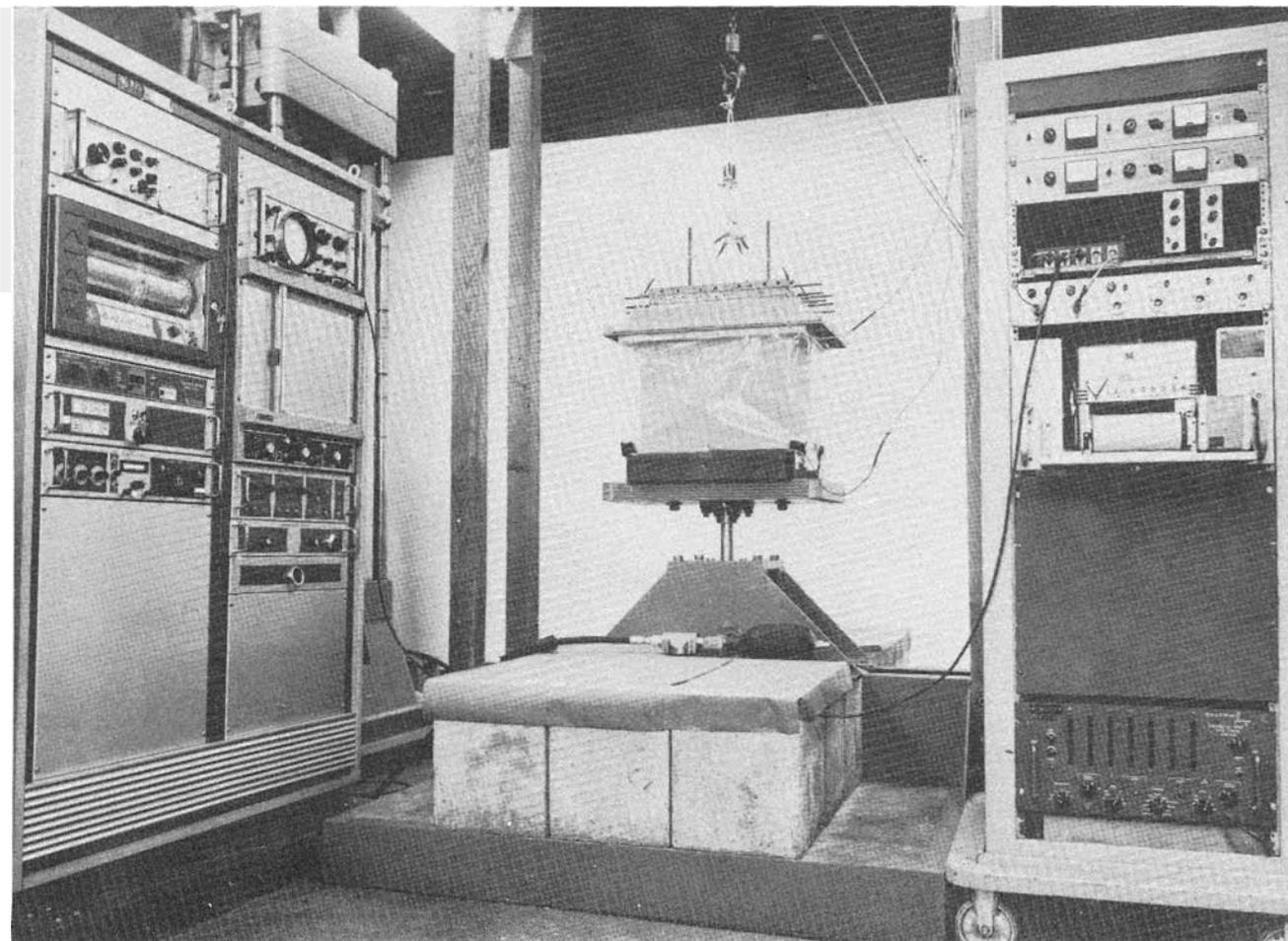

FREQUENCY RESPONSE, DAMPING, AND TRANSMISSIBILITY CHARACTERISTICS OF TOP-LOADED CORRUGATED CONTAINERS

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ABSTRACT

During transportation, stacked corrugated containers receive dynamic loading forces from vibration which, added to the weight of the stacked load, may cause failure of the lower containers in the stack. This study, in cooperation with the Fibre Box Association, experimentally verified that top-loaded containers are frequency-sensitive systems with resonant frequencies ranging from 8.4 to 18.2 cycles per second. Transmissibility amplification ratios as high as 6.7 were found at resonance, with a representative value of system damping of 0.115. These resonant responses occur within the range of frequencies likely to be present in common carrier transportation vehicles. Thus consideration must be given to these dynamic effects when selecting or designing corrugated containers.

FREQUENCY RESPONSE, DAMPING, AND TRANSMISSIBILITY CHARACTERISTICS OF TOP-LOADED CORRUGATED CONTAINERS

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INTRODUCTION

Corrugated fiberboard containers are used almost universally for the packaging and shipment of a wide variety of products. Corrugated containers are economical, light in weight, and use a minimum amount of material to perform their function because of their efficient structural design. However, they do not possess the same inherent compressive strength of heavier and more costly containers. This is an important criterion of performance because it represents a limiting factor in the utilization of corrugated fiberboard containers.

A shipping container necessarily experiences top-to-bottom compressive loading. In shipment and storage, containers must be stacked on top of each other, often to considerable heights, to efficiently utilize available space. Stacking thus imposes large compressive loads on the lower containers. In many instances the packaged item may be fragile or unable to support a load without suffering damage; therefore, the container itself must be capable of supporting most or all of the load. Thus containers must be designed and selected to withstand the rigors of the storage

and shipment environment. Storage stacking loads are static "dead" loads which do not vary as a function of time, and failure of containers in storage is related primarily to the creep characteristics of the container material. Relationships have been developed^{2,3} which can be used to predict the static load-carrying ability of containers, expressed as a ratio of the top-to-bottom compressive strength as determined in a testing machine.

During transportation the stacking loads may not be as great, because stack height is limited by the vehicle, but the Containers experience dynamic loading forces from the up-and-down vibration of the vehicle on its suspension system in addition to the existing stacking load.

A previous study,⁴ conducted at the Forest Products Laboratory in cooperation with the Fibre Box Association, investigated the effects of transportation vibration on the stacking loads which can be safely supported by corrugated containers. Vibration transmissibility theory was used to analyze a vertical stack of loaded containers as a simplified spring-mass system with vibration excitation applied to the base of the stack. This analysis showed that such a system was sensitive

¹Maintained at Madison, Wis., in cooperation with the University of Wisconsin.

²Kellicutt, K. Q., and Landt, E. F. Safe Stacking Life of Corrugated Boxes. *Fibre Containers* 36(9): 28-29. 1951.

³Moody, R. C., and Skidmore, K. E. How Dead Load, Downward Creep Influence Corrugated Box Design. *Package Engineering* 11(8): 75-81. 1966.

⁴Godshall, W. D. Effects of Vertical Dynamic Loading on Corrugated Fiberboard Containers. U.S.D.A. Forest Serv. Res. Pap. FPL 94. 1968. Forest Products Lab., Madison, Wis.

to certain frequencies. The dynamic loads imposed on the bottom container could be several times greater than the static stacking load when the frequency of the applied vibration was at or near the resonant frequency of the system. Calculations based on static repeated loading data obtained from a universal testing machine predicted that the resonant frequencies of the load and container would range between 10 and 17 Hz (Hertz) (cycles per sec.), depending on load and acceleration levels,

A survey of environmental data showed that the range of the most probable vibrations experienced in rail or truck shipment was from 3 to 20 Hz at acceleration levels between 0.1 and 0.8 G. Thus the range of probable vibration inputs included the range of resonant responses of the load-container systems; it was highly probable that the containers would receive large dynamic loading forces. This would cause an effective reduction in the actual stacked load a corrugated container could safely sustain.

An experimental test program verified the conclusions reached in the preceding analysis. The experimental procedure used was a highly simplified simulation of the transportation environment. Single empty containers were top-loaded with weights to various loads ranging between 10 and 70 percent of maximum compressive strength of the box, as determined by previous tests in a universal testing machine. Containers were then subjected to a sinusoidally varying vibration along the vertical axis at discrete frequencies ranging from 4 to 10 Hz at peak acceleration levels ranging from 0.2 to 0.8 G.

This experimental test program showed that the load-container systems were indeed frequency sensitive and that the load a container could sustain was drastically reduced as the exciting frequency of vibration approached the calculated resonant frequency of the system. A summary of the test results is shown in figure 1. The load, frequency, and vibration level conditions below and left of a frequency curve represent the safe loading conditions at which the containers experienced at least 21,600 cycles of loading without failure. The areas above and to the right of a frequency curve represent the loading conditions which produced failure,

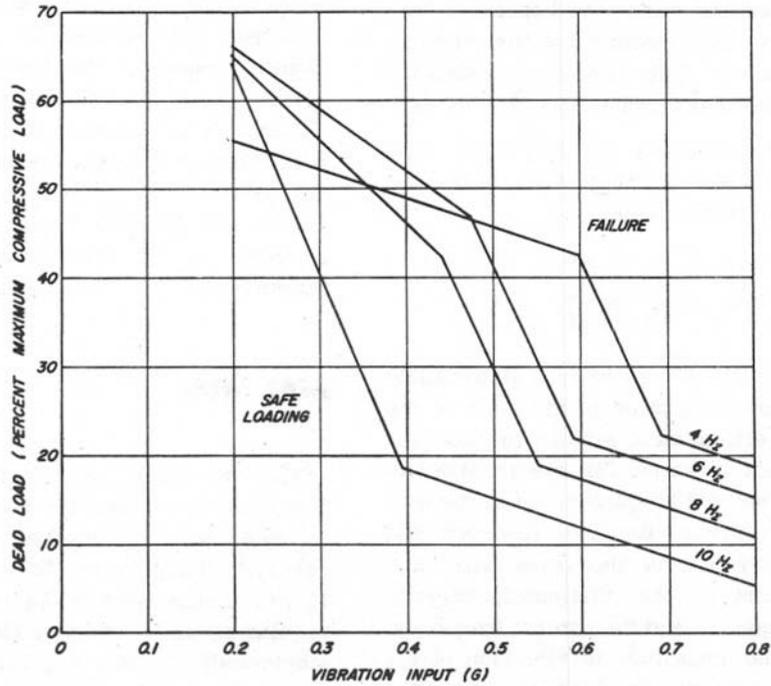
The capabilities of the mechanically driven vibrator limited the experimental data to frequencies of 10 Hz or less. This frequency range

includes a substantial portion of the range expected in transportation, but not the range of predicted resonant frequencies, from 10 to 17 Hz. This machine also restricted the experimental program to discrete fixed frequency and amplitude levels. Nevertheless, the experimental data conclusively supported the behavior predicted by the theoretical analysis, within the range of operation of the vibration machine.

Since the response characteristics of top-loaded containers depend on the frequency of the vibration input, the desirability and usefulness of more completely determining those characteristics of the load-container system over the entire range of frequency of possible interest is apparent. Therefore, a second phase of the study was initiated, again in cooperation with the Fibre Box Association, to determine those dynamic characteristics of top-loaded corrugated containers which are of the greatest value and interest. Those characteristics include the actual resonant frequency (f_r), the maximum transmissibility (T_r) (which occurs at resonance), and the effective damping coefficient (c/c_c) of the system. Knowledge of these dynamic characteristics will greatly aid in the design and selection of containers to withstand expected transportation environments. Additional objectives of this study were to further confirm the applicability of the analysis used, and to determine how accurately resonant frequencies could be calculated from machine compression test data using the previously developed method⁴ of analysis.

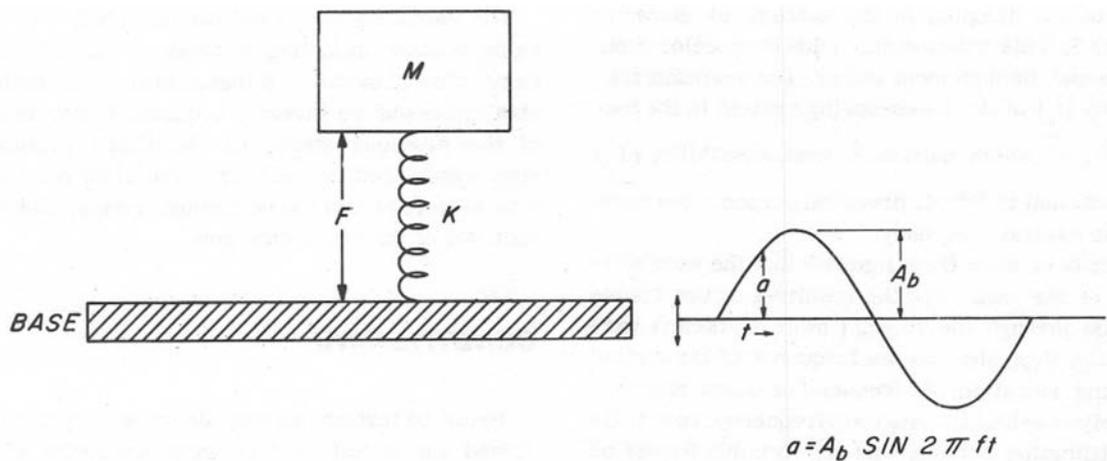
Analysis of Stacked Containers as a Mechanical System

For vibration analysis, any mechanical system can be represented by one or more spring-mass systems, as shown in figure 2. Actually a stack of loaded containers is a complicated system, but for the purposes of our analysis we have represented it as a single linear spring-mass system without damping, with the bottom container acting as an elastic spring, and the upper containers acting as a unified mass. The mass exerts a compressive force on the spring equal to its weight, which is the force due to the earth's gravitational acceleration, as stated by Newton's First Law: Force = Mass x Acceleration. This is a static loading, such as would be applied to a container in storage.



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Figure 1.--Summary plot of safe loading limits as determined in first phase of study. The area below and to the left of a frequency line defines the combination of loading conditions that will not cause failure of the container at that specified frequency. The area above and to the right of that frequency line defines the conditions under which the container will fail.



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Figure 2.--Simple spring-mass system with sinusoidal acceleration applied to the base; F = force transmitted through the spring.

If some additional force were applied and then suddenly removed, the mass would oscillate vertically about its static resting position at some particular frequency. This frequency of oscillation is called the natural frequency (f_n) of the system, and is determined by the magnitude of the mass (M) and the stiffness (K) of the spring, as expressed by the relationship:

$$f_n = \frac{1}{2\pi} \sqrt{\frac{K}{M}}$$

A different action occurs when a sinusoidally varying vibration is applied to the base of the system, in the vertical axis, similar to the actual transportation environment. The mass will oscillate, not at the natural frequency, but at the frequency of the forcing vibration. However, the amplitude of vibration of the mass will vary greatly, dependent on the relationship between the forcing frequency and the natural frequency. The ratio of the amplitude of vibration of the mass (A_m) to the amplitude of the forcing vibration (A_f) is called the transmissibility ratio (T) and is expressed by the relationship

$$T = \frac{A_m}{A_f}$$

The transmissibility is a function of the ratio of the forcing frequency to the natural frequency and of the damping in the system, as shown in figure 3. This relationship holds for acceleration, force and displacement ratios. The resonant frequency (f_r) of the mass-spring system is the frequency at which maximum transmissibility (T_r) occurs, and is for all practical purposes the same as the natural frequency.

It may be seen from figure 3 that the acceleration of the mass and the resulting forces transmitted through the spring (lower container) vary greatly, dependent on the frequency of the applied forcing vibration. At frequencies below approximately one-half the resonant frequency, very little amplification occurs, and the dynamic forces on the lower container are small.

At forcing frequencies between about 0.5 and 1.4 times the natural frequency, large amplifications of the vibratory input occur producing dynamic

forces, which added to the static loadweight, may cause compressive overloading and failure of the container. At frequencies above 1.4 times the natural frequency, the vibration input is attenuated and the dynamic loading forces become insignificant. The amount of amplification or attenuation is inversely related to the damping in the system.

Thus the dynamic characteristics of greatest interest are the resonant frequency, maximum transmissibility, and damping coefficient.

MATERIAL

The corrugated fiberboard containers used for this study were from the same production run as the containers evaluated in the first phase of this study.² They were 18-9/16 inches long by 12-3/8 inches wide by 14 inches high, and were regular slotted containers (RSC) of 200-pound test single-wall, C-flute. A glued manufacturer's joint was used, with a crushed tab inside the container. The containers were manufactured according to normal commercial practice except that a particularly high level of quality control was maintained to obtain uniformity throughout the production run of 2,100 containers.

SAMPLING PROCEDURE

The sampling plan used for this study was the same blocked sampling procedure adopted earlier? This plan assured that data and conclusions obtained could be directly compared with those of the previous study. Of the three replicate specimens used for each test condition, one each was selected from the beginning, center, and end sections of the production run.

CONDITIONING

Prior to testing, all containers were preconditioned and conditioned in accordance with ASTM Method D 641-49. The hocked-down containers were stored in a preconditioning chamber at 80° F. and 30 percent relative humidity. They were set up and closed by stapling, top and bottom while

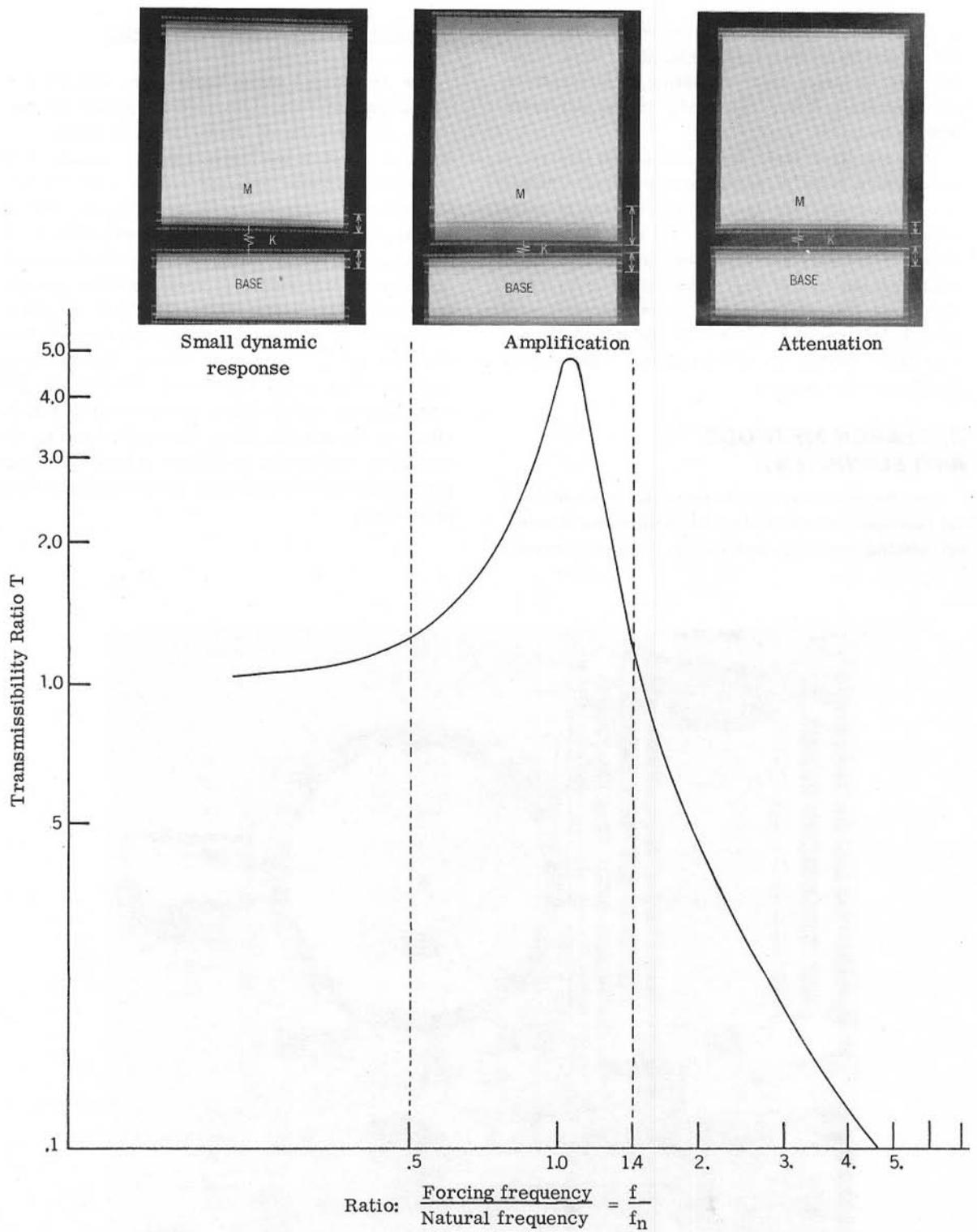


Figure 3.--Pictorial representation showing how the response of a single degree of freedom system varies as a function of the ratio of the forcing frequency to the natural frequency of the system.

in this condition. Then they were conditioned at 73° F. and 50 percent relative humidity for a minimum of 48 hours prior to testing. This condition was maintained for the tests in the universal testing machine.

The vibration tests were conducted in a room conditioned to 68° F. and 65 percent relative humidity. The containers were sealed in a bag of polyethylene film while at 73° F. and 50 percent relative humidity and were transported and tested while sealed in the bag. Since the vibration test durations ranged from only a few seconds to a maximum time of 1-1/2 hours, the moisture content of the containers did not change significantly during the test period.

RESEARCH METHODS AND EQUIPMENT

Two separate evaluation procedures were used for this study--repeated cyclic loading in a universal testing machine and sweep frequency tests.

Repeated Compressive Loading Tests

The repeated loading tests were conducted to obtain data that could be used to predict the resonant frequencies of load-container systems at various levels of loading and at several vibration amplitudes. Single containers were loaded in top-to-bottom compression in a universal testing machine (fig. 4) but were not loaded directly to failure, as in a conventional test. Instead, the container was first loaded to some selected percentage of the previously determined average maximum compressive strength. Second, the container was then cycled five times between the maximum and minimum loads that would result from this selected load weight being accelerated to a given vibration amplitude. After the cyclic loading, the container was loaded to failure. A load-deformation graph was recorded for the complete loading procedure.

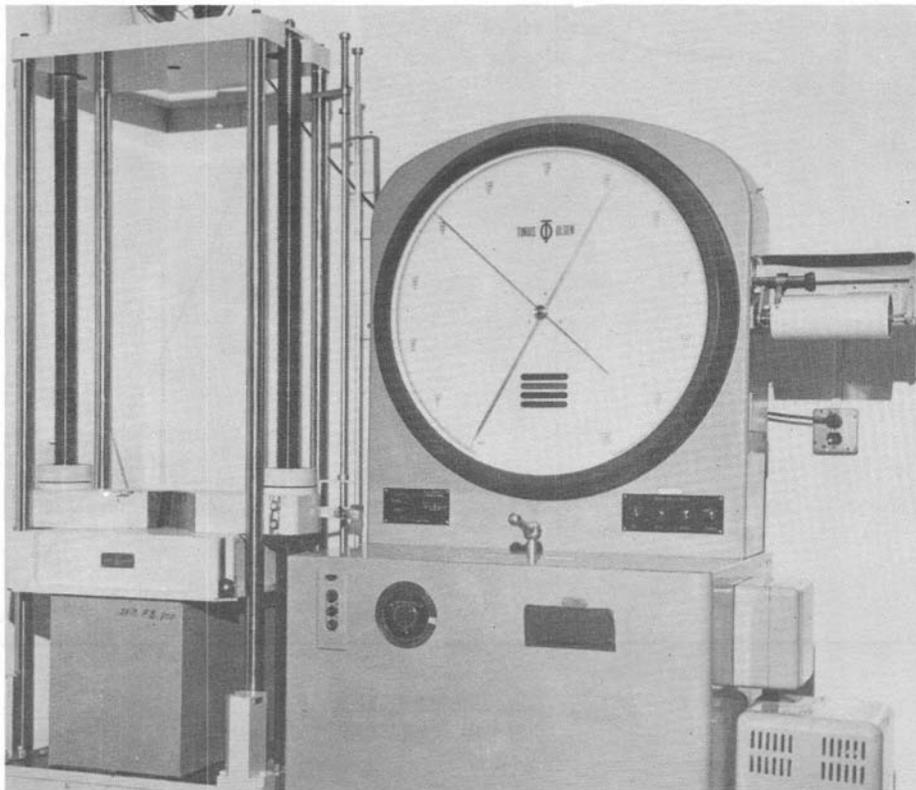


Figure 4.--Repeated loading top-to-bottom compression tests in a universal testing machine.

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Sweep Frequency Tests

Because of the limits of the vibration testing equipment then available, the frequency range investigated in the first phase⁴ did not fully cover the range of probable transportation vibration, nor did it cover the range of predicted resonant frequencies for the load-container systems. Therefore, to accomplish the objectives of this study, a vibration exciter was obtained which had sufficient frequency range and the capability to maintain constant acceleration levels while being swept continuously across the entire range of frequencies of interest. This was accomplished by converting an existing electrohydraulic structural testing system into a vibration exciter. A hydraulic piston actuator was mounted vertically on a large inertial mass, and a table large enough to hold the test containers was mounted on the end of the

piston. The experimental arrangement is shown in figure 5.

The vibration system employs a closed electrohydraulic servoloop. A hydraulic power supply provides the basic source of power for the double-acting piston. This hydraulic power is controlled by an electromagnetic servovalve which is driven by a servoamplifier. Programmed command signals are compared with a feedback signal from a displacement transducer attached to the piston; the difference of these two signals is applied to the servovalve, which regulates the flow of hydraulic fluid in the direction to reduce the difference signal to zero. While the system controls in the displacement mode, control signals are programmed in the acceleration mode and are then converted to equivalent displacement commands by double integration. This allows acceleration levels to be held at selected constant values

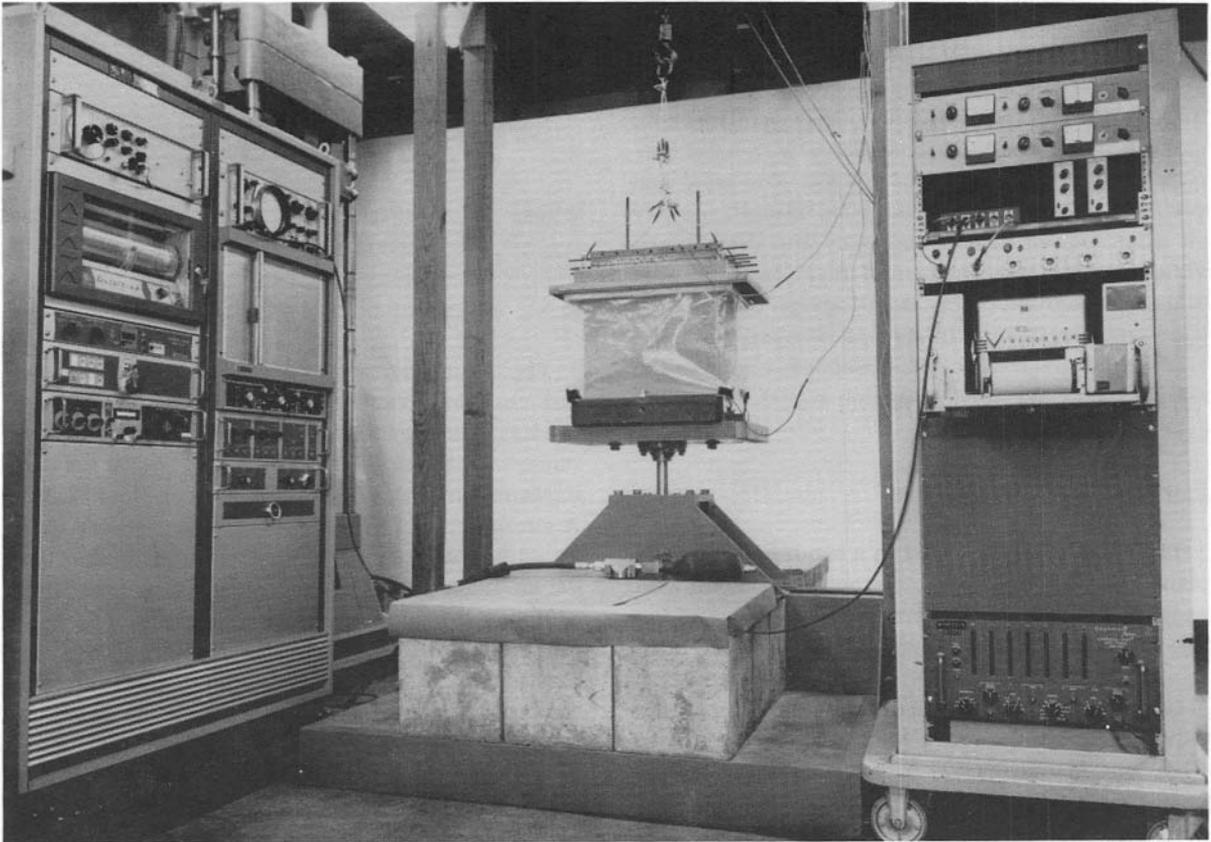


Figure 5.--Experimental arrangement for sweep frequency transmissibility tests of containers on electrohydraulic vibration machine.

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over any range of frequency sweep desired. A function generator provided sine-wave command signals which could be manually swept across the desired range of frequencies,

An electric hoist handled the loading weights and restrained the load upon failure of the container. A steel plate weighing approximately 500 pounds was fastened to the vibration table to provide a more suitable operating load for the hydraulic system and to improve the waveform of the vibration applied to the test containers.

Instrumentation

The vibration input applied to the container and the dynamic response of the load were measured by a separate instrumentation system, independent of the vibrator control system. Two identical strain gage accelerometers with a maximum range of 15 G and a flat frequency response from d.c. to 250 Hz were used. One accelerometer was mounted on the vibrating base, and the other was mounted on the load. The output signals from the accelerometers were amplified by wide-band d.c. amplifiers and recorded by an optical oscillograph, using fluid-damped galvanometers with a flat frequency response from 0 to 600 Hz. The input and response signals were recorded simultaneously as a function of time. Frequency was determined visually from the calibrated function generator dial and recorded on the oscillograph chart by the operator. Transmissibility was determined as the ratio of the magnitude of the response signal to the magnitude of the input vibration signal.

Dynamic Test Procedure

The container, enclosed in a vapor barrier, was placed on the vibration table and top-loaded with weights to some predetermined percentage of the top-to-bottom compressive strength of the container. The vibration exciter controls were set to produce a specified peak acceleration level, and vibration was begun at the low-frequency limit of 4 Hz. The frequency was then slowly swept upward through resonance until a frequency was reached at which substantial attenuation occurred. The frequency was then swept downward until resonance was reached and vibration was continued at the resonant frequency until failure of the con-

tainer occurred or until an arbitrary time limit of 1-1/2 hours was reached. Because in many instances the resonant frequency of the load-container system shifted with time, the input frequency was readjusted to maintain resonance. These sweep frequency tests were conducted at 23 different combinations of dead load and acceleration input levels to determine the response over the entire range of probable loading conditions.

ANALYSIS OF RESULTS

Repeated Compressive Loading Tests

The resonant frequencies of the top-loaded containers were computed from data obtained in the repeated compressive loading tests performed in a universal testing machine. The analysis was made by considering the loaded container to be a simple single-degree-of-freedom system, and computing the resonant frequency from the relationship

$$f_n = \frac{1}{2\pi} \sqrt{\frac{K}{M}}$$

Where f_n = natural frequency of vibration of the system

K = spring factor of the container

M = mass (W/g) of the load

The spring factor was determined from the load-deformation graph obtained from the repeated loading tests by drawing tangents to the loading curves at the dead load value. A more detailed explanation of this technique is given in FPL 94.⁴ A summary plot of the resonant frequencies computed is shown in figure 6. It can be seen that they range from 10.4 Hz to 17.8 Hz and are functions of both the input acceleration level and of the supported dead load. The resonant frequency decreases as the amplitude of the input vibration increases, and tends to increase as the dead load percentage increases. These results differ from those which might be expected of a linear system, and result because a corrugated container has nonlinear load-deformation characteristics. The container acts as a soft spring with small loads and stiffens as the load increases. The calculated spring factor varies from 850 pounds per inch at

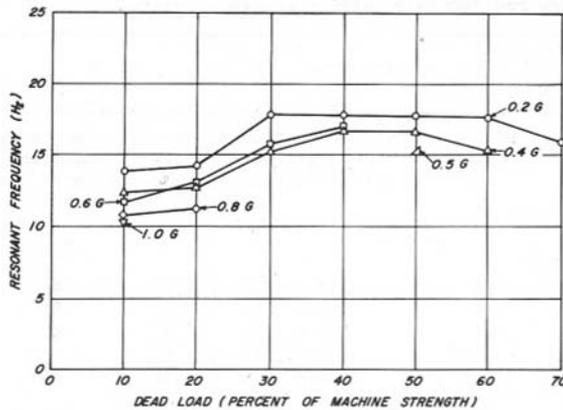


Figure 6.--Summary of resonant frequencies of loaded containers, computed from repeated compressive loading tests, for various vibration input levels. M 139 076

a load of 10 percent of maximum compressive strength to 14.030 pounds per inch at a load of TO percent of maximum compressive strength.

Resonant Frequencies

The resonant frequencies of the load-container systems as determined by the sweep frequency tests are shown in figure 7. Resonant frequencies could be determined at only 16 of the 23 test conditions. The other seven test conditions were severe enough that the container failed before the resonant frequency was reached.

A comparison of the predicted and experimentally determined resonant frequencies can be obtained from table 1, which summarizes results. Reasonably good agreement is obtained, with the predicted values running slightly above the initial experimental values. However, for test conditions that did not severely load the container, a gradual increase of the resonant frequency with time was noted. This appeared to be due to working and stiffening of the corners of the boxes, causing the spring factor or stiffness of the box to increase. For most of these instances, the predicted resonant frequency fell between the initial and final values of the actual resonant frequency. Thus the simplified analysis, using spring factors easily obtained from the repeated loading tests, is capable of accurately predicting the resonant frequencies of top-loaded corrugated containers.

The resonant frequency could not be determined

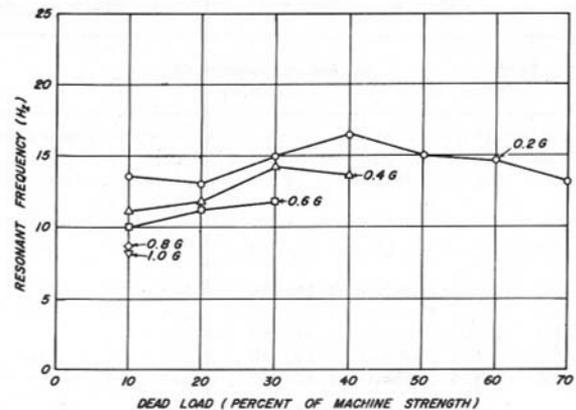


Figure 7.--Summary of resonant frequencies of loaded containers as determined experimentally by sweep frequency tests for various vibration input levels. M 139 074

at some of the scheduled test conditions, because the severity of the test condition caused failure of the container before resonance was reached, as the input frequency was slowly swept upward toward the resonant frequency. These failure conditions were at the same levels as previously determined in the first phase of this study¹ with fixed vibration frequencies.

Nonlinear systems are known to exhibit frequency instability, where the resonant amplitude will shift up or down, depending on whether resonance is approached from a higher or from a lower frequency. This phenomenon was noted only a few times in these tests, and the change in amplitude was not large. However, at a number of the different test conditions, a low-level resonant response was noted at approximately one-half the resonant frequency. Within the limits of the frequency range employed in these tests, no multi-order harmonic responses were noted,

Transmissibility

The maximum transmissibility found at each test condition is given in table 1. Values range from 1.65 to 6.70, but do not necessarily represent the transmissibility at resonance because, at many test conditions, the container failed before resonance was reached. Because of the high transmissibilities, peak dynamic loads sufficient to cause the containers to fail were reached even though the static load weight was relatively small.

Table 1.--Summary of frequency response, damping, and transmissibility characteristics¹

Test condition		Repeated compressive loading				Dynamic Data					
Static load input	Vibration weight	Static load	Spring factor K	Calculated natural frequency f _n	Experimental resonant ₂ frequency f _r	Ratio f _n /f _r	Cycles to failure ³	Maximum transmissibility (c/c _c)	Damping ratio	Calculated maximum dynamic load	
Pct.	G	Lb.	Lb./in.	Hz	Hz					Pct.	
70	+0.2	543.0	14,030	15.9	13.2	1.20	220	1.76	0.28	94	
60	+2	466.0	15,330	17.6	14.7	1.19	570	2.39	.21	89	
60	+4	466.0	11,210	15.3	--	--	140	1.85	.27	104	
50	+2	388.0	12,370	17.6	15.0	1.17	620	2.86	.17	78	
50	+4	388.0	11,030	16.7	--	--	210	1.79	.28	86	
50	+5	388.0	9,270	15.3	--	--	135	1.65	.30	91	
40	+2	311.0	9,980	17.8	16.5	1.08	>60,000	2.25	.22	58	
40	+4	311.0	8,200	16.6	13.6	1.22	560	3.69	.14	99	
40	+6	311.0	8,870	16.7	--	--	180	2.45	.20	99	
30	+2	232.0	7,520	17.8	15.0 + 18.2	1.19 + 0.98	>60,000	3.82	.13	53	
30	+4	232.0	5,400	15.2	14.3	1.06	610	5.41	.09	95	
30	+6	232.0	5,900	15.8	11.9	1.33	470	3.71	.13	96	
30	+7	232.0	5,970	15.9	--	--	210	2.20	.23	76	
20	+2	155.0	3,150	14.1	13.0 + 16.8	1.08 + 0.84	>60,000	5.83	.09	43	
20	+4	155.0	2,530	12.6	11.8 + 15.5	1.07 + 0.81	>60,000	4.79	.10	58	
20	+6	155.0	2,550	12.7	11.3	1.12	460	4.37	.11	72	
20	+8	155.0	2,010	11.3	--	--	170	4.67	.11	95	
20	+9	155.0	2,030	11.3	--	--	140	3.80	.13	88	
10	+2	77.5	1,490	13.7	13.6	1.01	>52,450	3.90	.13	18	
10	+4	77.5	1,180	12.2	11.1 + 12.6	1.10 + 0.97	>44,660	6.60	.08	36	
10	+6	77.5	1,090	11.7	10.0 + 11.3	1.17 + 1.04	>39,140	5.17	.10	41	
10	+8	77.5	890	10.8	8.5 + 12.2	1.27 + 0.89	>50,340	6.70	.07	63	
10	+1.0	77.5	850	10.4	8.4 + 9.2	1.24 + 1.13	1,800	5.37	.09	64	

¹All values given are averages for 3 containers.

²In some instances, the resonant frequency shifted during the test period as indicated.

³Tests were terminated at either 60,000 cycles or 90 minutes, whichever occurred first.

This may be seen from the calculated maximum dynamic loading forces given in table 1. These values were calculated using the relationship

$$F_m = W(1 + G_i T_m)$$

where F_m = maximum dynamic loading force

W = static load weight

G_i = input acceleration level G's

T_m = maximum transmissibility ratio

and F_m and W are expressed as a percentage of the machine compressive strength of the container.

The containers failed at all test conditions where the maximum dynamic load exceeded

70 percent of the machine compressive strength of the container, and did not fail (with one exception) when the maximum dynamic load was less than 70 percent. The exception occurred when the input acceleration level was 1.0 G, which allowed the container to leave the vibrator bed and impact against it. Previous studies^{2,3} on static stacking strength of corrugated containers have shown that containers can sustain about 70 percent of their machine compressive strength for periods of a day or two.

Effective System Damping

The transmissibility of a vibrating spring-mass system is a function of the damping characteristics of the system. An undamped spring-mass system theoretically would have a transmissibi-

lity of infinity at resonance. The higher the damping in a system, the lower the resonant response will be. Since the actual transmissibilities have been determined experimentally, approximate values of equivalent system damping can be determined, using the relationships

$$\frac{1}{T} = \eta = 2\left(\frac{c}{c_c}\right) \quad \text{Or} \quad \left(\frac{c}{c_c}\right) = \frac{1}{2T}$$

where η = damping factor

$\left(\frac{c}{c_c}\right)$ = fraction of critical damping, or damping ratio

T = transmissibility ratio, or amplification factor.

These relationships are strictly valid only for linear single-degree-of-freedom systems with linear viscous damping, but nevertheless are sufficiently applicable to nonlinear systems to pr—

vide a useful estimate of the damping characteristics of corrugated fiberboard containers.

Damping ratios for each of the 23 load-container systems evaluated are given in table 1. Values range from 0.07 to 0.30. Highest values were obtained from containers which were dynamically overloaded and failed in a relatively low number of cycles. The transmissibilities recorded for these containers were not obtained at resonance, because failure occurred before resonance was reached; thus the highest range of damping ratios cannot be considered typical of corrugated containers. The average of the damping ratios for all of the containers which did not fail during the scheduled test period is 0.115 and this value can be considered more representative. This estimate of the damping ratio of corrugated containers is useful for prediction of the response of load-container systems at other service conditions.

CONCLUSIONS

This study has experimentally proven that top-loaded containers may be subjected to large dynamic loading forces, in addition to the static stacking loads, when they are subjected to vibration, such as in transportation. The high dynamic loads occur when the input vibration is at or near the resonant frequency of the load-container system. The resonant frequencies for the containers evaluated ranged from 8.4 to 18.2 Hz at various loadings. Transmissibilities (amplification factors) as high as 6.3 were found at resonance.

It was confirmed that simple linear vibration theory can be utilized with data from compression tests, which can be easily performed in universal testing machines, to satisfactorily predict resonant frequencies of load-container systems.

Because the resonant frequencies of the load-container systems fall within the range of probable transportation vibration frequencies, stacking load weights must be reduced in shipment, or stronger containers must be used, to avoid container failure from the large dynamic loading forces which may occur.