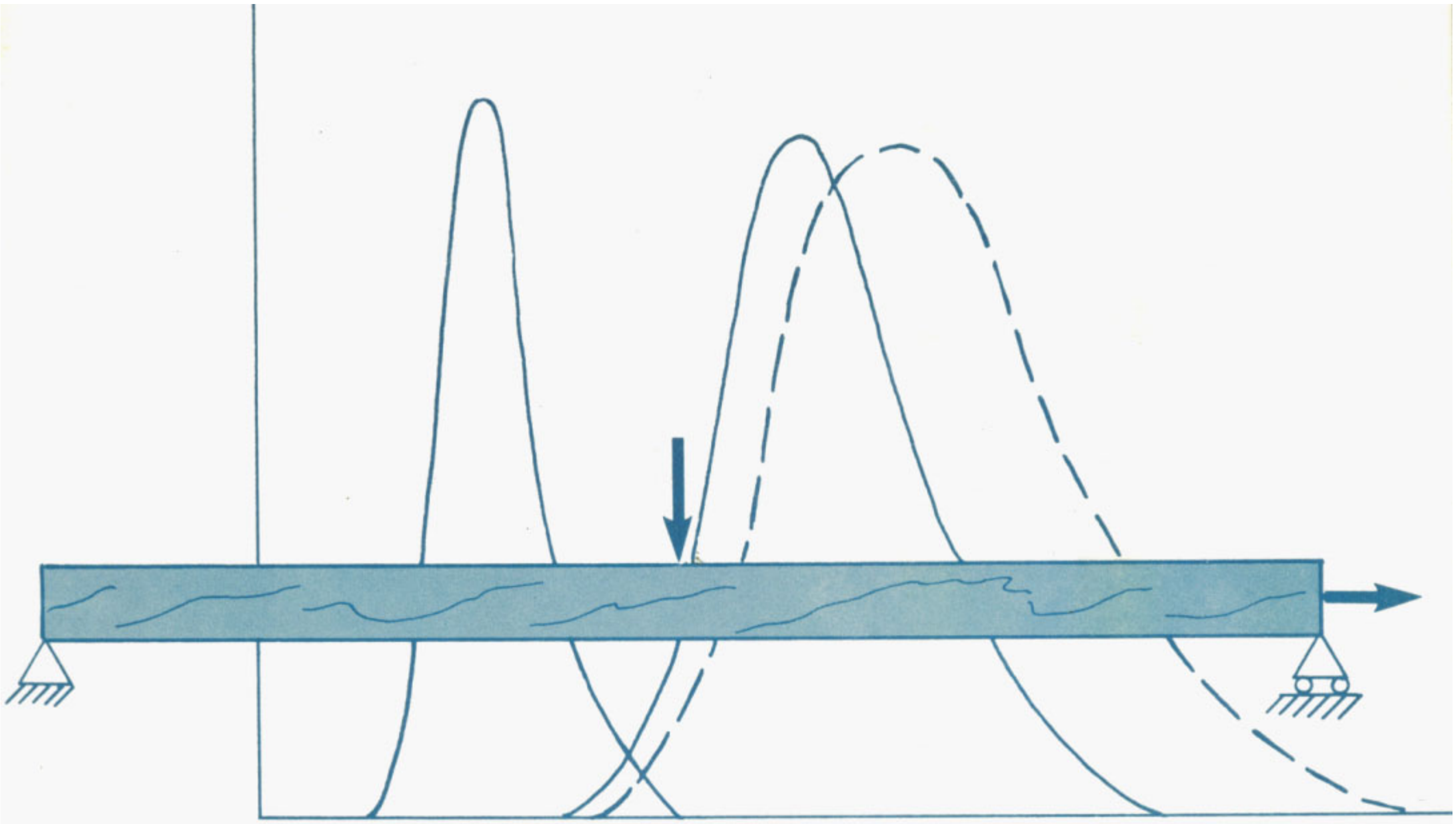

DIFFERENTIAL RELIABILITY:

PROBABILISTIC ENGINEERING APPLIED TO WOOD MEMBERS IN BENDING/TENSION

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ABSTRACT

Reliability analysis is a mathematical technique for appraising the design and materials of engineered structures to provide a quantitative estimate of probability of failure. Two or more cases which are similar in all respects but one may be analyzed by this method; the contrast between the probabilities of failure for these cases allows strong analytical focus on the case differences. This comparative procedure is known as differential reliability analysis. The technique is demonstrated by means of an example involving a simple truss member.

Applications of reliability analysis important to truss design are discussed. Differential reliability analysis is shown to be of value for code calibration purposes—that is, for evaluating new products or structural systems in terms of the prevailing practice. Reliability analysis can also be valuable for predicting future design-and-use payoff for investments in material properties research.

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DIFFERENTIAL RELIABILITY:

PROBABILISTIC ENGINEERING APPLIED TO WOOD MEMBERS IN BENDING/TENSION

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INTRODUCTION

Reliability methods of design are of increasing interest today in structural engineering for all materials, including wood. Reliability based design considers the probability that the structure will last a given length of time against the agents that can cause it to fail. The principal agent is. of course, load but others such as fire or decay can be dealt with using the same theory.

Probabilistic approaches are a natural way to solve structural design problems. The design process always concerns itself with structures yet to be fabricated and yet to be subjected to loads. These events lie in the future, and the systematic way to appraise the future is via application of probability concepts.

A recent paper by Zahn *(9)* [3/] introduces the subject of reliability analysis for wood and covers the fundamentals of both philosophy and method. The present paper continues the development with further explorations into the realm of wood truss engineering.

This research is part of a series of studies designed to explore the sensitivity of product performance to variations in material properties. Emphasis is placed on the wood truss because It is the component of the light frame house most consistently subject to engineering analysis.

THE CONCEPT OF DIFFERENTIAL RELIABILITY

The reliability analysis of a structure, while lucid in theory, can becomplicated in actual detail. Such analysis as presented here consists of a load distribution which Is mathematically associated with a resistance or strength distribution to produce a single result called the probability of failure. This number may be based on a series of necessary assumptions: once calculated. its magnitude can have great value, but only to an engineer who understands probability analysis. One advantage is that the process used is realistic as well as multifaceted: The precision of the estimated probability of failure is limited only by data and not by lack of processes for mathematical assembly of the answer. Nevertheless, the contemporary application of reliability will include many technical problems inherent in the introduction of a new concept.

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2/ Maintained at Madison, Wis., in cooperation with the University of Wisconsin.

3/ Numbers in parentheses and italics refer to literature cited at the end of the paper.

Specifically, incomplete information on both load and resistance will suggest caution In assessing the significance of unique calculations of the probability of failure. On the other hand, incomplete data need not retard immediate application of reliability where the technique can focus on differences between elements of the design process.

When studies of two or more cases are made, the contrast between the probabilities of failure for these cases allows strong analytical focus on the case differences. This strong analytic advantage occurs because all of the assumptions involved in one case can also be carried through for the others in a. completely formal way.

As an example of this concept, consider a structural component built with two alternative materials–perhaps a low variability lumber In one case and high variability lumber in the second—eachsubjected to reliability analysis. The necessary assumptions of integrity of the samples in representing such factors as population. consistent quality of fabrication, distribution of loads, and time scale can be identical tor both cases. The identity between cases of all but the focus variable gives special significance to the comparison of probabilities of failure calculated for each case. These probabilities are measures of relative safety which open the way for new quantitative comparisons. A particular case can be chosen as a benchmark of proven acceptable safety and all the other cases ranked against it. One might name the procedure more formally as differential reliability analysis. Two cases having equal failure probability have the same margin of Safety under the analytic constraints used for both cases.

One immediate use for the methodology of differential reliability is to test the degree to which failure probability is sensitive to changes In characteristics of the variables that comprise the ingredients of an analysis. This approach makes It possible to identify which variable Characteristics are of major importance and which are of minor importance in terms of result. More efficient use of research funds can result from such knowledge. Also, it is possible to quantitatively appraise the effects of quality control In materials preparation. fabrication. and structural maintenance, a vitally needed measure impossible to attain until now.

Differential reliability is also of value for code calibration *(1,8,9).* New methods of engineering or new structural systems, believed to represent improvements, must be evaluated by comparison with methods or products currently in use. The new method must result in performance or safety at least as good as the performance of the original. A current example in engineering analysis is the movement toward techniques involving limit states design *(1).* These techniques differ sharply in some respects from those used in traditional deterministic design. The new factors and considerations in limit states design must be properly evaluated so that designs by the new process are equivalent to old designs for the same end use. At the same time, treatment of alternative materials must be equitable. Concepts of differential reliability offer one method of evaluating such alternatives.

Application to a Member Subjected to Combined Loads

The potential of differential reliability is best shown by an example which also illustrates the process of producing compatible resistance and load functions for an actual case relevant to truss design. The sample structure is a bending member having a single concentrated load. *P,* also subjected to a tensile load. *Q* (fig. 1). The design criterion of interest in this example is the well-known interaction equation which requires the following:

$$\frac{f_b}{F_b} + \frac{f_t}{F_t} \leq 1 \qquad (1)$$

where

$f_b = \frac{3PL}{2bh^2}$ = the bending stress resulting from the load *P.*

F_b = the allowable bending stress for the structural lumber, which, in this example, is estimated as a fifth percentile value from a simulated distribution of bending test strength divided by 2.1 = 2.899 pounds per square inch (lb/in.^{2}). The 2.1 factor is an adjustment for load duration and safety factor *(2).*

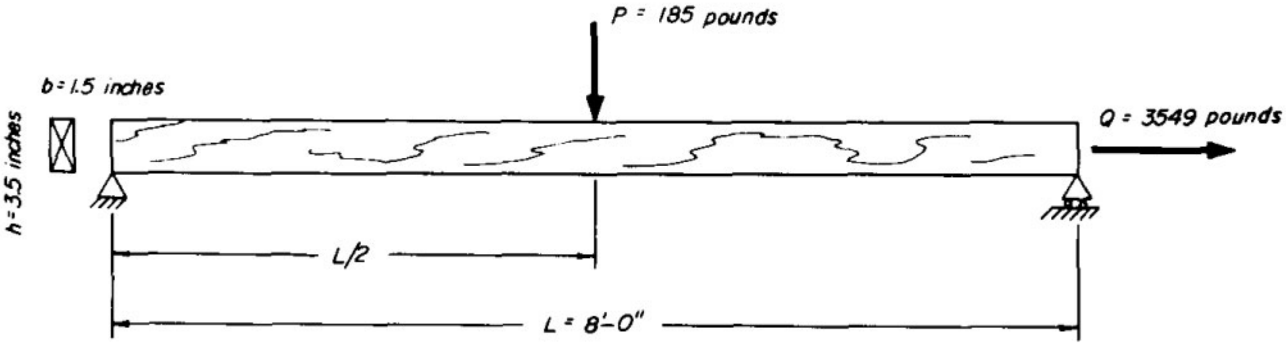


Figure 1.–This structure, which serves as an example in the analysis. is a lumber bending member bearing a single concentrated load, *P.* while also being subjected to a tensile load. *Q.* The loads have been proportioned so that both the bending stress and tensile stresses are one-half of their respective allowable values. This structure simulates a member from a more generalized structure such as a truss.
(M 145658)

f_t = $\frac{Q}{bh}$ the tensile stress resulting from the load *Q.*

F_t = allowable tensile stress for the structural lumber. which, in this example, is estimated as a fifth percentile value from a simulated distribution of tensile test strength divided by 2.1 = 1,352 lb/in.^{2}

The details of arriving at allowable stresses (2.899 lb/in.^{2} for F_b and 1.352 lb/in.^{2} for F_t) will be discussed later. The parameters of this problem have been worked out so that the bending stress and the tensile stress are equally important. These requirements are met by

$$\frac{f_b}{F_b} = \frac{f_t}{F_t} = \frac{1}{2} \qquad (2)$$

which will produce a structure that is exactly at the traditional design limit. In summary, the case shown in figure 1 represents the combined load effects found in structures such as trusses where the members have been sized properly. The questions now are: What engineering qualities can reliability analysis add to the present knowledge of this structure, and what new information may be needed to accomplish this?

To begin a reliability approach, a criterion of failure is needed. One such criterion can be derived from the historical way that combined stresses are handled. This is expressed in formula (1). the traditional interaction formula. This expression becomes a failure criterion when modified by inserting actual bending strength and actual tensile strength in the corresponding denominators in place of the allowable values, F_b and F_t. A strength fraction results which may be expressed algebraically as follows:

$$I' = \frac{f_b}{B} + \frac{f_t}{T} \qquad (3)$$

where

B = modulus of rupture of the lumber and
T = tensile strength of the lumber.

B and ***T*** are seen to be random variables. **I'** is also a random variable with a failure criterion at unit value. When I' is one or more, the structure can be considered as having failed.

Equation (3) offers a somewhat simplistic failure criterion but is easily understood, and its shortcomings when compared with more sophisticated theory of failure are offset to a degree in differential reliability applications. Two or more cases will always be under study, and the application of the same failure criterion to both makes it more of a standard reference point than an absolute level of failure.

Equation (3) reveals that f_b and f_t can also be random variables if the load Is a random variable. For the moment, we shall hold the load constant at the figure 1 values and develop a distribution for **I'** under those loads. Later considerations will lead to the more classic form of solution Involving resistance and load distributions.

Monte Carlo Analysis

A Monte Carlo simulation is used in this development because it is ideally suited for application in indeterminate structures such as wood trusses and frames. Monte Carlo techniques are well known and have been used extensively in cases where more formal mathematical procedures become intractable. The procedure generates a distribution for I' using a large number of repeated solutions of the sample structure; in each a typical pair of random variable values, *B* and *T,* are used. The results of these multiple trials can be collected together into a histogram to which an, appropriate probability density function can be' fitted.

The simple task of Monte Carlo trials becomes more complex when one considers that the *B, T* pair must relate to the same lumber piece and that Band Tare mutually exclusive properties. If the bending strength of a piece is known, it is impossible to determine its tensile strength and vice versa. This problem is serious where combined stresses are utilized. It will be considered more carefully as the technical detail of this analysis is developed.

Strength Prediction Model

The *B, T* pair needed for input must realistically characterize the lumber being modeled in the analysis. If the lumber is the product of a grading system in which *B, T* pair values are assigned, presumably the analytic or predictive model of the grading system could be used to assign pair values for simulation. Unfortunately. such an approach would be very complex, particularly for visually graded lumber, and could constitute a research program in itself. This is because the grading system is complex and is very difficult to reduce to an analytic model. Further, it may be desirable when using differential reliability to compare lumber from different sorting systems or lumber units known beforehand to have characteristic differences. A common prediction system is therefore necessary to most easily make differential reliability comparisons.

Research interests in recent years have led to the accumulation of data relating modulus of elasticity. *E.* to *B* and independently to *T.* While this. of course, is a major element in mechanical grading, a useful correlation between *E, B,* and *T* often can be shown in data derived by other sorting means. The choice of *E,* as a predictor, therefore, permits equal differential reliability treatment of data from differing sources.

In a fully developed analysis, a unique *E* value (measured by the same technique in all cases) would be used to predict both *B* and *T.* Further, this value ideally should be an *E* cross-referenced to a common base through appropriate standards procedures (e.g., ASTM D 2915 *(2)).* This initial differential reliability analysis has relied upon an available data set which does not meet ideal criteria but does permit relating bending *E* (E_b) to *B* and tensile *E* (E_t) to *T.*

Figure 2 shows a plot of the E_b versus *B* data along with the weighted lines which bound 99 percent of the residuals. For Monte Carlo simulation of lumber *B* values, the regression line and residual characteristics are necessary. The repeated process is one of randomly selecting an E_b value from a suitable distribution. calculating the corresponding *B* value given by the regression, and then, with a

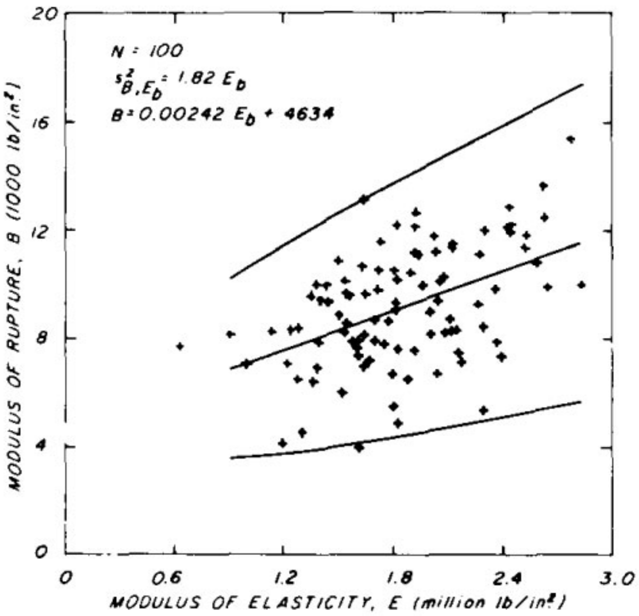


Figure 2.–E_b versus *B* data along with the weighted least squares regression line. The assumptions of the regression model are that the residual variance. s^2_{B,E_b}. is proportional to E and that the residuals are normally distributed. Under these assumptions the curved lines bound 99 percent of the residuals. (M 145655)

dom generation process for *B* values. The resulting distribution of generated *B* values can only be realistic if it resembles the original distribution determined by tests (fig. 3). Visual resemblance was chosen as the criterion for acceptance of the regression model.

To produce a *B* distribution judged appropriate, the proper statistical representation of the residual variance was necessary. For this data set the premise adopted is that residual variance Is proportional to *E.* Comparison of the histogram of actual *B* values with the histogram of generated *B* values supports the choice of statistical treatment for the data.

Figure 4 is similar to figure 2 but relates to the tensile strength analysis. The treatment of these two figures is paralles except that it was found necessary to make a logarithmic transformation on tessile strength to obtain realistic generation results. Again, simulated

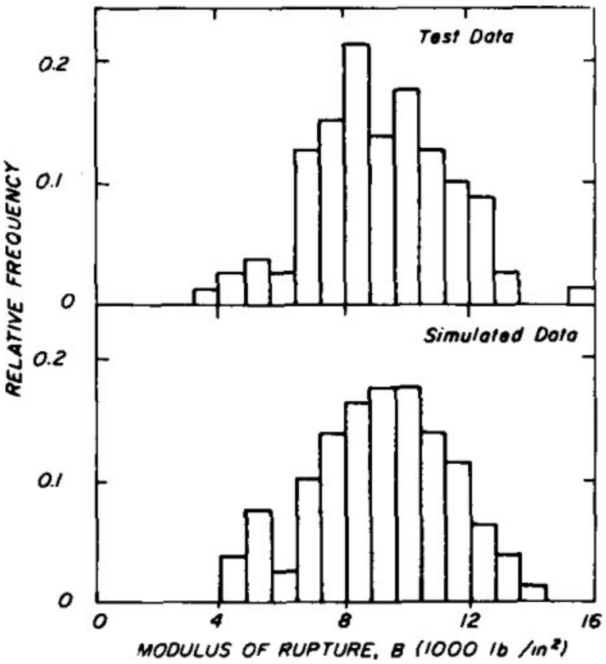


Figure 3.–The lower histogram was formed from the 100 test data. With the test data and its companion E_b, an *E-B* strength model was developed. From this model 100 simulated *B* data were generated, and these are shown in the upper histogram. (M 145 659)

tensile data (fig. 4) mimicked the experimental data (fig. 5).

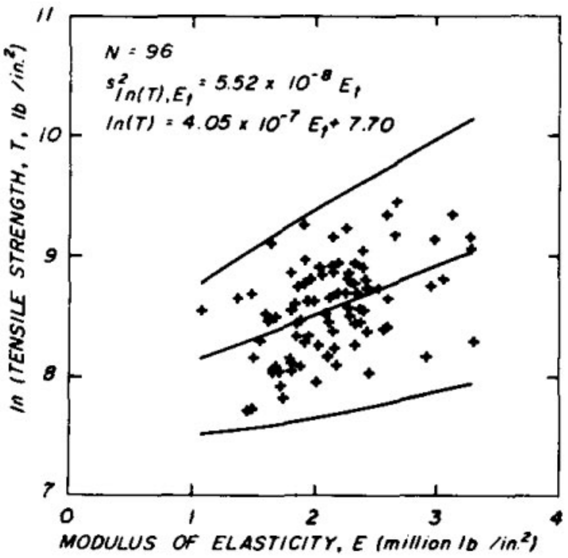


Figure 4.–Et versus the natural logarithm of data, *T,ln(T),* along with the weighted least squares regression line. The assumptions of the regression model are that the *ln(T)* residual variance, $s^2_{ln(T),E_t}$, is proportional to E_t and that the residuals are normally distributed. Under these assumptions the curved lines bound 99 percent of the residuals.

(M 145 656)

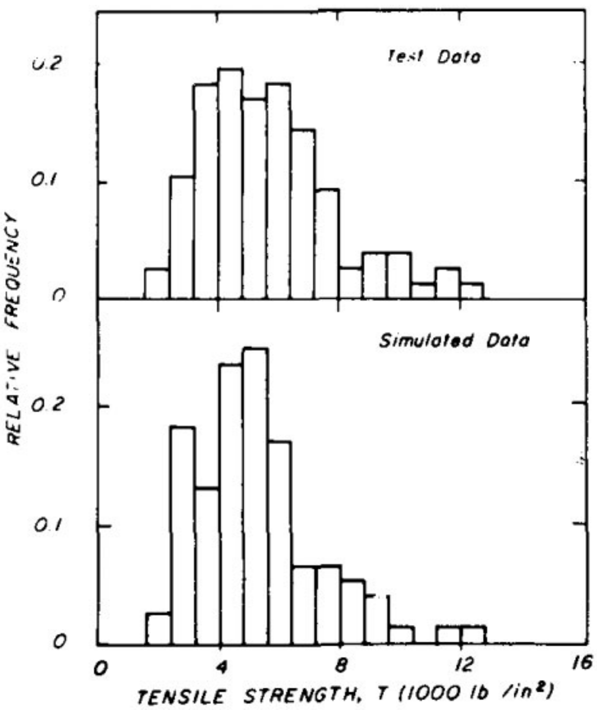


Figure 5.–Simulated *T* data closely resemble the skewed test data. From the 96 test data an *E-T* model was developed to simulate 96 data points, shown in the upper histogram. (M 145 661)

CONSTRUCTION OF A SIMULATED STRESS GRADE

Each piece of lumber must be assigned a single *E* value. However, as would be expected, the average *E* values and coefficients of variation (cv_E) of the two data sets shown in figures 2 and 4 are not the same. The average value for the E_b data was 1.85 million Ib/in.^{2} with a cv_E of 0.221. The average value of the E_t data was 2.13 million Ib/in.^{2} with a cv_E of 0.203. For simulation it is reasonable to pick a working *E* distribution with an average lying between the averages of the two E data sets. This choice Insures reasonable subsequent use of the regression equations which can only be valid in the range from which they were derived. The variance of the selected distribution must also be representative of both the E_b and E_t data sets. These considerations lead to the selection of an E distribution with an average of 2 million Ib/in.^{2} and cv_E of 0.21 for use in the simulation study. This cv_E corresponds closely to that recommended in the National Design Specification *(5).* The distributional form was chosen to be lognormal.

By the random selection of many E values, a corresponding number of tensile and bending strength values can be calculated from the regression equations (figs. 2 and 4). The 5-percent exclusion limits can be estimated from the data sets by methods of ASTM D-2915. By simulating 5,000 *E* values, the 5-percent exclusion limit for bending strength was 6,030 lb/in.^{2} and for tensile strength 2,812 Ib/in.^{2} These exclusion limits were further divided by 2.1. This placed the simulated population on a basis consistent with that applied to commercial truss lumber *(2);* the resulting stress grade for this sample set has allowable stresses of 2,899 Ib/in.^{2} In bending and 1,352 Ib/in.^{2} in tension. These were the values used in setting up the structure in figure 1.

GENERATION OF PROBABILITY OF FAILURE

Conditional Probability of Failure at Design Load

The procedures developed—lumber grade simulation, strength prediction modeling, the modified interaction formula *(r),* and the Monte Carlo procedure—werenow combined to calculate probability of failure. The process began with generation of piece properties to be used for a member subject to combined stress. In review, the first step was random selection of *E* followed by a random generation of Busing the relationship shown in figure 2. Then, again using the chosen *E,* a *T* value was generated in the same way using the relationship shown in figure 4. Such a process generates uncorrelated *B* and *T* values. The lack of correlation implies an assumption which can be difficult to accept; the effect of this assumption will be examined later.

The computer generates 5,000 sets of *E, B, T* combinations representing 5,000 pieces of typical lumber. On a repeated basis, each of the 5,000 lumber bending tensile members is subjected to analysis using formula 3, resulting in 5,000 values of the strength fraction, I', which can be assembled in histogram form and a curve fitted to the data (fig. 6). The area to the right of I' = 1 is the probability of failure of the structure for the given loads. This only reflects variability in the structural resistance under design load and does not yet reflect load variability. As the reliability procedure is further broadened, load variability will be considered.

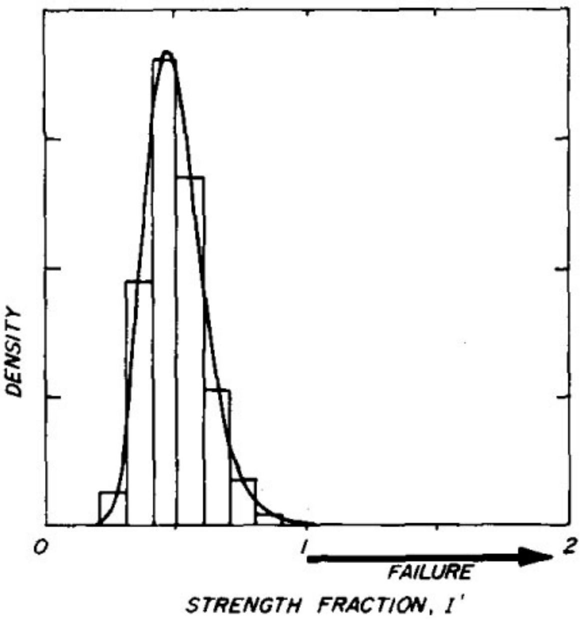


Figure 6.–The histogram of calculated I' values for 5,000 lumber bending-tensile members. For this illustration the cv_E = 0.21 and the residual correlation equals zero. The area to the right of I' = 1 under the fitted density curve is the probability of failure of the structure under the constant loads *P* and *Q.* (M 145 663)

Formulation of A Generalized Probability of Failure

A linear analytic system, common to most applications in wood frame engineering, is used for the structure of figure 1. if this structure were part of a truss, changes in the external load system would result in proportional effects on *P* end *Q.* If formula 3 is expressed in terms of basic quantities,

$$I' = \frac{3PL}{2bh^2B} + \frac{Q}{bhT} \tag{4}$$

and, if *P* and *Q* are changed proportionally by the load system to *nP* and *nQ,* then I' becomes *n*I' in direct proportion to the load change factor, *n.* if the distribution of load is known, it can be related to an n scale with n = 1 being the level that produces the original design values of *P* and *Q* (fig. 7). When n = 2. both *P* and *Q* are doubled; when n = 1/2. both *P* and *Q* are halved. The load is now expressed as a random variable, n, and the criterion of failure is characterized by *n*I' at unity or greater. The probability of failure, p_f, can be characterized in the expression

$$p_f = \text{Prob}(nI' \geq 1). \tag{5}$$

Operating on the expression within the bracket produces a more familiar form

$$p_f = \text{Prob}(n \geq \frac{1}{I'}) \tag{6}$$

which corresponds to the classical formulation for probability of faliure *(4)* which is stated in standard notation as:

$$p_f = \text{Prob}(S \geq R). \tag{7}$$

where

S = the random load variable and

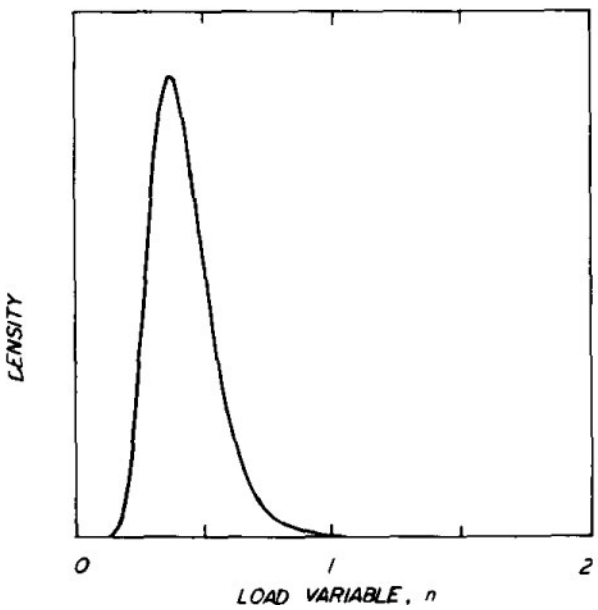


Figure 7.–The random load variable *n* is shown. The value of "1" on the abscissa corresponds to design load on the structure, "2" corresponds to 2 times design load, and so forth. Loads less than design load are most common while loads greater than design load are unlikely.
(M 145 662)

R = the random resistance variable in dimensions relevant to the load variable such as pounds, pound-inches, or lb/in.[2]

Note that while *n* corresponds to *S* and *1/*I' corresponds to *R, n* and *1/*I' are dimensionless variables. Probability statements pertinent to *S* and *R* distributions are equally pertinent to n and *1/*I' distributions.

Given formula 7, probability of failure is traditionally formulated as

$$p_f = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{s} f_R(r)dr \right] f_S(s)\, ds \qquad (8)$$

where

$f_R(r)$ = the probability density function of resistance and

$f_S(s)$ = the probability density function of load.

With certain types of probability density functions for *S* and *R,* the integration can be performed in closed form. in other instances, computer based numerical methods are readily available. A more detailed explanation of the probability of failure integral is given in the appendix.

In the case under study, the lognormal distribution was found to fit well to the calculated distribution of I' (fig. 6). When I' is lognormally distributed, *1/*I' is also lognormally distributed (fig. 8). Because of this, and because other studies (e.g., Corotis *(3)*) have found it to be a logical choice, the lognormal distribution was also selected to represent load. This relatively simple choice was made because this report Involves many complexities which, at least in early stages, could only be met with estimates and approximations.

The load distribution (fig. 7) can be shifted along the horizontal axis according to the judgment of the engineer and the data he may possess. This is a matter of estimating the likely occurrence of loads equal to or greater than design load. In a recent study of floor loads, Corotis *(3)* concluded that the present design load levels are close to the 99.9 percent cumulative levels for observed floor loads. To agree with this, the figure 7 load curve should be located so that the area under the curve to the right of 1 on the horizontal axis totals 0.001. The coefficient of variation of the load was taken to be 0.30 as recommended for live load by Siu et ai. *(8).*

Lognormais and other nonnegative distribution functions make good sense in applications of load and resistance in which negative quantities have no meaning. Nevertheless. the direct applicability of many classical distribution functions should be further considered. Good approximation methods currently available make it possible to examine logical boundaries on random variables other than zero and infinity. For instance, it appears illogical that lumber of near-zero strength can survive the stresses of milling and handling. Although it was not done in this example, minimum bounds on strength of materials could be independently estimated and a corresponding lower limit on resistance could result. The left tail of the resistance function would then be truncated and its area reallocated to the remainer of the function. On the other hand, upper extremes of load are often identified as "disaster" levels beyond the practical interest of the engineering design; upper limits can be set on the load distribution and it too can be truncated. (The influence of load truncation on results will be discussed later.)

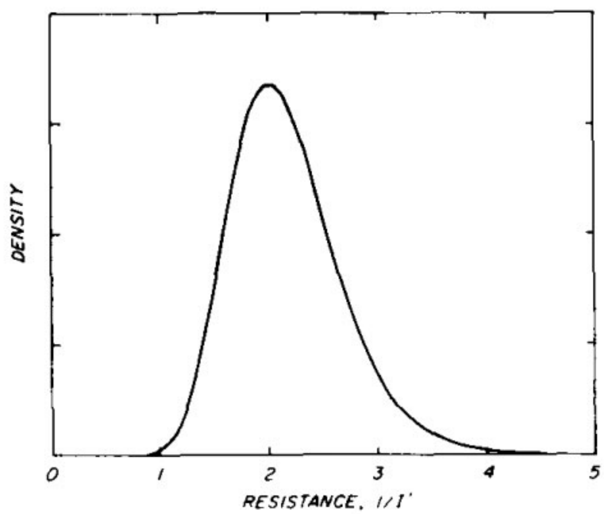


Figure 8.–The random variable *1/*I' is a resistance variable. The distribution is defined by the reciprocals of I' values shown in figure 6. The I' data were fitted by the lognormal distribution and consequently *1/*I' is fitted equally well by the lognormal distribution. (M 145 657)

In summary, actual test data on fundamental strength properties were used in this example, and the correlations between *E* and *B* and between *E* and *T* were calculated from these data. The generation of *E, B* pairs assumes a distribution of residuals about the regression line (figs. 2 and 4). This residual distribution was assumed to be normal and its variance proportional to E. It is also necessary to estimate the unknown correlation between *E, B* residuals and *E, T* residuals. The initial example calculation assumes it to be zero, but the influences of higher levels are examined in later results.

When all of the detailed input information is assembled for the example and processed into the load and resistance distributions (figs. 7 and 8). application of the integral formula (7) yields a probability of failure for these distributions of 0.565×10^{-5}. This figure is based on the structure in figure 1 and was calculated with all of the input conditions that have been imposed. If this structural situation were to be taken as a standard, then the probability of failure equal to 0.565×10^{-5} could be assigned the role of a base value—anexpression of belief that the structure will fail in service under the variable loading imposed. The magnitude of this probability is difficult to Interpret in any exact sense without a background of experience with other similar numbers. This difficulty is further compounded by the limited data base and related uncertainties such as the accuracy of the strength model. However, we believe that the errors in the system are stable or have the same bias in one application as in another. Thus, comparison among cases can have meaning in spite of the limited value of individual answers.

APPLICATION OF DIFFERENTIAL RELIABILITY TO ANALYSIS OF THE COMBINED LOAD MEMBER

The reliability methodology developed above was extended through the differential reliability concept to examine the influence of (a) lumber property variability, (b) the concomitance of *B* and *T,* (c) the probability of exceeding design load, and (d) the effect of load truncation. Increased insight into all of these concerns helps to focus the research on planned reliability design of trusses. For example, demonstrable effects of lumber properties reinforce and guide the search for lumber property data being shared by the U.S. Forest Products Laboratory (FPL). University of Wisconsin, University of Illinois. and Washington State University as part of their joint Truss Lumber Program.* (Specifically, concomitance between *B* and *T* is being investigated thoroughly for the first time by FPL and Washington State researchers.) Because many light frame structures need not be designed to the "disaster" level, efficiency may be enhanced by considering load truncation and the probability of exceeding design loads.

Variability of E

The structure shown in figure 1 was analyzed differentially with lumber representing two different distributions in initial E input. Both E distributions were lognormal with a mean of 2.0×10^6 lb/in.^{2}. The coefficient of variation was 0.21 in the first case and 0.1 in the second. These differences in cv_E bear some relationship with reality because they are probably the differences that would be observed between visually graded and machine stress rated lumber. Table 1 gives resulting probabilities of failure as a function of variability in *E.*

The probabilities of failure increase in going from the cv_E of 0.1 to 0.21. This result is expected because strength values *B* and *T* are positively correlated to E; hence, larger variation in E implies a larger variation in *B* and *T,* and the result is a more variable resistance function.

Probability of the Load Exceeding the Design Load

Three different levels of probability of exceeding design load were used. The load curve was located so that the area under the load curve to the right of 1 on the horizontal axis was 0.001, 0.01, and 0.1. These areas represent probabilities designated as P_S in table 1 and represent different horizontal scale

*As this paper goes to press, the following reference has become available: William L. Galligan and Edwin Kallio. 1977. Example of Integrated Research: Influence of Lumber Properties on Truss Performance. Forest Prod. J., 27 (11), 12-15.

Table 1.–Failure probabilities multiplied by 10^5 for a single beam under combined stress 1/

Failure probabilities x 10^5			
P_S	Residual r^2	Coefficient of variation in E	
		0.1	0.21
0.001	0.0	0.196	0.565
	.5	.945	2.08
	1.0	1.64	3.36
			7.46
			19.7
	1.0	16.3	28.2
1	.0	87.9	147
	.5	180	267
	1.0	233	333

1/ The load function is taken as lognormal with cv = 0.3. P_S is the probablility that the load will exceed design load. The resistance function is lognormal and has been derived from formula (3) by simulation.

locations of the load distribution with respect to the design load.

As P_S increases in table 1, so do the failure probabilities. This follows logically because P_S is the area under the load function to the right of design load. Increased chances of high loads should increase the structural p_t.

Concomitance of B and T

The concomitance, or cofunctioning of *B* and *T,* in the simple beam is not well understood, nor is the correlation between the two strength properties for the same piece. Previous research at Purdue University on combined bending and tension stress demonstrated the difficulty of an empirical approach to property interaction *(6,7).* Concomitance, therefore, becomes a subject of study within the research program. The procedure, then, is to choose a level of correlation between residuals in the *E, B* and *E, T* relationships and to use this correlation in a random process to make the final selection of *T* for the given *E.* The influence of several chosen levels of correlation between residuals upon the probability of failure helps determine the Importance of further study on concomitance.

For each value of P_S in tabla 1, the correlation between the *B* regression residual and the *T* regression residual values was set at three levels as designated by the square of the correlation coefficient. This represented three different levels of *B, T* correlation—the extreme correlations of 0 and 1, and one intermediate value, r^2 = 0.5.

Table 1 illustrates that the probability of failure increases with increasing residual correlation in the *B* and *T* regression residuals. When r^2 = 1, the resistance distribution has. higher variation and thus higher values of p_f result. This increase of probability of failure may not seem immediately convincing; note, however, that the drawing of a piece with an extreme *B* value when r^2 = 1 will cause the companion *T* value to be an extreme also. This situation leads to extension of the resistance distribution In both the high and low directions. On the other hand, when r^2 = 0 and an extremely good or poor value of *B* is drawn, it is most unlikely that a corresponding low or high value of *T* will be drawn. This in turn results in smaller variation in the resistance function.

The magnitude of the concomitance effect is shown in table 1 to be a function of the coefficient of variation of *E* and the probability of the load exceeding design load, represented by P_S. The concomitance, therefore, is a concern in research on both design and materials. The sections that follow include illustrations of some potential implications of this effect.

Design Reflections from the Differential Reliability Analyses

From the preceding analysis a logical focus from an engineering standpoint is the effect of cv_E. As an example of the differential approach, consider the first row of table 1 where P_S = 0.001, the load function is not truncated, and r^2 = 0.0. The probability of failure when cv_E is equal to 0.21 is 2.88 times greater than when cv_E equals 0.1. This number, 2.88, may be termed the probability ratio. if cv_E = 0.21 is taken as a standard, then pf = 0.565 x 10^{-5} is a benchmark probability. The cv_E of 0.1 can then be thought of as representing a new structural system. The immediate question is, how much higher could the design loads P and *Q* in figure 1 be for the cv_E = 0.1 case and have equal p_f as the standard benchmark situation? Under the assumptions of lognormal load and resistance of table 1, it can be shown that *P* and *Q* could be increased as much as

Table 2.–The increase in load carrying capability with equal reliability.[1/]

P_S	Residual r^2	increase in load capability
0.001	0.0	1.0835
	.5	1.0722
	1.0	1.0689
.01	.0	1.0771
	.5	1.0626
	1.0	1.0597
.1	.0	1.0567
	.5	1.0494
	1.0	1.0472

1/ The model structure was built with lumber with a cv_E of 0.1 versus lumber with a cv_E of 0.21. This is an alternative analysis of the data presented in table 1.

Table3. – Failure probabilities multiplied by 10^5 for a single beam under combined stress. [1/]

		Failure probabilities x 10^5	
P_s [2/]	Residual r^2	Coefficient of variation in E	
		0.1	0.21
0.001	0.0	0.159	0.515
	.5	.891	2.02
	1.0	1.59	3.29
.01	.0	2.21	6.00
	.5	9.26	18.0
	1.0	14.7	26.3
.1	.0	36.7	84.1
	.5	116	194
	1.0	165	257

1/ The input conditions are identical lo those of table 1 except the load distribution has been truncated at 1.5 times design load.

2/ After truncation of the load these numbers are slightly less but not enough to affect the results.

8.35 percent (table 2). Stated differently, for the case of $cv_E = 0.1$ the allowable design stresses for the lumber could be increased 8.35 percent with the same safety as the benchmark. It should be remembered that these conclusions result from a pilot trial of an entire system constructed from limited input information. Further consideration and study would be required to reach conclusions suitable for design purposes.

A similar analysis can be performed for the case of the truncated load distribution where loads of "disaster" level are not of interest (table 3). The probability ratio, based on the first row of table 3, is 3.24. This number should not be directly compared to the former ratio of 2.88 because, in addition to the focus variable, the load model has been changed. However, by repeated numerical integration of the probability equation and by graphing the results, it was found that *P* and *Q* could be increased 8.50 percent when $cv_E = 0.1$. Although the probability ratio differs between the cases depending on whether or not the load distribution is truncated, the engineering result is very much the same.

An analysis of increase in load carrying capacity for equal reliability was not carried out for the remaining truncated load cases represented by table 3. Nevertheless, results of tables 2 and 3 illustrate the importance of probabilistic load data.

Considering table 2 from a design perspective, an increase in load carrying capacity of less than 10 percent may not be impressive in view of other design uncertainties. It should be noted, however, that small increases in load carrying assignment (allowable properties) can have a marked effect if such changes result in qualification of otherwise ineligible lumber species/grade combinations for a crucial use such as the common 28-foot house truss. It is also important to reemphasize that this study employed a simplified structural model: an analysis for a truss may produce different results.

Many other design analyses of this type can be made. incorporating a variety of comparisons: decisions then become possible based on the quantified estimates of a probabilistic approach to engineering.

Applications of Differential Reliability to Materials Engineering

Concerns for proper focus in materials research elicited this research into truss lumber. The problem is common to all materials engineering: The difficulty of predicting future design-and-use payoff for research investments in material properties. Questions often asked regarding materials research include: What is the value of materials quality control? Will more accurate assignment of properties significantly improve design, especially where other factors are imperfectly known? The reliability research reported herein demonstrates the usefulness of probabilistic efforts tor materials research.

This research, while not evaluating an actual truss design, did demonstrate procedures for such an evaluation. Three materials-related influences on truss performance are of particular Importance: (1) material property variation; (2) differing methods of probabilistic property characterization; and (3) the relation between the several mechanical properties presumed to be interactive in members making up the wood truss. These three concerns will be reflected carefully in the next step—the combining of better data and more property, distribution insights with actual truss design.

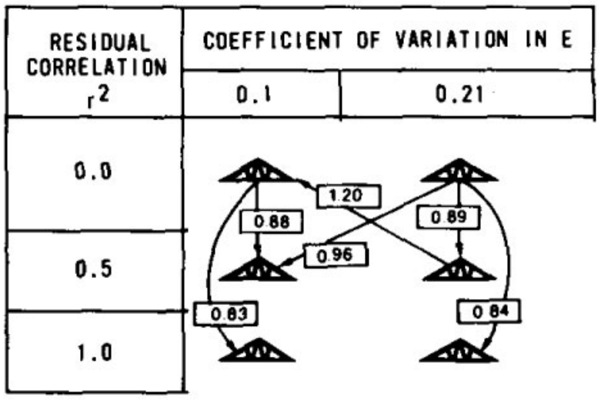
Of major significance to materials research is the sensitivity study on concomitant properties previously discussed. The comparisons of table 2 assume that the degree of residual correlation was equal for both levels of lumber cv_E. So little is known of this correlation. however, that one must consider the possibility that grading systems producing different levels of cv_E also may produce both cv_E correlation interactions and effects of "grade" level. Table 4, based on the data analysis that led to table 1, permits some insights into these possibilities.

in table 4 the Interaction between grading systems and concomitance, as represented by residual correlation, Is Illustrated by diagonal arrows at r^2 levels of 0.5. The numbers within the arrows are the relative change in load carrying capacity, where the benchmark is the lumber with cvE of 0.21. Combinations of cv_E and correlation can result in negative or positive effects, with a positive difference in load-carrying capacity up to approximately 20 percent.

Different levels of concomitance within a grading system are simulated by the vertical arrows in table 4. With r^2 = 0.0 as the base, the Increased levels of correlation result In reductions in load-carrying capacity for equal reliability.

This analysis with a simple one-member structure has suggested that correlation between strength properties can be important. It also suggests, however, that the effects are not so great that in a multiple member system such as a truss the effects may not be highly significant. Because research to quantify concomitance within and between different grading systems can be difficult. the degree of concomitance should be examined in analysis of a typical truss.

Table 4.–The difference in load carrying capability in the model structure as a result of concomitance [1/]



1/ The arrows express a change (increase or decrease) in load-carrying capability as a result of differences in combinations of r^2 and cv_E, where r^2 is residual correlation and cvE is the coefficient of variation in E. The basis lor the change is the load-carrying capability at cv_E of 0.21 lor diagonal comparisons and at r^2 = 0.0 for vertical comparisons.

CONCLUSIONS

Reliability-based design contrasts strongly with the existing deterministic design process, which has sharp zones of demarcation between tabulated allowable material values, tabulated design loads, and the analysis of the structure. Reliability analysis, on the other hand, can deal simultaneously with the variable characteristics of all of these three principle phases of the design process.

Basic notions of structural reliability have existed for some time but their application to wood structural design problems is relatively new. The concept of differential reliability is a potentially powerful tool penetrating many presently difficult problems that relate to calibration between old and new practice. New materials and new engineering methods can be quantitatively compared with accepted materials and methods by comparing the predicted failure probabilities of structures.

The research reported herein demonstrates a need to know more about the correlation between bending and tensile strength. It confirms previous indications that decreased variability in materials properties could lead to increased reliability or—for equal reliability—increased load-carrying capacity. In addition, it illustrates the mathematics required to generate and deal with frequency distributions of resistance and load. Furthermore, differential reliability has been applied to a truss design format.

Reliability studies can serve materials research through assessing the sensitivity of failure probabilities or probability ratios to varying levels of input variables. Reliability procedures can identify which variables need more study and to what level of precision – i.e., the effectiveness of materials research can be increased through improved identification of research targets and scaling of the depth of study.

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APPENDIX:
DERIVATION OF PROBABILITY OF FAILURE

A clearer picture of probability of failure can be gained by an assembly process using the concept of integral. Figure A-1 shows a load distribution while a resistance distribution is given below in figure A-2. Note that the horizontal axis has been labeled x in both cases to clarify the necessity that the load variable, *s.* and the resistance variable, *r.* must be measured in identical units. Both curves can actually be plotted on axes of the same density–the *x* axes–but are shown separately in figures A-1 and A-2 to permit exposition of further details.

The process of calculating a probability of failure can be visualized as a stepwise procedure of calculating each infinitesimal increment of failure probability, dp_1. at each value of x and associated infinitesimal element *dx,* and then summing (integrating) to obtain the total

$$p_f = \int dp_f \qquad \text{(1a)}$$

where

dp_f = (the probability of load occurring in the interval *x, x + dx*) times (the probability that the resistance is less than *x*).

The probability of load occurring at the interval *x, x +* dx is the area of the shaded vertical bar shown in figure A-1 and is

$$f_S(x)dx. \qquad \text{(2a)}$$

The probability that the resistance is less than x is the area under the figure A-2 curve to the left of x (shown as the shaded portion) and is

$$\int_{\infty}^{x} f_R(x)dx \qquad \text{(3a)}$$

Then

$$dp_f = \left[\int_{\infty}^{x} f_R(r)dr\right] f_S(s)ds \qquad \text{(4a)}$$

or, in standard notation

$$dp_f = \left[\int_{\infty}^{x} f_R(x)dx\right] f_S(x)dx \qquad \text{(5a)}$$

Summing dp, for each associated *x* value is a second integration process and produces formula 8.

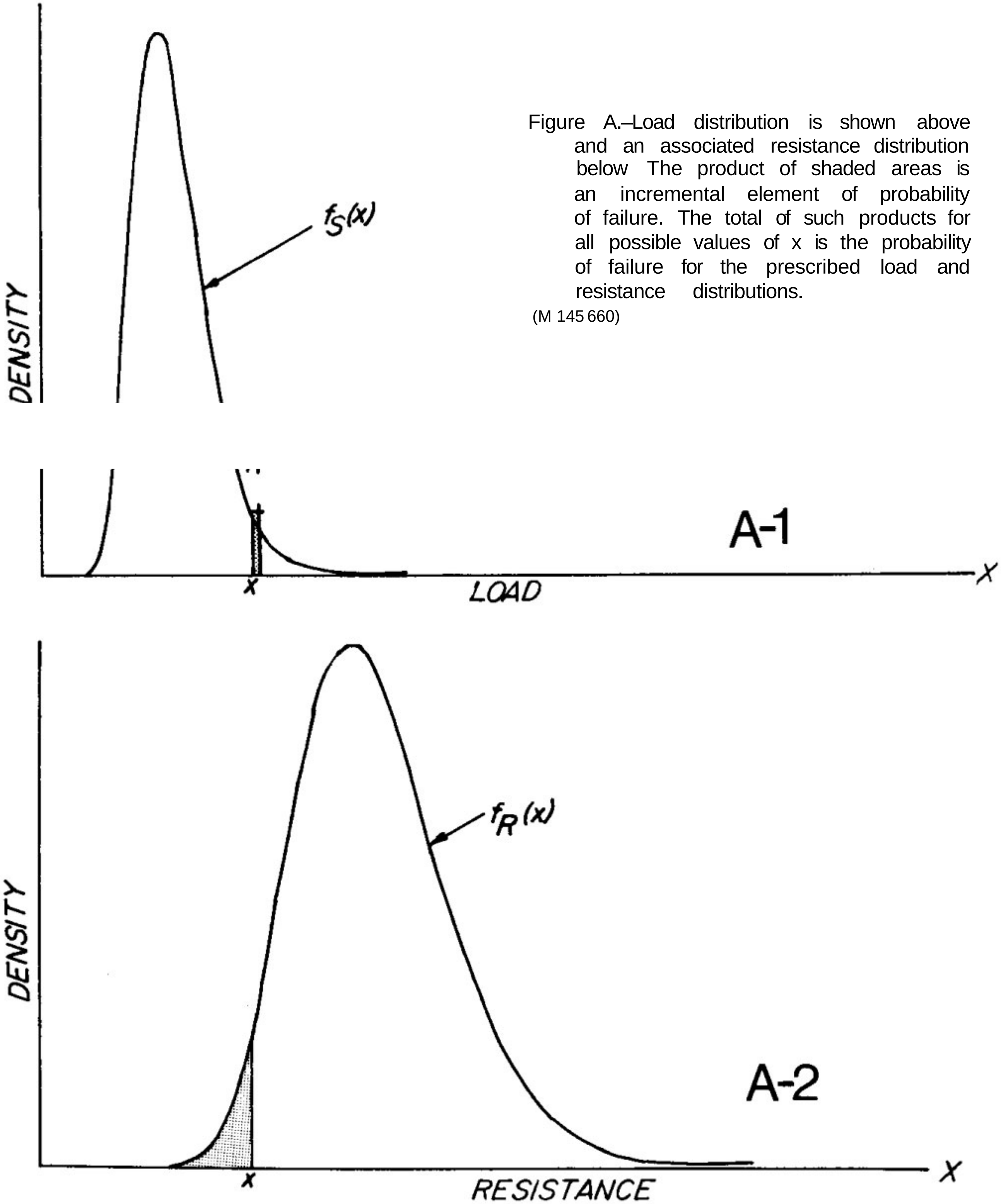

Figure A.–Load distribution is shown above and an associated resistance distribution below The product of shaded areas is an incremental element of probability of failure. The total of such products for all possible values of x is the probability of failure for the prescribed load and resistance distributions.
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