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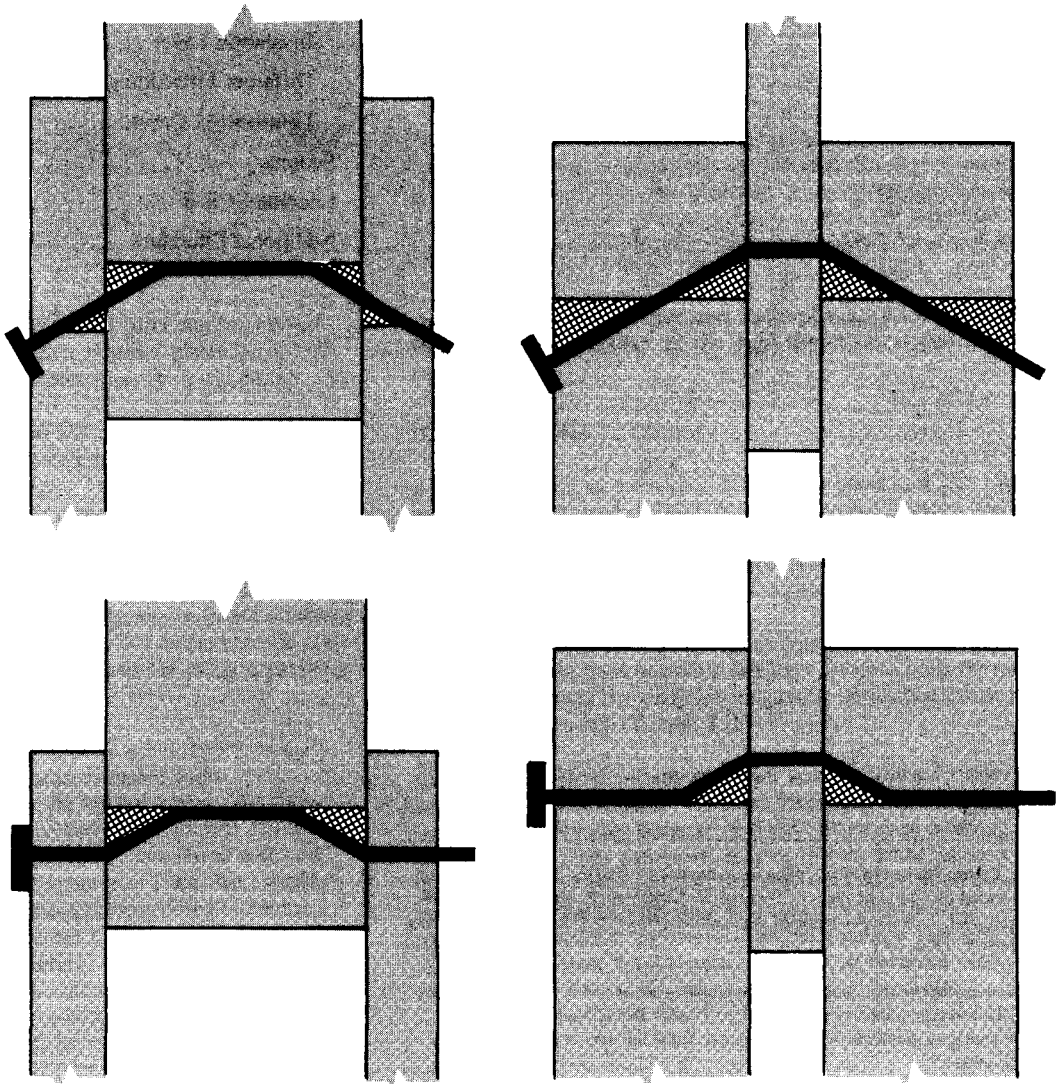
Lateral Load-Bearing Capacity of Nailed Joints Based on the Yield Theory

Theoretical Development

Petter Aune
Marcia Patton-Mallory



Abstract

A European-based yield theory using a material science approach predicts the ultimate lateral load of nailed joints with different joint configurations. The original theory is presented here and extended to include steel-to-wood joints and joints with a layer of insulation (or gap) between joint members. Formulas provide ultimate load for wood-to-wood and steel-to-wood joints and joints with unequal wood strength, A final formula represents the relationship between the wood's embedding stress and deformation not on the traditional assumption of plasticity, but as a fourth-root curve.

Keywords: Nail, yield theory, nail joints, lateral load, ultimate load

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Lateral Load-Bearing Capacity of Nailed Joints Based on the Yield Theory

Theoretical Development

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Background

The ultimate lateral load of a nailed timber joint can be derived by use of a theory of "yielding" which assumes plasticity in both the wood and the fastener; this we refer to as the yield theory.

The general yield theory has existed since the 1940's. It is currently used in Europe where it provides a rational basis for setting design criteria for nailed joints. The validity of the method of derivation has been confirmed by experimental investigations (Aune 1966; Meyer 1957; Möller 1950; Siimes and others 1954). In several European countries (CIB 1983), the code values for lateral load-bearing capacity are based on the yield theory. Use of the yield theory by code writers and researchers in North America would promote international adoption of a consistent method and promote use of a uniform method of analysis for joints containing nails, bolts, or lag screws.

The purpose of this report is to present the yield theory as a rational basis for predicting yield load of nailed joint configurations commonly found in the United States. We show how the yield theory applies to two- and three-member joints with either all-wood members or wood members combined with steel members or wood-based sheathing materials. Then, we extend application to wood members of unequal embedding strength. Here, for the first time, we present the formulas for joints having an interlayer gap (or layer of insulation) between surfaces, and for steel-to-wood joints. Finally, we substitute a fourth-root curve for the original plastic embedding relationship. Derivation of this last formula makes possible an extension of the yield theory to joint deformations.

A companion report (Aune and Patton-Mallory 1986) provides experimental verification of the yield theory developed in this report.

Nailed Joints

Although the use of nails goes far back in history, it was after the industrial manufacture of nails began in the last century that nails became one of the most common fasteners in wooden houses, structural timber constructions, and wooden assemblies of all kinds.

Improved methods of engineering design soon showed the need for research into joint behavior and joint design. Tests on nailed joints were among the early investigations performed at the Forest Products Laboratory (Wilson 1917).

Since the mid-1950's many countries have conducted extensive research in nail fasteners. Particular investigations were required in particular countries because of the differences in building practices, nail types and wood species in use. Until recently even the test methods used in these investigations were often unique to each country. Various theories have been used to derive nailed-joint behavior; the result is an even greater variation in design criteria than the differences in the test methods alone could justify.

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In 1979, Ehlbeck made a comprehensive survey of the status of nailed joint design in wood and in wood-based products. He summarized the research of the previous 20 to 30 years concerning different theories of joint performance, design criteria, national codes, and test methods. An extensive reference list and a list of additional relevant literature make his investigation complete and useful.

Ehlbeck states that research should now be aimed at "uniform conclusions instead of performing isolated research within the national boundaries." Uniform worldwide standards and codes "based on international agreement in the light of the advanced thinking in materials and construction" may result from this approach. Now that the need for standardization is appreciated, we believe the yield theory can contribute substantially to this goal.

Earlier Work on the Yield Theory

As early as 1941, the yield theory originated with the work of the Danish scientist, K. W. Johansen, who first applied the theory to timber fasteners (Johansen 1941). Eight years later he published a shortened version of his original paper in English (Johansen 1949). Some of his investigations on nailed joints were unfinished, and for years the results remained unpublished; eventually, Larsen (1977) completed and published this work.

Referring to his studies of bolted joints, Johansen stated that load-bearing behavior is composed of two "effects:"

1. The "dowel effect" of the bolt which depends on the bolt's resistance to bending and the resistance of the wood to crushing.

2. The "tensional effect" of the bolt which depends on the bolt's resistance to tension and the presence of friction between abutting surfaces.

Johansen analyzed the dowel joint (fig. 1) in detail. He assumed that no axial tension occurred in the fastener and, thus, no frictional contribution to the lateral load-bearing capacity. He also assumed the stress-deflection relationships of the dowel in bending and of the wood in embedding were ideally plastic, as shown in figure 2. Based on these assumptions, Johansen obtained the load-bearing capacity for single- and double-shear joints. The expressions are rather simple and each is related to a particular mode of failure. He verified both the assumptions and the formulas by tests.

Möller (1950) applied the basic yield theory to nailed joints in single and double shear. His investigation included symmetrical and nonsymmetrical two-member joints and symmetrical three-member joints. He also considered the effect of having joint members of different embedment strengths. Although the principles had been introduced by Johansen 10 years earlier, the yield theory concerning nailed joints is quite often referred to as "Möller's theory."

Subsequently, Siimes et al. (1954), Mack (1960), and Aune (1966) have verified the yield theory for nailed joints by experiments.

Meyer (1957) discussed the effect of friction. The contribution of friction is not usually taken into account in joint design.

Larsen and Reestrup (1969) investigated lag screw joints. They found conditions in a laterally loaded screw joint differed slightly from those in a bolted joint because of different embedding values along the screw. Furthermore, the yielding might occur in the shank as well as in the threads, so that different values of the yield moment M_y had to be considered. Formulas for the load-bearing capacity were derived and verified by tests.

Norén (1974) summarized the different formulas developed for different kinds of timber connectors. Larsen (1973) offered a rationale supporting the use of the yield theory in the codes of the Scandinavian countries. Later (1979), he reported the theoretical background and the approximations for bolted joints given in the proposal for the CIB Structural Timber Design Code (1983).

Most recently, McLain and Thangjitham (1983) surveyed the yield theory as it applies to bolted joints typical of American construction practices. Their analysis includes a model modified to account for the lack of end fixity in bolted joint tests.

The work of the scientists and engineers discussed above contributed to the yield theory presented in this paper. In addition to the list of actual citations, we include a supplementary list of additional literature on the subject of nailed joints.

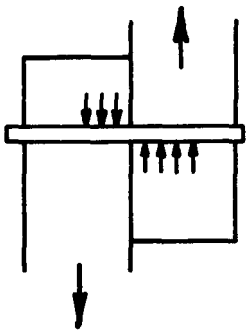


Figure 1.—The generalized dowel joint which forms the basis for the yield theory. (ML85 5235)

Development of the Yield Model

The yield model assumes a behavior in the yielded joint similar to one of the basic modes of joint failure depicted in tables 1 through 4. The model also assumes that both the nail subjected to bending and the wood under embedding stress yield by ideally plastic deformation. The curve of bending moment versus rotation (fig. 2a) approximates the behavior of a steel nail quite well and, although the load-embedment relationship assumed for the wood (fig. 2b) is less certain, research (Aune 1966) indicates the approximations are adequate to predict joint yield.

In predicting joint yield the basic yield model assumes there is no limitation on joint deformation. The yield model assumes that the joint does not fail because of insufficient spacing or end distances at loads below fastener yield. Finally, the yield model ignores friction because it is difficult to estimate accurately and in many joints does not exist.

For all practical purposes, the yield load is considered to be a nailed joint's ultimate or failure load. Some joint configurations may eventually reach a higher load after experiencing large deformations.

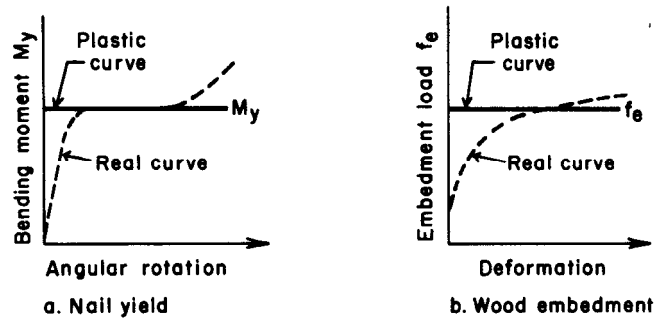


Figure 2.—(a) Nail yield and (b) wood embedment both are assumed ideally plastic in the generalized yield theory. (ML85 5236)

Wood Members with Similar Embedding Strengths

Two-Member Joints

Six basic failure modes exist for two-member joints (tables 1, 2, and 5):

1. Wood yield (slotting) in side member only (mode 1.1).

2. Wood yield (slotting) in main member only (mode 1.1A).

3. Wood yield (slotting) in both members (modes 1.2 and 1.2S).

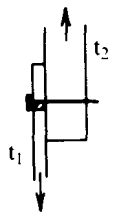
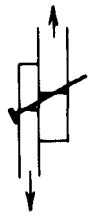
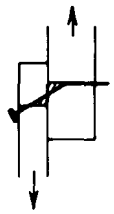
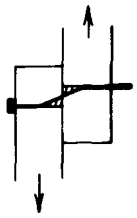
4. Wood yield and one-point nail yield in main member only (modes 1.3 and 1.3S).

5. Wood yield and one-point nail yield in side member only (mode 1.3A).

6. Wood yield and two-point nail yield (modes 1.4 and 1.4S).

The failure modes for joints having two wood members are shown in tables 1 and 5. Corresponding modes for joints having one steel plate member are shown in table 2.

Table 1.— Formulas and failure modes for two-member joints with both members of the same wood embedding strength. Member thickness conditions determine the failure mode.

Mode of failure number	Failure geometry	Number of nail yield points	Yield load F_u (lb)	Thickness conditions
1.1		0	$F_u = fe \cdot t_1$	$\frac{t_1}{\sqrt{\gamma}} \leq 1$ $\alpha \geq 3$
1.2		0	$F_u = \frac{f_e \cdot t_1}{2}\left[\sqrt{3\alpha^2 + 2\alpha + 3} - (1 + \alpha)\right]$	$\frac{t_1}{\sqrt{\gamma}} < \frac{4}{(1 + 3\alpha) - \sqrt{3\alpha^2 + 2\alpha + 3}}$ $1 < \alpha < 3$
1.3		1	$F_u = \frac{f_e}{3}\left[2\sqrt{t_1^2 + 3\gamma} - t_1\right]$	$1 < \frac{t_1}{\sqrt{\gamma}} < 2 + \sqrt{2}$ $t_2 \geq \frac{2}{3}\sqrt{t_1^2 + 3\gamma} - \frac{1}{3}t_1 + 2\sqrt{\gamma}$
1.4		2	$F_u = \sqrt{2f_e \cdot M_y}$	$\frac{t_1}{\sqrt{\gamma}}$ and $\frac{t_2}{\sqrt{\gamma}} \geq 2 + \sqrt{2}$

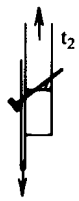
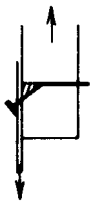
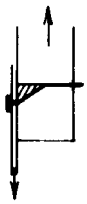
f_e = wood embedding strength (lb/in)

M_y = nail yield moment (lb in)

α = t_2/t_1

γ = M_y/f_e

Table 2.—Formulas and failure modes for two-member joints with one steel member. Member thickness conditions determine the failure mode

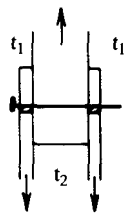
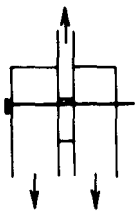
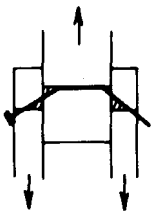
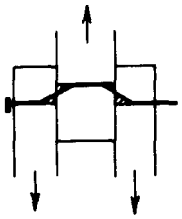
Mode of. failure number	Failure geometry	Number of nail yield points	Yield load F_u (lb)	Thickness conditions
1.2S		0	$F_u = f_e \cdot t_2 (\sqrt{2} - 1)$	$\frac{t_2}{\sqrt{\gamma}} < 2 + \sqrt{2}$
1.3S		1	$F_u = \sqrt{2f_e \cdot M_y}$	$\frac{t_2}{\sqrt{\gamma}} \geq 2 + \sqrt{2}$
1.4S		2	$F_u = \sqrt{2}\sqrt{2f_e \cdot M_y}$	$\frac{t_2}{\sqrt{\gamma}} > 4$

f_e = wood embedding strength (lb/in)

M_y = nail yield moment (lb in)

$\gamma = M_y/f_e$

Table 3.—Formulas and failure modes for three-member joints with all members of same wood embedding strength. Member thickness conditions determine the failure mode

Mode of failure number	Failure geometry	Number of nail yield points	Yield load F_u (lb)	Thickness conditions
2.1		0	$F_u = 2f_e \cdot t_1$	$0 < \frac{t_1}{\sqrt{\gamma}} \leq 1$ $\alpha > 2$
2.2		0	$F_u = f_e t_2 = f_e \cdot \alpha t_1$	$\alpha < 2$ $\frac{t_2}{\sqrt{\gamma}} < 2\sqrt{2}$
2.3		2	$F_u = \frac{2}{3} f_e \left[2\sqrt{t_1^2 + 3\gamma} - t_1\right]$	$1 < \frac{t_1}{\sqrt{\gamma}} < 2 + \sqrt{2}$ $t_2 \geq \frac{2}{3}\left[2\sqrt{t_1^2 + 3\gamma} - t_1\right]$
2.4		4	$F_u = 2\sqrt{2f_e \cdot M_y}$	$\frac{t_1}{\sqrt{\gamma}} \geq 2 + \sqrt{2}$ $\frac{t_2}{\sqrt{\gamma}} \geq 2\sqrt{2}$

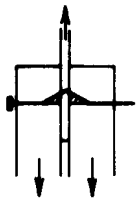
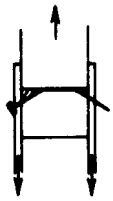
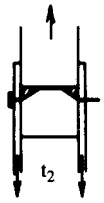
f_e = wood embedding strength (lb/in)

M_y = nail yield moment (lb in)

$\alpha = t_2/t_1$

$\gamma = M_y/f_e$

Table 4.—Formulas and failure modes for three-member joints with a steel center member, or steel side members. Member thickness conditions determine the failure mode. All wood members have the same embedding strengths

Mode of failure number	Failure geometry	Number of nail yield points	Yield load F_u (lb)	Thickness conditions
2.2SA		[1]2	$F_u = 2f_e\left[\sqrt{2}\sqrt{t_1^2 + 2\gamma} - t_1\right]$	$\sqrt{2} < \frac{t_1}{\sqrt{\gamma}} < 4$
2.2SB		[1]4	$F_u = 4\sqrt{f_e M_y}$	$\frac{t_1}{\sqrt{\gamma}} \geq 4$
2.3S		2	$F_u = 2\sqrt{2f_e M_y}$	$\frac{t_2}{\sqrt{\gamma}} \geq 2\sqrt{2}$
2.4S		4	$F_u = 4\sqrt{f_e M_y}$	$\frac{t_2}{\sqrt{\gamma}} \geq 4$

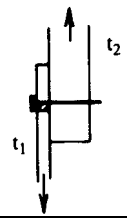
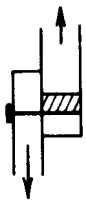
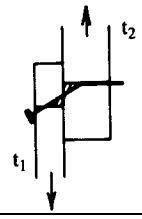
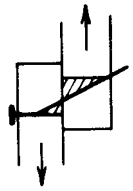
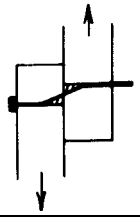
[1]The middle yield point is regarded as two yield points in the ultimate load formula.

f_e = wood embedding strength (lb/in)

M_y = nail yield moment (lb in)

$\gamma = M_y/f_e$

Table 5.—Formulas and failure modes for two-member wood joints. Wood members (t_1 and t_2) have unequal embedding strengths (f_e and γf_e lb/in respectively). Member thickness conditions determine the failure mode

Mode of failure number	Failure geometry	of nail yield points	Yield load F_u (lb)	Thickness conditions
1.1		0		$\frac{t_1}{\sqrt{\gamma}} < \sqrt{\frac{2\beta}{1+\beta}}$ $t_2 > \frac{t_1}{\beta}$
		0	$F_u = \beta f_e t_2$	$\frac{t_2}{\sqrt{\gamma}} < \sqrt{\frac{2}{\beta(1+\beta)}}$ $t_1 > \beta t_2$
1.2		0	$F_u = f_e \cdot t_1 \left[\frac{\sqrt{\beta + 2\beta^2(1+\alpha+\alpha^2) + \alpha^2\beta^3} - \beta(1+\alpha)}{1+\beta} \right]$	$\frac{t_1}{\sqrt{\gamma}} < \frac{(1+\beta)}{(2\beta+1+\alpha\beta) - \sqrt{\beta + 2\beta^2(1+\alpha+\alpha^2) + \alpha^2\beta^3}}$ $\frac{1}{\beta} < \alpha < \frac{1+\sqrt{2(1+\beta)}}{\beta}$
1.3		1	$F_u = f_e \left[\sqrt{\frac{2\beta(1+\beta)t_1^2}{(2+\beta)^2} + \frac{4\beta\gamma}{(2+\beta)}} - \frac{\beta t_1}{(2+\beta)} \right]$	$\frac{t_1}{\sqrt{\gamma}} > \sqrt{\frac{2\beta}{1+\beta}}$ and $2 - 2\sqrt{\frac{\beta}{1+\beta}} < \frac{t_1}{\sqrt{\gamma}} < 2 + 2\sqrt{\frac{\beta}{1+\beta}}$ $t_2 \geq \sqrt{\frac{2(1+\beta)t_1^2}{\beta(2+\beta)^2} + \frac{4\gamma}{\beta^2(2+\beta)}} - \frac{t_1}{(2+\beta)} + \frac{2\sqrt{\gamma}}{\sqrt{\beta}}$
1.3A		1	$F_u = f_e \cdot \beta \left[\sqrt{\frac{2\alpha^2(1+\beta)t_1^2}{(1+2\beta)^2} + \frac{4\gamma}{\beta(1+2\beta)}} - \frac{\alpha t}{(1+2\beta)} \right]$	$\frac{t_2}{\sqrt{\gamma}} = \frac{\alpha t_1}{\sqrt{\gamma}} > \sqrt{\frac{2}{\beta(1+\beta)}}$ and $\frac{1}{\sqrt{\beta}}\left[2 - 2\sqrt{\frac{1}{(1+\beta)}}\right] < \frac{\alpha t_1}{\sqrt{\gamma}} < \frac{1}{\sqrt{\beta}}\left[2 + 2\sqrt{\frac{1}{(1+\beta)}}\right]$
1.4		2	$F_u = \sqrt{\frac{\beta \cdot 4f_e \cdot M_y}{(1+\beta)}}$	$\frac{t_1}{\sqrt{\gamma}} \geq 2 + 2\sqrt{\frac{\beta}{1+\beta}}$ $\frac{t_2}{\sqrt{\gamma}} \geq \frac{1}{\sqrt{\beta}}\left[2 + 2\sqrt{\frac{1}{(1+\beta)}}\right]$

Nail yielding at one point.—The yield load F_u for mode 1.3 (table 1) can be derived using the virtual displacement method. Consider the deformed nail in figure 3. The yield load and failure mode for the nailed joint shown in figure 3 depends on the relative thicknesses and relative embedding strengths of the members. When both members have equal embedding strength f_e the work done by nail embedding and nail yield is:

$$W = f_e \cdot \int \eta d\xi + \Sigma M_y \Theta$$

where

f_e = embedding strength for both wood members (load per unit length) (lb/in)(see fig. 2)
M_y = yield moment of the nail (lb in) (see fig. 2)
W = work done by joint (lb in)
ξ, η = integration variables for area of wood experiencing

Θ = angular rotation of yielded nail

Using small angle approximation, and considering the one yield-point of mode 1.3

$$\tan \Theta = \frac{1}{a} \doteq \Theta$$

$$W = f_e \int \eta d\xi + M_y \cdot \frac{1}{a}$$

$$= f_e \, \Sigma A + M_y \cdot \frac{1}{a} \qquad (1)$$

where

a = $x_1 + (t_1 - x)$
t_1 = thickness of side member (in)
x, x_1 = lengths of nail embedment (in) defined by figure 3
ΣA = shaded area in figure 3 representing embedment of nail (in^2)

Substituting

$$\Sigma A = \frac{1}{2}\left[\frac{x^2}{a} + \frac{(t_1 - x)^2}{a} + \frac{x_1^2}{a}\right]$$

and equating external work to internal work

$$F \cdot 1 = W = \frac{f_e}{2a}\left[x^2 + (t_1 - x)^2 + x_1^2\right] + \frac{M_y}{a}$$

where $F \cdot 1$ = lateral force on the nail undergoing unit displacement. Setting the areas equal for the embedment stress yields

$$x_1 = t_1 - 2x$$

Let

$$\gamma = \frac{M_y}{f_e}$$

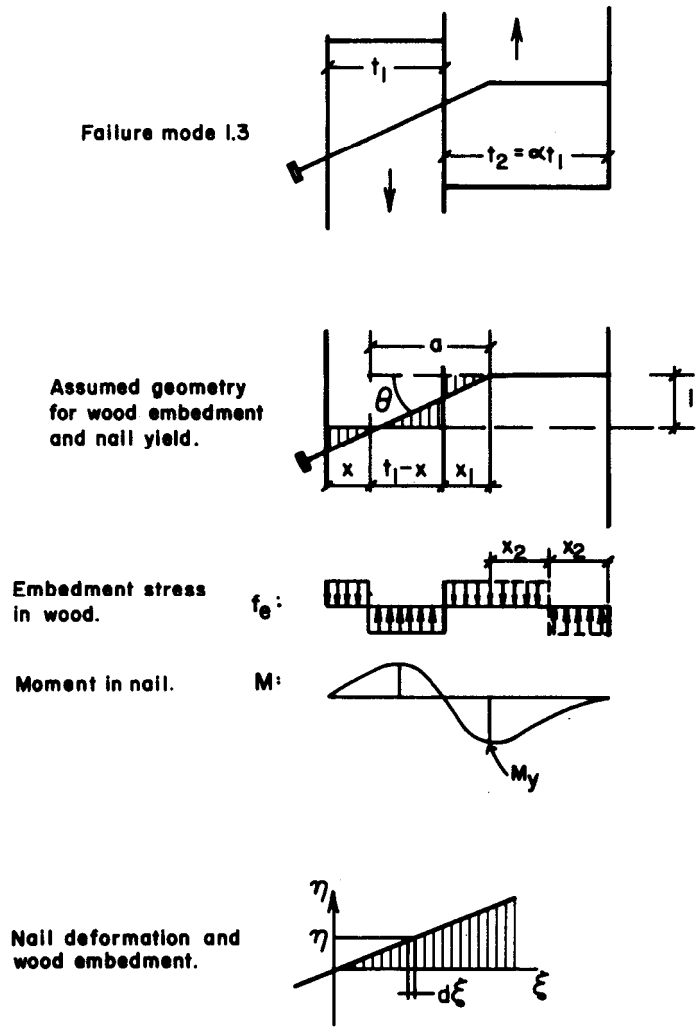


Figure 3.—Geometry of deformed nail in a two-member joint having one-point nail yield and wood crushing (mode 1.3, table 1). (ML85 5237)

Substituting for γ and for

$$a = x_1 + (t_1 - x) = 2t_1 - 3x$$

results in

$$F = \frac{f_e}{(2t_1 - 3x)}\left[3x^2 - 3t_1x + t_1^2 + \gamma\right]$$

which, for dF/dx = 0 gives

$$x = \frac{2}{3}t_1 \pm \frac{1}{3}\sqrt{t_1^2 + 3\gamma}$$

(It can be shown that $3x - 2t_1 < 0$, therefore + sign has no interest.) Since $\frac{d^2F}{dx^2} > 0$ the formula gives a minimum value

$$F_u = \frac{f_e}{3}\left[2\sqrt{t_1^2 + 3\gamma} - t_1\right] \qquad (2)$$

The thickness requirements can be derived that assure that failure mode 1.3 (table 1) shall occur.

(1) No slotting (mode 1.1) in the t_1 member gives

$$x > 0,\ t_1 > \sqrt{\gamma}$$

(2) No nail yielding in the t_1 member gives

$$f_e x^2 < M_y$$

$$2 - \sqrt{2} < \frac{t_1}{\sqrt{\gamma}} < 2 + \sqrt{2}$$

(3) Nail yielding in the t_2 member gives

$$f_e x_2^2 > M_y,\ x_2 = \frac{\alpha t_1 - x_1}{2}$$

$$\frac{t_1}{\sqrt{\gamma}} < \frac{4}{(1 + 3\alpha) - \sqrt{3\alpha^2 + 2\alpha + 3}}$$

where $\alpha = t_2/t_1$ (fig. 3).

Other failure modes.—The yield loads for modes of failure 1.1, 1.2, and 1.4 were derived in a similar way. The yield loads and the corresponding thickness requirements assuring the particular failure shall occur are given in table 1.

Steel side member.—By substituting one wood member in the joint by a steel plate, the formulas given in table 2 can be derived for failure modes 1.2S, 1.3S, and 1.4S. In this case, the steel plate thickness is assumed negligible compared to the wood thickness and the nail length.

Three-Member Joints

Four basic failure modes exist for three-member joints (tables 3, 4, and 6):

1. Wood yield (slotting) in both side members (mode 2.1).

2. Wood yield (slotting) in the main member (mode 2.2).

3. Wood yield and two-point nail yield (modes 2.3 and 2.3S).

4. Wood yield and four-point nail yield (modes 2.4 and 2.4S).

When the wood main member of mode 2.2 is replaced with a steel plate, two additional failure modes are possible (table 4):

5. Wood yield and one-point nail yield (mode 2.2SA).

6. Wood yield and three-point nail yield (mode 2.2SB).

The failure modes for wood joints having three wood members are shown in tables 3 and 6. Corresponding modes for joints having either steel side plates or a steel center member are shown in table 4.

AU-wooden members.—The modes of failure for three-member joints (table 3) ate identical to those for two-member joints with the one exception of failure mode 2.2. For this case the yield load is proportional to the central member thickness.

The yield load formulas for three-member wood joints and corresponding thicknesses required to assure that a failure mode shall occur are shown in table 3. They are derived in a manner similar to that used for two-member joints.

Steel side or center member.— The yield load formulas and corresponding required thicknesses for three-member joints with either steel side or steel center member are shown in table 4.

Wood Members With Different Embedding Strengths

The load-bearing capacity of a bolt, dowel, or screw joint depends on the angle between the load and the wood's grain. For nailed joints, the effect of grain angle is rather small and it is usually neglected. However, joints made from different species of wood might require different embedment strengths for the wood members similar to different embedment strengths for different angle-to-grain loads. For such cases, the introduction of different f_e values for each wood member is useful.

Two-Member Joints

Two additional failure modes are necessary to describe joints having unequal embedment strength (table 5):

1. Wood yield (slotting) in main member only (mode 1.1A).

2. Wood yield and one-point nail yield in side member only (mode 1.3A)

First we consider the general case of two wood members having embedment strength related by β (fig. 4):

$$\beta = \frac{f_e \text{ (main member)}}{f_e \text{ (side member)}}$$

where f_e = embedment strength of wood members (lb/in).

The yield load for one-point nail yield (mode 1.3) can be derived using virtual displacements.

$$W = f_e \Sigma A + M_y \cdot \frac{1}{a}$$

$$a = \frac{t_1 (1 + \beta) - x(2 + \beta)}{\beta}$$

(Symbols are defined under equation (l).)

Substituting

$$\Sigma A = \frac{1}{2}\left[\frac{x^2}{a} + \frac{(t_1 - x)^2}{a} + \frac{\beta x_1^2}{a}\right]$$

and equating external work to internal work

$$F \cdot 1 = W = \frac{f_e}{2a}\left[x^2 + (t_1 - x)^2 + \beta x_1^2\right] + \frac{M_y}{a}$$

where F = lateral force on the nail (1 b).

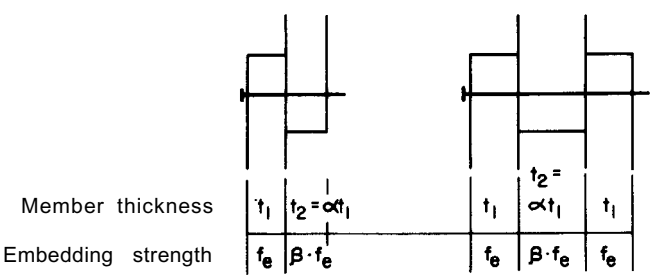


Figure 4.—Two- and three-member joints with unequal embedding strengths. Embedding strength (βf_e) of the main member (t_2) is a function of the embedding strength (f_e) of the side member. (ML85 5238)

Substituting

$$x_1 = \frac{(t_1 - 2x)}{\beta}$$

$$\gamma = \frac{M_y}{f_e}$$

$$a = \frac{t_1(1 + \beta) - x(2 + \beta)}{\beta}$$

results in

$$F = \frac{f_e[2(2 + \beta)x^2 - 2t_1 x(2 + \beta) + t_1^2(1 + \beta) + 2\beta\gamma]}{2[t_1(1 + \beta) - x(2 + \beta)]}$$

which, for dF/dx = 0 gives

$$x = \frac{2t_1(1 + \beta) \pm \sqrt{2\beta(1 + \beta)t_1^2 + 4\beta(2 + \beta)\gamma}}{2(2 + \beta)}$$

(+ sign is of no interest) and

$$F_u = f_e\left[\sqrt{\frac{2\beta(1 + \beta)t_1^2}{(2 + \beta)^2} + \frac{4\beta\gamma}{2 + \beta}} - \frac{\beta t}{2 + \beta}\right] \qquad (3)$$

Again, the thickness requirements that assure failure mode 1.3 shall occur can be derived.

(1) No slotting in the t_1 member gives

$$x > 0, \frac{t_1}{\sqrt{\gamma}} > \sqrt{\frac{2\beta}{1 + \beta}}$$

(2) No nail yielding in the t_1 member gives

$$f_e x^2 < M_y$$

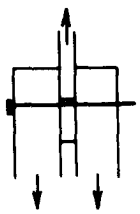
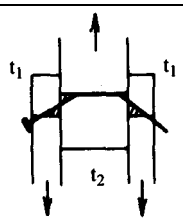
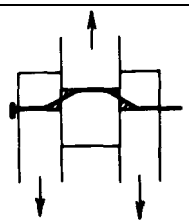
$$2 - 2\sqrt{\frac{\beta}{1 + \beta}} < \frac{t_1}{\sqrt{\gamma}} < 2 + 2\sqrt{\frac{\beta}{1 + \beta}}$$

The yield loads for other possible failure modes are derived in a similar way. The ultimate load formulas and the corresponding thickness requirements are given in table 5.

Three-Member Joints

In the three-member joints, only the modes of failure 2.2, 2.3, and 2.4 (table 6) are of interest. Different relative wood embedding strengths do not change the conditions for failure mode 2.1. Modes 2.3 and 2.4 are identical to modes 1.3 and 1.4 (table 1) in the two-member joint. The yield load formulas and corresponding thicknesses are given in table 6.

Table 6.—Formulas and failure modes for three-member wood joints. Wood members (t_1 and t_2) have unequal embedding strengths (f_e and βf_e lb/in respectively). Thickness conditions determine the failure mode

Mode of failure number	Failure geometry	Number of nail yield points	Yield load F_u (lb)	Thickness conditions
2.2		0	$F_u = \beta f_e t_2 = \beta f_e \alpha t_1$	$t_1 > \beta t_2$ $\frac{t_2}{\sqrt{\gamma}} < \frac{4}{\sqrt{\beta(1+\beta)}}$
2.3		2	$F_u = 2f_e\left[\sqrt{\frac{2\beta(1+\beta)t_1^2}{(2+\beta)^2} + \frac{4\beta\gamma}{(2+\beta)}} - \frac{\beta t_1}{(2+\beta)}\right]$	$\frac{t_1}{\sqrt{\gamma}} < 2 + \sqrt{\frac{\beta}{1+\beta}}$ $t_2 \geq \frac{2\sqrt{2\beta(1+\beta)t_1^2 + 4\beta(2+\beta)\gamma} - 2\beta t_1}{\beta(2+\beta)}$
2.4			$F_u = 2\sqrt{\frac{\beta 4 f_e \cdot M_y}{(1+\beta)}}$	$\frac{t_1}{\sqrt{\gamma}} \geq 2 + 2\sqrt{\frac{\beta}{1+\beta}}$ $\frac{t_2}{\sqrt{\gamma}} \geq \frac{4}{\sqrt{\beta(1+\beta)}}$

f_e = wood embedding strength (lb/in)

M_y = nail yield moment (lb in)

$\alpha = t_2/t_1$

$\gamma = M_y/f_e$

Sheathing and Insulation Members

Insulation material used between sheathing and wood members has negligible resistance to nail slotting. Insulation does have significant stiffness, however, to keep the distance between sheathing and the other joint member constant, and acts as a gap between joint members.

Four failure modes are theoretically possible for joints consisting of a wood main member, a layer of insulation, and an outer sheathing member (table 7):

1. Thin sheathing: Wood yield and one-point nail yield in main members only (mode 3.3A).

2. Thin sheathing: Wood yield and two-point nail yield (mode 3.4A).

3. Thick sheathing: Wood yield and one-point nail yield in main members only (mode 3.3B).

4. Thick sheathing: Wood yield and two-point nail yield (mode 3.4B).

The yield load, F_u, for failure mode 3.3A (table 7) can be derived using the virtual displacement method. Consider the deformed nail in figure 5a:

$$W = f_e \Sigma A + M_y \cdot \frac{1}{a}$$

where a = e + x and e = insulation thickness (in). (Other symbols are defined under equation (l).) Substituting

$$\Sigma A = \frac{x^2}{2a}$$

$$a = e + x$$

and equating external and internal work,

$$F \cdot 1 = W = \frac{f_e x^2}{2(e + x)} + \frac{M_y}{(e + x)}$$

where F = lateral force on the nail (1b)

Substituting

$$\gamma = \frac{M_y}{f_e}$$

results in

$$F = \frac{f_e(x^2 + 2\gamma)}{2(e + x)}$$

which, for dF/dx = 0 gives

$$x = -e + \sqrt{e^2 + 2\gamma}$$

$$F_u = f_e \sqrt{e^2 + 2\gamma} - e \qquad (4)$$

According to figure 3, yielding of the nail in the main member (t_2 member) requires:

$$f_e x_2^2 > M_y$$

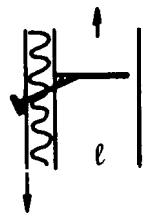
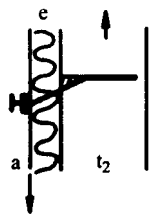
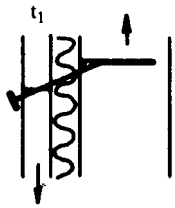
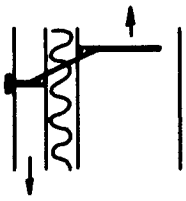
The minimum length (ℓ) of the nail should be approximately:

$$\ell = e + x + 2x_2$$

$$\ell = \sqrt{e^2 + 2\gamma} + 2\sqrt{\gamma}$$

Failure mode 3.4A (table 7) may occur if a bushing or hardboard cleat (fig. 5b) is used under the nailhead, inducing a second yield-point in the nail. Thicker sheathing may also induce a second yield-point in the nail. Table 7 shows loads and corresponding requirements concerning nail length and sheathing thickness for failure modes 3.3A, 3.3B, 3.4A, and 3.4B. Although the sheathing may have an embedding strength different from the wood member, the major contributions to the yield load are nail bending and main member embedment. The error introduced by assuming equal embedment in the members is insignificant for this situation.

Table 7.—Formulas and failure modes for two-member sheathing to wood joints with an intermediate layer of insulation (e). Wood has embedding strength f_e. Thickness conditions and theoretical nail length determine the failure mode

Mode of failure number	Failure geometry	Number of nail yield points	Yield load F_u (lb)	Failure mode determinants
3.3A		1	$F_u = f_e \sqrt{e^2 + 2\gamma} - e$	Theoretical length of nail: $\ell = \sqrt{e^2 + 2\gamma} + 2\sqrt{\gamma}$
3.4A		2	$F_u = f_e \left[\sqrt{e^2 + 4\gamma} - e\right]$	Theoretical length of nail: $\ell = \sqrt{e^2 + 4\gamma} + 2\sqrt{\gamma} + a$
3.3B		1	$F_u = \frac{f_e}{3}\left[2\sqrt{t_1^2 + et_1 + e^2 + 3\gamma} - (t_1 + 2e)\right]$	Thickness conditions: $\frac{\sqrt{e^2 + 4\gamma} - e}{2} < t_1 < \frac{\sqrt{e^2 + 8\gamma} - e + 4\sqrt{\gamma}}{2}$ Theoretical length of nail: $\ell = \frac{2t_1 + e}{3} + \frac{2}{3}\sqrt{t_1^2 + et_1 + e^2 + 3\gamma} + 2\sqrt{\gamma}$
3.4B		2	$F_u = \frac{f_e}{2}\left[\sqrt{e^2 + 8\gamma} - e\right]$	Thickness conditions: $t_1 \geq \frac{\sqrt{e^2 + 8\gamma} - e + 4\sqrt{\gamma}}{2}$ Theoretical length of nail: $\ell = \sqrt{e^2 + 8\gamma} + 4\sqrt{\gamma}$

a = thickness of cleat under nailhead (in)

e = thickness of intermediate layer of insulation (in)

f_e = wood embedding strength (lb/in)

M_y = nail yield moment (lb in)

γ = M_y/f_e

Embedding Stress-Deflection Relationship

The assumption of plastic behavior of materials in the embedding stress-deformation relationship has proved adequate for the yield model even though the wood behavior does not fit the assumed straight line. However, the yield model is not limited to this assumption. By substituting a fourth-root curve for the assumed plastic embedment behavior we are able to expand the yield model to include joint deformation.

Aune (1966) combined the general yield theory and the fourth-root curve relationship between embedding stress and deflection.

We derive the ultimate load for a three.-member joint with a steel center member, failure mode 2.2SB (table 4), using the fourth-root curve and symbols defined in figure 6.

Using virtual work

$$F \cdot \delta = \delta \cdot 2 \int_0^x f_e \cdot \frac{\xi}{x} d\xi + \frac{4M_y \cdot \delta}{x}$$

where

f_e = embedding stress $= f_1 \frac{\eta^{1/4}}{\eta_1^{1/4}}$

f_1, η_1 = corresponding embedding stress and deformation in fourth-root relationship (lb/in, in)

M_y = yield moment of nail (lb in)

x = length of wood embedment (in)

δ = joint deflection (in)

ξ, η = integration variables for area of wood experiencing fastener embedment (in, in)

Substituting

$$\eta = \frac{\xi}{x} \delta$$

$$\eta^{1/4} = \left[\frac{\xi}{x} \cdot \delta\right]^{1/4}$$

$$f_e = \frac{f_1}{\eta_1^{1/4}} \left[\frac{\xi}{x} \cdot \delta\right]^{1/4}$$

Rearranging

$$F = \frac{2 f_1 \delta^{1/4}}{\eta_1^{1/4} x^{5/4}} \int_0^x \xi^{5/4} d\xi + \frac{4M_y}{x}$$

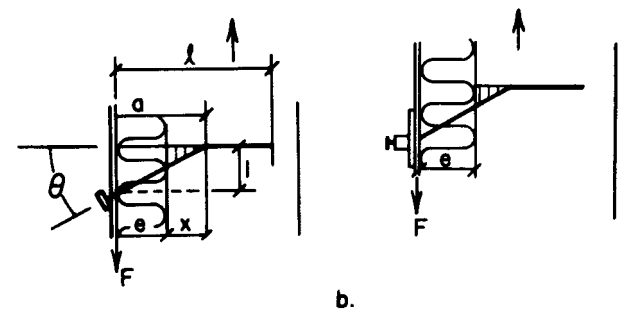
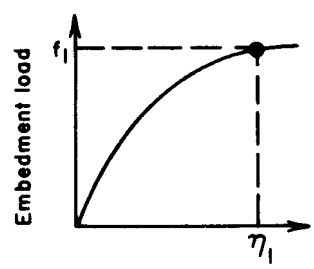
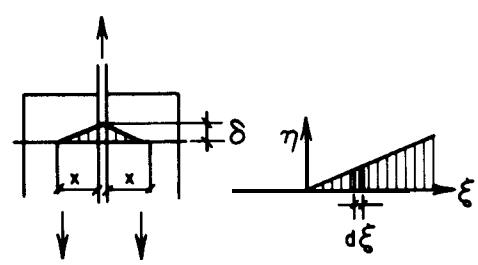


Figure 5.—(a) Geometry of deformed nail with an intermediate layer of insulation of thickness e. Yield load is a function of e. (b) Nailhead fixity results in an additional nail yield-point. (ML85 5239)



a. *Deformation*



b.

Figure 6.—(a) The relationship between wood embedding stress and deformation assumed to be a fourth-root curve. (b) Geometry of deformation for a three-member joint with a steel center member (failure mode 2.2SB, table 4) which we use to exemplify the fourth-root embedment relationship in the yield theory. (ML85 5240)

Now, substituting

$$\int_0^x \xi^{5/4} d\xi = \frac{4}{9} x^{9/4}$$

$$F = \frac{8 f_1 \delta^{1/4} x}{9 \eta_1^{1/4}} + \frac{4 M_y}{x}$$

which, for dF/dx = 0 gives

$$x = \frac{3}{2^{1/2}} \left(\frac{M_y}{f_1} \right)^{1/2} \cdot \left(\frac{\eta_1}{\delta} \right)^{1/8}$$

$$F_u = \frac{8}{3} (2 f_1 M_y)^{1/2} \cdot \left(\frac{\delta}{\eta_1} \right)^{1/8} \qquad (5)$$

Similar formulas can be derived for all modes of failure in which the nail forms plastic hinges. For other cases such as failure modes 1.2, 1.3, and 2.3 (tables 1 and 3) the ultimate load can be obtained by successive approximation (Aune 1966).

The yield load formulas for two joint configurations are shown in table 8. For comparison the corresponding loads F_u are shown for the general case (tables 3 and 4) with $\eta_\lambda = \delta$. It is not possible to derive formulas for required wood thickness with the fourth-root embedding stress assumption.

Table 8.—Formulas and failure modes for three-member joints with steel and wood members, comparing estimates of yield loads assuming a plastic relationship (F_u) or assuming a fourth-root curve (F_{uo}) between wood embedding stress and deformation

Mode of failure number	Number of nail yield points	Estimates of yield load		Ratio
		F_{uo}	F_u (table A4)	F_{uo}/F_u
2.2SB	4	$\frac{8\sqrt{2 f_1 M_y} \cdot (\delta)^{1/8}}{3(\eta_1)^{1/8}}$	$4\sqrt{f_e M_y}$	0.94
2.4	4	$\frac{8\sqrt{f_1 M_y} \cdot (\delta)^{1/8}}{3(\eta_1)^{1/8}}$	$2\sqrt{2 f_e M_y}$	0.94

f_1 = wood embedding stress at deformation η_1

f_e = wood embedding strength (lb/in)

M_y = nail yield moment (lb in)

F_u = yield load of joint assuming plastic wood embedment (lb)

F_{uo} = yield load of joint assuming fourth-root wood embedment (lb)

δ = joint deformation (in)

η = wood embedding deformation (in)

η_1 = wood embedding deformation corresponding to embedding stress f_1 (in)

Comparing formulas for different failure modes leads us to the following observations on the influence of different joint variables.

Member Thickness

Formulas for required member thickness can provide joint geometries which produce the highest yield load. Yield loads are a function of member thicknesses (t_1 and t_2) and the ratio of nail yield moment to wood embedment strength (γ). From the two-member joint formulas in table 1 (wood-to-wood joints), the highest yield load value occurs from two-point nail yield (failure mode 1.4); i.e., t_1 (and t_2) has to have a value at least $(2 + \sqrt{2})\sqrt{\gamma}$. A further increase in the member thickness has no effect on yield load. An identical yield load value is obtained in a steel plate joint by one-point nail yield (mode of failure 1.3S, table 2). In this case the connection between the plate and the nail is regarded as a hinge. The required minimum wood member thickness is the same in both cases.

Interestingly, joints of the 1.2S and 1.2 modes of failure (tables 1 and 2) have the same yield load when both members have the same thickness ($\alpha = 1$) in mode 1.2. When $a = 1$, modes 1.3S and 1.4 have the same yield load which is $\sqrt{2}$ times the yield load of mode 1.4S.

Nailhead Fixity

To show the effect of fixity in the nailhead we compare the yield loads of two-member joints with steel side plates (modes of failure 1.3S and 1.4S, table 2). The theoretical load increases 41 percent when a second yield point occurs in the nail at the steel side plate (mode 1.4S). This increase is attained with just 17 percent increase in wood thickness. This effect may occur in nailed connections where thick steel plates restrain rotation of the nailhead.

Steel Side Plates

By replacing the wooden side members by steel plates, two- and four-point nail yield in three-member joints (in failures modes 2.3S and 2.4S table 4) correspond to the previously mentioned one- and two-point nail yield in the two-member joints (in failure modes 1.3S and 1.4S, table 2). The yield loads in the three-member joints are double the yield loads in the two-member joints, but the required thicknesses are not doubled.

Concerning the three-member joint with steel center member (failure mode 2.2SA, table 4), the yield load always exceeds the highest yield load value for any three-member joint with all-wooden members (i.e., failure mode 2.4). The yield load value derived for the three-member joint with a steel center member and three-point nail yield (failure mode 2.2SB) is identical with that for three-member joint with steel side plates and four-point nail yield (failure mode 2.4S). This yield load value turns out to be 41 percent higher than the corresponding value in an all-wooden three-member joint. This advantage is realized in a combined nail/steel plate joint where one or several plates are placed in slots within wooden truss members. The higher yield load is obtainable only with an adequate distance between the slots or the plates.

Insulation Layer

From the formulas given in table 7, it is not possible to see the effect on the yield load either of the thickness of insulation, e, or of the mode of failure. These effects, however, can be analyzed by assuming certain ratios between insulation thickness, e, and the square root of the ratio of nail yield to wood embedment, $\sqrt{\gamma}$.

The yield load is greatly decreased by having an intermediate layer of insulation instead of a plain connection. The trend is the same for all three modes of failure (fig. 7). Failure mode 3.4A gives the highest yield load, which is 40 to 90 percent higher than the corresponding load with failure mode 3.3A and 5 to 40 percent higher than the load with failure mode 3.4B (see fig. 7). This effect needs to be. taken into account in designing nailed joints that have a layer of insulation.

Different Embedding Strengths

To demonstrate the influence of different embedding strengths in a joint, we compare the formulas in table 5 with the corresponding formulas in the general case (tables 1-4). The complexity of some of the formulas makes it difficult to compare the whole range of failure modes. Table 9 shows the comparison between a two-member joint with two-point nail yield (failure mode 1.4, the maximum obtainable ultimate load) and a two-member joint with no nail yield, only crushing in both wood members (failure mode 1.2). The analysis assumes member thicknesses are equal.

The formula for failure mode 1.4 shows that yield loads are not very sensitive to increases in embedding strength of one joint member. This may be seen by comparing yield loads of joints with different ratios of main-member to side-member embedding strengths (β). The yield load increases 9 percent for $\beta = 1.5$ and 15 percent for $\beta = 2.0$. Möller (1950) suggests using an average embedding strength value when calculating the yield load of joints with unequal embedding strengths. Doing so leads to a marked reduction in the required thickness of the member having the highest embedding strength (t_2). Required thickness is reduced to 78 percent for $\beta = 1.5$ and to 65 percent for $\beta = 2.0$. Failure mode 1.4 occurs in a thinner member that has a higher embedding strength.

Assuming failure mode 1.2 occurs in a joint and that member thicknesses are equal, table 9 gives some indication of the effect of increasing the embedding strength of one member. In this case, in contrast to the case previously discussed, the yield load depends entirely on the embedding strength. For this reason, it is logical to expect a relatively larger increase in the ultimate load by the 1.2 mode of failure than by the 1.4 mode when different embedding strengths are considered. Changing member thickness would make little change in results.

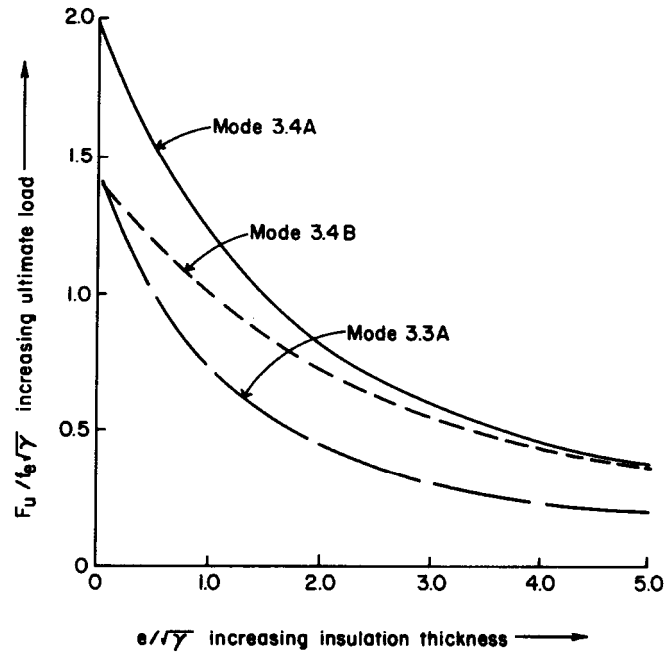


Figure 7.—A dimensionless form of the Field load

$$\left(\frac{F_u}{f_e\sqrt{\gamma}}\right)$$

is used to compare the effect of increased insulation thickness $(e/\sqrt{\gamma})$. *Mode 3.4A (hardboard cleat under the nailhead) gives the highest yield load throughout the range on insulation thicknesses. (ML85 5241)*

Table 9.—Comparison between two-member joint with two-point nail yield (failure mode 1.4) and a two-member joint with no nail yield (failure mode 1.2) for wood members with unequal embedding strengths

Mode of failure number	β	Increase in maximum load	Change in required member thickness	
			t_1	t_2
			Pct	
1.4	1.0	0	0	0
	1.2	4	+2	-11
	1.5	9	+4	-22
	2.0	15	+6	-35
1.2	1.0	0		
	1.2	10		
	1.5	24		
	2.0	47		

Fourth-Root Curve

Comparing the ratio of yield loads demonstrates the effect of using a fourth-root curve in place of the plastic embedding relationship (table 8).

The fourth-root curve method gives a yield load 6 percent lower than that obtained in the general case assuming plasticity under embedding stress behavior. The reduction caused by changing the assumed distribution of embedding strength is less than expected.

It is not possible to derive formulas for the required wood thickness using this fourth-root curve method. Comparisons with the general case discussed above, however, will give some indication of required wood thickness. The distance x between the abutting surfaces and the yield point in the nail appears to be approximately 6 percent higher using the general case than by the fourth-root curve method. A total increase of 6 percent in the required wood thickness seems therefore to be a reasonable prediction for the modes of failure given in table 8. Deviations of that order might well be ignored.

By using the fourth-root curve method, it should be possible to predict a load-deflection curve for the entire joint. In addition to assumptions about the embedding stress distribution, we make a fundamental assumption about the nail's moment-rotation curve. The nail is generally assumed to be plastic, as shown in figure 2. This assumption is correct except for the initial part of the moment-rotation curve. In fact, the predicted curve should be corrected from the origin to a point where the bending moment of the nail approaches M,. Aune (1966) has shown how to correct the predicted curve by using a method of successive approximation. The moment-rotation curve for the nail bending should therefore be recorded during testing.

Summary

The European-based yield theory provides yield load formulas for two- and three-member nailed wood joints. Additional yield theory formulas address joints with steel plate members, unequal wood embedding strength, and a gap made by an insulation layer. Final formulas include a fourth-root wood embedding stress-deformation relationship.

If we control the relative member thicknesses, we can theoretically control the yield mode. Certain yield modes have more nail yield points which increase a joint's theoretical ultimate lateral load.

In many cases steel plate members theoretically increase the ultimate lateral load over all-wood member joints. The increase in yield load is partially due to additional nail yield points. Additional nail yield points when steel side plates are used account for increased load caused by nailhead fixity.

An intermediate layer of insulation theoretically reduces the yield load depending on the thickness of the insulation.

In terms of the yield theory, we can account for increased embedding strength in one wood member of a two-member joint whenever wood crushing rather than nail yield is the predominant failure mode.

Use of a fourth-root embedding curve has little effect on the theoretical ultimate lateral load. The fourth-root curve, however, might make it possible to predict load-deformation behavior of a nailed joint near the yield point.

Experimental verification of the yield theory developed in this report has been provided by the authors in a companion research report (Aune and Patton-Mallory 1986).

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List of Symbols

a, x, x_1 = distance along the nail embedding the wood (in)
e = insulation thickness, gap (in)
f_1 = wood embedding stress at deformation η_1 (lb/in)
f_e = wood embedding strength (lb/in)
f_y = yield strength of the nail (lb/in*)
F = lateral force on nail (lb)
F_u = computed ultimate load, yield load (lb)
ℓ = nail length (in)
M_y = yielding moment of the nail (lb in)
t_1, t_2 = wood member thickness (in)
W = work done by joint (lb in)
$\alpha = t_2/t_1$, ratio of *main* member thickness to *side* member thickness
β = ratio of joint embedding strengths
$\gamma = \frac{M_y}{f_e}$
δ = joint deformation (in)
η, η_1 = wood embedding deformation (in)
θ = angle
θ_1 = angular deflection
ξ = coordinate for integration

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