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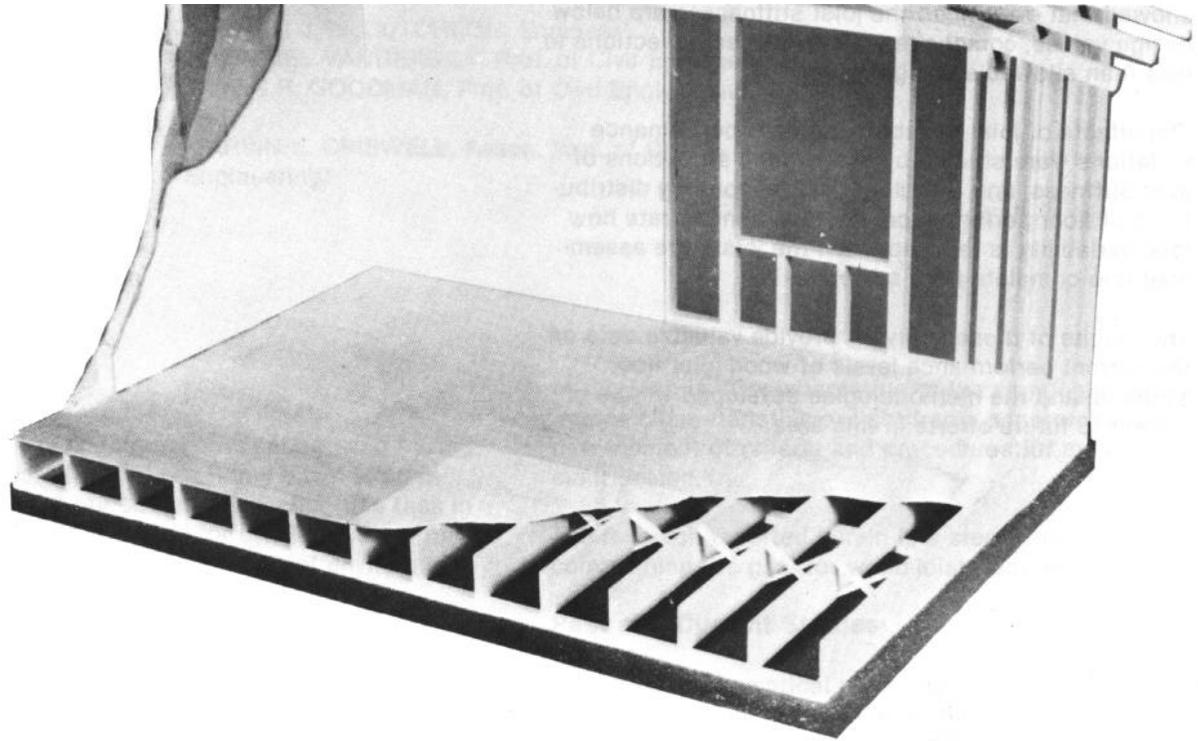
Forest Service

Forest  
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Research  
Paper  
FPL 405



# Wood Joist Floors: Effects of Joist Variability on Floor Stiffness



## **Abstract**

A theoretical study was conducted of the performance (deflection) of wood-joist floors subjected to distributed loads.

Eleven "benchmark" floors, typical of current construction practice, were analyzed. Results demonstrated that composite action, ignored by current design methods, can be substantial.

Performance distributions were calculated for five floors from available joist stiffness data. Results showed that even when the joist stiffnesses are below design values, composite action reduces deflections to less than allowable design levels.

The effects of joist variability on floor performance variations were studied by assuming distributions of joist stiffness, and calculating corresponding distributions of floor performance. Results demonstrate how joist variability is reduced when the joists are assembled into complete floor systems.

The results of these analyses provide valuable data on the current performance levels of wood-joist floor systems, and the methodologies developed will be of benefit to future efforts in this area.

## **Acknowledgment**

The authors express their thanks to Dr. Michael R. O'Halloran of the American Plywood Association for his assistance in selecting the floors to be analyzed and appropriate sheathing properties, and to James H. Haskell, Mathematical Statistician at the Forest Products Laboratory, for his help and guidance with the statistical aspects of this study.

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# Wood Joist Floors: Effects of Joist Variability on Floor Stiffness-

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## Introduction

The challenge of providing economical housing having the quality expected by today's home buyer is an increasing concern of our society. Achieving this task in ways consistent with the need for efficient use of our natural resources and the preservation of environmental quality is of increasing importance.

In spite of the wide use and economic importance of wood construction in housing, current methods of design and analysis lag behind the modern methods used for other materials. Wood-joist floor systems are generally designed by assuming the joists act alone as simple beams. This conservative design procedure neglects many factors which contribute to the strength and stiffness of the floor. It also neglects variations in joist properties in that all joists are assumed to be identical, with strength and stiffness properties equal to code-prescribed values.

As the result of a planning conference held at the Forest Products Laboratory (FPL), a long-range plan for modernizing the design of light-frame structures was developed. Entitled "Five-Year Action Plan for Light-Frame Construction Research"<sup>3</sup>; one primary objective

of the plan is "Documentation of the composite performance characteristics of light-frame construction and development of criteria and procedures for more efficient design."

The research reported herein is a step toward accomplishing the goal for wood-joist floor systems.

## Past and Current Studies

A verified mathematical model which properly assesses the static behavior of wood-joist floor systems is now operational at Colorado State University (CSU) (25)<sup>4</sup>. This computer-aided method of analysis includes the effects of such variables as the degree of composite behavior between the joist and sheathing components, sizes and properties of the joists and sheathing, spacing of joists, presence of gaps between pieces of sheathing, and variable material and connector properties. The model has been used extensively to study the effects of various parameters (3, 6, 8, 9, 10, 26, 27, 28). Monte Carlo simulation procedures have been used to assess the effects of material variability on the performance of floor systems (5). Research is continuing at CSU on the development of simplified design concepts, development of an ultimate strength procedure, and the evaluation of floor performance using ingrade lumber data.

Research at FPL has resulted in an approximate method for computing T-beam deflections (16), and is continuing on methods for simplified floor design. In addition, recent data have been collected and analyzed for the ingrade stiffness of typical joist lumber (7). These data provide a source of "calibration" for proposed design techniques through the use of simulation

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<sup>2</sup> Colorado State University, Ft. Collins, Colo. 80523.

<sup>3</sup> Hans, G. E. 1977. Five-year action plan for light-frame construction research. USDA For. Serv., For. Prod. Lab., Madison, Wis.

<sup>4</sup> Italicized numbers in parentheses refer to literature in the list of references at the end of the report.

methods for assessing the performance of wood floors constructed of typical materials.

Other important complementary research includes work on slip modulus and sheathing gap parameters by FPL and CSU researchers, work on “limit states” design concepts being conducted by FPL staff, and other reliability-based design studies now proceeding in Canada and elsewhere.

### Objectives and Scope

The CSU and FPL study comprised three main areas of investigation:

(1) To quantify current performance levels, 11 wood-joint floors were selected which are typical of current construction practice. The deflections of these “benchmark” floors were calculated and compared with current design criteria.

(2) Five “benchmark” floors were selected for more intensive study. Using existing joist stiffness data, distributions of floor performance were calculated. These results and those from area (1) above provide information which can serve as a baseline for the calibration of new methods of floor design.

(3) To further quantify the effects of joist variability on floor performance, five other floors were analyzed for various assumed distributions of joist stiffness.

## Benchmark Floors

### Floor Selection

The 11 floors selected are representative of current construction practice and include single- and double-sheathed floors with nailed and glued connections. Nailed floors were designed on a bare joist basis to satisfy National Design Specification (NDS) (19) stress criteria for a uniform load of 50 pounds per square foot (40 lb/ft<sup>2</sup> live + 10 lb/ft<sup>2</sup> dead load) without exceeding a deflection under 40 pounds per square foot live load of joist span/360. Glued floors were designed following American Plywood Association (APA) recommendations (1) which consider composite action in calculating stiffness, but compute strength on the basis of the bare joists.

A summary of the assumed joist data is given in table 1. Sheathing and connector properties and sheathing gap stiffnesses are listed in table 2. The meanings of most of the symbols given in the tables are explained in figure 1. In figure 1,  $k$  represents the assumed linear slip modulus between connector load and deformation. Thus  $k_{SS}$  is the slip modulus for the connectors between sheathing layers while  $k_S$  is for the joist-to-sheathing connectors.

Table 1.—Joist and connector data, benchmark floors<sup>1</sup>

Floor	Joist			Modulus of elasticity <u>10<sup>6</sup> lb/in.<sup>2</sup></u>	Connection <sup>2</sup>	Sheathing <sup>2</sup> thickness and type <u>In</u>
	Size	Spacing <u>In.</u>	Span			
1	2 x 8	16	13 ft-1 in.	1.7	8d nails	19/32 plywood
2	2 x 8	16	11 ft-9 in.	1.3	8d	19/32 plywood
3	2 x 8	16	13 ft-10 in.	1.7	Glue	19/32 plywood
4	2 x 8	16	13 ft-1 in.	1.7	8d plywood to joist 6d sheathing	5/8 plywood 5/8 particleboard
5	2 x 8	16	13 ft-10 in.	1.7	Glue, plywood to joist 6d sheathing	5/8 plywood 5/8 particleboard
6	2 x 8	24	11 ft-3 in.	1.7	8d	3/4 plywood
7	2 x 12	16	18 ft-3 in.	1.3	8d	19/32 plywood
8	2 x 12	24	17 ft-6 in.	1.7	8d	3/4 plywood
9	2 x 12	24	17 ft-6 in.	1.7	Glue	3/4 plywood
10	2 x 12	24	17 ft-6 in.	1.7	8d plywood to joist 6d sheathing	3/4 plywood 5/8 particleboard
11	2 x 12	24	14 ft-10 in.	1.3	8d	3/4 plywood

<sup>1</sup> Sizes, spacings, spans, and connectors were chosen as typical commercial practice for floor built with lumber having the indicated modulus of elasticity. Dry American Lumber Standard sizes were assumed.

<sup>2</sup> See table 2 for connector and sheathing details.

Table 2.—Sheathing and connector data, benchmark floors<sup>1</sup>

Floor	Sheathing	Bending stiffness parallel to face grain	Axial stiffness parallel to face grain	Bending stiffness perpendicular to face grain	Axial stiffness perpendicular to face grain	Sheathing thickness t	Connector spacing s	Connector stiffness k	Gap <sup>2</sup>
		Lb/in. <sup>2</sup>	Lb/in. <sup>2</sup>	Lb/in. <sup>2</sup>	Lb/in. <sup>2</sup>	In.	In.	Lb/in.	Lb/in. <sup>2</sup>
1	19/32 Underlayment	1,142,000	825,000	317,000	525,000	0.5782	6.7	17,500	1,000
2	19/32 Underlayment	1,142,000	825,000	317,000	525,000	.5782	6.7	17,500	1,000
3	19/32 Underlayment	1,142,000	825,000	317,000	525,000	.5782	1.0	54,000	1,000
4	Particleboard 5/8 unsanded	250,000 1,193,000	250,000 854,000	250,000 283,000	250,000 504,000	.6250 .6095	6.7 6.7	3,800 24,000	1 500
5	Particleboard 5/8 unsanded	250,000 1,193,000	250,000 854,000	250,000 283,000	250,000 504,000	.6250 .6095	6.7 1.0	3,800 54,000	1 500
6	3/4 Underlayment	1,192,000	830,000	504,000	807,000	.7345	5.8	25,000	1,000
7	19/32 Underlayment	1,142,000	825,000	317,000	525,000	.5782	6.7	17,500	1,000
8	3/4 Underlayment	1,192,000	830,000	504,000	807,000	.7345	5.8	25,000	1,000
9	3/4 Underlayment	1,192,000	830,000	504,000	807,000	.7345	1.0	54,000	1,000
10	Particleboard 3/4 unsanded	250,000 1,300,000	250,000 871,000	250,000 451,000	250,000 774,000	.6250 .7345	5.8 5.8	4,000 25,000	1 500
11	3/4 Underlayment	1,192,000	830,000	504,000	807,000	.7345	5.8	20,000	1,000

<sup>1</sup> Sheathing values were selected to be representative of average values of material which is produced.

<sup>2</sup> All gap element lengths = 0.10 in.

Locations of each joist, gap between pieces of sheathing, and sheathing strip used in the computer-assisted analyses are given in figures A1 through A10 of Appendix A.

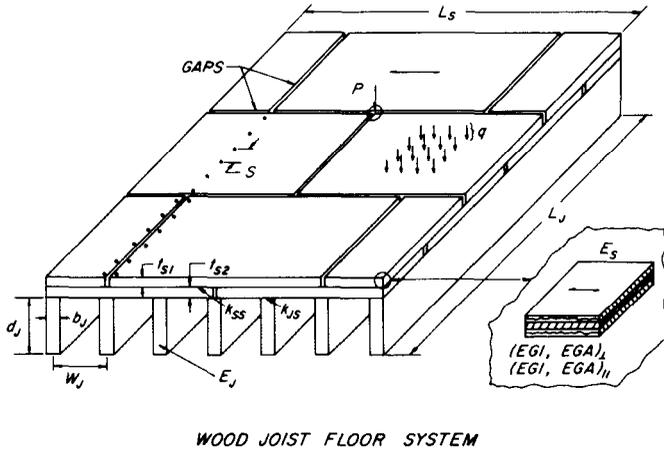
### Benchmark Analyses (CSU)

Each of the benchmark floors was analyzed by using the mathematical model developed at CSU and embodied in computer program FEAFL0 (24, 25). Each floor contained 11 joists, with rigid supports beyond the first and eleventh joists.

There is no standard way of reporting floor performance or of comparing performances of different floors. Dawson (3) reported maximum joist deflections. Fezio, et al. (6) reported maximum joist deflections, maximum joist tensile stresses, and maximum interlayer shear force. In Appendix A of his report (5), Fezio also compared the mean values for each floor with the maxima and standard deviations.

For a uniformly loaded rectangular floor with identical joists, all joists except the two adjacent to the ends of the sheathing strips (joists 1 and 11 in figs. A1-A10) deflect by nearly the same amount. For example, the midspan joist deflections for floor No. 1 are given in table 3.

For uniformly loaded floors, the average of midspan joist deflections appears to be a good measure of floor performance and this measure is used herein. For floors with variable component properties, use of the average floor deflection appears to be a reasonable measure of performance, since any practical design method which accounts for variability will probably be based on average performance rather than individual joist performance. This topic is further discussed later. An average floor deflection probably is not the best measure of response to concentrated loads and a different technique will have to be devised for defining acceptable performance of floors under point loads.



M151-002

Figure 1.—Symbols defining material and connector properties and floor geometry.

**Benchmark Analyses (FPL)**

Eight of the benchmark floors were analyzed using the method presented in Research Paper FPL 289 (16). Since the method applies only to two-layer beams (i.e., joists plus a single layer of sheathing), it was not possible to analyze the three-layer floors, Nos. 4, 5, and 10.

The FPL method can be used to compute the deflection of a T-beam with a joist web and sheathing flange and includes the effects of open gaps in the flange. Because the gaps must be either completely open (i.e., transmit no axial force) or closed (i.e., nonexistent, with continuous sheathing), it was necessary to approximate the effect of the “flexible” gaps specified in table 2. For the nailed floors, this was accomplished by empirically doubling the distance between gaps from 48 to 96 inches and considering them open. For the glued floors (Nos. 3 and 9), the greater disruption in composite behavior due to gaps was considered and the spacing was left at 48 inches. As is shown below, these assumptions gave good results.

The connector stiffness values presented in FPL 289 do not agree with those in table 2. The values in table 2 were used in the FPL calculations so that the results of the FPL and CSU calculations may be compared.

Table 3.—Midspan deflections of Floor No. 1

Joist No.	Deflection	Relative deflection
	In.	
1, 11	0.2422	0.726
2, 10	.3373	1.012
3, 9	.3433	1.030
4, 8	.3314	.994
5, 7	.3259	.978
6	.3247	.974
	Avg = 0.3334	

**Benchmark Results**

Results of the benchmark floor analyses are presented in table 4 and figures 2 and 3. In table 4, the results of the CSU computer analyses and the FPL computations are identified. Deflections and span/deflection ratios are given for three cases: (a) the bare joist which represents current design practice ( $\Delta_1$ ), (b) the complete floor with the finite k listed in table 2 ( $\Delta_k$ ), and (c) an “infinite” k value ( $k = 10^8$ ) which represents rigid glue ( $\Delta_\infty$ ). All computations were made using average modulus of elasticity (MOE) values (table 2). Table 4 also lists the degree of composite action achieved with each construction, as defined by the table, footnote 4.

Figure 2 shows the ratio of each of the three computed deflections to allowable deflection (span/360) for the CSU simulations. (A plot of the FPL results would be almost identical.) The calculated stiffness of the benchmark floors is better than the assumed design criterion of span/360 when average material values are used. The ratio of predicted deflection,  $\Delta_k$ , to span/360 ranged from 0.515 to 0.777 with an average of 0.694. This is equivalent to stating that the ratio of  $\Delta_k$ /span ranged from 1/700 to 1/460 with an average of 1/520.

Table 4 shows the amount of composite action that was developed in each floor. If sheathing and joist are unconnected, k is very low and no composite action is developed. If rigid glue is used ( $k \geq 10^8$ ), essentially 100 percent of the potential composite action is developed. Nails and glue provide k values usually in the range  $10^3$  to  $10^5$  pounds per inch and the resulting floors exhibit incomplete composite action. For the nailed floors, the computed percent of composite action ranged from 42.5 to 57.8 percent with an average of 53.6 percent; for the glued floors with higher k the range was 65.2 to 82.2 percent with an average of 74.8 percent. The improvement in floor performance, which can be obtained through the use of glue, can be demonstrated by examining floors No. 8 and 9, which are identical except

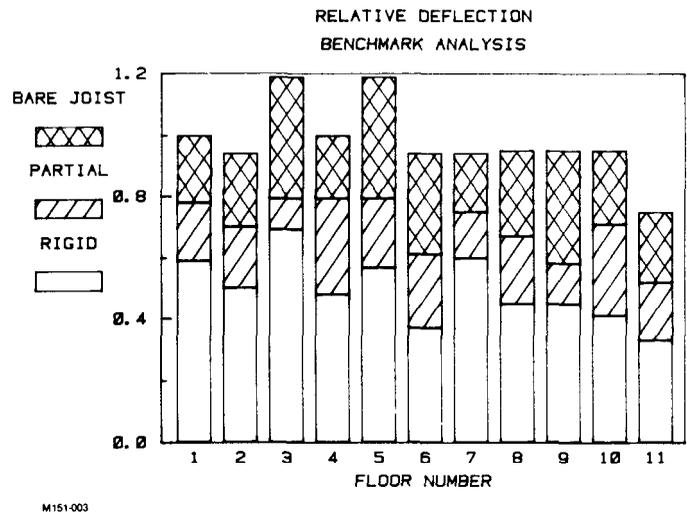


Figure 2.—Relative deflections of benchmark floors ( $L/360 = 1.0$ ).

**Table 4.—Deflections of benchmark floors under uniform load of 40 pounds per square foot**

Floor	Span, L	Analyzed by	$D_1^1$	$L/D_1$	$\Delta_{\infty}^2$	$L/\Delta_{\infty}$	$D_k^3$	$L/D_k$	Composite action <sup>4</sup>
			In.	In.	In.	In.	In.	Pct	
1	157	CSU	0.4342	362	0.2552	615	0.3334	471	56.3
		FPL	.4338	362	.2512	625	.3389	463	51.9
2	141	CSU	.3694	382	.1978	713	.2729	517	56.2
		FPL	.3689	382	.1949	724	.2718	519	55.8
3	166	CSU	.5426	306	.3186	521	.3584	463	82.2
		FPL	.5421	306	.3139	529	.3486	476	84.8
4	157	CSU	.4342	362	.2072	758	.3378	465	42.5
5	166	CSU	.5426	306	.2591	641	.3578	464	65.2
6	135	CSU	.3562	379	.1391	971	.2307	585	57.8
		FPL	.3543	381	.1376	981	.2330	579	56.0
7	219	CSU	.5753	381	.3663	598	.4567	480	56.8
		FPL	.5751	381	.3593	610	.4689	467	49.2
8	210	CSU	.5580	376	.2676	785	.3910	537	57.5
		FPL	.5572	377	.2620	802	.4183	502	47.0
9	210	CSU	.5580	376	.2659	790	.3334	630	76.9
		FPL	.5572	377	.2620	802	.3400	618	73.6
10	210	CSU	.5580	376	.2422	867	.4182	502	44.3
11	178	CSU	.3766	473	.1644	1,083	.2546	699	57.5
		FPL	.3760	473	.1609	1,106	.2645	673	51.8

<sup>1</sup>  $\Delta_1$  is deflection of joists alone.

<sup>2</sup>  $\Delta_{\infty}$  is deflection of joists with sheathing rigidly attached.

<sup>3</sup>  $\Delta_k$  is computed deflection of complete floor assembly.

<sup>4</sup> Defined as  $100 \times \left[ \frac{\Delta_1 - \Delta_k}{\Delta_1 - \Delta_{\infty}} \right]$

for type of connector. The use of glue for floor No. 9 reduced deflections by 15 percent compared to the deflections for nailed floor No. 8. Although this is a substantial increase in performance, it should be noted that the long-term behavior of glued floors has not been completely quantified.

Figure 3 shows the increase in performance which is obtained with partial composite action and with rigid fasteners. In this figure, the reference stiffness (1.0 on the vertical scale) is that of the bare joists.

**Simulation Studies**

The benchmark analysis demonstrate the theoretical performance of floors, assuming that all properties are constant. To determine how distributions of joist properties affect floor performance, five floors (Nos. 1, 2, 3, 4, and 6) were selected for more intensive study. These five represent a wide variety of floor types, including both high and low joist MOE, nailed and glued sheathing, two- and three-layer floors, and two different joist spacings.

Data on the distributions of joist stiffnesses are available from a study conducted by FPL and Purdue

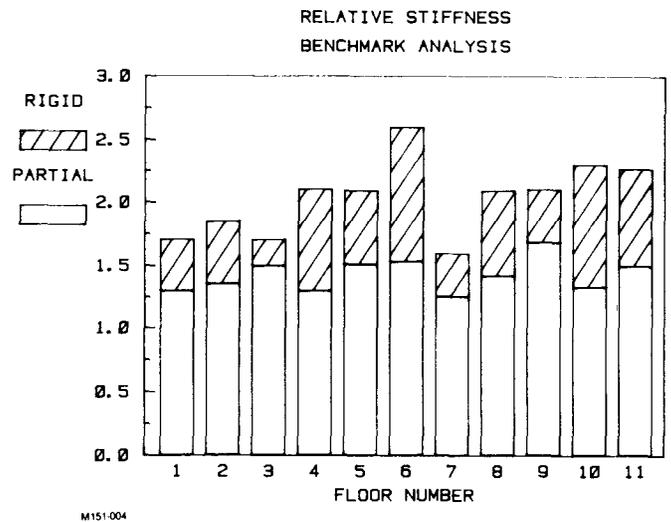


Figure 3.—Relative stiffnesses of benchmark floors (bare joists = 1.0).

University (7) and are referred to herein as the "Purdue data." Two hundred lots of 10 consecutive joists each were sampled. Table 5 presents the estimates for joist EI (expressed nondimensionally as the ratio of actual EI to the EI given in the NDS (19)). These were assumed to follow normal distributions. In general, the data are well represented by normal curves. Upper and lower truncation points were chosen to coincide with the maximum and minimum values observed in the sample.

In computing floor performance from the Purdue data, two techniques were used:

(1) The joists which constitute each floor were assumed to all be from the same lot. This represents what usually happens in actual practice.

(2) The individual joists for each floor were selected from the entire joist distribution.

The methods for effecting these computations are discussed below.

### Simulation Analyses (CSU)

To determine the distributions of floor performance, 440 individual floors were analyzed: 40 replications of each floor times 5 floor configurations times 2 sampling methods, plus an additional 40 floors of type 2. The input data to the FEAFL0 computer program were generated by the Monte Carlo procedure shown in figure 4.

For sampling by lots (indicated by arrows on left of fig. 4), first a lot mean EI was randomly selected from the lot distribution ( $\beta_1$  in fig. 4), and then the individual joist stiffnesses were determined by randomly selecting modifiers ( $\beta_2$  in fig. 4) from the within-lot distribution. These modifiers were multiplied by the lot mean to obtain individual joist values. For each floor, a new lot EI was selected.

For sampling by individual joists (right side of fig. 4), the procedure was the same except that a new "lot" EI was selected for each joist and not just for each floor. This is equivalent to assuming all 2,000 Purdue joists were collected into one 2,000-member lot.

The results of these simulations are presented in figures 5 through 9, where performance level, as de-

Table 5.—Parameters of truncated normal distributions which define the "Purdue" data'

	Lots	Joists within lot
Mean	0.88	0.88
Standard Deviation	.11	.16
Minimum	.6220	.2923
Maximum	1.1674	1.6657

<sup>1</sup> Actual bending stiffness (EI) divided by NDS bending stiffness.

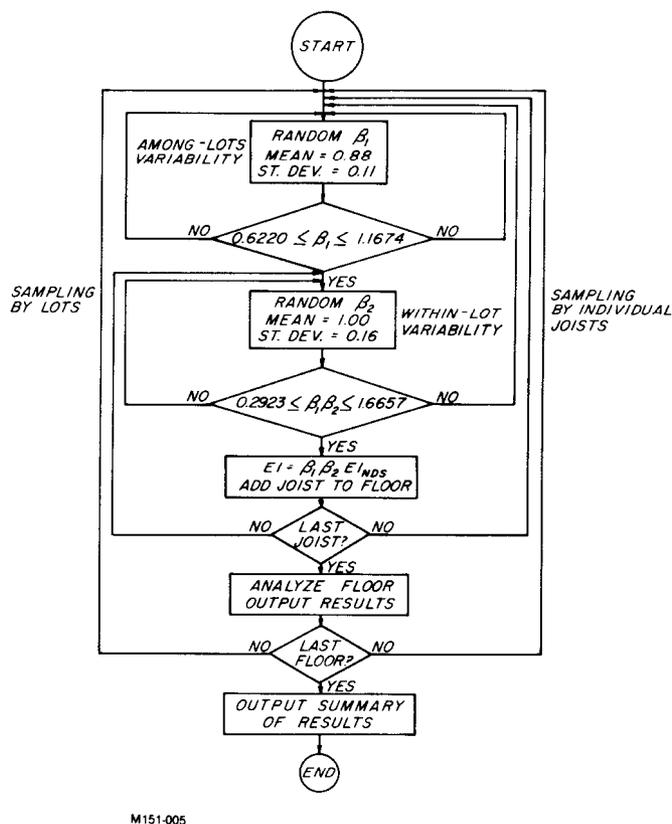


Figure 4.—Flow diagram of CSU Monte Carlo procedure.

finied by the span/deflection ratio, is plotted as a cumulative distribution function (CDF). In each plot, the 40 CSU floors (80 for floor No. 2 sampled by lots, top of fig. 6) are represented by the dots; the solid curves are the results of the FPL analyses, discussed below. The top plot in each figure is for sampling by lots; the bottom plot is for sampling by individual joists. The span/deflection ratios corresponding to  $\Delta_1$ ,  $\Delta_k$ , and  $\Delta_\infty$ , as computed in the benchmark analyses, are also indicated.

### Simulation Analyses (FPL)

For the FPL analyses, nondimensionalized joist EI values, as determined from the Purdue data, are also assumed to be defined by two truncated normal distributions. The properties of a truncated normal distribution are defined (12), and its CDF can be readily calculated.

The analytical procedure of FPL 289 was developed for single T-beams. To permit simulation of floor behavior using a beam model, it is necessary to combine the within- and among-lot variabilities so that selection of a single joist MOE is influenced by both variabilities. This can be accomplished by using a combined variance. For sampling by lots this combined variance is given by

$$\sigma_L^2 \doteq \sigma_A^2 + \sigma_W^2/9 = (0.11)^2 + (0.16)^2/9 \quad (1)$$

where

- $\sigma_L^2$  = derived variance for sampling by lots,
- $\sigma_A^2$  = variance for among-lot distribution,
- $\sigma_W^2$  = variance for within-lot distribution,

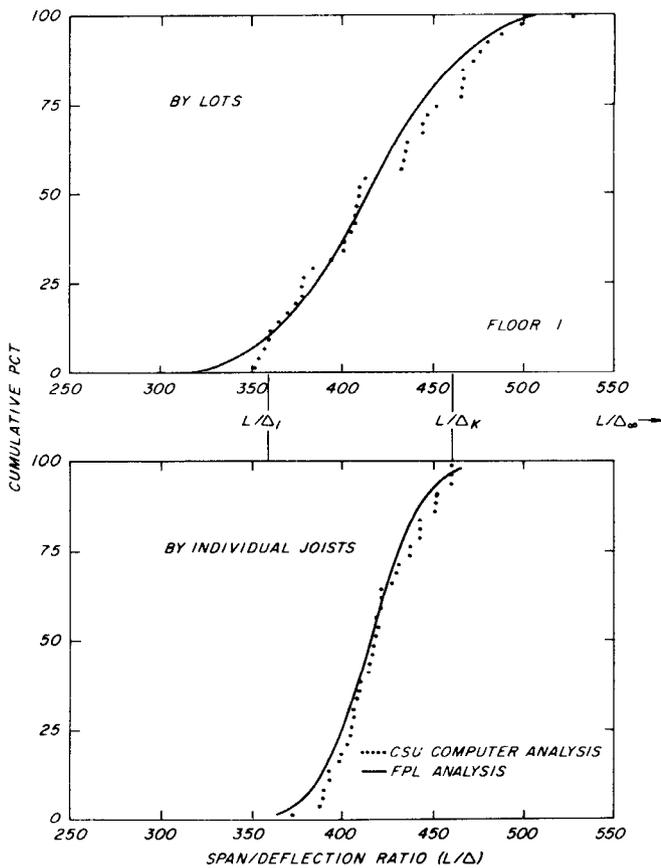
In equation (1), the divisor 9 results from selecting the average deflection of the 9 interior joists as the measure of floor deflection, and the 5 sign appears because the distributions are truncated. Equation (1) would be exact if the full normal distributions were used.

For sampling by joists, the combined variance is given by

$$\sigma_j^2 = (\sigma_A^2 + \sigma_W^2)/9 \quad (2)$$

where

$\sigma_j^2$  = derived variance for sampling by joists.



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Figure 5.—Results of floor 1 simulations with Purdue data. Top plot shows results for sampling by lots, bottom for sampling by individual joists. Dots are results of CSU computer analyses; solid curves are results of FPL analyses.

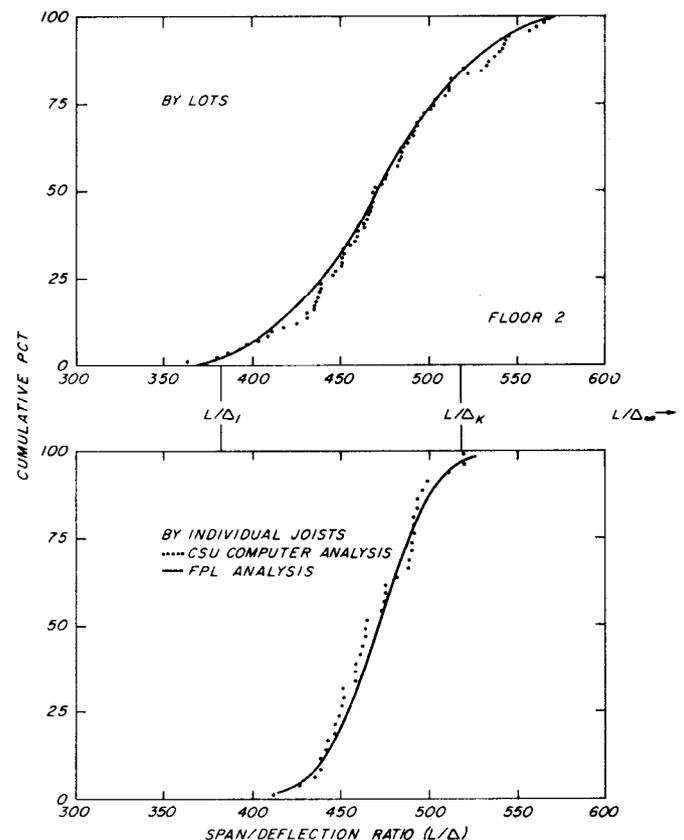
Thus, for the FPL analyses, the Purdue distributions of table 5 were modified (equations (1) and (2)) to obtain the two truncated normal distributions of table 6.

Distributions of floor performances were determined by calculating the CDF for each of the truncated normals in table 6 (at the 0, 2-1/2, 5, 10, 15, ..., 85, 90, 95, 97-1/2, and 100 percentile points), analyzing the corresponding T-beams, and plotting the results in figures 5 through 9. The solid curves indicate the results of these analyses.

### Simulation Results

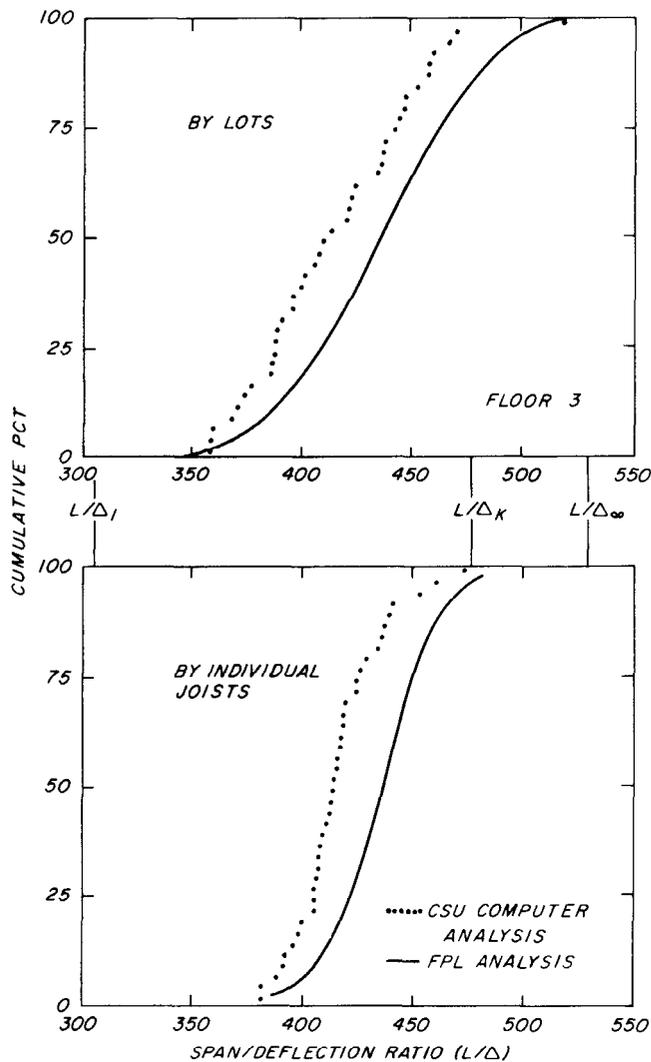
It can be seen from figures 5 through 9 that there is good agreement between the CSU and FPL computations, with the possible exception of floor No. 3 (fig. 7), where the FPL model predicts a slightly (approximately 4 pct) stiffer floor. From these results, the following observations can be made:

- (1) Predicting the deflection of a uniformly loaded floor on the basis of the bare joists (represented by



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Figure 6.—Results of floor 2 simulations with Purdue data. Top plot shows results for sampling by lots, bottom for sampling by individual joists. Dots are results of CSU computer analyses; solid curves are results of FPL analyses.



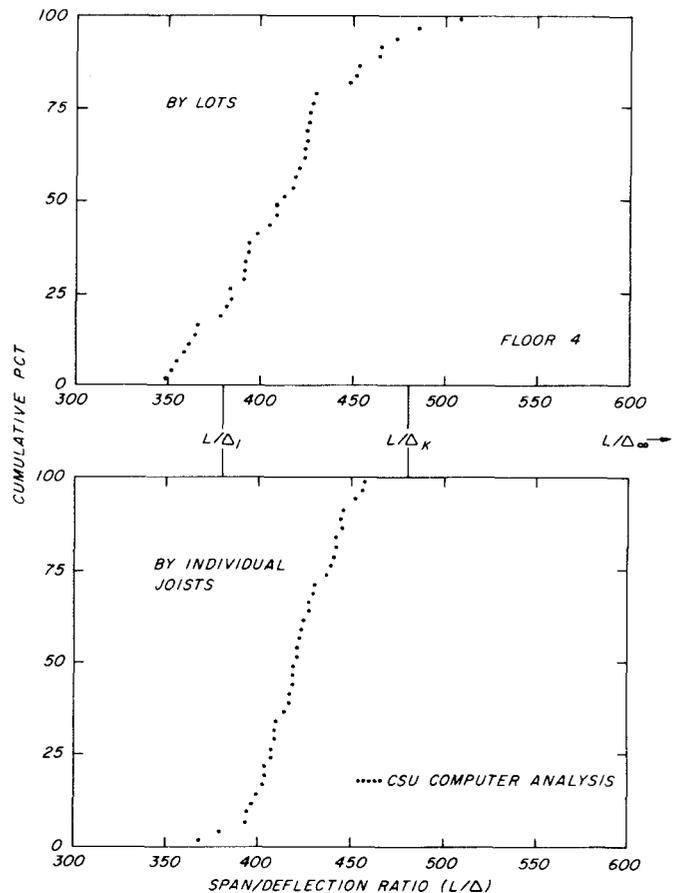
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Figure 7.—Results of floor 3 simulations with Purdue data. Top plot shows results for sampling by lots, bottom for sampling by individual joists. Dots are results of CSU computer analyses; solid curves are results of FPL analyses.

$L/\Delta_1$ ) is conservative. The current design procedure, based on average joist stiffness, should, in concept, predict a computed stiffness at about the 50 percentile point on the CDF. However,  $L/\Delta_1$  is consistently below this point.

(2) Conversely, if floors were designed with proper consideration to interaction and with the full NDS values for joist stiffness (represented by  $L/\Delta_k$ ), the resulting designs would be considerably more "liberal." The  $L/\Delta_k$  points are consistently above the 50 percentile point. This is as expected, since such analyses benefit from the increased stiffness due to interaction but do not take into account that the joist stiffnesses may be lower than assumed in NDS, as indicated by the Purdue data.

(3) Span/deflection ratios for real floors (sampling by



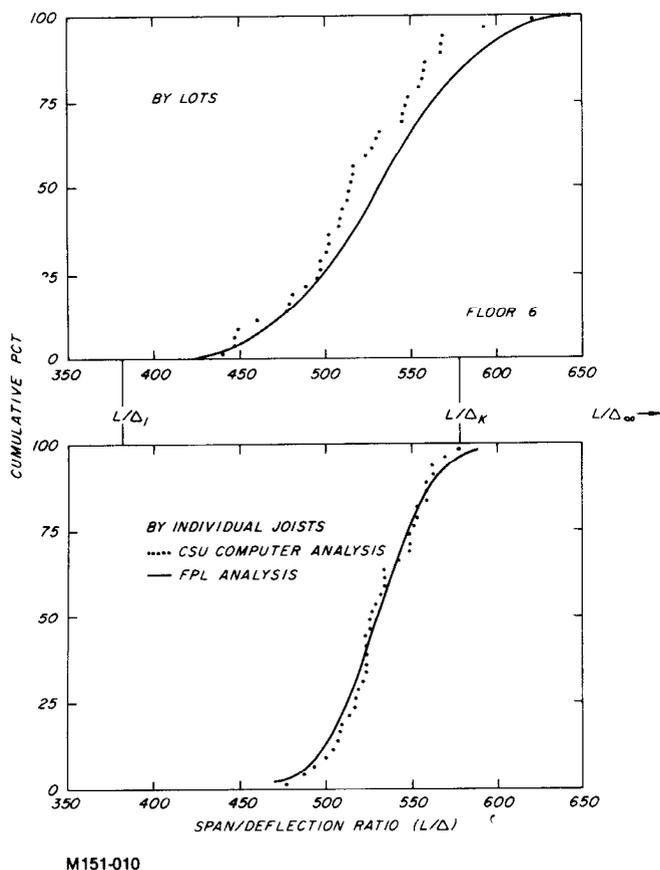
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Figure 8.—Results of floor 4 simulations with Purdue data. Top plot shows results for sampling by lots, bottom for sampling by individual joists. Dots are results of CSU computer analyses.

lots) can be determined for any desired exclusion limit. For example, the values in table 7 were obtained for the 5 percent limit and the 50 percent limit (median). It is interesting to note that for the 5 percent limit (which is used in deriving allowable strength properties), three of the five floors' span/deflection ratios are very close to the current criterion of 360.

### Effect of Joist Variability

Floors No. 2, 3, 5, 7, and 10 were selected (see table 1) in the study on effects on floor performance of varying joist MOE. Joist MOE values were assumed to be normally distributed about the mean values listed in table 1 with a coefficient of variation (COV) of 0.2 for the first simulation and a COV of 0.4 for the second. A COV of 0.2 corresponds closely to the value of 0.25 used in the National Design Specification (19) for visual grading while 0.4 might correspond to natural variation with no grading. While the previous simulations involved both among- and within-lot variations, the joist values for this study were selected using single distributions to define joist stiffness. Lower and upper cutoff limits of



M151-010

Figure 9.—Results of floor 6 simulations with Purdue data. Top plot shows results for sampling by lots, bottom for sampling by individual joists. Dots are results of CSU computer analyses; solid curves are results of FPL analyses.

0.3 and 1.7 times the mean MOE were assumed for the distribution of joist properties. Cutoff limits were used to prevent the occurrence of absurdly high or low (even negative) joist stiffnesses; the values selected are approximately equal to the maximum and minimum values observed in the Purdue data (table 5). The limiting MOE values for each case were:

Floors No.	Mean MOE $\times 10^6$ lb/in. <sup>2</sup>	Lower limit, $\times 10^6$	Upper limit, $\times 10^6$
2, 7	1.3	0.39	2.21
3, 5, 10	1.7	.51	2.89

These cutoff values resulted in reduced variation when compared to the original input values (table 9, columns 2, 3, and 4).

Fezio et al. (6) have shown that joist variability is the major source of floor deflection variability, being more important than either sheathing or connector variability for uniformly loaded floors.

### Variability Analyses (CSU)

The same procedure was used as in the simulation

study, except that among-lot variation was not considered. The results are plotted in figures 10 through 14 as dots.

### Variability Analyses (FPL)

The distributions of joist properties were again assumed to be defined by truncated normal curves. For each of these, the expected value ( $E_T$ ) and the reduced standard deviation ( $\sigma_T$ ) can be computed (12). Since the deflection reported for each floor is the average for the nine interior joists, the distribution of floor properties can be approximated by dividing  $\sigma_T$  by 3 (square root of 9). Thus, the floor properties were assumed to be defined by truncated normal distributions with a mean of  $E_T$  and a standard deviation of  $\sigma_T/3$ . The resulting values are shown in table 8.

For these derived truncated normals the CDF's were computed, the corresponding T-beams were analyzed, and the results plotted in figures 10 through 14 by solid curves. Additional statistics derived from these analyses are given in table 9.

### Variability Results

Agreement between the CSU and FPL computations was good. The largest discrepancy was for glued floor No. 3 (fig. 11), where the FPL procedure again predicted about 4 percent more stiffness than the CSU mathematical model.

The summary given in table 9 includes two columns (Nos. 5 and 6) giving the deflection COV's of the in-

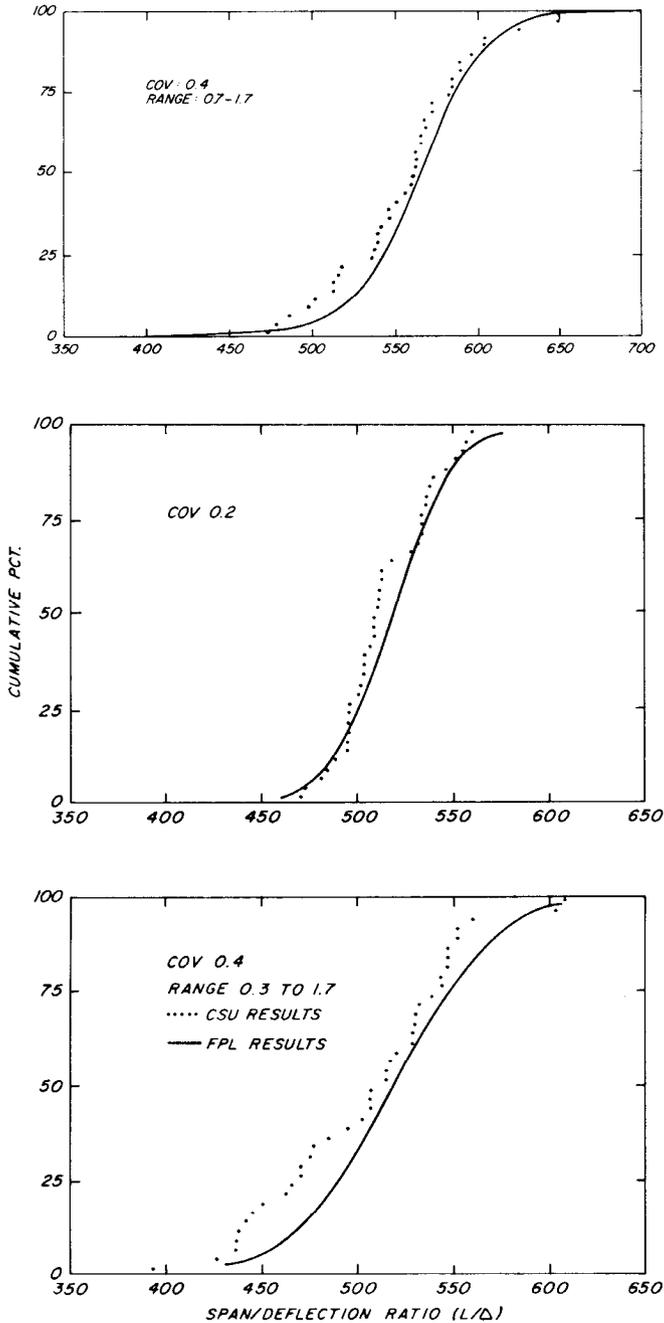
Table 6.—Parameters derived for joist stiffness distributions used in FPL analyses<sup>1</sup>

	Lots	Joists
Mean	0.88	0.88
Standard Deviation	.1222	.065
Minimum	.6220	.2923
Maximum	1.1674	1.6657

<sup>1</sup> Actual bending stiffness (EI) divided by NDS bending stiffness.

Table 7.—Span/deflection ratios at 5 percent and 50 percent limits

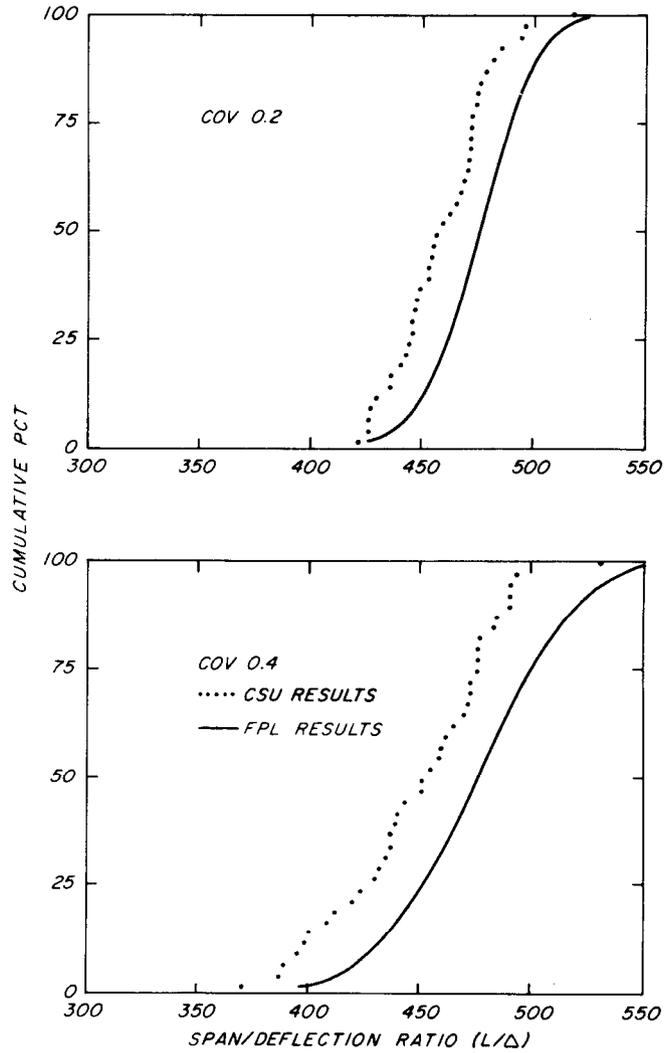
Floor	5 percent (sampling by lots)		5 percent (sampling by joists)		50 percent (median)	
	CSU	FPL	CSU	FPL	CSU	FPL
1	360	350	393	379	414	419
2	395	397	430	428	470	471
3	360	372	385	399	412	436
4	353	—	388	—	411	—
6	446	452	490	485	514	531
Average	383	393	417	423	444	464



M151-012

Figure 10.—Results of floor 2 joist variability simulations. First plot shows results for input COV of 0.2, second for COV of 0.4; for both, the range of E is 0.3 to 1.7 times the mean MOE. Third plot is for COV of 0.4 with a range of 0.7 to 1.7 times the mean MOE.

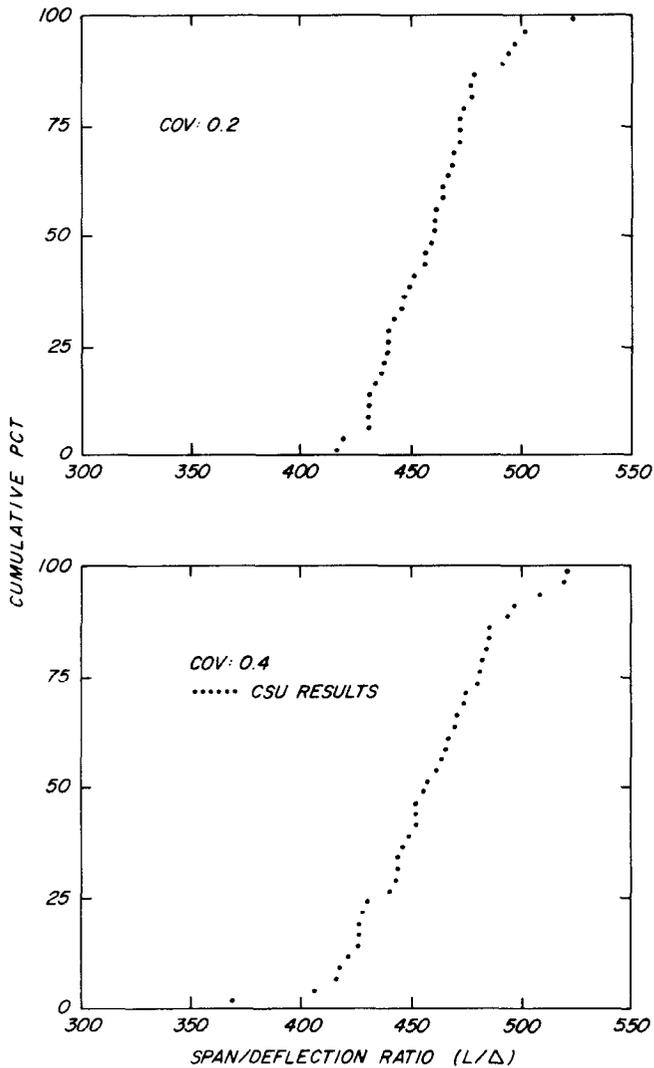
dividual joists. These values for the FPL and CSU computations are not in agreement because the CSU computer model accounts for two-way action in the floors (due to sheathing stiffness perpendicular to the joist span), while there is no provision for this in the FPL T-beam model.



M151-013

Figure 11.—Results of floor 3 joist variability simulations. Top plot shows results for input COV of 0.2, bottom for COV of 0.4; for both, the range of E is 0.3 to 1.7 times the mean MOE.

The floor system acts as a "filter" to remove a substantial portion of the variation in the input joist values as may be seen by comparing the input COV's (i.e., the COV's of the assumed distributions of joist MOE) with the COV's of the individual joist deflections and mean floor deflections. The approximate relative COV values given in table 9 show that the variation in average floor deflection is about one-fourth of the joist MOE variation. In general, the amount of filtering of joist MOE variation will depend upon the contribution of joist stiffness to total system stiffness, and on the method selected for defining overall floor performance. Thus hypothetical floors, consisting of joists only, will show zero percent filtering, i.e., variation of input joist MOE data is undiminished in the output. Floors in which the sheathing provides all the stiffness will produce 100



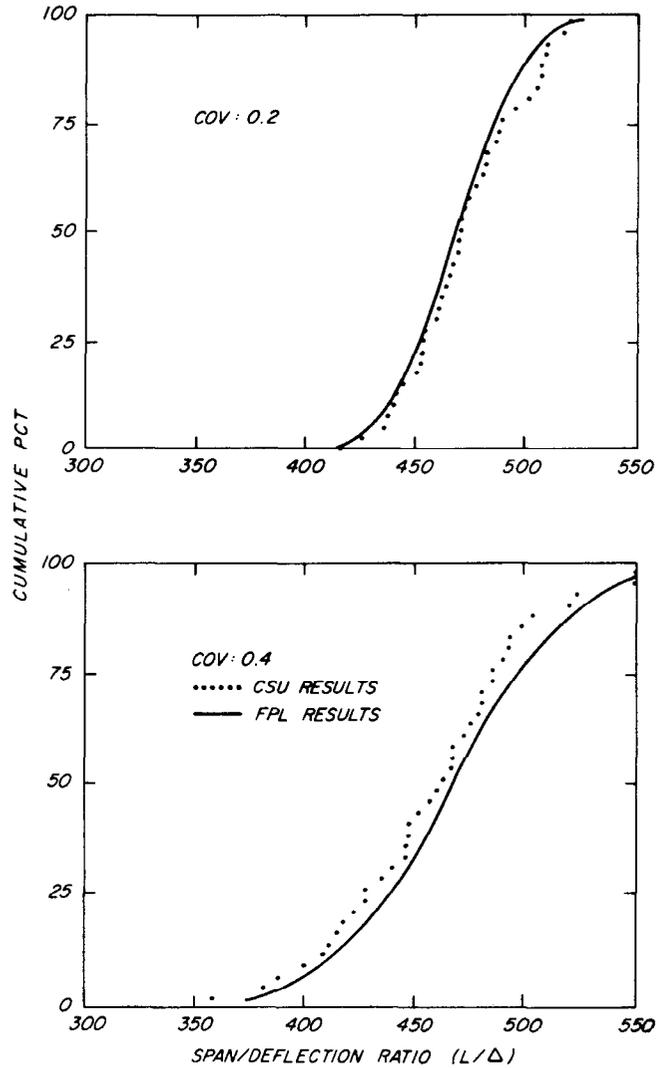
M151-014

Figure 12.—Results of floor 5 joist variability simulations. Top plot shows results for input COV of 0.2, bottom for COV of 0.4; for both, the range of E is 0.3 to 1.7 times the mean MOE.

percent filtering, i.e., none of the joist variability will be present in the system response variability.

In this study, the average deflection of nine joists was selected as the measure of floor performance. Thus, no filtering should yield 33 percent of the input joist variability in the output. The filtering effect reduced this value to about 25 percent.

The effects of the floor behavior in “filtering” the variation are clearly demonstrated in figure 15. As compared to the 45 degree line representing a one-to-one correspondence between input and output, figure 15 quantifies the positive effect of composite and two-way action in floor systems in reducing the input variation. As can be seen, the floor type was not a strong influence since all floors exhibited a nearly similar sensitivity to



M151-015

Figure 13.—Results of floor 7 joist variability simulations. Top plot shows results for input COV of 0.2, bottom for COV of 0.4; for both, the range of E is 0.3 to 1.7 times the mean MOE.

input. As noted previously, the types of floors were markedly different; thus, this result is encouraging as to the possible use of results such as figure 15 in quantifying joist floor sensitivity to input MOE variation.

The effect of the cutoff level of MOE is to reduce the effective variation in the floor joists chosen by the simulation procedures. This effect has obvious implications for grading methods. As shown in table 9 and figure 15 for floor No. 2, increasing the lower cutoff level from 0.3 to 0.7 of the mean MOE has a pronounced effect on the resulting COV of floor results. For this case, the input COV for MOE of 0.4 is reduced to 0.2307 by the cutoff of 0.7 as compared to 0.3230 with the 0.3 lower cutoff. This study shows the type of sensitivity to input variations which could be expected for changes in grading procedures. Further studies of

this type could define a grading procedure which could optimize the utilization potential of the softwood dimension lumber produced for use in the light-frame joist market.

## Discussion

### Computational Procedures

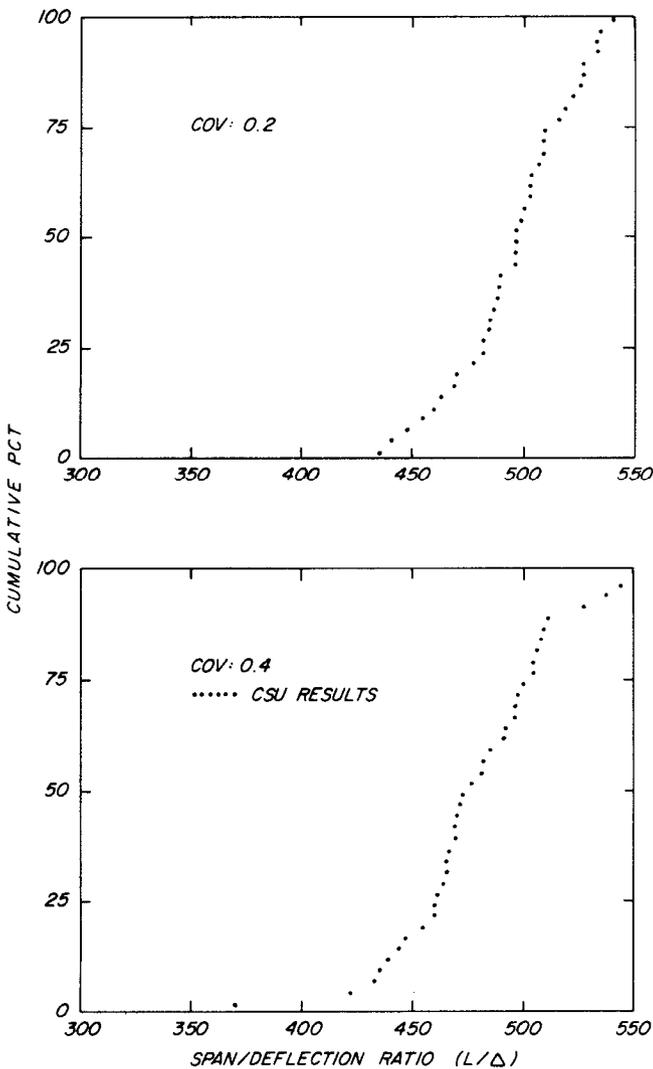
The calculations in this cooperative study were carried out by two different methods. The CSU floors were analyzed by means of a mathematical model of the complete floor (25) which has been proven to be a flexible research tool. The program can analyze two- or three-layer floors subjected to distributed or concentrated loads with proper consideration of composite action, two-way action, and the effects of gaps in the sheathing. A T-beam analysis (16) was used in the FPL computations. This simplified model considers composite action between the joist and a single layer of sheathing with open gaps. The method is limited because there is currently no predictive technique to define the value to use for  $L'$  (the effective distance between gaps) when flexible gaps are present. The T-beam analysis is well suited to the loading (distributed) and the stiffness criterion (average deflection) selected for this study. Agreement between CSU and FPL analyses was generally good.

In the simulation studies with the Purdue joist data and in the study of joist variability, two different methods were employed to generate the joist properties. CSU used a Monte Carlo procedure, while FPL computed the properties from derived distributions. The Monte Carlo method is a powerful tool for studying the influence of input variation in material properties on the behavior of structural systems; but, it may be possible in future studies to effect savings in the number of floors needed to define the performance CDF's by selecting joist lots close together at known points on the tails of the lot stiffness distribution, and further apart near the middle.

### Results of Analyses

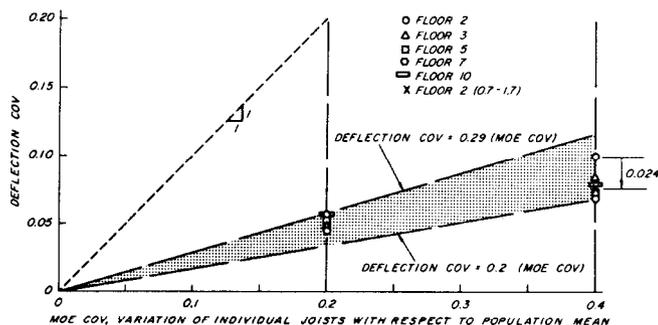
The analyses of the benchmark floors illustrate the large amount of composite action which is ignored in the current practice of designing on the basis of joists acting alone (fig. 2). Table 4 and figure 3 present the amount of composite action which is achieved with many different types of construction. These measures of composite behavior provide a tool for improving floor design through the improvement of interlayer connections, such as gluing.

The influence of composite action is sufficient to reduce the mean simulated deflections of floor systems to less than the current design levels, even when considering some reduction in joist stiffnesses below design levels and the effects of variability. Span/deflection ratios were around 390 at the 5 percent limit and 450 at the median (table 7). These results demonstrate the level of conservative design currently being used,



M151-016

Figure 14.—Results of floor 10 joist variability simulations. Top plot shows results for input COV of 0.2, bottom for COV of 0.4; for both, the range of  $E$  is 0.3 to 1.7 times the mean MOE.



M151-017

Figure 15.—Mean floor deflection variation as a function of joist MOE variation.

Table 6.—Derived floor properties<sup>1</sup> used in FPL variability analyses

Distribution of joist stiffness				Derived properties				
Mean	Standard deviation	Truncation		Expected value	Joists		Floors	
		Lower tail	Upper tail		Standard deviation	Mean	Standard deviation	
1.00	0.20	0.30	1.70	1.0000	0.1994	1.0000	0.0665	
1.00	.40	.30	1.70	1.0000	.3278	1.0000	.1093	
1.00	.40	.70	1.70	1.1172	.2531	1.1172	.0844	

<sup>1</sup> Actual EI divided by NDS EI.

Table 9.—Sensitivity of floor deflection to joist variability

Floor (1)	Input joists (COV) (2)	Coefficient of variation of modulus of elasticity for joists selected <sup>2</sup>		Deflection variation (COV)			
				Individual joists		Floor <sup>1</sup>	
				FPL (3)	CSU (4)	FPL (5)	CSU (6)
2	<sup>3</sup> 0.2	0.1994	0.1896	0.1555	0.0904	0.0510	0.0454
3	<sup>3</sup> .2	.1994	.1935	.1429	.0921	.0466	.0471
5	<sup>3</sup> .2	—	.1971	—	.0840	—	.0512
7	<sup>3</sup> .2	.1994	.2030	.1687	.0985	.0552	.0558
10	<sup>3</sup> .2	—	.1990	—	.1002	—	.0567
2	<sup>3</sup> .4	.3278	.3230	.2781	.1775	.0838	.0993
3	<sup>3</sup> .4	.3278	.3357	.2573	.1621	.0768	.0823
5	<sup>3</sup> .4	—	.3084	—	.1285	—	.0715
7	<sup>3</sup> .4	.3278	.3461	.3012	.1718	.0909	.0966
10	<sup>3</sup> .4	—	.3313	—	.1642	—	.0781
2	<sup>4</sup> .4	.2265	.2307	.1843	.1265	.0573	.0757
Approximate relative COV values		1.00	1.00	.80	.50	.25	.25

<sup>1</sup> Average floor deflection using 9 joists (interior joists in CSU analyses).

<sup>2</sup> The COV obtained is less than the input COV because of the truncation due to the upper and lower limits chosen.

<sup>3</sup> Lower and upper cutoff limits of 0.3 and 1.7 times the mean MOE.

<sup>4</sup> Lower and upper cutoff limits of 0.7 and 1.7 times the mean MOE.

wherein an average span/deflection ratio of 360 is assumed. The study of joist variability illustrates how a complete floor system subjected to uniform loads tends to filter out the variability in joist properties. In these simulations, the variation in floor deflection was only about 25 percent of the variation in joist stiffness (table 9 and fig. 15) as compared to 33 percent which would be expected without any filtering.

The joist variability analyses also illustrate the marked effect which a cutoff level of joist stiffness can have on floor performance (see floor No. 2, table 9). Increasing the lower cutoff level, such as by a simple testing procedure, can improve performance. The means for evaluating the benefits associated with improved material grading have been demonstrated in this study.

### Improved Design Procedures

The work reported herein is part of an effort to develop

new design methodologies for wood-joist floors which will properly incorporate the many factors known to affect floor performance.

A T-beam model of floor stiffness has been developed (76) and was used for the FPL analyses in this study. The procedure accurately computes the partial composite action between a joist and a single layer of sheathing, but it is currently limited in applicability to the design of individual beams since it does not account for two-way action. It also requires an empirical modification of the basic method to handle flexible gaps in the sheathing.

The three possible methods for floor design proposed by CSU (22, 23) are:

- (1) Direct use of the computer program FEAFFLO,
- (2) Use of dimensionless charts (termed R-charts), and
- (3) Use of assembly tables.

The use of computer programs such as FEAFL0 will continue to serve the research community as efforts continue to refine basic methods of analysis. In its present form, however, FEAFL0 has limited usefulness as a practical tool for individual designers except perhaps for large prebuilt housing manufacturers. FEAFL0's primary contribution to designers may be in its use as the backbone of computer-derived design charts and assembly tables, and in the simulation studies needed for reliability-based design.

An R-chart is a dimensionless chart which shows how the effective stiffness of a two-layer T-beam varies with slip modulus and gap stiffness. Sample charts have been presented for uniformly loaded floors with one layer of sheathing. The concept may also be applicable to other constructions and loadings (22, 23).

The simplest design method, from the user's standpoint, is an assembly table. As the name implies, an assembly table provides a design for a complete assembly of joists, sheathing, and connectors. To use an assembly table the designer need only pick out an assembly which will span the needed distance. Most currently-used span tables, which are based on bare joist design, are an embryonic form of assembly tables. Assembly tables for inclusion in model building codes and other design aids can realistically be constructed using the most powerful and accurate analyses available; the individual designers need not even be aware of the computational procedures entailed in the necessary analyses used in determining assembly table values.

If the assembly table concept is adopted as a design procedure, it may still be desirable to have available a supplemental method. Because a set of tables can consider only a finite number of floor configurations, loads, and performance criteria, an alternate procedure could provide the designer with a means for assessing the suitability of floor designs not contained in the tables. The alternate need not be as comprehensive as the assembly tables, and might be based on a T-beam model, such as that presented in FPL 289 (16), or on the R-chart concept.

## Summary and Conclusions

A cooperative research program between Colorado State University and the Forest Products Laboratory was undertaken to examine the theoretical performance of typical wood-joist floors using computer-based and equation-based methods of analysis.

The benchmark floor analyses revealed that the usual practice of designing on the basis of bare joists ignores considerable amounts of beneficial composite action between the floor joists and the sheathing materials.

Using real joist data, the simulation analyses showed that even when joist stiffnesses are below their design

values, composite action reduces floor deflections to less than allowable design values.

The joist variability simulations demonstrate how variability in joist properties is reduced when the joists are assembled into complete floors. These simulations also show how truncating the lower tail of the joist distribution can greatly affect floor performance.

Future studies of wood-joist floor design, including reliability-based design, will benefit from the results of the studies and methodologies presented herein. Implications for improved utilization of wood in light-frame structures are clearly evidenced in this study and implementation of these research results in design offers much promise for the future of wood and wood-based products.

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# Appendix

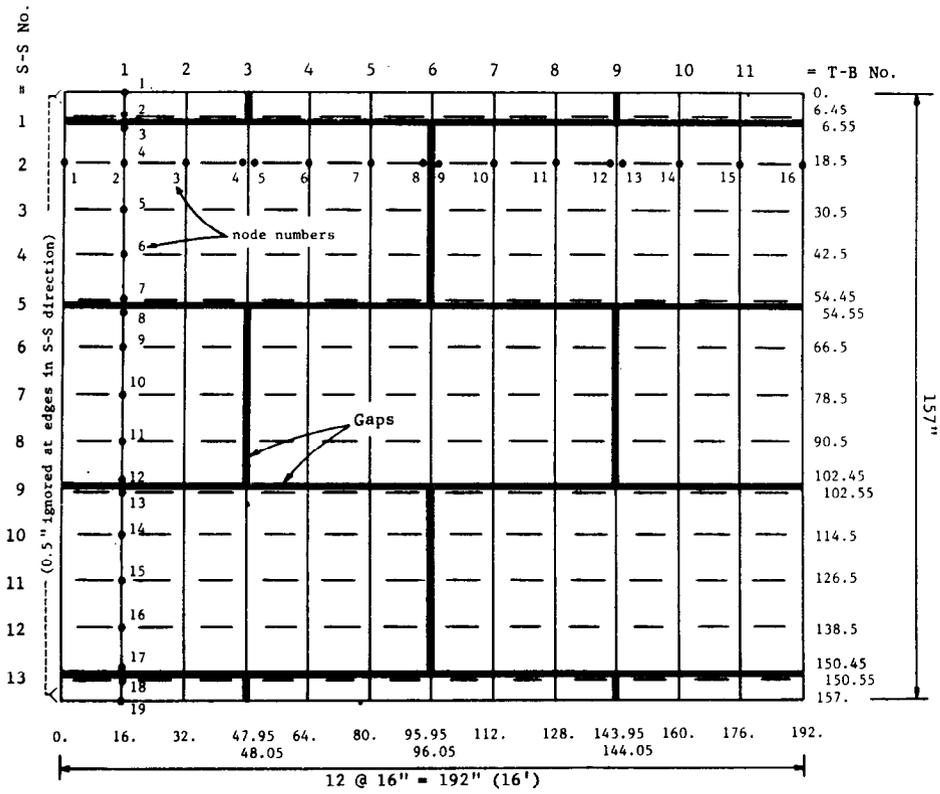


Figure A1.—Details of CSU floor analyses, floor 1.

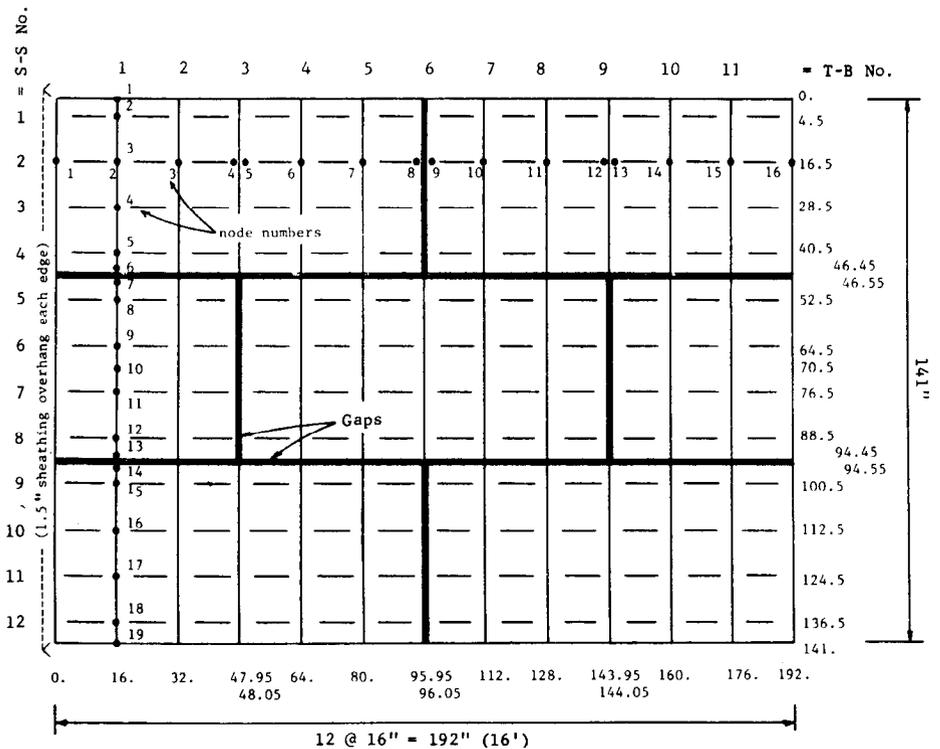


Figure A2.—Details of CSU floor analyses, floor 2.

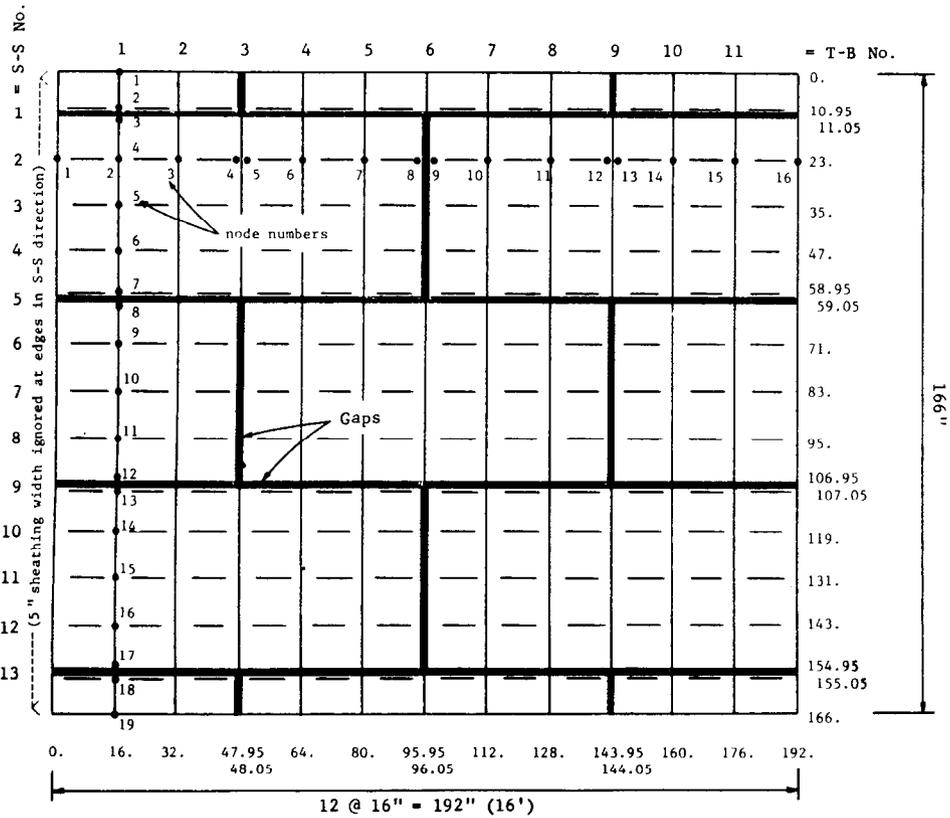


Figure A3.—Details of CSU floor analyses, floor 3.

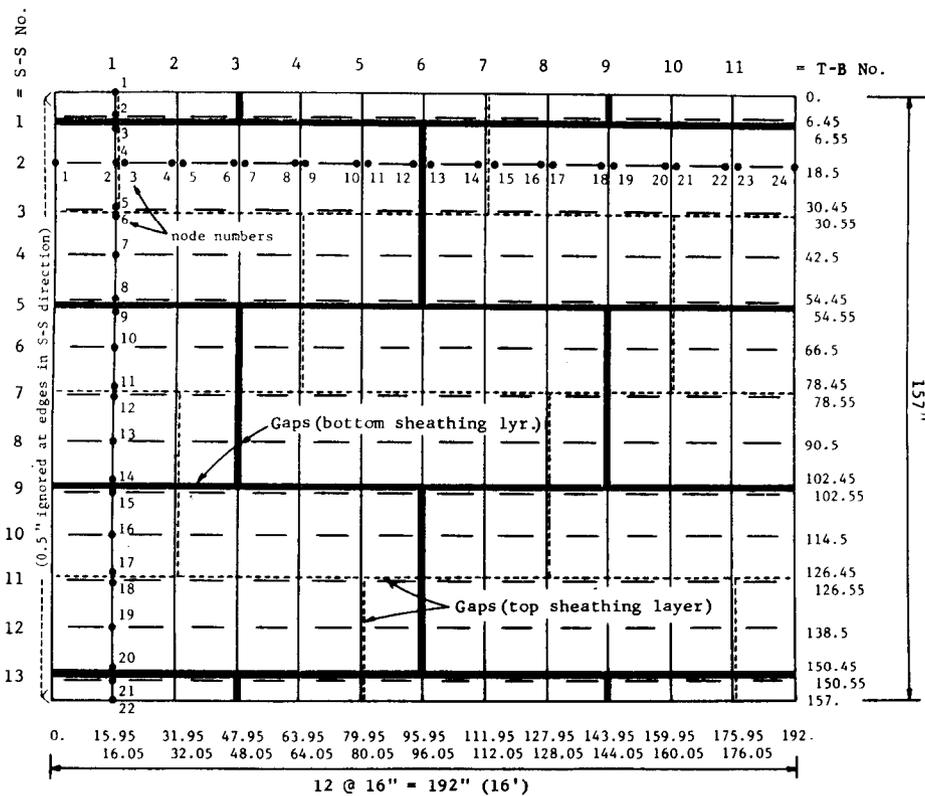


Figure A4.—Details of CSU floor analyses, floor 4.

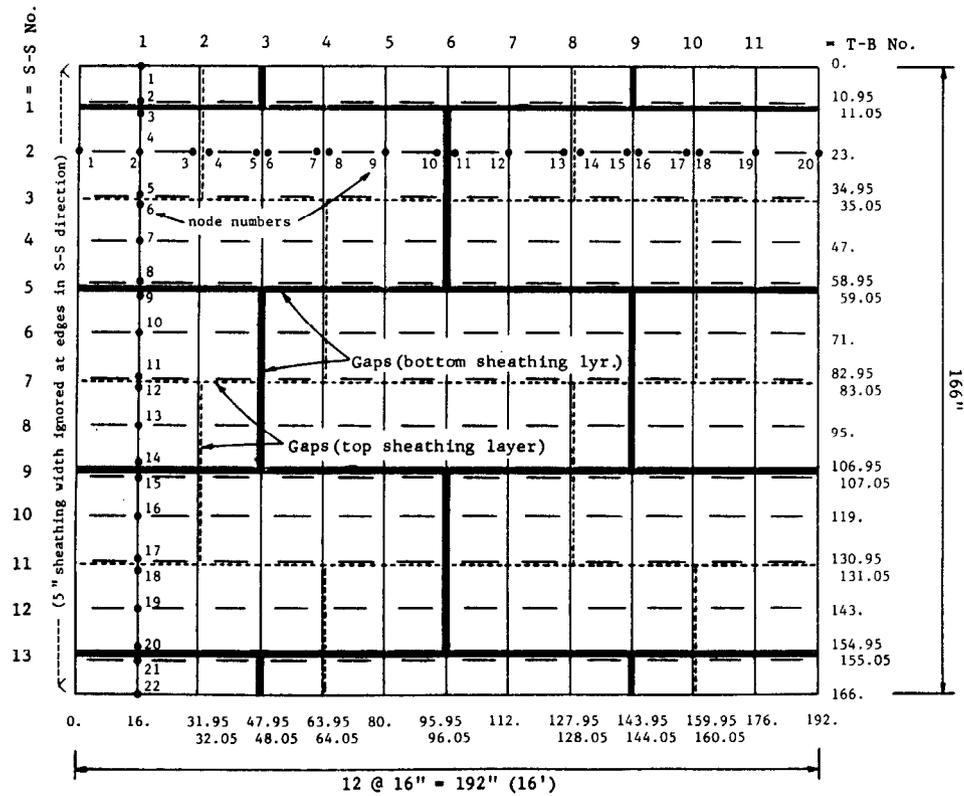


Figure A5.—Details of CSU floor analyses, floor 5.

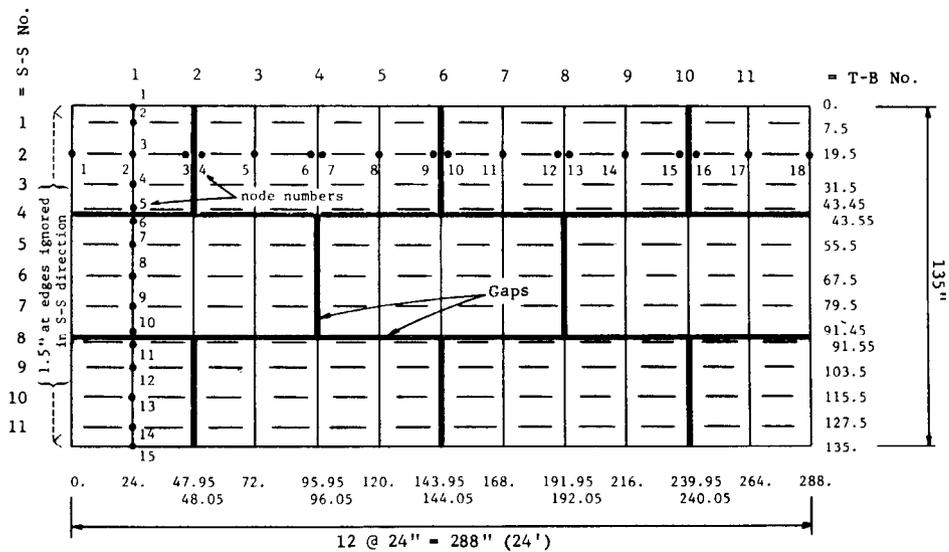


Figure A6.—Details of CSU floor analyses, floor 6.

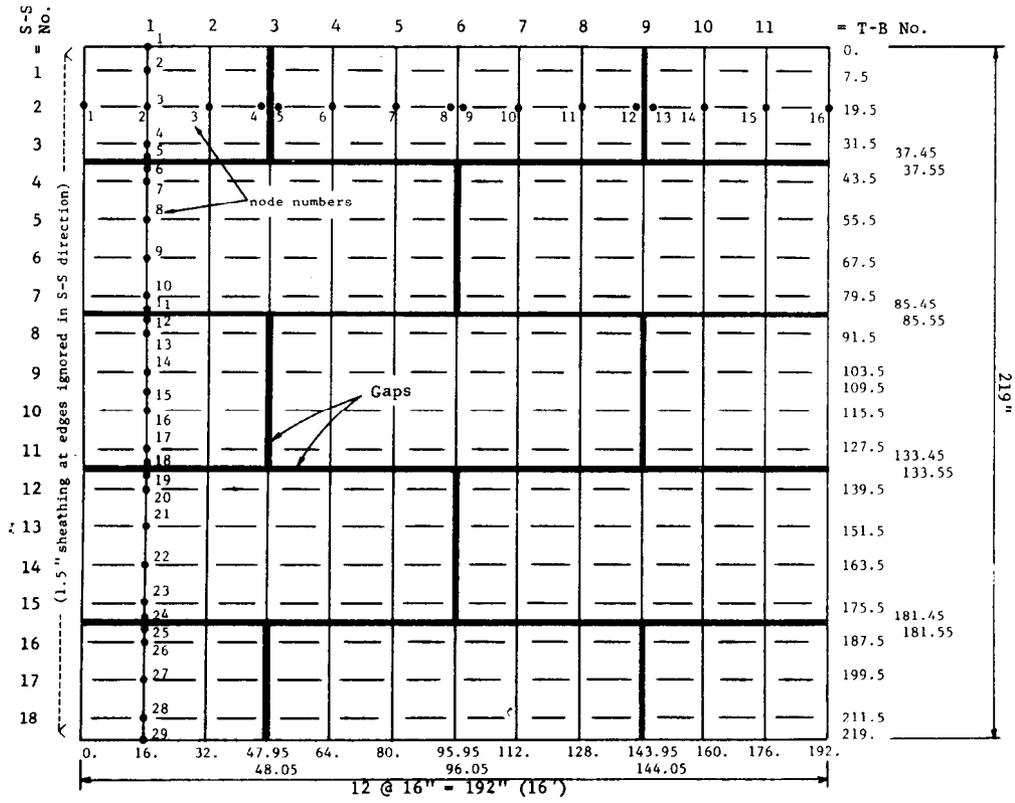


Figure A7.—Details of CSU floor analyses, floor 7.

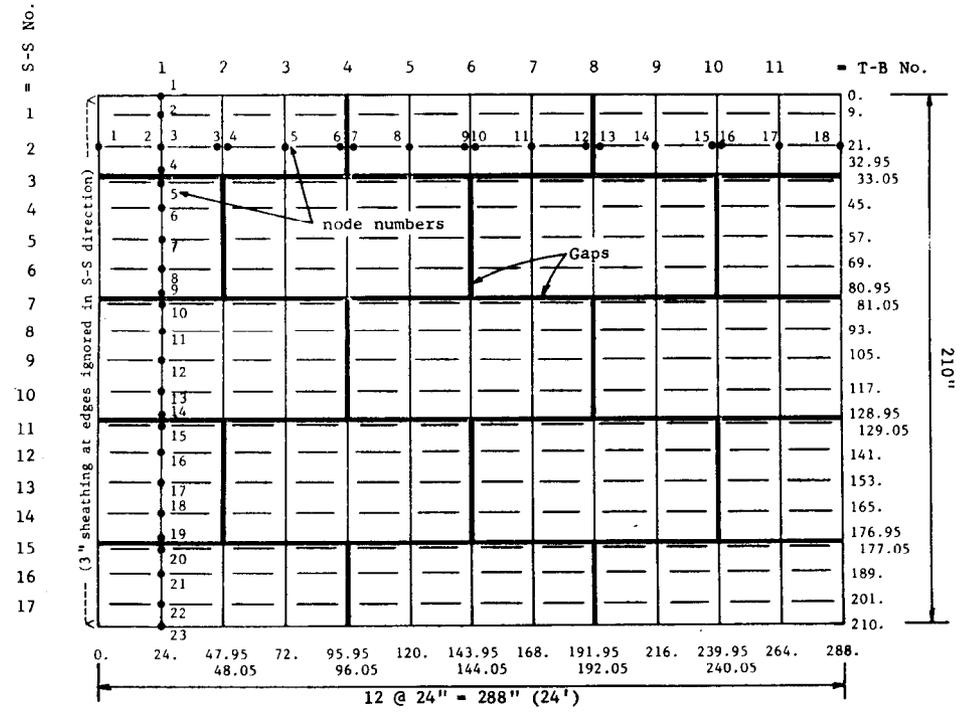


Figure A8.—Details of CSU floor analyses, floors 8 and 9.

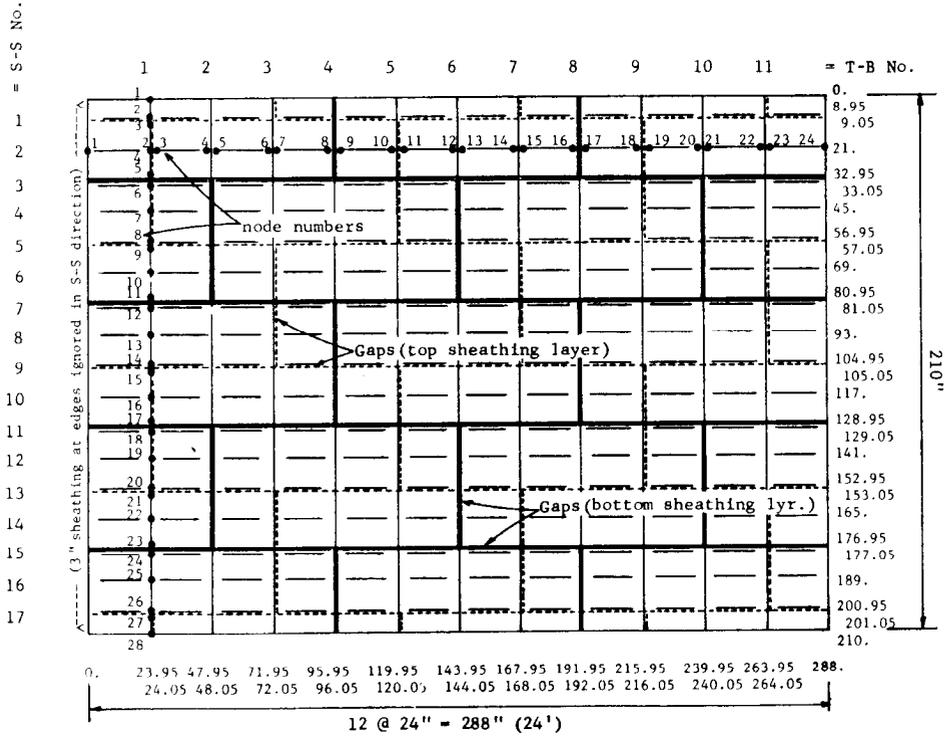


Figure A9.—Details of CSU floor analyses, floor 10.

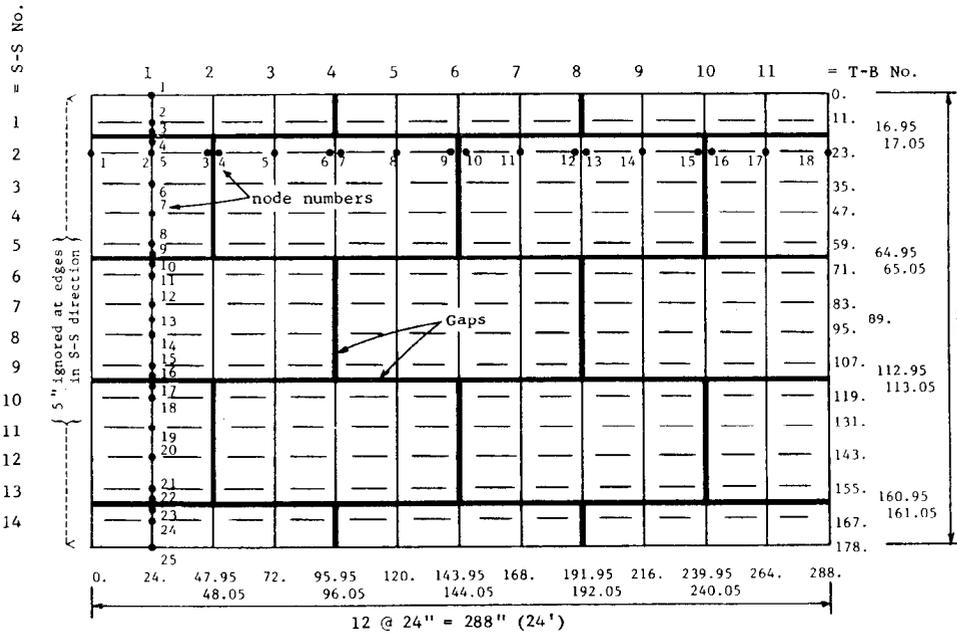


Figure A10.—Details of CSU floor analyses, floor 11.

U.S. Forest Products Laboratory

Wood Joist Floors: Effects of joist variability on floor stiffness, by W. J. McCutcheon, M. D. Vanderbilt, J. R. Goodman, and M. E. Criswell, Madison, Wis., FPL 1981.

21 p. (USDA For. Serv. Res. Pap. FPL 405).

Report on a theoretical study of the performance (deflection) of wood-joist floors subject to distributed loads. Results demonstrate how joist variability is reduced when the joists are assembled into complete floor systems.

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