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Deflections of Uniformly Loaded Floors

A Beam-Spring analog

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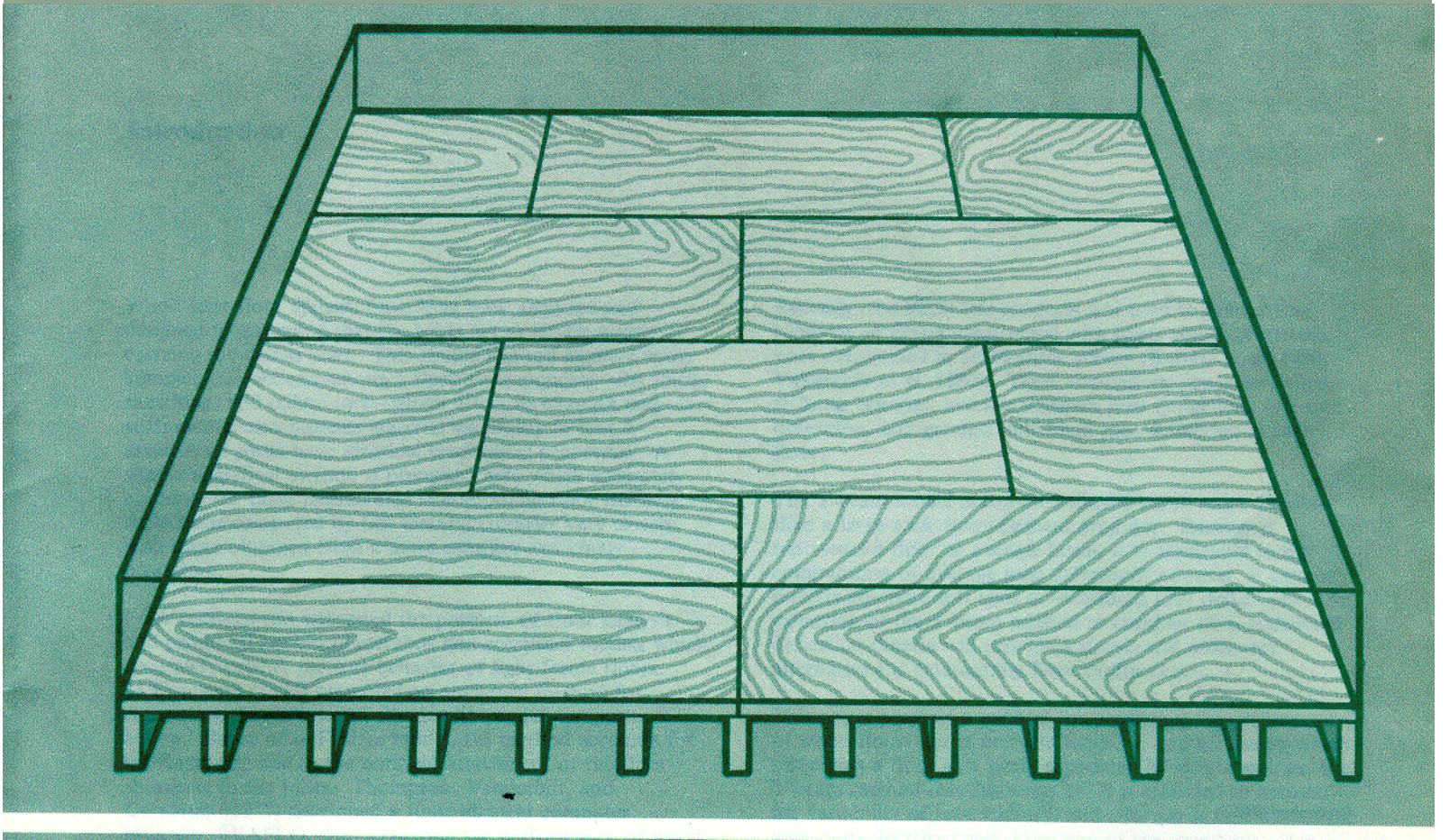

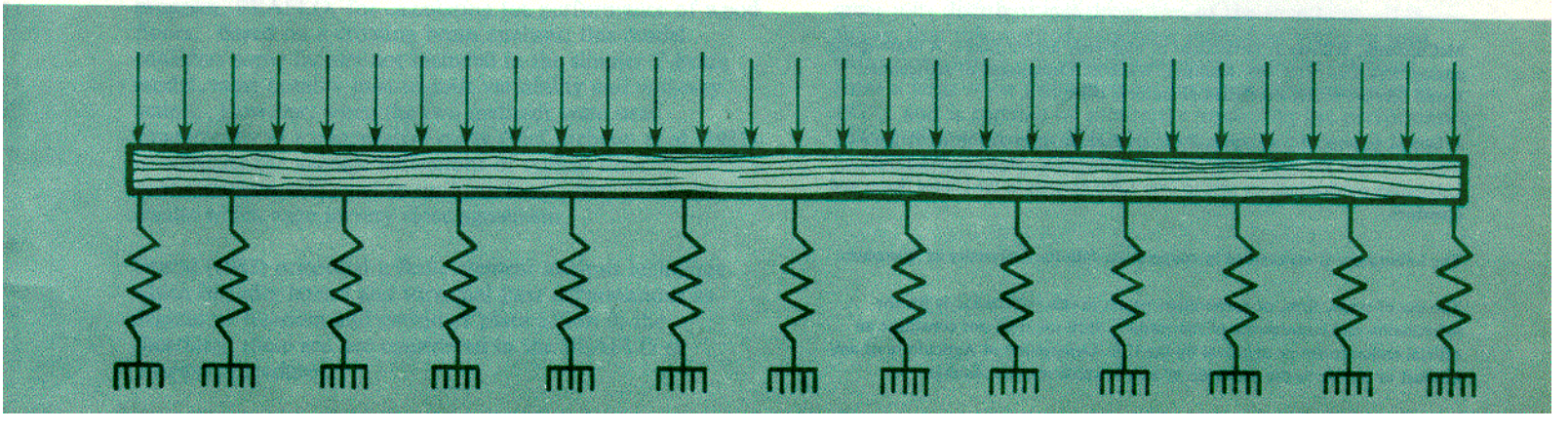

Abstract

A new method for computing the performance of uniformly loaded wood floors is presented. The procedure presents a floor as a simple structure consisting of a beam supported by elastic springs. The method computes midspan joist deflections which are virtually identical to those obtained from a large-scale finite element program, but at a fraction of the computational effort. Also, computations agree very closely with laboratory results. A simple BASIC program is presented for implementing the procedure.

Keywords: Floors, deflections, wood-joist, uniform load

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Deflections of Uniformly Loaded Floors

A Beam-Spring Analog

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Introduction

Wood-joist floor systems have long been analyzed and designed by assuming that the joists act as simple beams in carrying the design load. This simple method neglects many complex interactions among a floor's components-the joists, sheathing, and connectors-which affect its strength and stiffness. It also neglects material variability in that it assumes all joists have identical mechanical properties. A simple analytical method is presented here which accurately accounts for these interactions in computing floor stiffness; deflections computed by this method compare very closely with those obtained from a more complex and costly procedure.

Prior research has addressed the problem of floor analysis and produced analytical methods and computer models which accurately predict the true behavior of light-frame floors. McCutcheon (1977) presented an analytical method for computing the stiffness of floors with partial composite action. Based on a T-beam model, the method accounts for interlayer slip due to the nonrigid attachment of the floor sheathing to the joists. Thompson, Vanderbilt, and Goodman (1977) developed a finite element computer program, FEAFLO, for calculating the performance of wood floors. Based on a crossing beam analysis, this model considers some factors not included in the simpler T-beam model, most notably joist-to-joist variability and two-way action. However, when the two methods were used concurrently in a cooperative study (McCutcheon et al. 1981) to compute the average deflections of typical floor configurations subjected to uniform loads, they produced results which were in very close agreement.

Foschi (1982) presented a finite element analysis technique which includes lateral and torsional joist deformations as degrees of freedom and considers plate action in the sheathing; these are not considered in the FEAFLO or T-beam procedures.

Recently, the FEAFLO program was used to predict the behavior of floors constructed with joists whose properties were determined in an in-grade survey (Bufano et al. 1980; Vanderbilt et al. 1980). These simulations considered three stiffness criteria: average midspan joist deflection, soft-spot deflection, and greatest individual joist deflection in each floor.

The T-beam model is computationally much more efficient than a large-scale finite element program. Because it represents a floor as a single beam, it is applicable to the computation of average joist deflection; however, it cannot directly compute soft-spot or individual joist deflections. An extension of the T-beam method to consider such additional deflection criteria will provide a simple and economic alternative for determining floor performance.

This paper presents a method for predicting the performance of wood floors under uniform loads. Using an analog which represents a floor as a beam supported by elastic springs, the T-beam method (McCutcheon 1977) is extended to account for variability of joist stiffness and two-way action due to the cross-joist distributional properties of the sheathing. It is shown that this method computes individual midspan joist deflections which are virtually identical to those obtained from a large-scale finite element program (Thompson et al. 1977), and is capable of accurately predicting the performance of real floors. A simple BASIC program is presented for implementing the procedure.

Methodology

A light-frame floor system consists of multiple parallel wood joists to which are fastened a sheathing made of plywood or other sheet material. In order to use the T-beam model to account for composite joist behavior and to also consider two-way action, it is necessary to conceptually simplify a floor in two steps. At the first level of abstraction, the sheathing is compressed into a narrow beam which spans across the simply-supported joists (fig. 1). This beam distributes the load among the joists. At the second level the joists, which act as leaf springs in supporting the sheathing distribution beam, are replaced by simple coil springs (fig. 2). These spring stiffnesses account for joist-sheathing composite action. The resulting structure is a beam supported by elastic springs. The ends of the beam may be either simply supported (fig. 2a) or spring supported (fig. 2b), depending upon whether the end joists on the floor are fully supported along their lengths or are free to deflect, respectively.

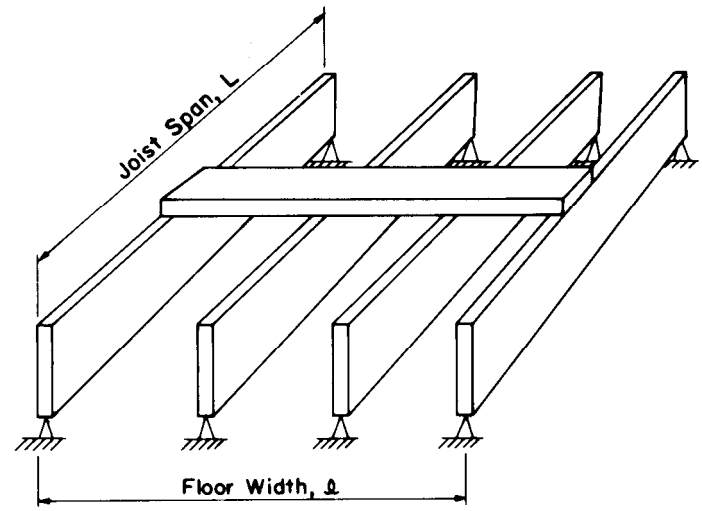

Figure I.--Floor system with sheathing "distribution beam" supported by simply-supported joists. (ML84 5180)

Joist "Spring" Properties

Each spring in the analog represents a floor joist, whose bending stiffness is increased by partial interaction with the sheathing. The aforementioned T-beam analysis (McCutcheon 1977) derived the following equations to define the composite bending stiffness, EI_j, of a joist:

$$EI_j = \frac{EI_R}{1 + f_\Delta \left(\frac{EI_R}{EI_U} - 1\right)} \tag{1}$$

$$EI_R = EI_U + \frac{(EA)_1(EA)_2}{(EA)_1 + (EA)_2} h^2 \tag{2}$$

$$f_\Delta = \frac{10}{(L'\alpha)^2 + 10} \tag{3}$$

$$\alpha^2 = \frac{h^2 k_n / s_n}{EI_R - EI_U} \left(\frac{EI_R}{EI_U}\right) \tag{4}$$

where

EI_j	= effective bending stiffness of the joist, including partial composite action
EI_R	= bending stiffness of the joist if the sheathing is rigidly attached
EI_U	= stiffness of the unconnected joist and sheathing, taken as the stiffness of the bare joist
$(EA)_1$, $(EA)_2$	= axial stiffness of the flange (sheathing) and web (joist) of the T-beam
h	= distance between centroids of the flange and web = ½ (t + d_j)
t	= sheathing thickness
d_j	= joist depth
L′	= distance between discontinuities (open gaps) in the sheathing in the direction of the joist span
k_n	= nail stiffness (load/slip ratio), assuming linear nail behavior
s_n	= average nail spacing.

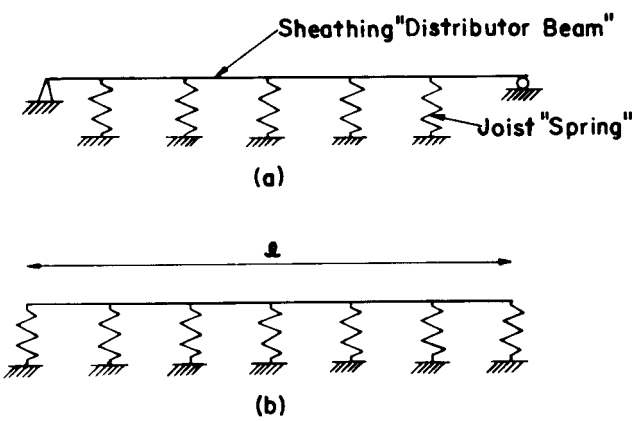

Figure 2.--Beam-spring analog structure with end joists fully supported (a) or free to deflect (b). (ML84 5181)

The midspan deflection, 4, of a uniformly loaded joist is

$$\Delta_j = \frac{5}{384} \frac{wL^4}{EI_j} \tag{5}$$

where

w = load per unit length
L = joist span.

The linear spring constant, k_j, required for the analog is the ratio of joist load to joist deflection:

$$k_j = \frac{wL}{\Delta_j} = 76.8 \frac{EI_j}{L^3} \tag{6}$$

Thus, the equivalent spring constant for each joist is computed by equation (6) after the joist's bending stiffness, EI_j, is computed by the T-beam method summarized by equations (14).

Sheathing "Beam" Properties

The bending stiffness of the analog beam is equal to the stiffness of the sheathing in the cross-joist direction. If there are no discontinuities in the sheathing in this direction, that is, if the sheathing is long enough to extend the full width of the floor, the analog beam stiffness, EI_b, is equal to $\frac{1}{12} E_s L t^3$. However, gaps are usually present and these disrupt the continuity of the sheathing and reduce its bending stiffness. The resulting reduced stiffness can be approximated by averaging out the effect of these discontinuities. For example, if the sheathing spans six joist spaces (as occurs with typical 16-in. spacing and 96-in. lengths of sheathing), a discontinuity occurs at every sixth joist in each row of sheathing. Thus, on the average, there is one-sixth of a gap at each joist crossing, and the average stiffness is reduced by one-sixth.

For the general case, the bending stiffness may be approximated by

$$EI_b = \frac{1}{12} E_s L t^3 \left(1 - \frac{s}{\ell'}\right) \tag{7}$$

where

EI_b = bending stiffness of analog beam
E_s = bending MOE of sheathing in cross-joist direction
s = joist spacing
ℓ' = length of sheathing (typically 96 in.)
L,t = previously defined.

Solution of Analog Structure

Analyzing a wood floor system with a finite element program involves the solution of a large number of simultaneous equations. A single joist may be broken into as many as 20 segments, and analysis of a lo-joist floor will therefore require the solution of up to 200 equations. The beam-spring analog involves just one degree of freedom per joist and, therefore, a lo-joist floor can be analyzed with just 10 simultaneous equations.

The actual equations which result from this method are derived in appendix A, and a simple BASIC program for implementing the method is presented in appendix B. Appendix C presents complete input and output for analyzing one floor.

Results

Comparison of Model with Finite Element Solution

We used the finite element program FEAFLO to determine the deflection characteristics and distributions of wood floors constructed with joists whose properties were determined from a large-scale sampling program (Bufano et al. 1980; Vanderbilt et al. 1980). Each floor comprised ten 2 by 8 joists which were free to deflect, plus an additional continually supported joist at each end (fig. 2a). Floor performance was characterized by three values: (1) the average midspan deflection of the eight interior joists, (2) the "soft-spot" deflection, defined as the highest average deflection of any three adjacent joists, and (3) the maximum deflection of any single joist. The "as-graded" joist data were used to analyze 138 floors for Douglas-fir (green) joists, and 107 floors for southern pine joists.

From each group (Douglas-fir and southern pine), 11 floors were analyzed by the beam-spring analog. These correspond to the stiffest floor, the floors at the 10th percentile of the stiffness distribution, the 20th percentile,. . . , 90th percentile, and the most limber floor.

The input data to the beam-spring model were the same values used in the FEAFLO analyses:

Joist spacing, s = 16 inches (ii.)
Joist span, L = 157 in. (Douglas-fir)
L = 154 in. (southern pine)
Sheathing thickness, t = 0.5782 in.
Sheathing axial modulus, along span, $E_a = 0.9404 \times 10^6$ lb/in.^{2}
Sheathing bending modulus, across span,
$E_b = 1.487 \times 10^6$ lb/in.^{2}
Nail stiffness (as assumed in FEAFLO analyses),
k_n = 30,000 lb/in.
Average nail spacing, s_n = 6.7 in.
Joist size, nominal 2 by 8, actual dimensions input for each joist
Joist modulus input for each joist
Length of sheathing, ℓ' = 96 in.

The T-beam model can account for the presence of complete discontinuities, i.e., open gaps, in the sheathing; but the FEAFLO analyses assumed flexible gaps between the 48-inch-wide pieces of plywood. Therefore, it was necessary to determine empirically an equivalent distance (greater than 48 in.) between open gaps, L ′. Previously McCutcheon et al. (1981) determined that L ′ = 96 inches can be used to account for flexible gaps with a stiffness of 10,000 lb/in.^{2}/in. In these analyses, a gap stiffness of 5,000 lb/in.^{2}/in. was used in the FEAFLO input, and L ′ = 75 inches gave good agreement between the two methods.

The FEAFLO and beam-spring analyses (table 1) give virtually identical results for all three deflection criteria--average interior deflection, soft spot, and maximum joist. Complete FEAFLO data were available for southern pine floor No. 74, and are compared to the beam-spring data in table 2. These results are presented graphically in figure 3, which also shows the results of a traditional bare joist analysis, which assumes no composite action and no two-way action.

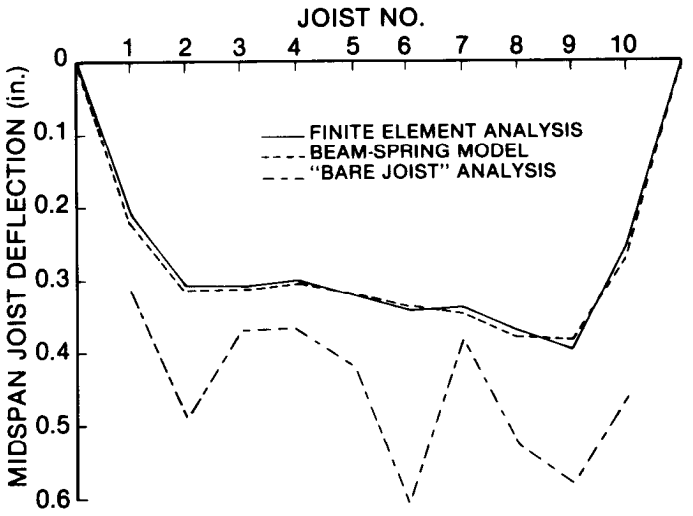

Figure 3.--Comparison of bare joist, finite element (FEAFLO). and beam-spring model analyses of southern pine floor No. 74. (ML84 5182)

Table 1.--Comparison of floor deflections as computed by finite element program (FEAFLO) and beam-spring analog

Floor No.	Average eight interior joists			Soft spot (three joist average)			Maximum (at joist number)		
	FEAFLO	Beam-spring	Difference	FEAFLO	Beam-spring	Difference	FEAFLO	Beam-spring	Difference
	In.	*In.*	*Pct*	*In.*	*In.*	*Pct*	*In.*	*In.*	*Pct*
					DOUGLAS-FIR				
82	0.2315	0.2331	+0.7	0.2673	0.2743	+2.6	0.3078 (2)	0.3142 (2)	+2.1
69	.2989	.3007	+0.6	.3109	.3119	+0.3	.3220 (8)	.3242 (8)	+0.7
103	.3091	.3111	+0.6	.3189	.3211	+0.7	.3273 (7)	.3285 (7)	+0.4
15	.3189	.3209	+0.6	.3355	.3383	+0.8	.3441 (3)	.3450 (3)	+0.3
111	.3296	.3310	+0.4	.3631	.3627	-0.1	.3755 (8)	.3787 (8)	+0.9
99	.3421	.3443	+0.6	.3521	.3554	+0.9	.3580 (3)	.3601 (3)	+0.6
130	.3483	.3504	+0.6	.3712	.3721	+0.2	.3758 (5)	.3749 (8)	-0.2
29	.3576	.3586	+0.3	.3730	.3720	-0.3	.3996 (9)	.3862 (9)	-3.4
55	.3735	.3756	+0.6	.4209	.4227	+0.4	.4460 (3)	.4401 (3)	-1.3
62	.4047	4059	+0.3	.4332	.4355	+0.5	.4410 (8)	.4489 (8)	+1.8
35	.4591	.4609	+0.4	.4979	.4990	+0.2	.5174 (4)	.5146 (4)	-0.5
					SOUTHERN PINE				
35	.2228	.2240	+0.5	.2366	.2390	+1.0	.2545 (2)	.2593 (2)	+1.9
91	.2814	.2797	-0.6	.2977	.3022	+1.5	.3419 (9)	.3218 (8)	-5.9
37	.2903	.2914	+0.4	.3106	.3102	-0.1	.3220 (7)	.3224 (7)	+0.1
90	.3021	.3063	+1.4	.3166	.3140	-0.8	.3438 (9)	.3301 (3)	-4.0
36	.3169	.3180	+0.3	.3472	.3484	+0.3	.3628 (6)	.3572 (6)	-1.5
24	.3244	.3247	+0.1	.3675	.3658	-0.5	.3822 (9)	.3810 (8)	-0.3
[1]74	.3377	.3383	-0.2	.3713	.3708	-0.1	.3964 (9)	.3842 (9)	-3.1
29	.3465	.3474	+0.3	.3653	.3643	-0.3	.3832 (9)	.3770 (8)	-1.6
31	.3545	.3549	+0.1	.3939	.3912	-0.5	.3993 (9)	.3989 (8)	-0.1
12	.3792	.3792	0.0	.4120	.4097	-0.6	.4700 (9)	.4508 (9)	-4.1
65	.4494	.4512	+0.4	.4852	.4876	+0.5	.4993 (3)	.4994 (3)	+0.0

[1]Complete analyses of this floor are presented in table 2 and figure 3.

Table 2.--Complete analysis of southern pine floor No. 74

Joist No.	Joist properties			Joist deflection			
	Width	Depth	Modulus of elasticity	FEAFLO	Beam-spring	Difference	Bare joist
	In.	*In.*	*10^6 lb/in^2*	*In.*	*In.*	*Pct*	*In*
1	1.501	7.250	2.152	0.2102	0.2222	+5.7	0.3173
2	1.493	7.188	1.446	.3108	.3165	+1.8	.4871
3	1.513	7.219	1.835	.3106	.3128	+0.7	.3739
4	1.486	7.219	1.887	.3008	.3019	+0.4	.3702
5	1.500	7.250	1.629	.3197	.3197	0.0	.4194
6	1.505	7.219	1.127	.3462	.3433	-0.8	.6121
7	1.545	7.281	1.699	.3439	.3484	+1.3	.3855
8	1.505	7.219	1.304	.3734	.3797	+1.7	.5290
9	1.510	7.250	1.159	.3964	.3842	-3.1	.5856
10	1.514	7.219	1.476	.2564	.2714	+5.9	.4645
	Average (8 interior joists)			.3377	.3383	+0.2	
	Soft spot (joists 7, 8, and 9)			.3713	.3708	-0.1	
	Maximum individual (joist 9)			.3964	.3842	-3.1	

Comparison of Model with Test Data

In developing the original T-beam analysis method (McCutcheon 1977), the performance of seven floors was evaluated experimentally. Of these seven, two floors were intentionally constructed with joists which had high degrees of variability in their stiffnesses. (The other five had nearly uniform joist properties.) Designated N-3 for the floor with nailed sheathing and G-3 for the floor with the sheathing attached by means of a rigid adhesive, these two are of the greatest interest because they are most useful in assessing the beam-spring model's ability to properly account for joist-to-joist variability.

The properties of the floors and the corresponding input data to the model were:

Number of joists, all free to deflect (fig. 2b), 9
Joist span, L = 144 in.
Joist spacing, s = 15.8125 in. (average of six spaces at 16 in. and two at 15.25 in.)
Joist size, 2 by 8 in. nominal, 1.5 by 7.25 in. actual
Joist moduli, determined individually
Sheathing thickness, t = 0.625 in.
Sheathing properties, determined experimentally:
axial modulus, along span, $E_a = 0.5 \times 10^6$ lb/in.^{2}
bending modulus, across span, $E_b = 1 \times 10^6$ lb/in.^{2}
Length of sheathing, $\ell' = 96$ in.
Width of sheathing, $L' = 48$ in. (N-3)
= 144 in. (G-3, glued tongue-and-groove edges)
(N-3) nail stiffness (as computed by McCutcheon 1977), $k_n = 9{,}400$ lb/in. average nail spacing, $s_n = 7.43$ in.
(G-3) rigid adhesive, $k_n = 1 \times 10^{25}$ lb/in.
$s_n = 1$

Except for end joists 1 and 9, the analog was an excellent predictor of floor N-3's performance, especially at the higher load (table 3, fig. 4). However, it is unlikely that the same nail-slip modulus would apply to both load levels. Use of a higher modulus at the lower load, to account for the steeper slope of a curvilinear load-slip curve, would bring the computed results into even closer agreement with the experimental.

The analog also did a good job of predicting the glued floor's performance (table 4, fig. 5), except for the anomaly at joists 7 and 8, where the experimental deflection of the apparently stiffer joist, No. 8, was greater than that of the more limber joist, No. 7.

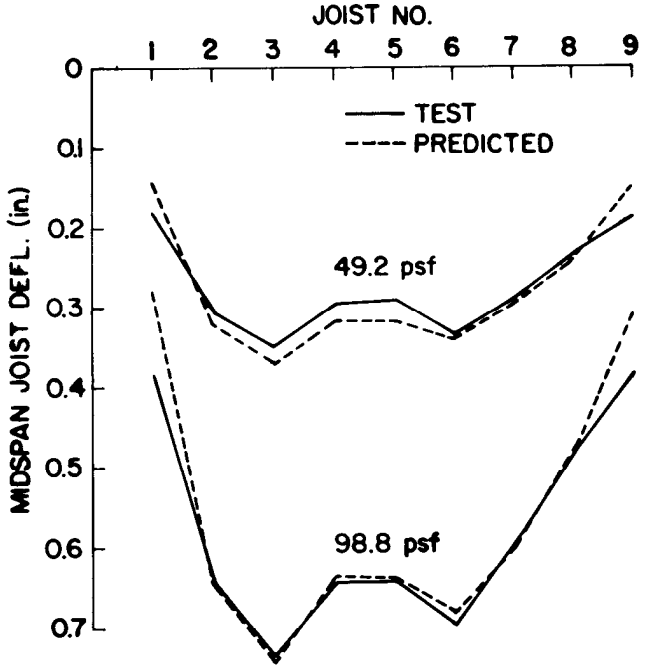

Figure 4.--Experimental and analytical midspan joist deflections for floor N-3. (ML84 5183)

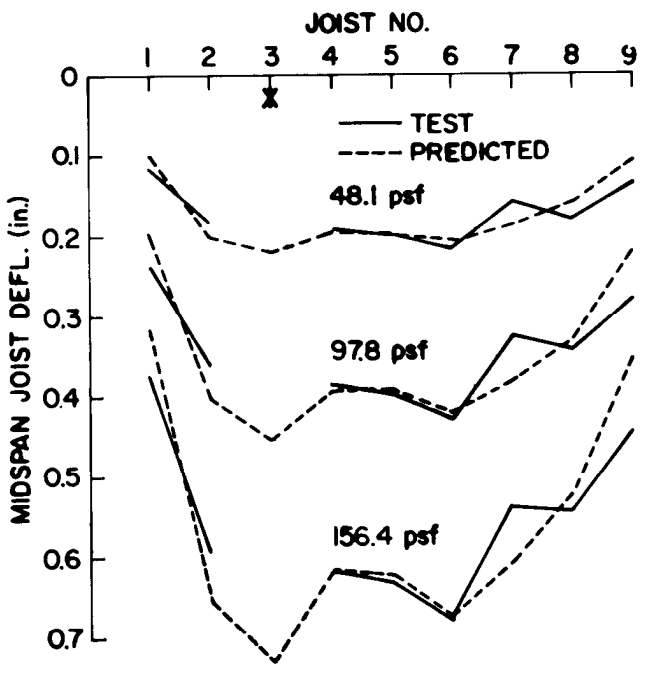

*Figure 5.--Experimental and analytical midspan joist deflections for floor G-3. (*Deflection gage No. 3 malfunctioned) (ML84 5184)*

Table 3.--Comparison of experimental and analytical performance of floor N-3

Joist No.	Modulus of elas-ticity	Joist deflection at 49.2 pounds per square foot			Joist deflection at 98.8 pounds per square foot		
		Test	Beam-spring	Differ-ence	Test	Beam-spring	Differ-ence
	10^6 lb/in.^{2}	*In.*	*In.*	*Pct*	*In.*	*In.*	*Pct*
1	2.32	0.183	0.139	- 24.0	0.386	0.279	- 27.7
2	1.85	.303	.320	+ 5.6	.639	.644	+ 0.8
3	1.29	.349	.370	+ 6.0	.734	.742	+ 1.1
4	2.40	.297	.318	+ 7.1	.646	.639	- 1.1
5	2.03	.290	.317	+ 9.3	.638	.636	- 0.3
6	1.42	.331	.339	+ 2.4	.692	.680	- 1.7
7	2.27	.291	.298	+ 2.4	.594	.599	- 0.8
8	2.67	.232	.240	+ 3.4	.483	.483	0
9	1.74	.190	.151	- 20.5	.381	.304	- 20.2

Table 4.--Comparison of experimental and analytical performance of floor G-3

Joist No.	Modulus of elas-ticity	Joist deflection at 48.1 pounds per square foot			Joist deflection at 97.8 pounds per square foot			Joist deflection at 156.4 pounds per square foot		
		Test	Beam-spring	Differ-ence	Test	Beam-spring	Differ-ence	Test	Beam-spring	Differ-ence
	10^6 lb./in.^{2}	*In.*	*In.*	*Pct*	*In.*	*In.*	*Pct*	*In.*	*In.*	*Pct*
1	2.34	0.115	0.097	- 15.7	0.232	0.196	- 15.5	0.376	0.314	-16.5
2	1.88	.182	.200	+ 9.9	.369	.408	+ 10.6	.593	.652	+ 9.9
3	1.26	([1])	.223	-	([1])	.454	-	([1])	.727	
4	2.39	.189	.191	+ 1.1	.384	.388	+ 1.0	.620	.620	0
5	2.05	.197	.191	- 3.0	.398	.389	- 2.3	.636	.622	- 2.2
6	1.40	.216	.207	- 4.2	.428	.422	- 1.4	.678	.675	- 0.4
7	2.22	.158	.188	+ 19.0	.325	.382	+ 17.5	.539	.611	+ 13.4
8	2.67	.178	.161	- 9.6	.344	.328	- 4.7	.544	.524	- 3.7
9	1.75	.137	.108	- 21.2	.277	.220	- 20.6	.443	.352	-20.5

[1]Deflection gage malfunctioned, floor G-3, joist 3.

Discussion

The input to this method consists of the physical dimensions and mechanical properties of the floor's components.

Joist bending stiffnesses can be easily determined from simple beam flexure tests, such as those specified by ASTM Standard D 198 (ASTM 1976). Similarly, sheathing properties (axial stiffness parallel to the joists and bending stiffness perpendicular) can be measured from axial and bending tests of appropriately oriented sheathing specimens. Standards for these tests may be found in ASTM D 3500, D 3501, and D 3043.

Appropriate nail stiffnesses may be more difficult to define. The performance of mechanical fasteners is the subject of continuing research by a number of investigators. As research in this area progresses, it should be possible to determine which test procedures are most appropriate for defining nail performance in various structural applications. The current state of the art for testing mechanical fasteners is given in ASTM D 1761.

However, a floor's performance is relatively insensitive to changes in the interlayer fastener stiffness. For example, if the nail stiffness k_n in experimental floor N-3 (table 3) is increased by a factor of 2.5 from 9,400 to 23,500, the computed joist deflections will decrease by about 6 percent. Similarly, if k_n in southern pine floor No. 74 (table 2) is reduced by 2.5 from 30,000 to 12,000, the deflections will increase by an average of only 16 percent.

Conclusion

A simple method has been presented for computing the midspan joist deflections in a uniformly loaded wood joist floor. A T-beam model accounts for the composite action between the joists and sheathing, and a beam-on-elastic-springs model accounts for joist variability and two-way action.

The method yields calculated results which are virtually identical to those obtained from a large-scale finite element analysis, but at a fraction of the computational effort, and results also compared closely with experimental data.

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Appendix A
Derivation of Beam-Spring Equations

Simply Supported Beam

A simply supported beam on elastic springs is shown in figure A-l. If Δ_w, is the deflection at node i due to the uniform load w, and Δ_{Fj} is the deflection due to spring force j, then the deflection at spring i, Δ_i, is:

$$\Delta_i = \Delta_w + \sum_{j=1}^{i-1} \Delta_{F_j} + \Delta_{F_i} + \sum_{j=i+1}^{n} \Delta_{F_j} \qquad \text{(A-1)}$$

or

$$-\sum_{j=1}^{i-1} \Delta_{F_j} + \Delta_i - \Delta_{F_i} - \sum_{j=i+1}^{n} \Delta_{F_j} = \Delta_w \qquad \text{(A-2)}$$

Expanding the expressions:

$$-\sum_{j=1}^{i-1} \frac{F_j a_j(\ell - a_i)}{6EI\ell}(2a_i\ell - a_i^2 - a_j^2) + \Delta_i$$

$$-\frac{F_i a_i^2(\ell - a_i)^2}{3EI\ell} - \sum_{j=i+1}^{n} \frac{F_j(\ell - a_j)a_i}{6EI\ell}(2a_j\ell - a_i^2 -$$

$$= \frac{wa_i}{24EI}(\ell^3 - 2\ell a_i^2 + a_2^3) \qquad \text{(A-3)}$$

But:

$a_i = is$
$a_j = js$
$\ell = (n+1)s$
$F_j = -k_j\Delta_j$

and, defining

$$k_j = \beta_j \frac{EI}{\ell^3} = \beta_j \frac{EI}{(n+1)^3 s^3}$$

$$\hat{i} = \frac{i}{n+1}$$

$$\hat{j} = \frac{j}{n+1}$$

Equation (A-3) becomes:

(A-4)

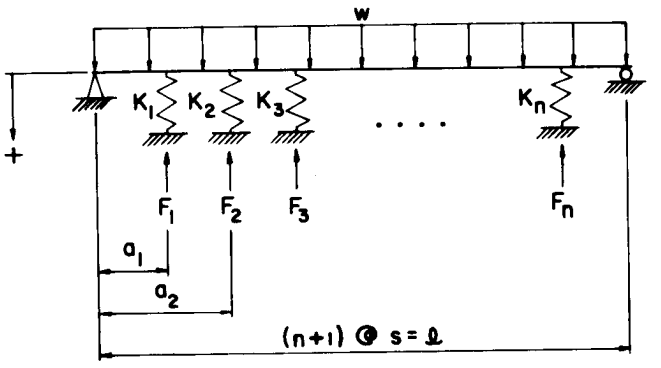

Figure A-1. --Beam on elastic springs with simply-supported ends. (ML84 5178)

Thus, from equation (A-4), the deflections of the beam (floor) and every spring (joist) can be determined by solving n simultaneous equations of the form:

$$[K]\{\Delta\} = \{W\}$$

where

$$K_{ij} = \beta_j\hat{j}(1-\hat{i})(2\hat{i} - \hat{i}^2 - \hat{j}^2) \qquad \text{for } i > j$$

$$= 2[3 + \beta_j\hat{i}^2(1-\hat{i})^2] \qquad i = j$$

$$= \beta_j(1-\hat{j})\hat{i}(2\hat{j} - \hat{i}^2 - \hat{j}^2) \qquad i < j$$

$$W_i = \frac{w\ell^4}{4EI}\hat{i}(1 - 2\hat{i}^2 + \hat{i}^3)$$

$$\hat{i} = \frac{i}{n+1};\ \hat{j} = \frac{j}{n+1};\ \beta_j = \frac{k_j\ell^3}{EI}$$

Beam with "Free" Ends

A beam with its ends spring-supported is shown in figure A-2. The derivation for this case is similar to that for the simply-supported beam except that additional terms are required to account for the movement of the beam ends. The deflection at node i is

$$\Delta_i = \Delta_w + \sum_{j=2}^{i-1} \Delta_{F_j} + \Delta_{F_i} + \sum_{j=i+1}^{n-1} \Delta_{F_j} + \frac{w}{2}\left[\frac{(\ell - a_i)}{k_1} + \frac{a_i}{k_n}\right]$$

$$+ \sum_{j=2}^{n-1} \frac{F_j}{\ell^2}\left[\frac{(\ell - a_i)(\ell - a_j)}{k_1} + \frac{a_i a_j}{k_n}\right] \qquad \text{(A-5)}$$

Expanding the expressions for Δ_{F_j} and substituting:

$$a_i = (i - 1)s$$
$$a_j = (j - 1)s$$
$$\ell = (n - 1)s$$
$$F_j = -k_j\Delta_j$$
$$k_j = \beta_j \frac{EI}{\ell^3} = \beta_j \frac{EI}{(n-1)^3 s^3}$$
$$\hat{i} = \frac{i-1}{n-1}$$
$$\hat{j} = \frac{j-1}{n-1}$$

gives:

$$\sum_{j=2}^{i-1} \beta_j \Delta_j \hat{j}(1 - \hat{i})(2\hat{i} - \hat{i}^2 - \hat{j}^2) + 2\Delta_i [3 + \beta_i \hat{i}^2 (1 - \hat{i})^2]$$

$$+ \sum_{j=i+1}^{n-1} \beta_j \Delta_j (1 - \hat{j})\hat{i}(2\hat{j} - \hat{i}^2 - \hat{j}^2)$$

$$+6 \sum_{j=2}^{n-1} \beta_j \Delta_j \left[\frac{(1-\hat{i})(1-\hat{j})}{\beta_1} + \frac{\hat{i}\hat{j}}{\beta_n}\right]$$

$$= \frac{w\ell^4}{4EI}\hat{i}(1 - 2\hat{i}^2 + \hat{i}^3) + 3\frac{w\ell^4}{EI}\left[\frac{(1-\hat{i})}{\beta_1} + \frac{\hat{i}}{\beta_n}\right] \qquad \text{(A-6)}$$

Again, the problem reduces to a set of n simultaneous equations of the form

$$[K]\{\Delta\} = \{W\}$$

where

$$K_{i1} = K_{in} = 0$$

except

$$K_{11} = K_{nn} = 6$$

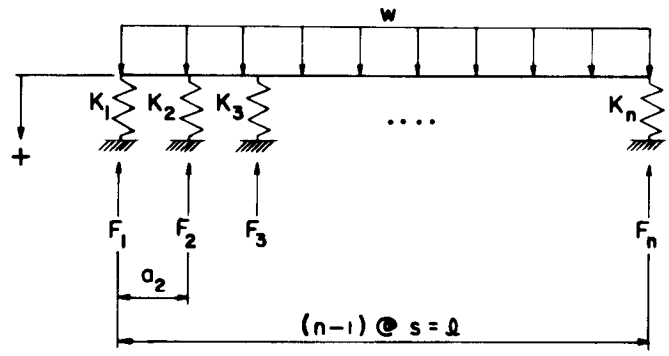

Figure A-2.--Beam on elastic springs with "free" ends. (ML84 5179)

and

$$K_{ij} = \beta_j \left\{\hat{j}(1 - \hat{i})(2\hat{i} - \hat{i}^2 - \hat{j}^2) + 6\left[\frac{(1-\hat{i})(1-\hat{j})}{\beta_1} + \frac{\hat{i}\hat{j}}{\beta_n}\right]\right\} \qquad \text{for } i > j,\ 2 \leq j \leq n-1$$

$$= 6 + 2\beta_i \left\{\hat{i}^2(1 - \hat{i})^2 + 3\left[\frac{(1-\hat{i})^2}{\beta_1} + \frac{\hat{i}^2}{\beta_n}\right]\right\}$$

$$\text{for } i = j,\ 2 \leq j \leq n-1$$

$$= \beta_j \left\{(1 - \hat{j})\hat{i}(2\hat{j} - \hat{i}^2 - \hat{j}^2) + 6\left[\frac{(1-\hat{i})(1-\hat{j})}{\beta_1} + \frac{\hat{i}\hat{j}}{\beta_n}\right]\right\} \text{ for } i < j,\ 2 \leq j \leq n-1$$

$$W_i = \frac{w\ell^4}{4EI}\left\{\hat{i}(1 - 2\hat{i}^2 + \hat{i}^3) + 12\left[\frac{(1-\hat{i})}{\beta_1} + \frac{\hat{i}}{\beta_n}\right]\right\}$$

for all i

$$\hat{i} = \frac{i-1}{n-1};\ \hat{j} = \frac{j-1}{n-1};\ \beta_j = \frac{k_j \ell^3}{EI}$$

Appendix B:
Basic Program for Floor Analysis

The following BASIC program implements the method presented. It is written to run on a Sharp model 1500 pocket computer but can easily be modified for other machines. The program prompts for input data and requires that all units be consistent; e.g., if span is in inches, then joist dimensions must also be in inches, and if joist E is in pounds per square inch, then uniform load must also be in pounds per square inch, not pounds per square foot.

The program is organized as follows:

Segment "F", statements 10-40
Initiate new problem (clear memory).
Input floor type (simply supported or "free").
Input number of joists.

"J", 100-170
Input joist data (spacing, span, widths, depths, elastic moduli).
Note: Input zero for properties which are not identical for all joists.

"S", 200-250
Input sheathing data (thickness, E_{axial}, $E_{bending}$, gap spacings).

"N", 300-320
Input nail data (stiffness, spacing).

"D", 400-790
Solution:
400-450 computes T-beam stiffnesses.
455 computes load vector.
460-495 assembles stiffness matrix.
500-507 modifies stiffness matrix for "free" ends.
510-740 solves simultaneous equations.
750-790 computes average, soft spot, and maximum deflection coefficients.

"L" 795
Input uniform load.

"A", 800-860
Display joist deflections.

999 End.

Program Listing

```
 10: "F":CLEAR :WAIT 0
 15: CLS :PRINT "FLOOR TYPE(SS or FRee)
     ";:INPUT F$
 16: IF F$ = "SS"GOTO 30
 17: IF F$ = "FR"GOTO 30
 20: GOTO 15
 30: CLS :PRINT "No. of JOISTS = ";:INPUT N
 40: DIM B(N),D(N),E(N),K(N,N),Q(N),BT(N)
100: "J":J$ = "JOIST":WAIT 0
102: CLS :PRINT J$ + " SPACING = ";:INPUT S
103: LL=(N-1+(F$="SS")*2)*S
105: CLS :PRINT J$+ " SPAN = ";:INPUT L
110: CLS :PRINT J$+ " WIDTH (all) = ";:INPUT B
115: CLS :PRINT J$+ " DEPTH (all) = ";:INPUT D
120: CLS :PRINT J$+ " MoE (all) = ";:INPUT E
125: USING "# # #"
130: FOR J = 1TO N
140: IF B>0LET B(J) = B:GOTO 150
145: CLS :PRINT J$;J;" WIDTH = ";:INPUT B(J)
150: IF D>0LET D(J)=D:GOTO 160
155: CLS :PRINT J$;J;" DEPTH = ";:INPUT D(J)
160: IF E>0LET E(J)=E:GOTO 170
165: CLS :PRINT J$;J;" MoE = ";:INPUT E(J)
170: NEXT J
200: "S":S$ = "SHTHNG":WAIT 0
210: CLS :PRINT S$+ "THICKNESS = ";:INPUT T
220: CLS :PRINT S$+ "E(axia1) = ";:INPUT EA
230: CLS :PRINT S$+ "E(bndg) = ";:INPUT EB
240: CLS :PRINT "GAP(along joist) = ";:INPUT LJ
250: CLS :PRINT "GAP(perp to joist) = ";:INPUT LS
300: "N":N$ = "NAIL":WAIT 0
310: CLS :PRINT N$+ "STIFFNESS = ";:INPUT KN
320: CLS :PRINT N$+ "SPACING = ";:INPUT SN
400: "D":IS=EB*T∧3/12*L*(1-S/LS)
405: FOR J = 1TO N:JJ =(J - 1 + (F$ = "SS"))*S/LL
410: A1 =EA*T*S:A2=E(J)*B(J)*D(J):IU=A2*
     D(J) ∧ 2/12
415: IF F$ = "FR"AND(J = 1 OR J =N)LET A1 = A1/2
420: H2 = (D(J) + T) ∧ 2/4:IR= IU + A1*A2/(A1+ A2)*H2
430: FD = 10/(H2*KN/SN*LJ ∧ 2*IR/IU/(IR - IU) + 10)
440: EI = IR/( 1 + FD*(IR/IU - 1))
450: BT = 76.8*EI/IS*(LL/L)∧3:BT(J) = BT
455: Q(J)=LL∧4*L/4/IS*JJ*(1-2*JJ∧2 + J J∧ 3 )
460: FOR I = 1TO N:II = (I - 1 + (F$ = "SS"))*S/LL
470: IF I>JLET K(I,J)=BT*JJ*(1 - II)*
     (2*II-II ∧ 2 - JJ ∧ 2)
480: IF I = JLET K(I,J) = 2*(3 + BT*II ∧ 2*(1-II) ∧ 2)
490: IF I< JLET K(I,J) = BT*(1 - JJ)*II*
     (2*JJ-II∧2-JJ∧2)
495: NEXT I:NEXT J
500: IF F$ = "SS"GOTO 510
501: FOR I = 1TO N:II = (I - 1)/(N - 1)
502: Q(I) = Q(I) + 3*LL ∧ 4*L/IS*((1- II)/BT(1) +
     II/BT(N))
503: FOR J=1TO N:JJ=(J - 1)/(N-1)
504: K(I,J)=K(I,J)+6*BT(J)*((1- II)*(1 - JJ)/BT
     (1) + II*JJ/BT(N))
505: IF J=1OR J=N LET K(I,J)=0
506: NEXT J:NEXT I
507: K(1,1) = 6:K(N,N) = 6
510: FORK=1TO N-1
520: FOR I=K+1TO N
530: IF K = 1GOTO 570
540: FOR J=1TO K-1
550: K(I,K) = K(I,K) - K(I,J)*K(J,K)
560: NEXT J
570: K(I,K) = K(I,K)/K(K,K)
580: NEXT I
590: FOR I=K + 1TO N
600: FOR J=1TO K
610: K(K+1,I)=K(K+1,I)-K(K+1,J)*K(J,I)
620: NEXT J:NEXT I
630: NEXT K
640: FOR I=2TO N
650: FOR K=1 TO I-1
660: Q(I) = Q(I) - K(I,K)*Q(K)
670: NEXT K:NEXT I
680: FOR I = NTO 1 STEP - 1
690: IF I = NGOTO 730
700: FOR K=I+1TO N
710: Q(I) = Q(I) - K(I,K)*Q(K)
720: NEXT K
730: Q(I) = Q(I)/K(I,I)
740: NEXT I
750: MX=0:SS=0:SI=0
760: FOR J=2TO N-1
770: IF Q(J) > MXLET MX = Q(J):AM = J
780: X = (Q(J - 1) + Q(J) + Q(J + 1))/3:IFX > SSLET
     SS=X:AS=J
790: SI = SI + Q(J):NEXT J:BEEP I
795: "L":WAIT 0:CLS:PRINT "UNIFORM LOAD =
     ";:INPUT Q
800: "A":U$ = " # # . # # # # ":WAIT :CLS
810: PRINT "AVE. DEFL = ";USING U$;SI/(N - 2)*Q
820: PRINT "SOFT SPOT = ";USING U$;SS*Q;
     " @ ";USING ;AS
830: PRINT "MAX. DEFL = ";USING U$;MX*Q;
     " @ ";USING ;AM
840: FOR I = 1TO N
850: PRINT "DEFL"; USING " # # # ";I; " = ";USING
     U$;Q(I)*Q
860: NEXT I
999: END
```

Appendix C:
Sample Input and Output

To demonstrate the use of the BASIC program, complete input and output are presented for experimental floor N-3 (table 3).

Program Prompt	Input	output
FLOOR TYPE (SS OR FR)?	FR	AVE. DEFL = 0.3145
NO. OF JOISTS = ?	9	SOFT SPOT = 0.3360 @ 3
JOIST SPACING = ?	15.8125	MAX. DEFL = 0.36% @ 3
JOIST SPAN = ?	144	DEFL 1 = 0.1388
JOIST WIDTH (all) = ?[1]	1.5	DEFL 2 = 0.3204
JOIST DEPTH (all) = ?[1]	7.25	DEFL 3 = 0.3696
JOIST MOE (all) = ?[1]	0	DEFL 4 = 0.3181
JOIST 1 MOE = ?	2.32E6	DEFL 5 = 0.3167
JOIST 2 MOE = ?	1.85E6	DEFL 6 = 0.3385
JOIST 3 MOE = ?	1.29E6	DEFL 7 = 0.2981
JOIST 4 MOE = ?	2.40E6	DEFL 8 = 0.2403
JOIST 5 MOE = ?	2.03E6	DEFL 9 = 0.1511
JOIST 6 MOE = ?	1.42E6	
JOIST 7 MOE = ?	2.27E6	
JOIST 8 MOE = ?	2.67E6	
JOIST 9 MOE = ?	1.74E6	
SHTHNG THICKNESS = ?	.625	
SHTHNG E (axial) = ?	.5E6	
SHTHNG E (bndg) = ?	1E6	
GAP (along joist) = ?	48	
GAP (perp to joist) = ?	96	
NAIL STIFFNESS = ?	9400	
NAIL SPACING = ?	7.43	
(Program computes for approximately 2 minutes)		
UNIFORM LOAD = ?	49.2/144	

[1]Enter zero if joist properties are not all identical; in this case, program prompts for individual entry.

Appendix D:
Program Variables

Program Variable	Text Variable	Meaning
F$		Floor type: SS for simply supported FR for "free"
N	n	Number of joists
B,B(J)		Joist widths
D,D(J)	d_j	Joist depths
E,E(J)		Joist modulus of elasticity
K(I,J)	K_{ij}	Stiffness matrix element
Q(J)	W_j	Load vector element (solution)
Q(J)	Δ_j	Displacements (output)
Q	w	Uniform load
BT,BT(J)	β_j	Nondimensional spring stiffness
S	s	Joist spacing
LL	ℓ	Floor width
L	L	Floor span
T	t	Sheathing thickness
EA	E_a	Sheathing axial modulus of elasticity, along joist
EB	E_b	Sheathing bending modulus of elasticity, across joist
LJ	L'	Sheathing gap, along joist
LS	ℓ'	Sheathing gap, across joist
KN	k_n	Nail stiffness
SN	s_n	Nail spacing
IS	EI_b,EI	Sheathing EI
II	$\hat{i}$	Nondimensional joist location
JJ	$\hat{j}$	Nondimensional joist location
A1	$(EA)_1$	Sheathing axial stiffness
A2	$(EA)_2$	Joist axial stiffness
H2	h^2	Square of distance between centroids of T-beam flange and web
IU	EI_U	Stiffness of bare joist
IR	EI_R	Stiffness of rigidly connected T-beam
EI	EI_j	Joist T-beam stiffness
FD	f_Δ	See equation (3)
MX		Maximum joist deflection coefficient
AM		Location (joist No.) of maximum deflection
SS		"Soft-spot" deflection coefficient
AS		Location (central joist No.) of soft spot
SI		Sum of interior joist deflection coefficients
X		Average deflection coefficient, three joists

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