confidence intervals have coverages approaching their nominal values. increasing the sample size by a factor of four decreases the standard deviations and standard errors by a factor of two, as expected. However, the standard deviation decreases at a slightly slower rate than the average standard error. This indicates that the distribution of the parameter estimate is not yet normal and some skewness exists.

Similar results for simulations using three constant load tests are given in table 3. Parameter estimates are based only on constant load failures with the simulations structured such that 50 percent of the specimens failed under the first ramp or constant load phase. The natural logarithm of the total time on test served as the dependent variable. Simulations were also done using total time on constant load as the dependent variable. The results using total time on constant load are not included in the tables, as the coverage probabilities were considerably lower than when using total time on test, although the average values of the parameter estimates were equally close to the true values.

Table 1 - Effect of error matching-three rates of ramp loading, lognormal errors

Matching	Parameter	n	True value	Average	Standard deviation	Average standard error of parameter estimates	Coverage of nominal confidence intervals			
							50	90	95	99
None r = 0	b	50	0.03	0.0310436	0.00604061	0.00434497	0.362	0.792	0.872	0.938
Part r = 0.67				.0302742	.00400448	.00404516	.500	.920	.954	.976
Full r = 1				.0300192	.00086344	.00390498	.996	1.000	1.000	1.000
None r = 0	c		.001	.0010000	.00003447	.00002502	.410	.778	.854	.936
Part r = 0.67				.0009988	.00002814	.00002500	.448	.860	.910	.978
Full r = 1				.0010000	.00002562	.00002495	.470	.896	.936	.984
None r = 0	w		.3	.3007051	.01734806	.01920324	.550	.932	.962	.992
Part r = 0.67				.3011068	.01875916	.01916177	.512	.892	.948	.984
Full r = 1				.3049999	.01753194	.01908649	.518	.920	.968	.996

Table 2 - Simulation results using three rates of ramp loading-part matched errors

Matching	Parameter	n	True value	Average	Standard deviation	Average standard error of parameter estimates	Coverage of nominal confidence intervals			
							50	90	95	99
Lognormal	b	50	0.03	0.0302742	0.00400448	0.00404516	0.500	0.920	0.954	0.976
		200		.0301704	.00199959	.00194499	.488	.904	.940	.988
	c	50	.001	.0009988	.00002814	.00002500	.448	.860	.910	.978
		200		.0010001	.00001412	.00001225	.460	.856	.916	.972
	w	50	.3	.3011068	.01875916	.01916177	.512	.892	.948	.984
		200		.3003685	.00836449	.00918694	.546	.926	.952	.994
Weibull	b	50	.03	.0306932	.00510527	.00487762	.464	.884	.928	.964
		200		.0300860	.00260538	.00220845	.458	.826	.900	.982
	c	50	.001	.0010010	.00003232	.00003053	.456	.876	.928	.988
		200		.0009993	.00001782	.00001458	.386	.824	.906	.966
	w	50	.3	.3058221	.02413786	.01749529	.376	.770	.844	.936
		200		.3020524	.01225947	.00811202	.340	.700	.798	.924

Table 3 - Simulation results using three constant load levels -part matched errors

Matching	Parameter	n	True value	Average	Standard deviation	Average standard error of parameter estimates	Coverage of nominal confidence intervals			
							50	90	95	99
Lognormal	b	50	0.03	0.0305090	0.00602165	0.00668390	0.548	0.914	0.948	0.988
		200		.0302290	.00317850	.00321861	.504	.918	.962	.988
	c	50	.001	.0009964	.00008594	.00009188	.538	.918	.956	.992
		200		.0009996	.00004359	.00004392	.512	.910	.956	.986
	w	50	.3	.3097154	.06145666	.06128619	.508	.916	.948	.976
		200		.3025515	.02747017	.02796832	.520	.916	.958	.986
Weibull	b	50	.03	.0303518	.00870141	.00889481	.554	.910	.946	.978
		200		.0300413	.00388621	.00430879	.534	.928	.966	.992
	c	50	.001	.0009933	.00011512	.00011628	.534	.898	.956	.986
		200		.0009978	.00005288	.00005592	.504	.920	.970	.994
	w	50	.3	.3075703	.08643013	.07536966	.476	.852	.894	.948
		200		.3006342	.03862518	.03440830	.442	.848	.920	.978

Example

Data from a recent study at the Forest Products Laboratory were used to demonstrate the use of these parameter estimates and variances. The data consisted of:

- 294 specimens of Douglas Fir 2 by 4 lumber each with an edge knot.
- Specimens were tested on edge, with the edge knot on the tension side stressed under maximum bending moment.
- The specimens were divided into six groups each with equal distributions of stiffness.
- The 6 groups of 49 specimens were assigned to one of the 6 tests:
  - fast, intermediate, or slow ramp loading.
  - high, intermediate, or low constant load.

For further details see Gerhards and Link (1983).

The cumulative damage model was fit to the ramp and constant load data separately. The program used to estimate these parameters (Link 1987) is available from the National Technical Information Service, 5285 Port Royal Road, Springfield, VA 22161. For the constant load tests, the parameters were fit using only constant load failures with the natural logarithm of total time on test serving as the dependent variable. The results for the lognormal distribution were

	Estimate (standard error)			Estimated correlations		
Tests	c	b	w	cb	cw	bw
Ramp	0.00142324 (0.00003067)	0.0664345 (0.0127301)	0.307520 (0.019881)	0.5362	–0.0304	–0.0669
Constant load	0.00119043 (0.00010598)	0.0293005 (0.0070679)	0.294694 (0.062319)	0.9766	–0.3810	–0.3717

Using a Hotelling's T^2 two-sample test statistic ($T^2 = 8.018$), the null hypothesis of equality of parameters for the two test groups is rejected ($p < 0.005$). Looking at one variable at a time with univariate two- sample z tests yields

H_o: Values equal for two test groups

Parameter	z	p
c	2.110	0.035
b	2.550	0.011
w	0.196	0.845

Fitting the cumulative damage model to both ramp and constant load data will yield estimates close to those using ramp load data only due to the down weighting of constant load failures.

The data and the fitted cumulative damage model for the ramp data are plotted in figure 1. The values plotted on the y axis are the approximate expected values of order statistics from a standard normal distribution. The natural logarithm of load (in pounds) is plotted along the x axis. Using the estimated variance-covariance matrix, 95 percent confidence bands for the expected values of order statistics are plotted as dashed bands around the fitted curve. To graphically compare the data to the fitted curve, Kolmogorov-Smirnov-type bands are given about the data (95 pct confidence bands indicated by +). Values to construct these bands if the lognormal distribution is assumed can be found in Conover (1980) (Lillifors test, table A15), or if the Weibull distribution is assumed, see Chandra et al. (1981). Note that the interpretation of these bands is quite different. The dashed band is a confidence band on the expected values of order statistics (produced horizontally). The Kolmogorov-Smirnov-type bands are a test of goodness of fit of the data to a fitted curve (produced vertically). The Kolmogorov-Smirnov bands give only an approximate goodness-of-fit test, as they are based on maximum likelihood estimation. We are not using maximum likelihood estimation, but asymptotically the bands are correct if asymptotically our estimates are equivalent to maximum likelihood estimates as Gupta's estimate σ^* is asymptotically equivalent to the maximum likelihood estimate of σ.

The data and the fitted cumulative damage model are plotted for the constant load tests in figure 2. The natural logarithm of total time on test (in minutes) is plotted on the x axis for failures under the preliminary uploading or constant load phases of the test. For the failures under the second ramp loading, the natural logarithm of time until failure under the second ramp loading is plotted. Confidence bands are only given for failures under constant load. Note that the fitted model in the region of failures under the second ramp loading continues the trend in the region of failures under the first ramp loading. The little curve at the beginning of failures under the second ramp loading results from the damage that the model predicts for the weaker surviving pieces, due to the prior constant load.

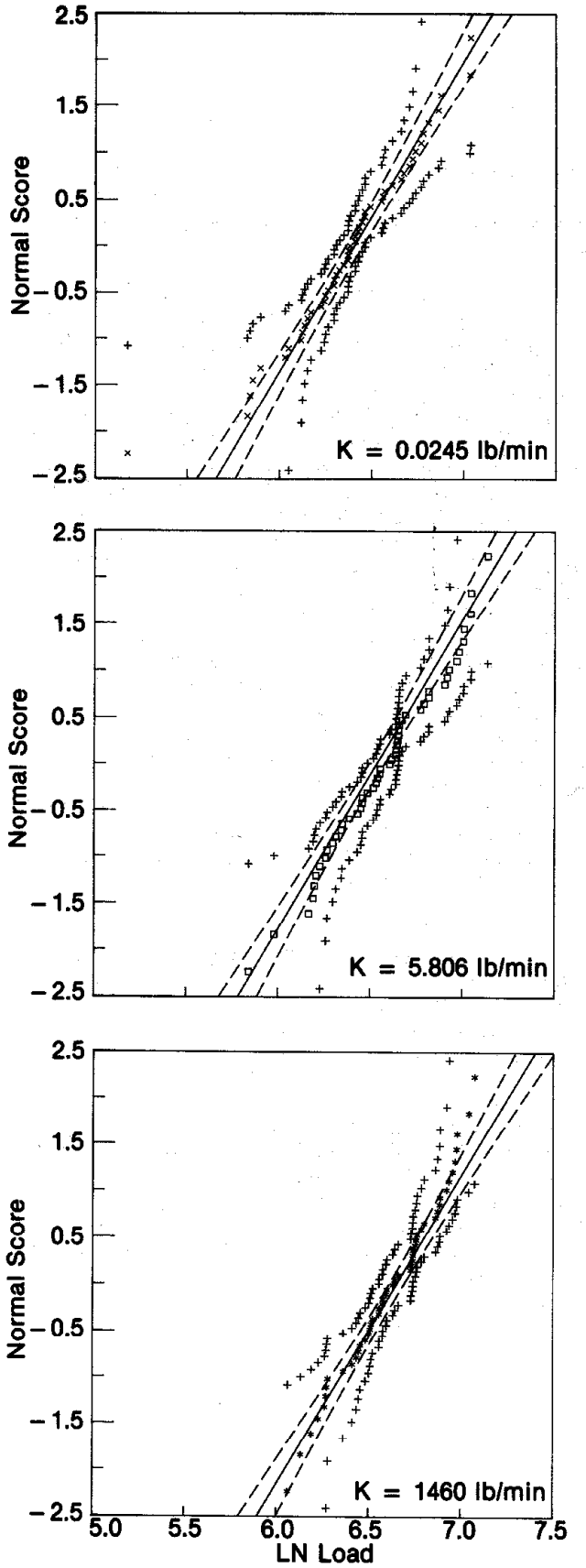

Figure 1 - Edge knot study ramp loading K = 1,460, 5.806, 0.0245 pounds per minute. (ML85 5188)

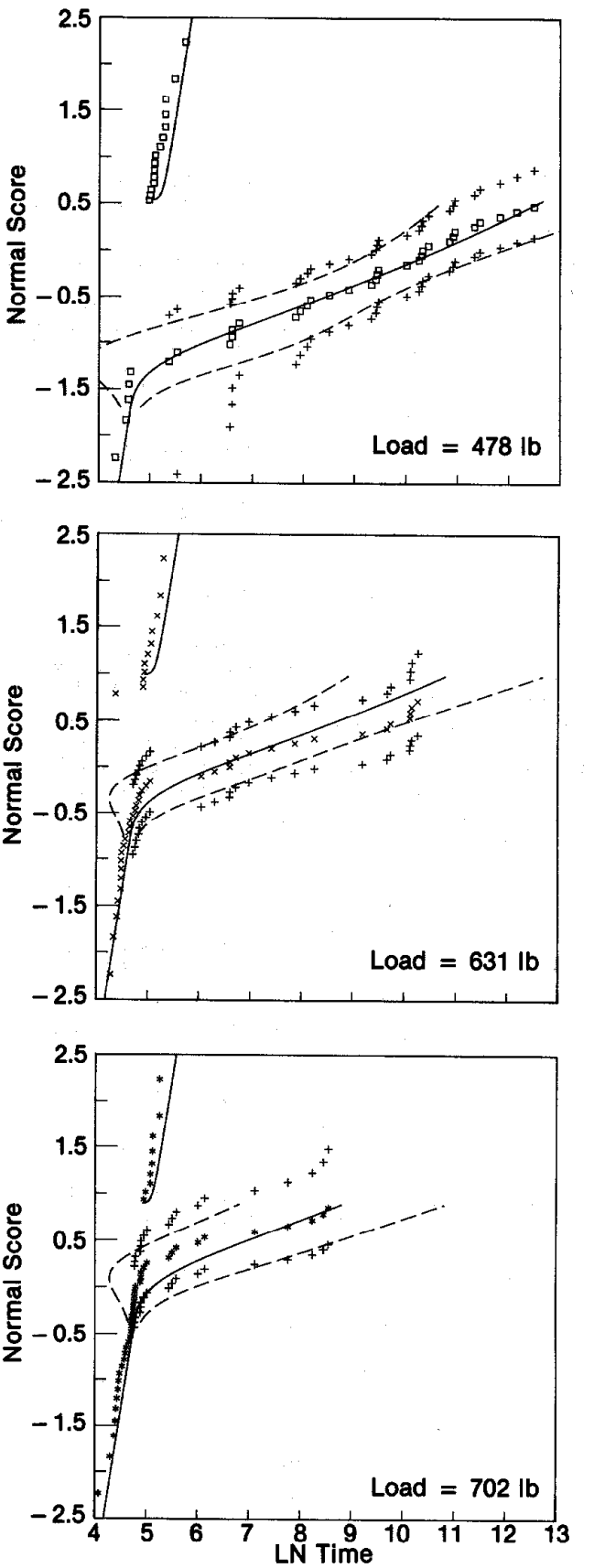

Figure 2 - Edge knot study constant load = 702, 631, 478 pounds. (ML85 5189)

Conclusions

Conclusions from this paper can be summarized as follows:

- The estimation and confidence interval procedures developed give relatively unbiased estimates and approximately correct confidence coverages for sample sizes of at least 50 per test.
- Matching of errors helps the estimation in small sample sizes but its influence diminishes as the sample size increases.
- The parameter estimates are not yet asymptotically normal for sample size of 200 but the approximation is adequate.

In practice

- both constant and ramp load tests
- at least three and preferably more rates of loading, k, or constant load levels

are needed to see if the cumulative damage model is appropriate.

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