BUCKLING COEFFICIENTS FOR SIMPLY SUPPORTED, FLAT, RECTANGULAR SANDWICH PANELS UNDER BIAXIAL COMPRESSION

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ABSTRACT

Presents the derivation of formulas for the buckling coefficients of simply supported, flat, rectangular sandwich panels under edgewise (in-plane) biaxial compression loads. Values of the coefficients are presented in graphs for sandwich with isotropic and orthotropic facings on isotropic and orthotropic (honeycomb) cores.
BUCKLING COEFFICIENTS
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INTRODUCTION

Structural sandwich components comprising thin, stiff facings bonded to both surfaces of a thick, lightweight core can provide highly efficient constructions for carrying various loads. It is essential that such sandwich carry in-plane or edgewise loads without buckling and that criteria for buckling account for the effects of low transverse shear rigidity of the sandwich core. Previous work for buckling of sandwich panels under uniaxial compression utilized an approximate energy method which is exact for panels with simply supported edges. The derivation here will also use an energy method to obtain a solution for the buckling coefficients of panels under biaxial compression.

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2 The author wishes to acknowledge the work of Gordon H. Stevens in checking the mathematical manipulations and Fred Rattner in computing values of the buckling coefficients.
3 Maintained at Madison, Wis., in cooperation with the University of Wisconsin.
**NOTATION**

a, b - Length of panel edge (see fig. 1)

c - Subscript denoting core

D - Bending stiffness or twisting stiffness, depending on subscripts (For sandwich with thin, equal facings, 
\[ D_x = \frac{G_{xy}}{2\lambda}, \quad D_y = \frac{E_{th}^2}{2\lambda}, \]

E - Young's modulus of elasticity of facing

G - Modulus of rigidity; for facings, G\(_{xy}\) is associated with shear distortion in the plane of the facing; for cores, G\(_{cz}\) and G\(_{cyz}\) are associated with shear distortion in the xz and yz planes, respectively.

H - Energy expressions

h - Distance between facing centroids

K - Buckling coefficient

m - Number of half waves in direction of x axis

n - Number of half waves in direction of y axis

R - Ratio core shear moduli; 
\[ R = \frac{G_{cz}}{G_{cyz}} \]

S - Shear load normal to surface of panel, per unit length of edge

t - Facing thickness

u - Transverse shear stiffness, 
\[ U_x = hG_{cz}, \quad U_y = hG_{cyz} \]

V - Parameter relating shear and bending stiffness

W - Special parameter relating shear and bending stiffness for sandwich with corrugated core

w - Deflection normal to sandwich panel

x - Axis; subscript denoting parallel to x axis (see fig. 1)

y - Axis; subscript denoting parallel to y axis (see fig. 1)

z - Axis; subscript denoting parallel to z axis (see fig. 1)

\( \mu \) - Facing Poisson's ratio; with subscripts \( \mu \) is the ratio of contraction in the y direction to extension in the x direction due to a tensile stress in the x direction

\[ \lambda = 1 - \mu_{xy} \]

**Figure 1. Notation**

M 137 511

\[ Z \text{ AXIS IS NORMAL TO PLANE OF SANDWICH} \]
DERIVATION OF BUCKLING LOAD FORMULA

The buckling load formula is derived by equating the strain energy due to shear and bending of the panel to the potential energy of the external loads after assumption of a suitable deflected surface of the panel. The resultant expression is minimized with respect to parameters defining core shear distortion to eventually obtain buckling coefficients.

The strain energy due to shear and bending of the panel is given by Libove and Batdorf\(^6\) as:

\[
H_1 = \frac{1}{2} \int_0^b \int_0^a \left\{ D_x \left( \frac{\partial^2 w}{\partial x^2} - \frac{S_x}{U_x} \right)^2 + 2\mu y x \left( \frac{\partial^2 w}{\partial x \partial y} - \frac{S_y}{U_y} \right) \left( \frac{\partial^2 w}{\partial x \partial y} - \frac{S_y}{U_y} \right) + D_y \left( \frac{\partial^2 w}{\partial y^2} - \frac{S_y}{U_y} \right)^2 \right\} \, dx \, dy
\]

\[
+ \frac{S_x^2}{U_x} + \frac{S_y^2}{U_y}
\]

(1)

Let \( g_x = \frac{\partial^2 w}{\partial x^2} - \frac{S_x}{U_x} \) and \( g_y = \frac{\partial^2 w}{\partial y^2} - \frac{S_y}{U_y} \)

then \( \frac{S_x^2}{U_x} = (1-g_x)^2 \frac{U_x}{\partial_x} \left( \frac{\partial w}{\partial x} \right)^2 \) and \( \frac{S_y^2}{U_y} = (1-g_y)^2 \frac{U_y}{\partial_y} \left( \frac{\partial w}{\partial y} \right)^2 \)

and substitution of these into (1) results in

\[
H_1 = \frac{1}{2} \int_0^b \int_0^a \left\{ D_x \left( \frac{\partial^2 w}{\partial x^2} \right)^2 + 2\mu y x \left( g_x g_y \frac{\partial^2 w}{\partial x \partial y} + \frac{\partial^2 w}{\partial x \partial y} \right)^2 + D_y \left( g_x g_y \right)^2 \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 \right\} \, dx \, dy
\]

\[
+ \mu y x \left( 1-g_x \right)^2 \left( \frac{\partial w}{\partial x} \right)^2 + \frac{U_x}{\partial_x} \left( 1-g_y \right)^2 \left( \frac{\partial w}{\partial y} \right)^2 \left( \frac{\partial w}{\partial y} \right)^2 \right\} \, dx \, dy
\]

(2)

For a very rigid core $U \to \infty$ and $g \to 1$ and expression (2) reduces to the usual one for plates.

The potential energy of the external loads is given by Libove and Batdorf\textsuperscript{6}

\[
H_2 = \frac{1}{2} \int \left[ \int_{0}^{b} \left[ N_x \left( \frac{\partial w}{\partial x} \right)^2 + N_y \left( \frac{\partial w}{\partial y} \right)^2 \right] dx dy \right]
\]

The deflection of a sandwich panel with simply supported edges is assumed to be

\[
v = v_0 \sin \frac{na}{a} \sin \frac{ny}{b}
\]

After substituting (4) into (1) and (2) and equating (1) and (2) the following expression is obtained after some manipulation:

\[
N_x = \pi^2 K \frac{\left( \frac{D}{D_y} \right)^{1/2}}{b^2}
\]

where the buckling coefficient $K$ is given by

\[
K = \frac{n^2}{1 + \left( \frac{na}{mb} \right)^2} \frac{\left( \frac{1}{D_x} \right)^{1/2}}{N_x} \left[ \frac{1}{D_y} \left( \frac{mb}{na} \right)^2 g_x^2 + 2 \frac{1}{D_{xy}} g_x g_y + \frac{1}{D_{xy}} \left( g_x + g_y \right)^2 \right] \]

\[
+ \left( \frac{1}{D_x} \right)^{1/2} \left( \frac{mb}{na} \right)^2 g_y^2 + \frac{1}{2} \left( 1 - g_x \right)^2 + \frac{1}{2} \left( 1 - g_y \right)^{2} \left( \frac{a}{b} \right)^2 \]

where

\[
v = \frac{\pi^2 \left( \frac{D}{D_y} \right)^{1/2}}{b^2 U_x}
\]

and

\[
R = \frac{U_x}{U_y}
\]
Formula (6) is then minimized with respect to $g_x$ and $g_y$ by equating partial derivatives to zero and solving the two resultant equations simultaneously for $g_x$ and $g_y$. The resultant formulas for $g_x$ and $g_y$ are given by

\[
g_x = \frac{1}{\beta} \left[ \left( \frac{na}{mb} \right)^2 \left( \frac{D_x}{D_y} \right) + \frac{D_{xy}}{(D_x D_y)^{1/2}} \right] m^2 R V + \left( \frac{a}{b} \right)^2 \left[ \mu \frac{D_x}{D_y} \right] n^2 \frac{(a/b)^2}{(d/b)^2} \right]
\]

\[
g_y = \frac{1}{\beta} \left[ \left( \frac{mb}{na} \right)^2 \left( \frac{D_x}{D_y} \right) + \frac{D_{xy}}{(D_x D_y)^{1/2}} \right] n^2 R V + \left( \frac{a}{b} \right)^2 \left[ \mu \frac{D_x}{D_y} \right] m^2 \frac{(a/b)^2}{(d/b)^2} \right]
\]

where

\[
\beta = \left[ \left( \frac{mb}{na} \right)^2 \left( \frac{D_x}{D_y} \right) + \frac{D_{xy}}{(D_x D_y)^{1/2}} \right] n^2 R V + \left[ \left( \frac{na}{mb} \right)^2 \left( \frac{D_x}{D_y} \right) + \frac{D_{xy}}{(D_x D_y)^{1/2}} \right] m^2 R V + \left( \frac{a}{b} \right)^2 \right]
\]

The critical buckling value of $N_x(N_{xc})$ is then obtained from (5) after substitution of (6), (7), and (8) and minimizing for integral values of the half waves $m$ and $n$ for various values of the ratios $N_y/N_x$, $a/b$, property values, and $V$.

**COMPUTATION OF BUCKLING COEFFICIENTS**

Buckling coefficients, $K_x$, were computed for sandwich with isotropic and orthotropic facings having cores of isotropic, orthotropic, or of corrugated material,

For isotropic facings it was assumed that $D_x/D_y = 1$, $\mu_{xy} = 0.25$, and $D_{xy}/(D_x D_y)^{1/2} = 0.375$.

For orthotropic facings it was assumed that $D_x/D_y = 1$, $\mu_{xy} = 0.2$, and $D_{xy}/(D_x D_y)^{1/2} = 0.21$.

For cores it was assumed that $R = 1$ for isotropic core and $R = 0.4$ or 2.5 for orthotropic honeycomb cores with hexagonal cells. Computation for sandwich with corrugated cores were carried out for $R = 0.01$ or 100 thus simulating the usual assumption for this core that it is infinitely stiff in shear parallel to corrugation flutes but has a finite shear stiffness perpendicular to the flutes. For flutes parallel to the $X$ axis the core shear parameter $V$ is replaced by

$$W = \frac{\pi^2 (D_x D_y)^{1/2}}{b^2 u_y}$$  \hspace{1cm} (10)

Curves of buckling coefficients for sandwich with isotropic facings and isotropic and orthotropic cores are given in figures 2, 3, 4, and 5. Coefficients for sandwich with orthotropic facings and isotropic and orthotropic cores are given in figures 6, 7, 8, and 9. Coefficients for sandwich with corrugated cores and isotropic and orthotropic facings are given in figures 10, 11, 12, and 13. These curves show the buckling coefficient $K$ as ordinate for various panel aspect ratios $(a/b)$ as abscissa. The abscissa scale is inverted at $(a/b = 1)$ so that the curves can be extended to the infinitely long panel $(a/b = \infty$ or $b/a = 0)$.

Computation of the buckling coefficients showed that minimum values were always obtained for $n = 1$ and for $m = 1, 2, 3$ for small values of $N_y/N_x$. For larger values of $N_y/N_x$ the value of $m = 1$ produced minimum buckling coefficients. The curves for which $m = 1, 2, 3$ produced minimums had the familiar cusped appearance indicated in the top dotted lines on figure 2. These cusps are omitted on the other curves and the envelope or asymptotes are shown for use in design.

**APPROXIMATE VALUES**

For square isotropic sandwich panels the buckling coefficient can be computed by the following approximate formula:

$$K = \frac{4}{(1+V)^2 \left(1+\frac{N_y}{N_x}\right)}$$  \hspace{1cm} (11)

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8These ratios were experimentally determined for aluminum honeycomb core in Forest Products Lab. Report 1849, "Mechanical Properties of Aluminum Honeycomb Cores," by E. W. Kuenzi.
Figure 2. Buckling coefficients for simply supported sandwich panels in biaxial compression. $V = 0$, $W = 0$. 

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Figure 3. Buckling coefficients for simply supported sandwich panels in biaxial compression. $V = 0.1$
Figure 4. Buckling coefficients for simply supported sandwich panels in biaxial compression. $V = 0.2$
Figure 5. Buckling coefficients for simply supported sandwich panels in biaxial compression. $V = 0.4$
Figure 6. Buckling coefficients for simply supported sandwich panels in biaxial compression. \( V = 0, \ W = 0 \)
Figure 7. Buckling coefficients for simply supported sandwich panels in biaxial compression. $V = 0.1$
Figure 8. Buckling coefficients for simply supported sandwich panels in biaxial compression. \( V = 0.2 \)
Figure 9. Buckling coefficients for simply supported sandwich panels in biaxial compression. $V = 0.4$

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Figure 10. Buckling coefficients for simply supported sandwich panels in biaxial compression.
Figure 11. Buckling coefficients for simply supported sandwich panels in biaxial compression.
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Figure 12. Buckling coefficients for simply supported sandwich panels in biaxial compression.
M 137 503
Figure 13. Buckling coefficients for simply supported sandwich panels in biaxial compression.