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Bending Strength of Vertically Glued Laminated Beams with One to Five Plies

Abstract

Properties of vertically laminated beams were studied to determine the effects of number and quality of component laminations on structural performance. Deterministic and probabilistic analyses were both used to show how performance varies with number and quality of laminations and with assumed loading parameters. Increase in allowable stress with number of laminations, currently recognized as a constant adjustment for three or more laminations, was significant for two-lamination beams as well. This increase proved to be inversely related to lumber quality. Results will be helpful to organizations publishing engineering design stresses in establishing a more efficient design of vertically laminated beams.

Abbreviations Used

β = safety index
LVDT = linear variable differential transducer
MOE = modulus of elasticity
$\overline{\text{MOE}}$ = mean modulus of elasticity
MOR = modulus of rupture
$\overline{\text{MOR}}$ = mean modulus of rupture
N = number of laminations
SR = bending strength ratio
Ω = coefficient of variation

BENDING STRENGTH OF VERTICALLY GLUED LAMINATED BEAMS WITH ONE TO FIVE PLIES[1]

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Introduction

Vertically glued, laminated beams are designed with stresses developed following ASTM D 245 *(2).*[3] This standard permits a 15 percent increase over single member bending stress for systems containing three or more members, provided they are spaced not more than 24 inches apart. This provision is directed specifically at repetitive member applications in light-frame components, such as roof or floor systems. However, it has also been applied to vertically glued, laminated beams consisting of three or more laminations. No provision is made in ASTM D 245 for two-lamination members and no further factors are applied to members with more than three laminations.

Hypothetically, the integral gluing of pieces of lumber might provide greater advantages than recognized by ASTM D 245. Also, the relative tensile strength of multiple laminated members is known to be related to grade, with lower grade lumber demonstrating greater advantages due to laminating *(19).*

The purpose of this study was to determine the strength and stiffness of vertically laminated beams and to show their relationship to number and quality of laminations. This information should contribute to more efficient design in vertically glued, laminated beams.

Background

A number of studies of vertically laminated beams have been made, but few have attempted to determine beam strength based on the mechanical properties of the component lumber. Wilson and Cottingham (20) suggested that the average strength ratio of the individual laminations represents a reasonable estimate of the strength ratio of the beams. However, their results were limited to a few tests of four-lamination beams.

McAlister *(10)* determined that two No. 2 southern pine boards laminated together were superior to No. 2 southern pine dimension lumber. In comparison with the dimension lumber, each of four systems used to laminate the two pieces together–(1) no fastening, (2) nailing with 6d common nails, (3) gluing with phenol resorcinol, and (4) gluing with neoprene-base mastic adhesive–resulted in higher allowable stresses due to decreased variability.

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[1] Research conducted in cooperation with the American Institute of Timber Construction (AITC).

[2] Maintained at Madison, Wis., in cooperation with the University of Wisconsin.

[3] Numbers in parentheses refer to literature cited at end of this report.

Nemeth *(13)* derived an equation to predict strength of three-ply beams based on knot distribution in single members. His work, however, was limited to one grade of material and his derivation to beams of three laminations. Bonnicksen and Suddarth *(9)* compared single- and three-ply nail-laminated beams as a means of introducing reliability-based theory to wood design. Reference (9) demonstrates the increased reliability of the three-ply members, but the study was not concerned with lumber quality or effects of the number of laminations.

These reports *(9,10,13)* indicate dependence of beam strength on both quality and number of laminations. However, no attempt has been made to define this relationship.

Research Materials

Effects due to material quality as well as to number of laminations were considered here by using three laminating grades of lumber and five beam sizes. The laminating grades were selected using the bending strength ratio (SR) (2) based on the maximum allowable edge knot size given in the standard grading rules *(15,18)* as an index of quality. Beam size categories included a control group (single 2 by 6's) and four beam groups of two, three, four, and five laminations respectively.

The lumber used consisted of L1 Douglas-fir (SR = 0.60), N2D southern pine (SR = 0.47), and L3 Douglas-fir (SR = 0.26). In each case, the material was selected at the laminating plant where the beams were manufactured. Table 1 lists the sample sizes selected for each grade. The smaller samples of N2D southern pine resulted from an insufficient inventory at the plant where these beams were fabricated. This slight modification of the intended sample sizes, however, had little effect on the sampling objective: To provide 80 percent confidence that the lower 95 percent tolerance limit on modulus of rupture (MOR) is within 10 percent of the true population value *(5).*

Table 1.–Number of samples

Grade and species	Total lumber sample (pieces 2 in. x 6 in. x 12 ft.)	Specimens for each number of laminations				
		1	2	3	4	5
L1–Douglas-fir	564	100	40	32	32	32
N2D–Southern pine	512	80	36	30	30	30
L3–Douglas-fir	564	100	40	32	32	32

Each piece of lumber was assigned to a beam size category—and given an orientation and location in a specific beam—by using a random number schedule. Moisture content was then determined using a power loss meter, and the weight and modulus of elasticity (MOE) were measured using an E-computer. Finally, knot sizes and locations were determined using the displacement method described in section 5.3.4 of ASTM D 245*(2).*

The lumber selected was 12 feet long with no end joints. The laminating process conformed to PS 56-73 *(16).* All beams were surfaced to a uniform depth of 5-1/18 inches and shipped to the U.S. Forest Products Laboratory (FPL) by truck.

Research Methods

Research methods conformed to established standards given in ASTM D 198 *(4).*

Conditioning

When the material was received at FPL, moisture contents were determined with a resistance-type meter. The Douglas-fir measurements appeared to be close to those taken in the field; thus, these beams were tightly stacked and wrapped until they were tested. The moisture content of southern pine, however, was found to be slightly higher than that measured in the manufacturing plant. To prevent a steep moisture gradient from developing, these pine beams were conditioned at 12 percent moisture content until time of test.

Procedure

The beams were tested in bending, using a two-point load on an 11-foot span. Load heads were placed 22 inches either side of midspan, and deflection readings included load head movement as well as the midspan deflection.

A special yoke was made, using clear plastic sides, to hold the two linear variable

differential transducers (LVDT's) used to measure beam deflection. These transparent sides permitted viewing fracture propagations during the test.

Before testing, each beam was marked, measured, and weighed. Because the beam failure sections were to be cut out and photographed after testing, orientation lines were drawn at 2-foot intervals and identified as to distance from the zero or numbered end. Section dimensions were then measured at the 4- and 8-foot marks.

Each beam was placed on test supports, and the special yoke was supported on nails placed along the beam's centroidal axis above each reaction point. The main body of the LVDT, which monitored beam deflection, was permanently attached to the yoke and its core arm extended downward to rest on a hinged angle placed at the centroidal axis of the beam. Lateral supports were used to keep the single members vertically alined during tests.

Test and recording equipment were zeroed using a 50-pound preload prior to each test. The load was then applied at a rate to cause failure of most specimens between 4 and 7 minutes. To prevent damage, the LVDT measuring full-span deflection was removed at about one-half the full scale load. The load head LVDT remained connected to detect any plastic behavior prior to failure. The test was stopped when the machine load dropped to 50 percent of maximum attained.

Notes were taken to record beam behavior under load. These included loads at which either audible or physical signs of distress were first noticed, the appearance of compression wrinkles, and estimates of the order of failure propagation.

After the test, sections were taken near the failure in each beam to determine moisture content and specific gravity by the ovendrying method.

Presentation and Analysis of Results

Tables 2 and 3 summarize physical and mechanical beam properties. Values listed in table 2 include distribution parameters for moisture content and specific gravity as well as allowable knot sizes and SR's determined according to ASTM D 245 *(2).* The original and adjusted mechanical property values are given in table 3.

Moisture content variations were small within any single group, but the differences between groups were significant. Thus, for purposes of analysis, values of MOR and MOE were adjusted to a common value of 12 percent *(3).*

Types of Failure

Most beam failures appeared to begin in tension. However, it was difficult to accurately classify the actual initiating factors. Many of the beams—more often for the Douglas-fir—gave audible signs of distress prior to any visual signs. Many beams also displayed compression wrinkles. These signs seemed to indicate continual stress redistribution throughout the test.

In some beams, weaknesses such as knots on the tensile or compressive face or local grain deviations provided rather obvious failure sources. However, in multiple member beams an edge defect on one member might be compensated by stronger, adjacent members. Failure types were classified into the general categories of tension, com-

Table 2.–Physical properties of material used

Grade and species	Moisture content		Specific gravity		Maximum allowable edge knot size[2]	Bending strength ratio from ASTM D 245
	Average	ΩMC[1]	Average	ΩSpG[1]		
	Pct	*Pct*	*Pct*		*Pct*	
L1–Douglas-fir	9.0	11	0.49	10	25	0.60
N2D–Southern pine	11.5	14	.52	11	31	.47
L3–Douglas-fir	8.3	12	.47	9	50	.26

[1] Ω = coefficient of variation.
[2] From standard grading rules.

Table 3.–Summary of modulus of elasticity and modulus of rupture results

Number of laminations	Number of Specimens	Modulus of elasticity				Modulus of rupture			
		Unadjusted values		Adjusted to 12 percent moisture content[2]		Unadjusted values		Adjusted to 12 percent moisture content[2]	
		Mean	Ω[1]	Mean	Ω[1]	Mean	Ω[1]	Mean	Ω[1]
		Million lb/in.^{2}	*Pct*	*Million lb/in.*^{2}	*Pct*	*Lb/in.*^{2}	*Pct*	*Lb/in.*^{2}	*Pct*
				L1–DOUGLAS-FIR					
1	100	2.17	17.2	2.03	17.5	7,130	36.8	6,520	37.3
2	40	2.22	10.6	2.10	11.3	7,140	25.4	6,620	26.5
3	32	2.25	9.8	2.15	9.8	7,860	19.7	7,340	20.1
4	32	2.27	8.1	2.17	8.5	7,910	16.0	7,400	16.3
5	32	2.27	8.1	2.18	8.4	8,250	16.9	7,750	17.2
3 + 4 + 5	96	2.26	8.6	2.17	8.9	8,000	17.5	7,500	17.9
				N2D–SOUTHERN PINE					
1	80	1.74	21.6	1.69	21.8	5,600	38.0	5,420	38.8
2	36	1.69	13.6	1.68	13.2	6,050	21.0	5,990	21.0
3	30	1.82	13.1	1.82	13.3	7,120	20.5	7,100	21.1
4	30	1.79	7.7	1.78	7.9	6,890	17.6	6,860	17.5
5	30	1.71	9.3	1.70	8.9	6,900	15.3	6,890	15.0
3 + 4 + 5	90	1.77	10.0	1.76	10.0	7,000	17.8	6,950	17.9
				L3–DOUGLAS-FIR					
1	100	1.61	23.6	1.50	24.0	3,910	51.8	3,520	52.6
2	40	1.84	13.8	1.73	14.3	4,980	31.9	4,550	32.4
3	32	1.77	10.0	1.67	10.3	5,070	14.1	4,660	14.2
4	32	1.81	7.9	1.72	8.3	5,080	16.1	4,700	16.5
5	32	1.76	7.9	1.67	8.0	5,140	17.3	4,740	17.8
3 + 4 + 5	96	1.78	8.6	1.68	8.9	5,100	15.7	4,700	16.2

[1] Ω = coefficient of variation.

[2] Adjustments according to ASTM D 2915, section 5.

Table 4.–Classification of beam failures

Number of laminations	Tension knots and deviation	Compression	Horizontal shear
	Pct	*Pct*	*Pct*
	L1–DOUGLAS-FIR		
1	82	15	[1]3
2	95	5	—
3,4,5	74	26	—
	N2D–SOUTHERN PINE		
1	80	17	[2]2
2	90	11	—
3,4,5	98	12	—
	L3–DOUGLAS-FIR		
1	92	8	—
2	97	3	—
3,4,5	100	0	—

[1] Three specimens failed at calculated stresses of 90, 152, and 224 lb/in.^{2}

[2] Two specimens failed at calculated stresses of 171 and 264 lb/in.^{2}

pression, and horizontal shear (table 4). The appearance of a compression wrinkle was taken to mean that the tensile edge must be stronger than the compression edge; thus, all beams displaying compression wrinkles are placed in the failure category of "compression." Horizontal shear failures occurred only in single member tests of L1 Douglas-fir and No. 2D southern pine. The lowest value calculated for maximum shear stress was 90 pounds per square inch. Inspection after testing indicated that this may have been due to the presence of end-grain seasoning checks.

Modulus of Elasticity

Table 3 lists values for mean MOE and the corresponding coefficients of variation (Ω) for each test sample.

As was expected, the number of laminations (N) had no significant effect on average MOE (i.e., $\overline{MOE}$) values. However, the variability decreased with an increase in N. This relationship is best represented by an averaging model which predicts no change in mean with a change in N,

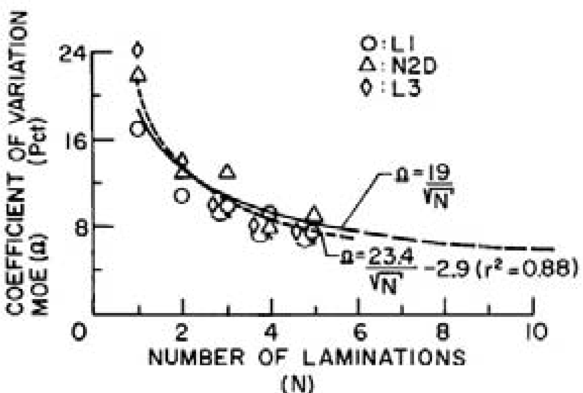
$$\overline{MOE}_N = \overline{MOE}_I \qquad (1)$$

and an inverse relation between Ω and the square root of N,

$$\Omega_N = \Omega_{I/\sqrt{N}} \qquad (2)$$

The best estimate of Ω_1, based on the single member MOE values, was 19 percent. In all cases except the L3 singles, this model provided estimates of the coefficient of variation within 2 or 3 percent of actual values (fig. 1). For the L3 singles, the predicted Ω was 5 percent lower than the measured value.

A comparison of average MOE with the MOE values published in AITC 117 *(1)* for use in design is shown in figure 2. The values published in AITC 117 are applicable to members with two or more laminations and are from two different editions of the standard (the 1976 edition and a proposed revision). The 95 percent confidence intervals on the mean of these groups either contain the published values or are quite close to them. If the three-, four-, and five-lamination beams were combined, the confidence interval would contain the published value for the L1 and N2D grade and exceed it for the L3. Thus, MOE of the material was quite close to average values published for the grade.

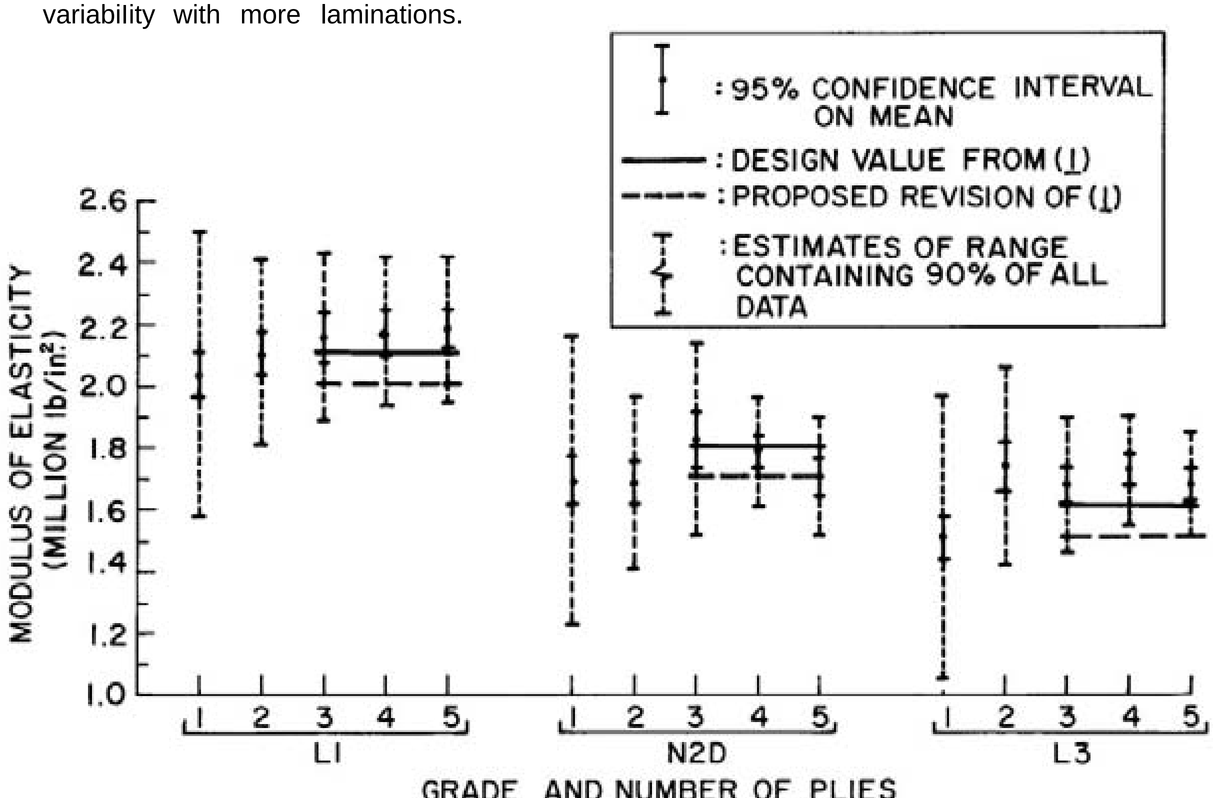


(M 146718)

Figure 1.–Decrease in modulus of elasticity variability with more laminations.

Modulus of Rupture

Average performance reflects general trends in the data while near-minimum strength forms the basis for present design stresses. Thus, both average and near-minimum strength will be discussed. New procedures are being applied to structural safety analyses that consider both load and resistance functions as random variables.



(M 146724)

Figure 2.–Comparison of test values with design modulus of elasticity *(1).*

These reliability-based design procedures, described by Zahn *(22),* will also be used to evaluate the data.

Average MOR

Average results together with their Ω values are summarized in table 3. There was no significant difference in the average strength of three-, four-, and five-ply specimens but all were significantly higher than the single-ply ones. These results differ from what Bonnicksen and Suddarth *(9)* found for mechanically laminated specimens–i.e., that the mean strength of one- and three-ply specimens does not differ significantly.

Two-ply specimens of the highest grade had strengths near the single-ply specimens, while two-ply specimens of the lowest grade resembled the group with three or more laminations. Variability decreased significantly with more laminations (fig. 3).

An empirical equation was developed to describe mean MOR as a function of quality (grade) and number of laminations using regression techniques. Knowing the MOR of a single ply sample, the mean MOR (i.e., $\overline{MOR}$) of a vertically glued, laminated sample would be as follows:

$$\overline{MOR}_N = (\overline{MOR}_I)(N^{\alpha}) \qquad (3)$$

where $\overline{MOR}_N$ = mean MOR for beams of N lamination

$\overline{MOR}_I$ = mean MOR for single lamination beams

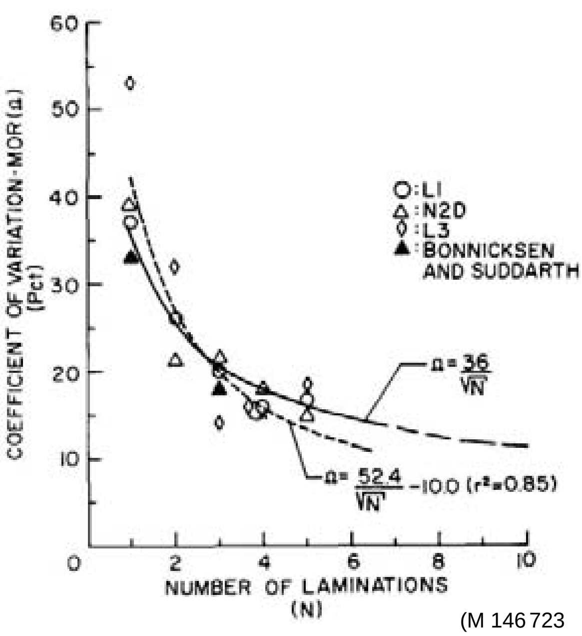
N = number of laminations

α = an empirical measure of the influence of lumber quality; i.e., α = 0.329 (1-1.049 SR), where

SR = minimum bending strength ratio of a single ply of a certain grade according to ASTM D 245 *(2).*

Using single member MOR values, the following expression was developed to model $\overline{MOR}_I$:

$$\overline{MOR}_I = MOR_C\ (SR)^{\gamma} \qquad (4)$$



(M 146 723

Figure 3.–Decrease in variability in modulus of rupture with more laminations.

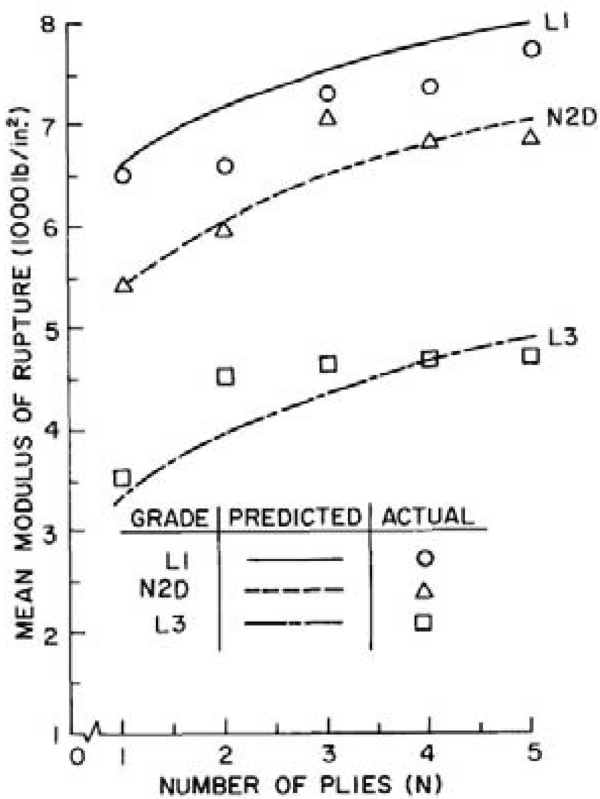


Figure 4.–Comparison of mean modulus of rupture values with those predicted by equations (3) and (4). (M 146 717)

where MOR_C = predicted average strength of defect-free material based on ASTM D 245 *(2)* and adjustments for size and method of loading. For this study, MOR, was taken as 10,000 pounds per square inch for both Douglas-fir and southern pine of the size and loading method used.

γ = 0.81, empirically determined constant.

Derived values are compared with actual mean MOR values in figure 4. Using an averaging model, the Ω was determined by

$$\Omega_N = \Omega_{I/\sqrt{N}} \qquad (5)$$

where

Ω_N = coefficient of variation of specimens with N plies

Ω_I = coefficient of variation of single-ply members which averaged 36 percent for the three grades.

The model and the data for Ω are compared in figure 3. This model also closely fits the Ω of one- and three-ply data obtained by Bonnicksen and Suddarth *(9).*

Near-Minimum Strength

One method of assigning design stresses to timber members is to determine the near-minimum strength of a representative sample and then apply adjustment factors *(3).* Thus, different methods of predicting near-minimum strengths can be evaluated by applying the adjustment values in reverse to the published design values. Generally, the near-minimum minimum strength is assumed to be near the fifth percentile of the population (appendix II).

Design values derived using standard procedures for the three grades of materials are given in table 5. Also included are estimated fifth percentile values for the test data by three methods. The purpose of the study was not to evaluate the appropriateness of single member design values. However, a comparison of the estimated near-minimum strength with the actual data showed a much larger portion of the single-ply pieces to be below the predicted value than the 5 percent expected. As shown in table 5 for the three grades, between 17 and 28 percent of the data were below, with the higher percentage in the higher grades. This suggests that significant adjustments are needed if the visual grading, strength ratio method is to accurately predict the lower fifth percentile of single members. The amount of adjustment needed is indicated by the various estimates of the fifth percentile listed. Tolerance limits estimated, assuming a lognormal distribution, indicate that reductions of 30 to 40 percent are appropriate with the higher reduction in the higher grade.

For two-ply members, the strength ratio method for single-ply members was slightly conservative in predicting the near-minimum test value for the N2D- and L3-grade samples. However, 18 percent of the L1 sample was below the predicted level, and a 10 to 20 percent reduction would be necessary to reach the estimated near-minimum strength.

For specimens containing three or more plies, the procedure for predicting near-minimum strength values was close for the L1 material and conservative for the N2D and L3 material. Based on the lognormal tolerance limits, increases of 10 to 20 percent for the N2D design values, and 70 to 80 percent for the L3, would approach the actual fifth percentile with estimated near-minimum strength values.

One method of predicting the near-minimum strength would be to use the model previously developed for predicting mean MOR and the Ω. An estimate of the fifth percentile can be made by subtracting 1.645 times Ω from 1 and multiplying this by the mean:

$$MOR_{0.05} = (MOR_C)(SR^{\gamma})(N^{\alpha})\left(1 - \frac{1.645\Omega_1}{\sqrt{N}}\right) \qquad (6)$$

where all terms are as previously defined.

Values thus calculated are compared with actual values estimated by nonparametric and lognormal tolerance limits in figure 5.

Reliability-Based Analysis

The previous deterministic analysis considered the approach suggested by ASTM D 245 *(2)* to evaluate the design properties of

Table 5.–Comparison of actual data with predicted near-minimum strength

Grade and species	Number of plies	Comparison with design values			Estimates of 5th percentile[3]					
					Nonparametric[4]		Tolerance limits (75 pct confidence)[5]			
		Estimated design stress[1]	Predicted near-minimun strength[2]	Portion of specimens less than predicted near-minimum[3]	Estimate	Percent of predicted	Normal distribution		Lognormal distribution	
							Estimate	Percent of predicted	Estimate	Percent of predicted
		Lb/in.[2]	*Lb/in.*[2]	*Pct*	*Lb/in.*[2]	*Pct*	*Lb/in.*[2]	*Pct*	*Lb/in.*[2]	*Pct*
L1–Douglas-fir	1	2,200	4,620	28	2,570	56	2,250	49	2,930	63
	2	2,210	4,650	18	3,930	85	3,400	73	3,920	84
	3	2,550	5,350	12	4,140	77	4,600	86	4,760	89
	4	2,550	5,350	6	4,590	86	5,160	96	5,280	99
	5	2,550	5,350	0	6,090	114	5,270	99	5,640	105
	3 + 4 + 5	[6]2,300	4,830	4	4,718	98	5,140	106	5,300	110
N2D–Southern pine	1	1,720	3,620	22	2,200	61	1,690	47	2,370	65
	2	1,740	3,640	3	3,680	101	3,670	101	3,950	109
	3	2,000	4,190	0	4,780	114	4,300	103	4,580	109
	4	2,000	4,190	0	4,730	113	4,620	110	4,880	116
	5	2,000	4,190	0	4,740	113	4,960	118	5,130	122
	3 + 4 + 5	[6]1,800	3,780	0	4,820	128	4,750	126	4,950	131
L3–Douglas-fir	1	820	1,720	17	1,290	75	270	16	1,210	70
	2	820	1,730	2	1,560	90	1,850	107	2,160	125
	3	950	1,980	0	3,530	178	3,430	173	3,570	180
	4	950	1,980	0	3,390	171	3,260	165	3,430	173
	5	950	1,980	0	3,490	176	3,170	160	3,380	171
	3 + 4 + 5	[6]900	1,890	0	3,520	186	3,380	178	3,400	180

[1] ASTM D 245 procedures used with clearwood stress values of 3,000 and 3,500 lb/in.[2] for medium grain and dense material, respectively, for 12-in -deep. uniformly loaded beams having a 21:1 span-to-depth ratio Values adjusted to conditions of test.

[2] Estimated design stress times 2.1 as suggested by ASTM D 2915 *(3).*

[3] All data adjusted to 12 pct moisture content.

[4] Values estimated using Lagrange interpolation *(21)* between nonparametric distribution function values *(6).*

[5] Factors for the multiple of the standard deviation to subtract from mean taken from Natrella *(12).*

[6] Published design values for members less than 12 in deep, AITC 117-76 *(1).* A slightly higher value of 2,400 lb/in.[2] has been proposed by AITC for L1 in a proposed revision.

the test samples. This analysis used established factors to account for the chances of possible overload and for load duration. A reliability analysis, on the other hand, considers a joint distribution of the member resistance (R) and the imposed loading (S). This enables the designer to have more control over the design safety, which depends upon the type of loading and use of the structure. A more detailed discussion of reliability-based design is given by Zahn *(22).*

Throughout the reliability analysis, the following equation (derived in appendix I), is used to relate the load and resistance distributions:

$$M_S = EXP\left[Ln(M_R) + Ln\frac{\Omega_S^2 + 1}{\Omega_R^2 + 1} - \beta\sqrt{Ln\,(\Omega_R^2 + 1)(\Omega_S^2 + 1)}\right] \qquad (7)$$

where

M_S = stress resulting from the mean imposed load (mean load induced stress),

M_R = mean MOR of the sample being considered,

Ω_S = assumed coefficient of variation of induced stress,

Ω_R = the coefficient of variation of MOR, and

β = the safety index desired.

The derivation of this equation assumes that both the load and resistance are lognormally distributed.

M_R and Ω_R are material properties determined from tests and are known. M_S, Ω_S, and β are the three remaining variables, and the purpose of subsequent analysis will be to examine their interdependence. Selection of the safety index, β, is dependent upon the desired reliability of the member to with-

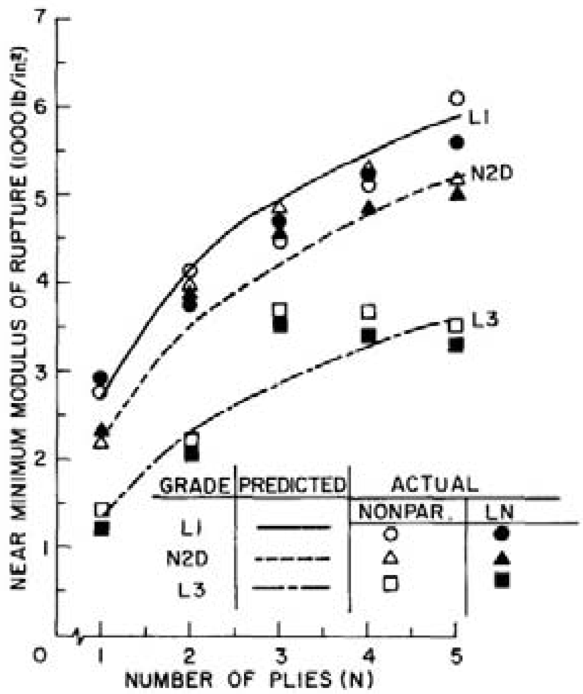


Figure 5. –Comparison of near-minimum strength estimated from the data and from equation (6). (M 146719)

stand the imposed load. Possible values of β and how they might be assigned are given by Zahn *(22).* The effect of changing both β and Ω_s on the mean load-induced stress, M_s, was first determined using the material properties previously discussed and equation (7). Note that load induced stresses must be on a 5-minute basis because strength values are based on a 5-minute test. Load-duration adjustments applicable to wood must be used to extrapolate to longer load durations.

Stress produced by the average imposed load would vary with both β and Ω_s for the grouping of three-, four-, and five-ply specimens of the three grades as shown in figure 6. This indicates the strong dependency of the mean load on both the β and the Ω_s. Similar information for the one- through five-ply specimens is given in appendix I.

To evaluate the present design stresses by this method, one further assumption about the loading was required: Current design loads were assumed to be at the 90th percentile of the loading distribution. Then, the dependency of β upon Ω_s was determined; the published design stresses (table 5) were adjusted to a 5-minute loading (divided by 0.62) to represent the load at the 90th percentile. Results are shown in figures 7, 8, and 9 for the three grades. The low values of β ($\approx$2.0) for single-ply members were expected, and reflect the unconservative nature of stresses assigned to them. The β for the three-, four-, and 5-ply group tended to increase as grade decreased, showing the greater benefit of laminating for low grade lumber.

Next, specific values were selected for both β and Ω_s to determine the 90th percentile of the load-induced stress. This stress might be interpreted as a “safe” design stress for these specific conditions. A β value of 3.0 was chosen as being applicable to some use conditions where a vertically laminated beam would be a single structural component. As Ω_s of 0.40 was selected, which is somewhat larger than suggested by Zahn (22). This higher Ω_s results in more conservative ratios between the multiple-ply and single-ply values (fig. 10). Also, these ratios appear to change little for higher values of Ω_s.

The 90th percentile of the load-induced stress distributions for the three grades is given in table 6. These values are obtained by multiplying M_s from equation (7) by (1 + 1.282 x Ω_s). This approach assumes that M_s is the mean and Ω_s the coefficient of variation of the parent population of induced stress. To make valid comparisons with deterministic design values (table 6, column 2), results must be adjusted to a 10-year load duration by the factor of 0.62. (For these adjusted values, see column 4 of table 6.) As expected, single member design stresses appear unconservative for all grades. Slight reductions are suggested for design values for multiple laminations of L1, while those of the N2D grade were all close to published values. Increases suggested for the multiple laminations of L3 (up to 49 percent) are somewhat less than the 70 to 80 percent values in table 5. Note that different results would be obtained for different values of β and Ω_s.

The purpose of this reliability-based design analysis was to appraise the design efficiency of the current recommended design stresses and also to compare results with the deterministic analysis. In general,

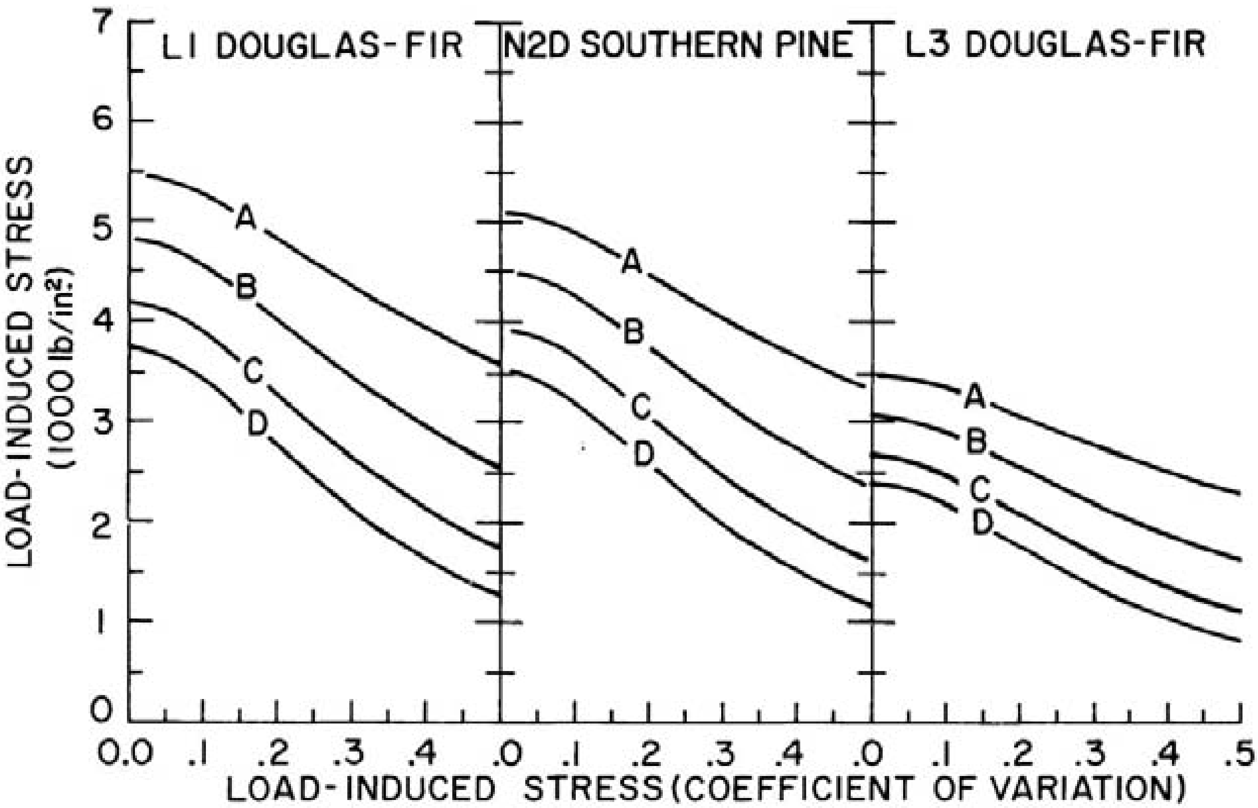
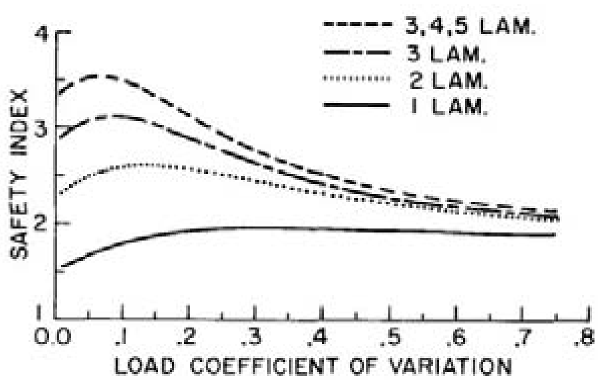


Figure 6.–Comparative reliability for the grouping of three-, four-, and five-ply beams at four safety levels (A-D) for three grades of material.

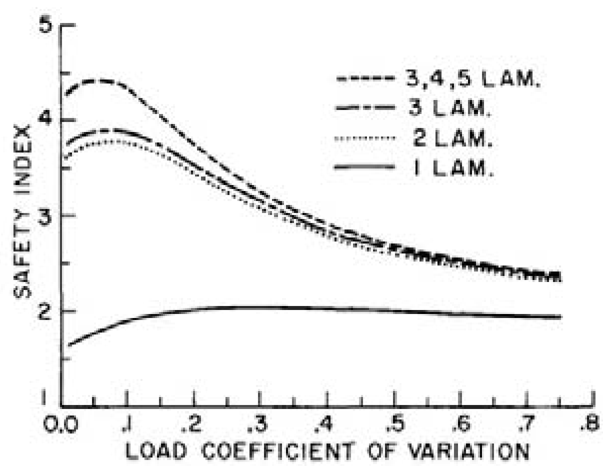
A Safety index β = 1.645 – reliability level = 0.95
B Safety index β = 2.33 – reliability level = 0.99
C Safety index β = 3.10 – reliability level = 0.999
D Safety index β = 3.75 – reliability level = 0.999

(M 146 710)



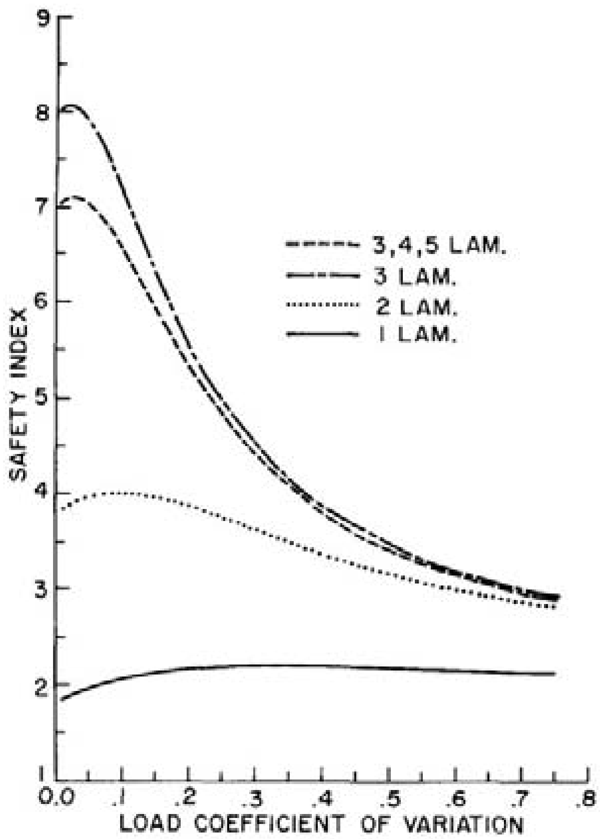
(M 146 722)

Figure 7.–Relative safety of current design versus load coefficient of variation for L1 Douglas-fir.



(M 146 721)

Figure 8.–Relative safety of current design values versus load coefficient of variation for N2D southern pine.



(M 146 720)

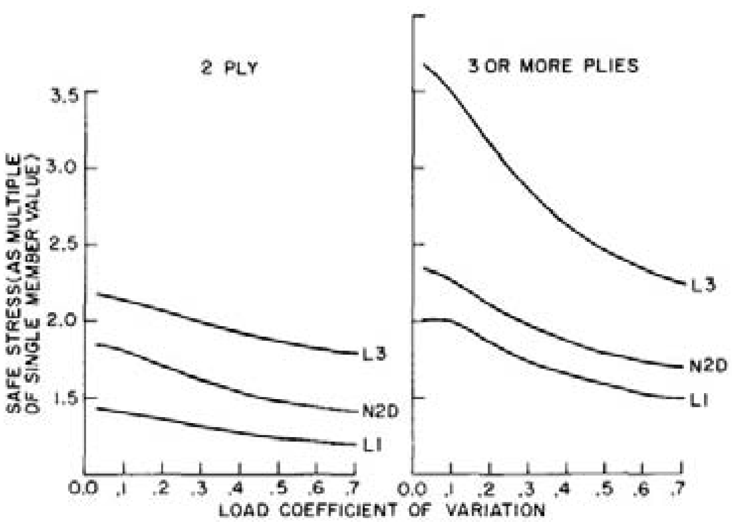
Figure 9.–Relative safety of current design values versus load coefficient of variation for L3 Douglas-fir.

the same trends were apparent by either analysis.

Effect of Number of Plies on Design

The effect of number of laminations is apparent when design stresses for beams of two or more laminations are expressed as multiples of the single member values. Table 7 shows that these ratios exceed the 1.15 "multiple member systems" factor described in ASTM D 245 *(2).* However, their use presumes that the near-minimum strength of single-ply members is adequately known or can be accurately predicted–a questionable assumption if using present procedures.

No definite trend appeared for any increase beyond three plies; the ratio for three-ply members appeared to be higher than for two-ply ones. For three or more plies, a ratio slightly over 1.5 appeared applicable to the highest grade, while a ratio between 2.5 and 3 appeared applicable to the lowest grade.



(M 146 716)

Figure 10.–Laminating effect at a safety index β = 3.0

Table 6.–Comparison of results of reliability analyses with estimated design stress

Grade and species	Number of plies	Estimated design stress for 10-year duration[1]	Results of reliability analysis for $\beta = 3.0$ and $\Omega_S = 0.40$ (90th percentile)		
			5-minute test	10-year duration[2]	Percent of design stress
		Lb/in.²	Lb/in.²	Lb/in.²	
L1 Douglas-fir	1	2,200	2,050	1,270	58
	2	2,210	2,640	1,640	74
	3	2,550	3,200	1,980	78
	4	2,550	3,410	2,110	83
	5	2,550	3,530	2,190	86
	[3]3 + 4 + 5	2,300	3,370	2,090	91
N2D Southern pine	1	1,720	1,670	1,040	60
	2	1,740	2,570	1,590	91
	3	2,000	3,050	1,890	94
	4	2,000	3,120	1,930	96
	5	2,000	3,220	2,000	100
	[3]3 + 4 + 5	1,800	3,120	1,930	107
L3 Douglas-fir	1	820	830	500	61
	2	820	1,590	990	121
	3	950	2,200	1,360	143
	4	950	2,170	1,350	142
	5	950	2,130	1,320	139
	[3]3+4+5	900	2,160	1,340	149

[1] ASTM D 245 procedures used with clearwood stress values of 3,000 and 3,500 lb/in.² for medium grain and dense material, respectively, for 12-in.-deep, uniformly loaded beams having a 21:1 span-to-depth ratio. Values adjusted to conditions of test

[2] Results of 5-min. test multiplied by a load duration factor of 0.62

[3] Values for design stress obtained from AITC 117-76

Table 7.–Ratio of multiple lamination "design stress" to that for single member

Grade and species	Number of plies	Ratio	
		Estimated fifth percentile using lognormal distribution[1]	Reliability analysis[2]
L1–Douglas-fir	2	1.34	1.29
	3	1.62	1.56
	4	1.80	1.66
	5	1.92	1.72
	3+4+5	1.81	1.64
N2D–Southern pine	2	1.67	1.54
	3	1.93	1.83
	4	2.06	1.87
	5	2.16	1.93
	3+4+5	2.09	1.87
L3–Douglas-fir	2	1.79	1.92
	3	2.95	2.65
	4	2.83	2.61
	5	2.79	2.57
	3+4+5	2.81	2.60

[1] ASTM D 245 procedures used with clearwood stress values of 3,000 and 3,500 lb/in.² for medium grain and dense material respectively for 12-in.-deep uniformly loaded beams having a 21:1 span-to-depth ratio Values adjusted to conditions of test

[2] Results of 5-min test multiplied by a load duration factor of 0.62

Conclusions

Bending tests on a sample of vertically laminated members with up to five plies of three grade (quality) levels revealed the following:

1. Average MOE was not affected by number of laminations. Also, average values were close to design values published for the different grades.
2. Up to three laminations, mean MOR increased with more laminations. However, there was no significant difference between the average strength with three-, four-, and five-ply members.
3. Variability in both the MOE and MOR results decreases with more laminations as measured by the coefficient of variation Ω. The relationship could be approximated using the inverse of the square root of the number of laminations (N) and values could be by

$$\Omega = \frac{19}{\sqrt{N}} \text{ for MOE and } \Omega = \frac{36}{\sqrt{N}} \text{ for MOR.}$$

4. The near minimum MOR, which forms the basis for present deterministic design stress, was strongly affected by N, with significant increases in the level between one and two and between two and three plies. Small differences between the three-, four-, and five-ply specimens suggested grouping them for design considerations.
5. The near-minimum strength of single members was considerably less than predicted using ASTM methods. Adjustments of 30 to 40 percent are suggested by both deterministic and reliability-based methods.
6. Procedures for estimating the near-minimum strength (and thus, for assigning design values of vertically laminated members) were found to vary in their conservatism depending upon quality and number of plies. The procedures when applied to two-ply members were slightly unconservative for the highest grade and somewhat conservative for the lowest grade. In addition, the procedures when applied to three-ply members were very Conservative for the lowest grade. It appeared that the lowest grade of lumber could justify design stresses up to 70 percent higher than now used.
7. Structural safety analysis conducted using reliability-based procedures yielded results consistent with the deterministic analysis of near-minimum strengths.
8. Assuming known strength properties of single members, the commonly used multiple-member factor of 1.15 is extremely conservative for two or more ply members, varying from 1.5 for the highest grade to over 2.5 for the lowest grade.

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Appendix 1. Reliability Model

The objective of a reliability analysis is to determine the probability of failure of a given member or system through jointly considering the distributions of resistance to load (R) and the imposed loads (S). For a structural component to fail, the load must exceed the resistance; therefore, the probability of failure is

$$P_f = P[R < S] \text{ or } P[R\text{-}S < 0]$$

For this analysis, both load and resistance are assumed to be distributed lognormally. This eliminates any consideration of values less than zero for load or resistance. The lognormal assumption says that the logarithms of R and S are normally distributed. The notation used to describe these variables will be as follows:

$$\lambda_R = \text{expected value or } E[\text{Ln } R]$$

$$\lambda_S = E[\text{Ln } S]$$

$$\mu_R^2 = \text{var}[\text{Ln } R]$$

$$\mu_S^2 = \text{var}[\text{Ln } S]$$

$$P_f = P[\text{Ln } R - \text{Ln } S < 0]$$

Normalizing this equation yields

$$P_f = P\left[\frac{\text{Ln}R - \text{Ln}S - (\lambda_R - \lambda_S)}{\sqrt{\mu_R^2 + \mu_S^2}} < \frac{-(\lambda_R - \lambda_S)}{\sqrt{\mu_R^2 + \mu_S^2}}\right]$$

The left side of the bracketed inequality is normally distributed with mean zero and standard deviation equal to 1. This is known as the standard normal distribution function and may be expressed as

$$P_f = \theta[-\beta] = 1 - \theta[\beta]$$

in which case θ designates the standard normal probability distribution function and

$$\beta = \frac{\lambda_R - \lambda_S}{\sqrt{\mu_R^2 + \mu_S^2}}$$

Using the following relationships

$$\lambda_i = \text{Ln}(M) - \frac{\mu_i^2}{2}$$

$$\mu_i^2 = \text{Ln}[\Omega_i^2 + 1]$$

Then, given a lognormal load distribution, and a specified coefficient of variation Ω_S and safety index β, the mean value of the load distribution may be found using the expression

$$M_S = \text{Exp}\left[\text{Ln}(M_R) + \text{Ln}\sqrt{\frac{\Omega_S^2 + 1}{\Omega_R^2 + 1}} - \beta\sqrt{\text{Ln}[(\Omega_R^2 + 1)(\Omega_S^2 + 1)]}\right] \quad \text{(I-1)}$$

For this study, parameters for the resistance distribution, M_R and Ω_R, were obtained from test results. Various values of the safety index β, depending upon the degree of safety desired, as well as values for load coefficient of variation Ω_S were used in equation (I-1) in order to derive the desired design load. Figures I-1 through I-5 indicate how design load values vary with reliability level and load coefficient of variation.

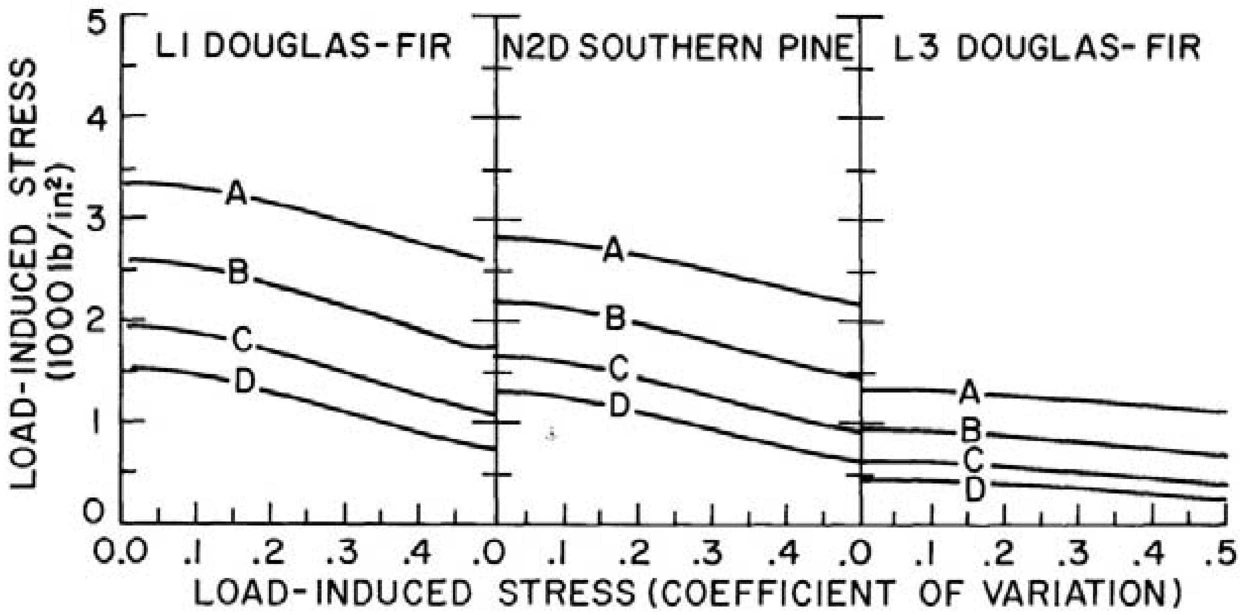

Figure I-1.–Comparative reliability plots for single member beams at four safety levels (A-D) for three grades of material quality. Loads were applied parallel to the wide face.
A Safety index β = 1.645 – reliability level = 0.9500
B Safety index β = 2.330 – reliability level = 0.9900
C Safety index β = 3.100 – reliability level = 0.9990
D Safety index β = 3.750 – reliability level = 0.9999

(M 146 712)

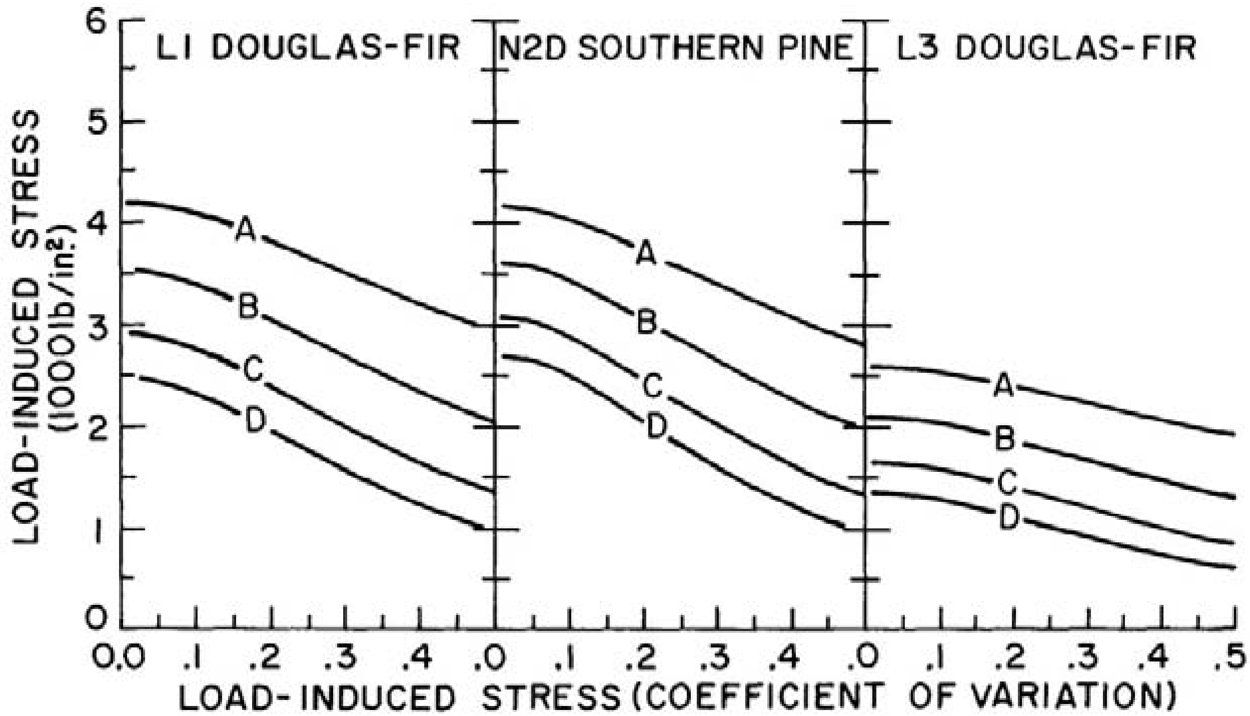

Figure I-2.–Comparative reliability plots for two lamination beams at four safety levels (A-D), for three grades of material quality (Safety index values as for fig. I-1).

(M 146 714)

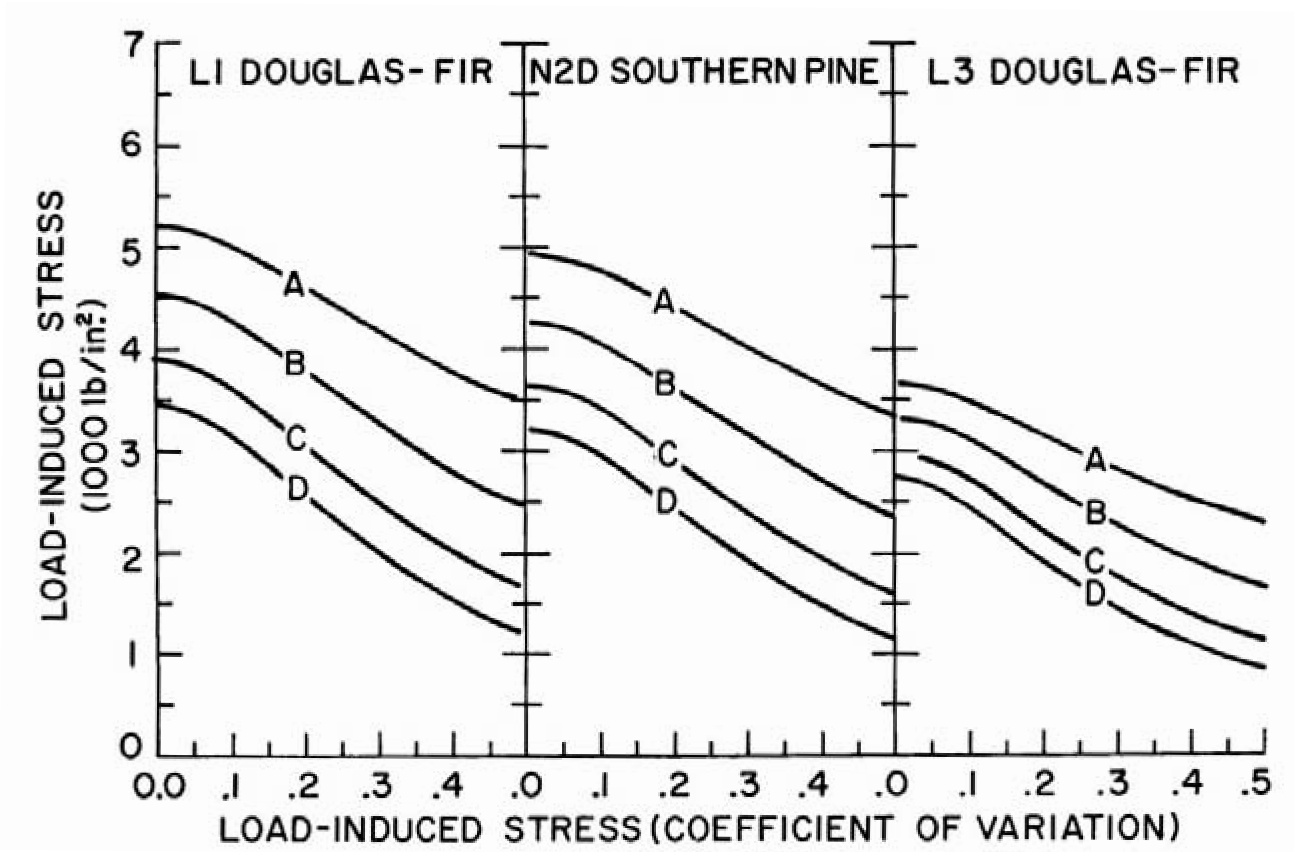

Figure I-3.–Comparative reliability plots for three-lamination beams at four safety levels (A-D), for three grades of material. (Safety index values as for fig. I-1). (M 146 715)

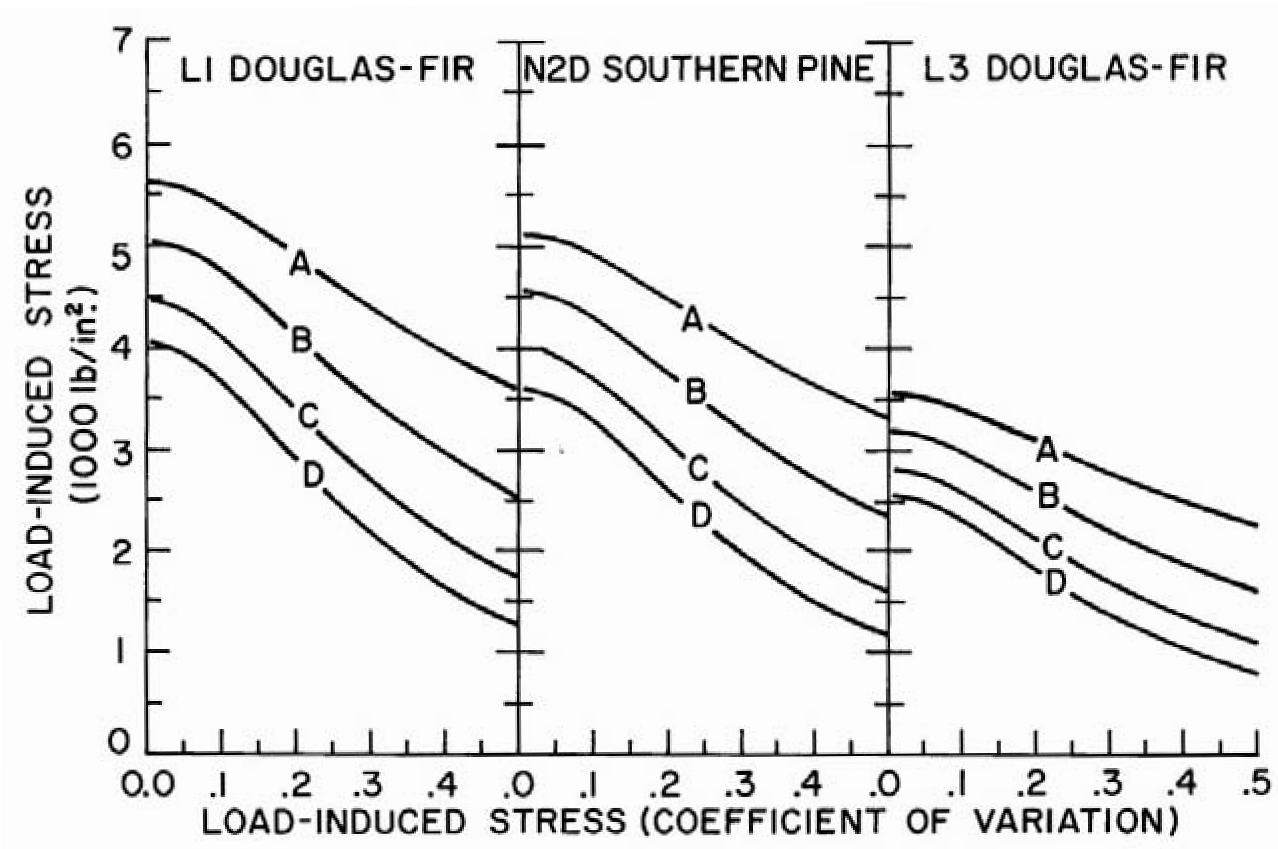

Figure I-4.–Comparative reliability plots for four-lamination beams at four safety levels (A-D), for three grades of material quality. (Safety index values as for fig. I-1). (M 146 711)

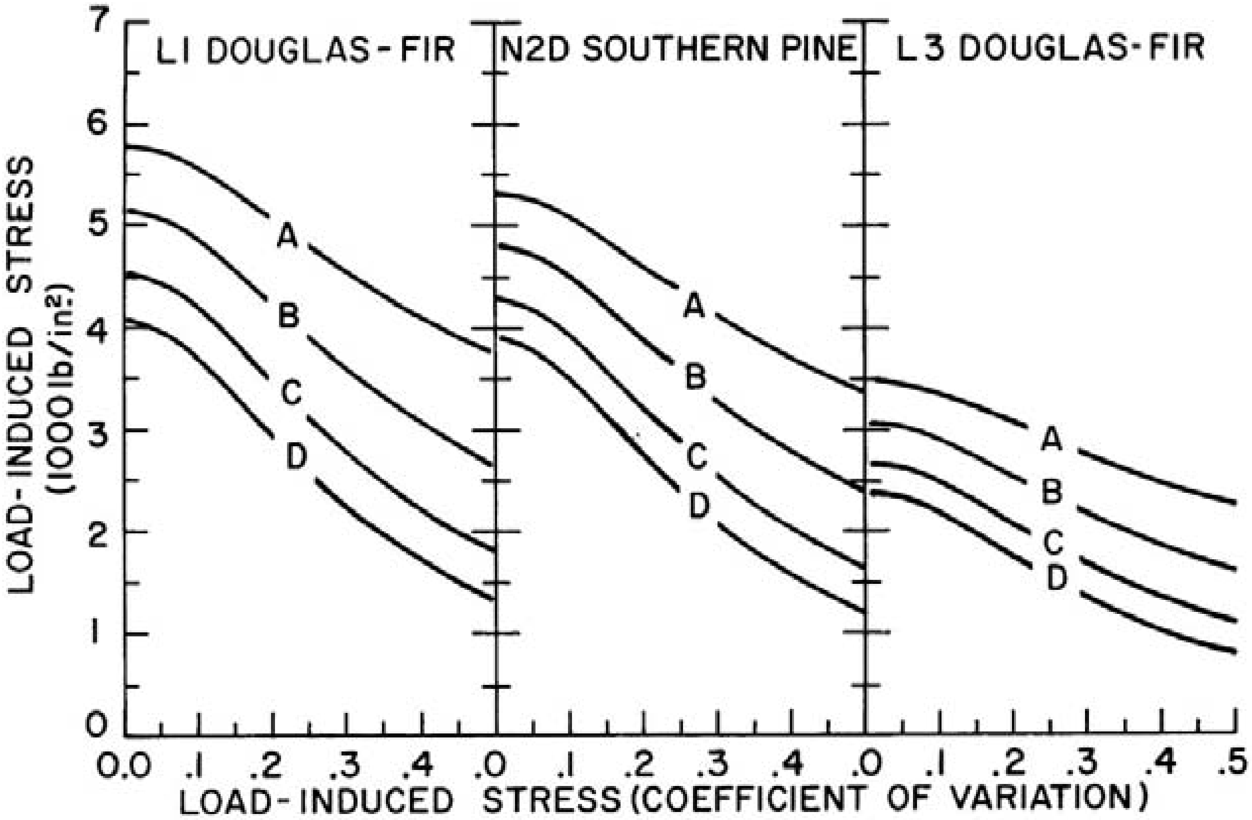

Figure I-5.–Comparative reliability plots for five-lamination beams at four safety levels (A-D), for three grades of material quality. (Safety index values as for fig. I-1). (M 146 713)

Appendix II. Analytical Concepts

A. Estimating the Fifth Percentile from Test Results

The fifth percentile of a distribution has also been referred to as the lower 5 percent exclusion limit. When some degree of confidence is associated with it, it is called the 95 percent tolerance limit. It is the value that would be expected to be exceeded by 95 percent of all values of the referenced population and is usually expressed in terms of the mean ($\bar{X}$) and the coefficient of variation, Ω:

$$X_{.05} = \bar{X}(1-K\Omega)$$

where

K = tolerance limit factor dependent upon sample size and degree of confidence desired (12).

B. Estimating the Expected Near-Minimum Strength from Design Values

Design values for the different number of laminations of the three grades can be estimated using procedures from ASTM D 245 *(2)* and a proposed ASTM standard for glulam timber. A clear-wood design stress for medium grain Douglas-fir and southern pine will be assumed to be 3,000 pounds per square inch and dense material of the same species as 3,500 pounds per square inch. These apply to 12-inch-deep beams at 12 percent moisture content with a 21:1 span-depth ratio. For beams of a specific grade and size, the proper strength ratio and size factor must then be applied. Then, for beams of three or more laminations, single member values are increased 15 percent (2).

Calculated design values are given in table II-1.

Table II-1 .–Estimated design values for grades and species evaluated

Gradeand species	Number of plies	Clearwood design stress	Bending strength ratio	Size factor	Multiple lamination factor	Design stress
L1 Douglas-fir	1	3,500	0.60	1.048	1.0	2,200
	2	3,500	.60	1.055	1.0	2,210
	3 or more	3,500	.60	1.055	1.15	2,550
N2D Southern pine	1	3,500	.47	1.048	1.0	1,720
	2	3,500	.47	1.055	1.0	1,740
	3 or more	3,500	.47	1.055	1.15	2,000
L3 Douglas-fir	1	3,000	.26	1.048	1.0	820
	2	3,000	.26	1.055	1.0	820
	3 or more	3,000	.26	1.055	1.15	950

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