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Simulations of Nonparametric Analyses of Predictor Sort (Matched Specimens) Data

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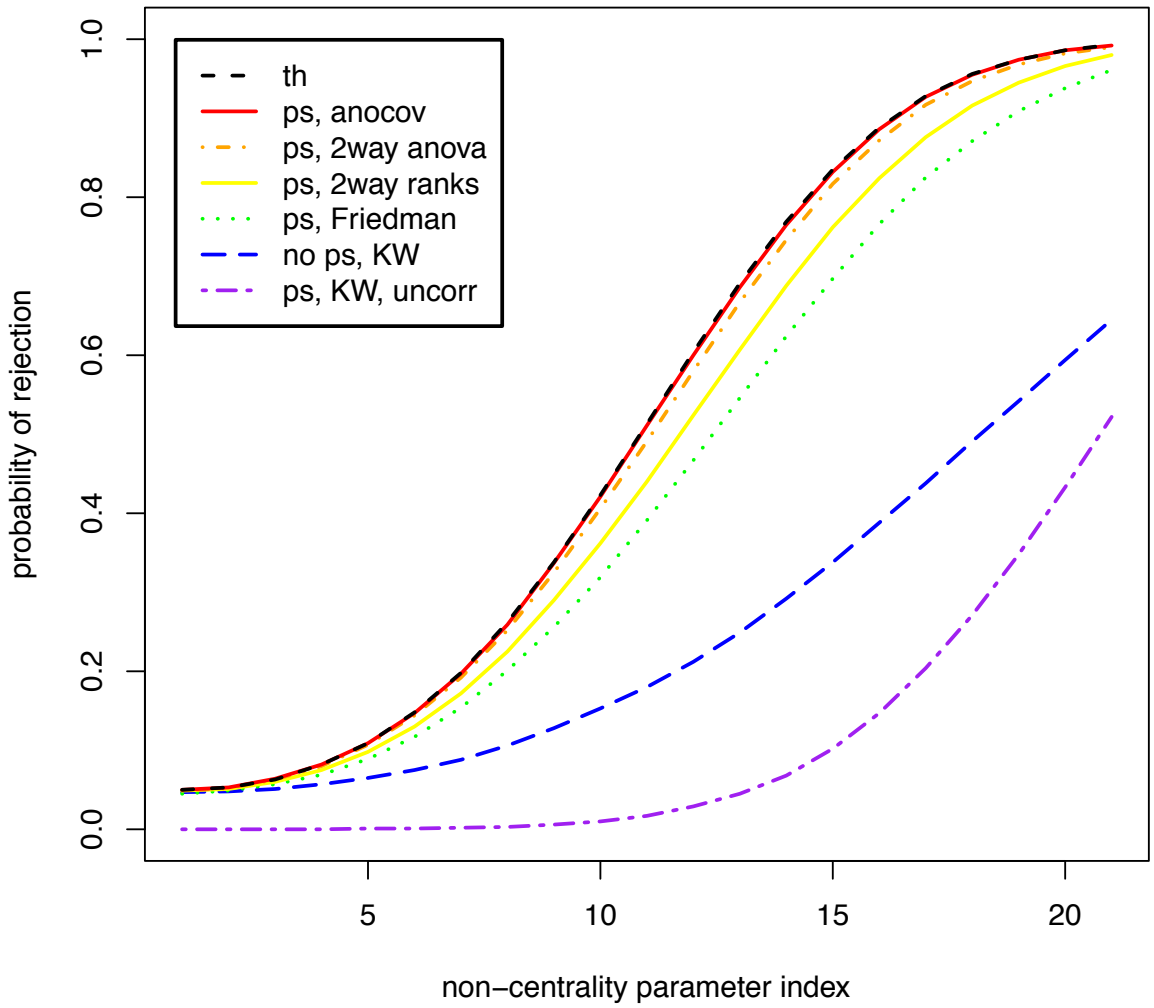

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Abstract

A type of blocked experiment has the potential of being poorly designed and analyzed. Several papers by Verrill and co-workers referred to such an experiment as a "predictor sort" experiment. It has also been referred to in other papers and textbooks as "artificial pairing" and "matched pair" or "matched subjects" design. The associated design process is also sometimes described as "forming blocks via a concomitant variable." In a wood research context, the response in such an experiment might be lumber strength after a treatment, and the predictor used to form blocks would be some combination of lumber stiffness, knot size, and slope of grain (all of which can be measured nondestructively prior to specimen allocation). Improperly designed or analyzed, predictor sort experiments can be associated with incorrect power calculations, inappropriate sample sizes, incorrect tests of hypotheses, and incorrect confidence intervals. In a 2017 paper, Verrill and Kretschmann reviewed the main results in the literature, added a section on multiple comparisons, and presented the results from power and confidence interval coverage simulations that emphasized the importance of proper design and analysis of predictor sort experiments; that work was based on the assumption that the predictor and the response have a bivariate normal distribution. This paper, which can be thought of as a companion to the 2017 paper, constitutes a nontheoretical, simulation-based first look at the effect of predictor sort allocation on nonparametric hypothesis tests (Kruskal–Wallis, one-way on ranks, one-way on Van der Waerden ranks, two-way on ranks, and Friedman tests) and nonparametric confidence bounds on quantiles. It also considers situations in which a bivariate normal assumption for the predictor–response pair does not hold.

Keywords: Predictor sort sampling, artificial pairing, matched pairs, matched subjects, concomitant variable, blocked ANOVA, analysis of covariance, nonparametrics, confidence bounds on quantiles

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Simulations of Nonparametric Analyses of Predictor Sort (Matched Specimens) Data

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1 Introduction

A type of blocked experiment has the potential of being poorly designed and/or analyzed. Verrill (1993, 1999), Verrill et al. (2004), and Verrill and Kretschmann (2017) referred to such an experiment as a "predictor sort" experiment. David and Gunnink (1997) spoke of "artificial pairing." In textbooks it is sometimes referred to as a "matched pair" or "matched subjects" design. The associated design process is also sometimes described as "forming blocks via a concomitant variable" (see, for example, Cox (1957)). In a wood research context, the response in such an experiment might be lumber strength after a treatment, and the predictor used to form blocks would be some combination of lumber stiffness, knot size, and slope of grain (all of which can be measured nondestructively prior to specimen allocation).

Improperly designed and/or analyzed, predictor sort experiments can be associated with incorrect power calculations, inappropriate sample sizes, incorrect tests of hypotheses, and incorrect confidence intervals. Verrill and Kretschmann (2017) reviewed the main results in the literature, added a section on multiple comparisons, and presented the results from power and confidence interval coverage simulations that emphasized the importance of the proper design and analysis of predictor sort experiments.

This work in Verrill and Kretschmann (2017) was based on the assumption that the predictor and the response have a bivariate normal distribution.

The current paper can be thought of as a companion piece to Verrill and Kretschmann (2017). It constitutes a nontheoretical, simulation-based first look at the effect of predictor sort allocation on *nonparametric* hypothesis tests (Kruskal–Wallis, one-way on ranks, one-way on Van der Waerden ranks, two-way on ranks, and Friedman tests) and nonparametric confidence bounds on quantiles. It also considers situations in which a bivariate normal assumption for the predictor–response pair does not hold.

2 Background

In the Introduction we loosely referenced "predictor sort sampling" and stated that the associated design process is also sometimes described as "forming blocks via a concomitant variable." We now provide additional details.

In a wood research context, Warren and Madsen (1977) described the specimen allocation procedure as follows:

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> One can take steps, however, to ensure that the inherent [initial] strength distributions of test and control samples are reasonably equivalent. Indeed, failure to do so can only throw doubt on the results.
>
> Specifically, then, all the boards in the experiment are ordered from weakest to strongest as nearly as can be judged from their moduli of elasticity, knot size, and slope of grain. To divide the material into J equivalent groups the first J boards, after ordering, are taken and randomly allocated one to each group. This is repeated with the second, third, fourth, etc., sets of J boards. The strength distributions of the resulting groups should then be essentially the same. (Warren and Madsen, 1977, Number 3, page 46)

Here the response is lumber strength after a treatment, and the predictor used to form blocks (of size J) would be some combination of lumber stiffness, knot size, and slope of grain (all of which can be measured nondestructively prior to specimen allocation).

In an agricultural context, the predictor variable might be, for example, animal age, initial animal weight, or plot fertility in a previous trial. In a behavioral or educational context, the predictor might be, for example, IQ or performance on a pre-test.

In this paper we refer to this type of design as a "predictor sort" design (because we sort specimens on the basis of a predictor that is correlated with the response, and then form blocks via collections of specimens with adjacent predictor values). To properly design experiments and analyze data, statisticians and experimenters *must* be able to recognize cases in which their specimens have been allocated by such a process.

Verrill (1999) cited discussions of this type of experiment in example 3.3 of Cox (1958), section 8.2 of Steel and Torrie (1960), section 5.1 of Kirk (1968), section 13.17 of Finney (1972), example 11.3 of Ostle and Mensing (1975), chapter 6 of Myers (1979), and example 6.13.1 of Snedecor and Cochran (1989). A more recent sampling of statistical texts found such experiments discussed in Kerlinger and Lee (1999), van Zutphen et al. (2001), example 5.1 of Toutenburg (2002), section 4.3 of Ruxton and Colegrave (2006), problem 3.8 of Casella (2008), Cozby and Bates (2011), Tuckman and Harper (2012), and section 8.1 of Kirk (2013).

Among the variables suggested as predictors to be used to form blocks were age, reaction time, initial weight, concentration of blood constituent, degree of disease, time since college, IQ, scores on a cognitive ability measure, grade point average, prior school performance, and pretest achievement.

As noted in Section 1, improperly designed and/or analyzed, predictor sort experiments can be associated with incorrect power calculations, inappropriate sample sizes, incorrect tests of hypotheses, and incorrect confidence intervals. These improper designs and analyses stem from an error in the implicit underlying probability model. In general, authors act as if the appropriate model in a "matched" one-factor case is

$$Y_{ij} = \mu_{\cdot j} + \mu_{i\cdot} + \sigma_Y \times \epsilon_{ij} \tag{1}$$

where the Y_{ij} are responses, $\mu_{\cdot 1}, \ldots, \mu_{\cdot J}$ denote the treatment effects, $\mu_{1\cdot}, \ldots, \mu_{I\cdot}$ denote the block effects, and the ϵ's are i.i.d. N(0,1)'s.

In fact, however, under a bivariate normal assumption, the correct probability model after a predictor sort with J treatments and I "blocks" is

$$Y_{ij} = \mu_{\cdot j} + \sigma_Y \left(\rho \left(X_{k(i,j),n} - \mu_X \right) / \sigma_X + \sqrt{1-\rho^2} P_{ij} \right) \tag{2}$$

where $\mu_1, \ldots, \mu_J$ denote the treatment effects; for a fixed i, the J $k(i,j)$'s are a randomization of the elements of $\{(i-1)J+1, \ldots, iJ\}$; $X_{l,n}$ denotes the lth order statistic among the X's (the predictors); and the P_{ij}'s are i.i.d. N(0,1)'s that are independent of the X's.

This follows from the fact that if the predictor X and the response Y have a bivariate normal distribution with correlation ρ and we do *not* use the predictor to form blocks, then we have

$$Y = \mu_{\cdot j} + \sigma_Y \left(\rho \left(X - \mu_X \right) / \sigma_X + \sqrt{1 - \rho^2} P \right) \tag{3}$$

where X and P are statistically independent normals, X has mean μ_X and standard deviation σ_X, and P is a N(0,1). Sorting the X, Y pairs by the X value and then forming blocks via J adjacent (and then randomized) X's then leads to (2).

The differences between models (1) and (2) (and, in particular, the fact that $X_{k(i,j_1),n} - X_{k(i,j_2),n}$ tends to be smaller than an arbitrary $X_1 - X_2$ and yet not equal to 0) are the source of both the advantages and the problems associated with predictor sort experiments.

Verrill (1993) and David and Gunnink (1997) focused on potential problems with hypothesis tests given a predictor sort design. Verrill (1999) focused on confidence intervals on the mean response associated with a treatment. Verrill et al. (2004) focused on confidence intervals on response quantiles associated with a treatment. Verrill and Kretschmann (2017) reviewed the earlier work, extended it to multiple comparison tests, and performed extensive simulations that documented the need for proper design and analysis of predictor sort experiments.

As noted in Section 1, this paper can be considered to be a companion piece to Verrill and Kretschmann (2017). It constitutes a nontheoretical, simulation-based first look at the effect of predictor sort allocation on *nonparametric* hypothesis tests (Kruskal-Wallis, one-way on ranks, one-way on Van der Waerden ranks, two-way on ranks, and Friedman tests) and nonparametric confidence bounds on quantiles.

Also, the predictor sort analyses described in Verrill and Kretschmann (2017) and earlier publications are based upon the assumption that prior to treatment, the predictor (that is used to perform the allocation) and the response have a bivariate normal distribution. In Section 3.1 we investigate the performance of nonparametric hypotheses tests of predictor sort data when the bivariate normal assumption holds. In later sections we consider the performance of parametric and nonparametric tests when the bivariate normal assumption does not hold.

Note on analyses of covariance

Given a predictor sort design, a scientist or statistician might assume that an analysis of covariance (with the predictor as the covariate) will yield perfectly satisfactory results. In some cases and for some purposes, it will. However, as was noted in Verrill (1993), if the relationship between the predictor and the response is not linear, a blocked analysis of variance (ANOVA) can yield better power than an analysis of covariance (ANCOVA). Also, as noted in Verrill and Kretschmann (2017), given a predictor sort design, a standard ANCOVA will yield *incorrect* confidence intervals on treatment means. Further, as established in the current paper, when the response is long-tailed (rather than normal), blocked ANOVAs on ranks and (sometimes) Friedman tests can perform much better than analyses of covariance. Finally, as noted in Verrill and Kretschmann (2017) and as further illustrated in the current paper, a predictor sort design yields relatively simple ANCOVA power calculations.

3 Simulations of Nonparametric Hypothesis Tests

We simulated the performance of parametric and nonparametric hypothesis tests under a variety of assumptions about the bivariate predictor–response distribution. In addition to a bivariate normal predictor–response distribution, we considered seven additional bivariate distributions. In all seven of these bivariate distributions, the predictor was taken to have a standard normal distribution,

while the response was taken to have one of the seven distributions listed in Table 1. (We realize that a normal assumption on the predictor is too restrictive. These simulations represent a *first* look at a generalization of our original bivariate normal predictor–response assumption.) Plots of the corresponding probability density functions are provided in Figures 1 – 3. Additional information about these seven distributions is provided in the Appendix.

In the simulations, to generate our bivariate predictor–response distributions, we take a Gaussian copula approach (see, for example, Nelsen (2006)). Thus, for example, to generate a Gaussian–Weibull with correlation parameter ρ, we set

$$y_i = \rho x_i + \sqrt{1-\rho^2} z_i \tag{4}$$

where x_i and z_i were independently drawn from a N(0,1) population. Thus x_i and y_i were N(0,1)'s with correlation ρ. x_i was taken as the ith value of the predictor (used in the predictor sort). The corresponding Weibull random variable, w_i, was obtained as

$$w_i = F_{\mathrm{W}}^{-1}(\Phi(y_i); \gamma, \beta)$$

where Φ denotes a N(0,1) cumulative distribution function (cdf) and $F_{\mathrm{W}}^{-1}(u; \gamma, \beta)$ denotes the inverse of the Weibull cdf with parameters γ and β (see the Appendix). (Note that the *generating* ρ in (4) will differ somewhat from the sample correlation between the x_i's and w_i's. See, for example, table 10 in Verrill and Kretschmann (2010).)

The inverses of the other distributions are provided in the Appendix.

In all cases, the parameters of the distributions were set so that variances were 1. This ensured that the powers associated with the different distributions would be at least roughly comparable.

In our simulations, we considered the one-factor case. As in Verrill and Kretschmann (2017), for each combination of

- predictor and response generating correlations, ρ, equal to 0.0, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95, and 0.99,
- number of treatments, J, equal to 2, 3, 5, 7, 9, 11, and 20,
- sample sizes, I, equal to 3, 5, 10, 20, and 40,
- and 21 noncentrality parameters,

we performed 40,000 trials. We created two versions of each of the resulting data sets. One version was created by allocating the specimens in a data set to the J treatment conditions via a standard randomization. The second version was created by allocating the specimens in a data set to the J treatment conditions via a predictor sort.

Distinct μ_j's that varied with the "noncentrality parameter index" (see appendix A of Verrill and Kretschmann (2017)) were added to the J groups of responses corresponding to the J treatments.

We then performed 10 hypothesis tests on the data sets, and two "theoretical power" calculations. The results from these simulations for the case in which the predictor–response distribution was Gaussian–Weibull (0.30 coefficient of variation), $\rho = 0.8$, $J = 5$, and $I = 10, 20$ are provided in Table 2. The full results of the simulation can be found at `www1.fpl.fs.fed.us/ps_nonpar_table2.html`. Columns 1 through 3 of these tables contain the ρ, J, and I values. Column 4 of these tables contains the "noncentrality parameter index" discussed in appendix A of Verrill and Kretschmann (2017). The contents of the remaining columns in these tables can be described as follows:

- Column 5: Standard allocation, 1-way anova. The estimated power of a standard one-way ANOVA on the standard randomization version of the data set.

- Column 6: Standard allocation, KW. The estimated power of a standard Kruskal-Wallis nonparametric test on the standard randomization version of the data set.
- Column 7: Predictor sort allocation, uncorr KW. The estimated power of a standard ("uncorrected") Kruskal-Wallis nonparametric test on the predictor sort version of the data set.
- Column 8: Predictor sort allocation, KW. The estimated power of a "corrected" Kruskal-Wallis nonparametric test on the predictor sort version of the data set. The corrected statistic is the standard KW statistic divided by $1-\rho^2$, where ρ is the (generating) correlation between the predictor and the response.
- Column 9: Predictor sort allocation, one-way ranks. The estimated power of a "corrected" one-way ANOVA on ranks (see, for example, Conover and Iman (1981)) on the predictor sort version of the data set. The corrected statistic is the standard F statistic divided by $1-\rho^2$.
- Column 10: Predictor sort allocation, Van der Waerden. The estimated power of a "corrected" Van der Waerden test on the predictor sort version of the data set (in this paper, a "Van der Waerden test" is a one-way ANOVA on the normal scores of the ranks of the data). The corrected statistic is the standard statistic divided by $1-\rho^2$.
- Column 11: Predictor sort allocation, 2-way anova. The estimated power of a two-way ANOVA on the predictor sort version of the data set. (The blocks are formed by specimens with adjacent [randomized within the block] values of the predictor.)
- Column 12: Predictor sort allocation, 2-way ranks. The estimated power of a two-way ANOVA on ranks (see Conover and Iman (1981)) on the predictor sort version of the data set.
- Column 13: Predictor sort allocation, Friedman. The estimated power of a Friedman test on the predictor sort version of the data set.
- Column 14: Predictor sort allocation, ancova. The estimated power of an ANCOVA on the predictor sort version of the data set.
- Column 15: "Theoretical," 1-way. The calculated "theoretical power" for a corrected one-way ANOVA on a predictor sort version of the data set. See expression (3) of Verrill and Kretschmann (2017) for a detailed discussion of the calculation.
- Column 16: "Theoretical," 2-way. The calculated "theoretical power" for a two-way ANOVA on a predictor sort version of the data set. See expression (4) of Verrill and Kretschmann (2017) for a detailed discussion of the calculation.

Listings of the simulation programs that produced the Table 2 power estimates can be obtained at `http://www1.fpl.fs.fed.us/ps_nonpar_powersim.html`.

For the eight bivariate distributions, plots that present a portion of the results of these simulations ($J = 2, 5$; $I = 10, 20$; $\rho = 0.8$) are provided in Figures 4–35. In these plots, the "noncentrality parameter index" is the m in column 4 of the corresponding power table. See appendix A of Verrill and Kretschmann (2017) for a discussion of this index. In these plots, "th" is the "theoretical power" calculated by (3) in Verrill and Kretschmann (2017) and presented in column 15 of the power table. "ps, ancova" is the estimated power of an ANCOVA after a predictor sort allocation (column 14 of the power table). "ps, 2way anova" is the estimated power of a blocked ANOVA after a predictor sort allocation (column 11 of the power table). "ps, 2way ranks" is the estimated power

of a blocked ANOVA on ranks after a predictor sort allocation (column 12 of the power table). "ps, Friedman" is the estimated power of Friedman's test after a predictor sort allocation (column 13 of the power table). "no ps, KW" is the estimated power of the Kruskal-Wallis test after a standard random allocation (column 6 of the power table). "ps, KW, uncorr" is the estimated power of a standard ("uncorrected") Kruskal-Wallis test after a predictor sort allocation (column 7 of the power table). We note that in these plots, lines sometimes overlie each other and can be difficult to discriminate.

Selected results from these simulations for the $J = 2, 5, 11$; $I = 10, 20, 40$; $\rho = 0.0, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95, 0.99$ cases are presented in Tables 3.1 to 3.24. These tables contain the same columns (with the same meanings) as those contained in Table 2. However, Tables 3.1 to 3.24 contain only a subset of the Table 2 rows. For each ρ, J, and I considered, only two rows are presented. The upper of these two rows is the first row in the corresponding Table 2 in which a power above 0.9 is attained for one of columns 5, 6, 11, 12, 13, or 14, or, if there is no such row, it is the $m = 21$ row. The lower of these two rows is the corresponding $m = 1$ row (so it contains the empirical sizes of nominal 0.05 tests). However, values in this lower row that lie between 0.048 and 0.052 (not significantly different from 0.05 for trials of size 40000) are left blank. These "streamlined" tables (together with the plots) permit one to more readily see the conclusions that we draw below.

A review of the full Table 2 results (`www1.fpl.fs.fed.us/ps_nonpar_table2.html`) demonstrates that in almost all cases where $I \geq 10$, test sizes reported in columns 5, 6, 11 – 14 are nominal (between 0.047 and 0.053) or conservative (below 0.047).

3.1 Nonparametric hypothesis tests when the predictor and the response have a bivariate normal distribution

See Tables 3.1 to 3.3 and Figures 4 to 7 for some of the simulation results. From the full simulation results, we see

1. The conclusions obtained in the parametric case (see Verrill and Kretschmann (2017)) continue to hold in the nonparametric case. In particular, large increases in statistical power and/or sample size reductions can be gained by performing a predictor sort allocation and analysis. To see this, compare the powers in columns 5 and 6 of Tables 3.1 to 3.3 with those in columns 11 to 14. The power improvements become larger as the generating correlation, ρ, between the predictor and the response increases.

   Also, if ρ is reasonably large, it is a statistical *blunder* to perform a predictor sort allocation and then follow the allocation with a standard Kruskal-Wallis test. Such an approach can considerably *reduce* power. (To see this, compare the values in column 7 of Tables 3.1 to 3.3 with the values in columns 5, 6, and 11 to 14.)

   The conclusions stated above *continue to hold* for bivariate Gaussian–Weibull, Gaussian–Uniform, Gaussian–right triangular, Gaussian–double exponential, and Gaussian-exponential data. We will not repeat these conclusions below.

2. In general, as one would expect, if the predictor–response relationship is truly bivariate normal, the power relationships are (see columns 11 to 14 of Tables 3.1 to 3.3)

$$\text{ANCOVA} > \text{two-way ANOVA} > \text{two-way on ranks} > \text{Friedman} \tag{5}$$

   (However, for very high ρ and small I, two-way ANOVA power can be reduced below nonparametric power.)

3. The theory in the parametric case leads (*very* roughly) to a conjecture that in the predictor sort allocation, nonparametric analysis case, one might be able to divide Kruskal-Wallis, one-way on ranks, and Van der Waerden test statistics by $1 - \rho^2$ to obtain tests that have approximately correct sizes and good power. The simulation results presented in the size rows of columns 8 to 10 of Table 3 permit us to evaluate this conjecture. They indicate that in the *known* ρ case, and for lower ρ's, actual sizes are not greatly inflated over nominal sizes and the resulting powers are comparable to those of a two-way ANOVA or an ANCOVA. This is especially true for the Van der Waerden test.

   However, as ρ increases, the inflation of nominal size over actual size for these modified one-way nonparametric tests becomes considerable. Thus a two-way ANOVA on ranks appears to be preferable to our "corrected" one-way nonparametric tests. However, the success of the correction factor for lower ρ's and higher I's suggests that efforts should be made to investigate the asymptotic distributions of suitably corrected one-way nonparametric test statistics given a predictor sort allocation.

4. The "theoretical power" obtained via (3) of Verrill and Kretschmann (2017) matches the simulation estimate for an ANCOVA. (Compare columns 14 and 15 of Tables 3.1 to 3.3.) Except in cases of high ρ and low I, the "theoretical power" obtained via (4) of Verrill and Kretschmann (2017) matches the simulation estimate for a two-way ANOVA. (Compare columns 11 and 16 of Tables 3.1 to 3.3.) In Verrill and Kretschmann (2017) we defined "high ρ" as (very roughly) greater than 0.80, and we defined "low I" as (very roughly) less than 10. For high ρ and low I, the "theoretical power" for a two-way ANOVA will overestimate the actual power of a two-way ANOVA. Because of inequality (5), in the bivariate normal case the powers of the nonparametric tests will be overestimated by results (3) and (4) of Verrill and Kretschmann (2017).

3.2 Nonparametric hypotheses tests when the predictor and the response have a bivariate Gaussian–Weibull distribution

At the Forest Products Laboratory we often work with strength properties (e.g., lumber modulus of rupture and modulus of elasticity) that have coefficients of variation that lie roughly between 0.2 and 0.4. Thus, for this simulation, we considered only Weibulls (other than the exponential distribution, see below) with coefficients of variation equal to 0.2, 0.3, and 0.4. The corresponding shape parameters, β, are given in Table 1. The inverse scale parameters, γ, that yield variances equal to 1 given the Table 1 shape parameters are also provided in Table 1. Further Weibull details are provided in Section 7.1. The Weibull distributions are plotted in Figure 1.

See Tables 3.4 to 3.12 and Figures 8 to 19 for some of the simulation results. From the full simulation results, we can conclude that for Weibull coefficients of variation between 0.2 and 0.4, analyses based on bivariate Gaussian–Weibull predictor/sort allocations behave in the same manner as analyses based on bivariate normal allocations, and the conclusions stated in Section 3.1 continue to hold.

3.3 Nonparametric hypotheses tests when the predictor and the response have a bivariate Gaussian–uniform distribution

A uniform distribution is an example of a symmetric, "short-tailed" (negative excess kurtosis) distribution. The nature of the uniform distribution used in our simulations is described in Table 1 and Section 7.2. The distribution is plotted in Figure 2.

See Tables 3.13 to 3.15 and Figures 20 to 23 for some of the simulation results. From the full simulation results we can see (using columns 11 to 15 of Tables 3.13 to 3.15):

1. The power ordering of the tests depends on the J, I, and ρ values. In general, for lower J, I, and ρ values, the order

$$\text{ANCOVA} > \text{two-way ANOVA} > \text{two-way on ranks} > \text{Friedman} \tag{6}$$

   which holds in the bivariate normal and bivariate Gaussian–Weibull cases continues to hold. However, for higher J, I, and ρ, we see orders such as

$$\text{Friedman} > \text{two-way ANOVA} > \text{two-way on ranks} > \text{ANCOVA} \tag{7}$$

   That is, we see a partial reversal of (6). The relative performance of the nonparametric tests improves.

2. Because we are no longer working with bivariate normal distributions, we can no longer expect the "theoretical power" reported in column 15 to match the empirical powers of the best performing tests. However, for ρ less than or equal to 0.80, the column 15 "theoretical power" still does a good job of approximating the power of the best performing test. Note that in Tables 3.13 to 3.15, there are three cases ($J = 5, I = 40$; $J = 11, I = 20$; $J = 11, I = 40$) in which, for larger ρ, the best performing test is the Friedman test and the empirical power exceeds the "theoretical power."

3.4 Nonparametric hypotheses tests when the predictor and the response have a bivariate Gaussian–right triangular distribution

A right triangular distribution is an example of a skewed, "short-tailed" (negative excess kurtosis) distribution. The nature of the right triangular distribution used in our simulations is described in Table 1 and Section 7.3. The distribution is plotted in Figure 3.

See Tables 3.16 to 3.18 and Figures 24 to 27 for some of the simulation results. From the full simulation results we can see (using columns 11 to 15 of Tables 3.16 to 3.18):

1. The power ordering of the tests depends on the J, I, and ρ values.

   For $J = 2$, $I = 10, 20, 40$, and most of the ρ's, the power order tends to be the same as the order in the bivariate normal case. That is,

$$\text{ANCOVA} > \text{two-way ANOVA} > \text{two-way on ranks} > \text{Friedman} \tag{8}$$

   For $J = 11$, $I = 10, 20, 40$, and $\rho \geq 0.60$, the power order

$$\text{Friedman} > \text{two-way on ranks} > \text{two-way ANOVA} > \text{ANCOVA} \tag{9}$$

   tends to hold. This is the reverse of the power order that tends to hold in the $J = 2$ case.

   For $J = 5$, the power ordering varies from (8) to (9) with (8) tending to hold for lower ρ and I and (9) tending to hold for higher ρ and I.

2. Because we are no longer working with bivariate normal distributions, we can no longer expect the "theoretical power" reported in column 15 to match the empirical powers of the best performing tests. However, for $J = 2$ and 5 and ρ less than or equal to 0.080, the column 15 "theoretical power" still does a good job of approximating the power of the best performing

test. (The approximation is good for all ρ's considered for $I = 40$.) For $J = 11$, the empirical power of the best performing test is generally *better* than the "theoretical power." However, the difference between empirical power and "theoretical power" is small except in the $I = 40$ case. (For $J = 11, I = 40$, the largest difference was between the 0.81 "theoretical power" and 0.92 maximum empirical power for $\rho = 0.90$.)

3.5 Nonparametric hypotheses tests when the predictor and the response have a bivariate Gaussian–double exponential distribution

A double exponential distribution is an example of a symmetric, "long-tailed" (positive excess kurtosis) distribution. The nature of the double exponential distribution used in our simulations is described in Table 1 and Section 7.4. The distribution is plotted in Figure 2.

See Tables 3.19 to 3.21 and Figures 28 to 31 for some of the simulation results. From the full simulation results we can see (using columns 11 to 15 of Tables 3.19 to 3.21):

1. The power ordering of the tests depends on the J, I, and ρ values.

   For $J = 2$, $I = 20, 40$, and most of the ρ's, the power order is

   $$\text{two-way on ranks} > \text{ANCOVA} > \text{two-way ANOVA} > \text{Friedman} \tag{10}$$

   For $J = 2$, $I = 10$, and $\rho \leq 0.60$, the power order is

   $$\text{two-way on ranks} > \text{ANCOVA} > \text{two-way ANOVA} > \text{Friedman} \tag{11}$$

   but for $J = 2$, $I = 10$, and $\rho \geq 0.70$, the power order is

   $$\text{ANCOVA} > \text{two-way on ranks} > \text{two-way ANOVA} > \text{Friedman} \tag{12}$$

   For $J = 5$ and 11, in almost all cases the power order is

   $$\text{two-way on ranks} > \text{Friedman} > \text{ANCOVA} > \text{two-way ANOVA} \tag{13}$$

   The exception is in the $J = 5, I = 10$ case in which the power order is primarily

   $$\text{two-way on ranks} > \text{ANCOVA} > \text{Friedman} > \text{two-way ANOVA} \tag{14}$$

   In any event, the two-way on ranks test dominates, and the Friedman test tends to be the runner-up except in the low J case.

2. In all cases other than the higher ρ cases for $J = 2$, $I = 10$, the "theoretical power" in column 15 *underestimates* the empirical power of the best performing test (a two-way on ranks test except in the higher ρ for $J = 2$, $I = 10$ cases mentioned above where an ANCOVA yields the maximum power). The largest differences between empirical power and "theoretical power" are on the order of 0.15.

3.6 Nonparametric hypotheses tests when the predictor and the response have a bivariate Gaussian–exponential distribution

An exponential distribution is an example of a skewed, "long-tailed" (positive excess kurtosis) distribution. The nature of the exponential distribution used in our simulations is described in Table 1 and Section 7.5. The distribution is plotted in Figure 3.

See Tables 3.22 to 3.24 and Figures 32 to 35 for some of the simulation results. From the full simulation results we can see (using columns 11 to 15 of Tables 3.22 to 3.24):

1. In all three of Tables 3.22 to 3.24, the most powerful test is a nonparametric test.
2. For the $J = 2$ and $J = 5$ cases (Tables 3.22 and 3.23), the most powerful test is the two-way on ranks test.
3. For the $J = 11$ case (Table 3.24) and lower I, the most powerful test is the two-way on ranks test. For higher I and higher ρ, the Friedman test is most powerful.
4. In all cases other than those in which ρ is high and $J = 2, I = 10$ or $J = 5, I = 10$, the "theoretical power" in column 15 *greatly underestimates* the empirical power of the best performing test. The largest differences between empirical power and "theoretical power" are on the order of 0.45.

4 Nonparametric Confidence Intervals on Quantiles after a Predictor Sort Allocation

Just as a predictor sort allocation can be used to reduce the sample sizes needed for parametric confidence bounds on treatment quantiles (see Verrill et al. (2004)), it can also be used to reduce the sample sizes needed for nonparametric confidence bounds on quantiles. (In this section we restrict ourselves to the case in which the predictor and the response have a bivariate normal distribution.)

For example, in the absence of a predictor sort, the probability that the kth order statistic from a random sample of size n from a population lies below the qth quantile of the population is given by (see, for example, David (1981))

$$\sum_{i=k}^{n} \binom{n}{i} q^i (1-q)^{n-i} \tag{15}$$

A web program to calculate this probability can be found at `http://www1.fpl.fs.fed.us/nonpar_ci_quant.html`. Using this program, one can see that in the absence of a predictor sort allocation, appropriate order statistics for nonparametric one-sided, lower 75% confidence bounds on fifth percentiles are 1, 2, and 3 for samples of size 28, 53, and 78, respectively. (These values are specified in table 2 of ASTM (2010) in association with the calculation of "allowable properties" for grades of structural lumber.) The exact coverages of such confidence bounds in the standard randomization case are provided in column 4 of Table 4. After a predictor sort allocation, the coverages of such nonparametric confidence intervals are elevated above the value provided by (15). These elevated values increase as ρ (the correlation between the predictor and the response in the predictor sort allocation) increases. See, for example, Table 4. Estimated actual coverages are reported for $\rho = 0.7$, 0.8, and 0.9. (A listing of the program that was used to perform the simulation that yielded Table 4 can be found at `http://www1.fpl.fs.fed.us/psnonpar_table4_code.html`)

Because of this elevation, the actual sample sizes needed to obtain the nominal coverages are reduced below the I values that would be determined from (15). In Table 5, we report the I's that are needed to obtain 75% coverage for J (the number of treatments) equal to 2, 3, or 5, and $\rho = 0.7$, 0.8, or 0.9 when we use the first, second, or third order statistic (OS) associated with a particular treatment as our nonparametric lower bound. These I's should be contrasted with 28 (in the OS = 1 case), 53 (in the OS = 2 case), and 78 (in the OS = 3 case). Permissible I's clearly decrease as ρ and J increase. For example, in the first order statistic, $J = 5$, $\rho = 0.9$ case, the necessary I declines from 28 to 21. (This 25% sample size reduction is the largest in the table. As indicated in Table 4, if we incorrectly ignore the predictor sort allocation, this 25% sample size reduction case corresponds to an actual coverage of 0.845 for a nominal 0.75 confidence interval.) (A listing of the program that was used to perform the simulation that yielded Table 5 can be found at `http://www1.fpl.fs.fed.us/psnonpar_table5_code.html`)

5 Summary

This paper represents a nontheoretical, simulation-based first look at the effect of predictor sort allocation on nonparametric hypothesis tests and confidence bounds on quantiles. For hypothesis tests, it also looks at the effects of replacing bivariate normal predictor/response distributions with bivariate normal–Weibull, normal–uniform, normal–right triangular, normal–double exponential, and normal–exponential distributions (representing "near normal," "symmetric short-tailed," "skewed short-tailed," "symmetric long-tailed," and "skewed long-tailed" response distributions).

Our simulations permit us to draw a number of conclusions:

- As in the parametric case, large increases in statistical power and/or sample size reductions can be gained by performing a predictor sort allocation and analysis. The power improvements become larger as the generating correlation, ρ, between the predictor and the response increases.

  Also, if ρ is reasonably large, it is a statistical *blunder* to perform a predictor sort allocation and then follow the allocation with a standard Kruskal–Wallis test. Such an approach can considerably *reduce* power (from that available from a standard random allocation followed by a standard Kruskal–Wallis test).

  These conclusions hold for bivariate Gaussian–Weibull, Gaussian–Uniform, Gaussian–right triangular, Gaussian–double exponential, and Gaussian–exponential predictor–response data, as well as for bivariate normal predictor–response data.

- The theory in the parametric case leads (*very* roughly) to a conjecture that in the predictor sort allocation, nonparametric analysis case, one might be able to divide Kruskal–Wallis, one-way on ranks, and Van der Waerden test statistics by $1 - \rho^2$ to obtain tests that have approximately correct sizes and good power. Our simulation results indicate that in the *known* ρ case, and for lower ρ's, actual sizes are not greatly inflated over nominal sizes and the resulting powers are comparable to those of a two-way ANOVA or an ANCOVA. This is especially true for the Van der Waerden test.

  However, as ρ increases, the inflation of nominal size over actual size for these modified one-way nonparametric tests becomes considerable. Still, the success of the correction factor for lower ρ's and higher I's suggests that efforts should be made to investigate the asymptotic distributions of suitably corrected one-way nonparametric test statistics (and two-way nonparametric test statistics) given a predictor sort allocation.

- If the predictor–response relationship is truly bivariate normal or bivariate normal–near normal (as in the case of the bivariate Gaussian–Weibull distributions considered in Section 3.2), the power relationships are generally

$$\text{ANCOVA} > \text{two-way ANOVA} > \text{two-way on ranks} > \text{Friedman} \tag{16}$$

- For uniform and right triangular response distributions (negative excess kurtosis or "short-tailed" distributions) and lower J, I, ρ values, the power relationships are generally well represented by (16), but for higher J, I, ρ values, the Friedman test tends to dominate.

- For double exponential and exponential response distributions (positive excess kurtosis or "long-tailed" distributions), the two-way on ranks nonparametric test dominates and the Friedman nonparametric test tends to be the runner-up. (In the $J = 2$ case, an ANCOVA and a two-way ANOVA perform better than a Friedman test.)

- The power ordering results tell us that if we perform a predictor sort allocation, our choice of hypothesis test should depend on some or all of J, I, ρ, and the distribution of the response.

- For bivariate normal distributions and the bivariate Gaussian–Weibulls that we considered (Weibull coefficients of variation equal to 0.2, 0.3, and 0.4), the "theoretical power" in column 15 (of Tables 3.1 to 3.12) matched the power of the best performing test (an ANCOVA except in a very few high ρ cases).

- For bivariate Gaussian–uniform distributions and $\rho \leq 0.80$, the "theoretical power" in column 15 (of Tables 3.13 to 3.15) matched the the power of the best performing test. For $\rho \geq 0.90$, the "theoretical power" in column 15 (of Tables 3.13 to 3.15) and the the power of the best performing test were still reasonably close.

- For bivariate Gaussian–right triangular distributions, $J = 2$ and 5, and ρ less than or equal to 0.80, the "theoretical power" in column 15 (of Tables 3.16 to 3.18) still does a good job of approximating the power of the best performing test. For $J = 11$, the empirical power of the best performing test is generally *better* than the "theoretical power." However, the difference between empirical power and "theoretical power" is small except in the $I = 40$ case. (For $J = 11, I = 40$, the largest difference was between the 0.81 "theoretical power" and the 0.92 maximum empirical power for $\rho = 0.90$.)

- For bivariate Gaussian–double exponential distributions, in all cases other than the higher ρ cases for $J = 2$, $I = 10$, the "theoretical power" in column 15 (of Tables 3.19 to 3.21) *underestimates* the empirical power of the best performing test. The largest differences between empirical power and "theoretical power" are on the order of 0.15.

- For bivariate Gaussian–exponential distributions, in all cases other than those in which ρ is high and $J = 2, I = 10$ or $J = 5, I = 10$, the "theoretical power" in column 15 (of Tables 3.22 to 3.24) *greatly underestimates* the empirical power of the best performing test. The largest differences between empirical power and "theoretical power" are on the order of 0.45.

- Just as we can reduce necessary sample sizes or narrow *parametric* confidence intervals on quantiles via a predictor sort allocation (see Verrill et al. (2004)), we can reduce necessary sample sizes or increase *nonparametric* confidence interval coverage of quantiles via a predictor sort allocation. In this paper we establish this result (via simulations) only for a bivariate normal predictor–response distribution, but we expect that it will also hold for other predictor–response distributions.

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7 Appendix — Distributions

We note that all the material in this section has previously appeared in the literature. We record it here for our future convenience and for the possible convenience of readers of this paper.

7.1 Weibull

7.1.1 pdf

For $w > 0$

$$f_{\mathrm{W}}(w;\gamma,\beta) = \gamma^{\beta}\beta w^{\beta-1}\exp\left(-(\gamma w)^{\beta}\right)$$

where β is the shape parameter and γ is the inverse of the scale parameter.

7.1.2 cdf

For $w > 0$

$$F_{\mathrm{W}}(w;\gamma,\beta) = 1 - \exp\left(-(\gamma w)^{\beta}\right)$$

7.1.3 Inverse of the cdf

For $y \in [0,1]$

$$F_{\mathrm{W}}^{-1}(y;\gamma,\beta) = (-\log(1-y))^{1/\beta}/\gamma$$

7.1.4 Moments

$$
\begin{aligned}
\mathrm{E}(w^k) &= \int_0^\infty w^k \gamma^\beta \beta w^{\beta-1} \exp\left(-(\gamma w)^\beta\right)\, dw \\
&= \gamma^\beta \beta \int_0^\infty w^{\beta+k-1} \exp\left(-(\gamma w)^\beta\right)\, dw \\
&= \gamma^\beta \beta \int_0^\infty w^{1+\frac{k-1}{\beta}} \exp\left(-\gamma^\beta w\right) w^{\frac{1}{\beta}-1}/\beta\, dw \\
&= \gamma^\beta \int_0^\infty w^{\frac{k}{\beta}} \exp\left(-\gamma^\beta w\right)\, dw \\
&= \int_0^\infty (w/\gamma^\beta)^{\frac{k}{\beta}} \exp(-w)\, dw \\
&= \frac{1}{\gamma^k} \int_0^\infty w^{\frac{k}{\beta}} \exp(-w)\, dw \\
&= \frac{1}{\gamma^k} \Gamma(1 + k/\beta)
\end{aligned}
$$

where $\Gamma()$ denotes the gamma function.

7.1.5 Mean, variance, coefficient of variation, skewness, excess kurtosis

$$\mu = \mathrm{E}(w) = \frac{1}{\gamma}\Gamma(1 + 1/\beta)$$

$$\sigma^2 = \mathrm{E}(w^2) - \mathrm{E}(w)^2 = \frac{1}{\gamma^2}\left(\Gamma(1 + 2/\beta) - \Gamma(1 + 1/\beta)^2\right)$$

$$\mathrm{cov} = \sigma/\mu = \sqrt{\Gamma(1 + 2/\beta) - \Gamma(1 + 1/\beta)^2}/\Gamma(1 + 1/\beta)$$

$$
\begin{aligned}
\text{skewness} &= \mathrm{E}\left((w - \mu)^3\right)/\sigma^3 \\
&= \mathrm{E}\left(w^3 - 3w^2\mu + 3w\mu^2 - \mu^3\right)/\sigma^3 \\
&= \frac{\Gamma(1 + 3/\beta) - 3\Gamma(1 + 2/\beta)\Gamma(1 + 1/\beta) + 2\Gamma(1 + 1/\beta)^3}{\left(\Gamma(1 + 2/\beta) - \Gamma(1 + 1/\beta)^2\right)^{3/2}}
\end{aligned}
$$

$$
\begin{aligned}
\text{kurtosis} &= \mathrm{E}\left((w - \mu)^4\right)/\sigma^4 \\
&= \mathrm{E}\left(w^4 - 4w^3\mu + 6w^2\mu^2 - 4w\mu^3 + \mu^4\right)/\sigma^4 \\
&= \frac{\Gamma(1 + 4/\beta) - 4\Gamma(1 + 3/\beta)\Gamma(1 + 1/\beta) + 6\Gamma(1 + 2/\beta)\Gamma(1 + 1/\beta)^2 - 3\Gamma(1 + 1/\beta)^4}{\left(\Gamma(1 + 2/\beta) - \Gamma(1 + 1/\beta)^2\right)^2}
\end{aligned}
$$

$$\text{excess kurtosis} = \text{kurtosis} - 3$$

7.2 Uniform

7.2.1 pdf

For $x \in [-a, a]$

$$f_{\mathrm{U}}(x; a) = 1/2a$$

7.2.2 cdf

For $x \in [-a, a]$

$$F_{\mathrm{U}}(x; a) = (x + a)/2a$$

7.2.3 Inverse of the cdf

For $y \in [0, 1]$

$$F_{\mathrm{U}}^{-1}(y; a) = 2ay - a$$

7.2.4 Moments

$$\mathrm{E}(X) = 0$$

$$\mathrm{E}(X^2) = a^2/3$$

$$\mathrm{E}(X^3) = 0$$

$$\mathrm{E}(X^4) = a^4/5$$

7.2.5 Mean, variance, skewness, excess kurtosis

$$\mu = 0$$

$$\sigma^2 = a^2/3$$

$$\text{skewness} = 0$$

$$\begin{aligned} \text{kurtosis} &= \mathrm{E}\left((X-\mu)^4\right)/\sigma^4 \\ &= \mathrm{E}\left(X^4\right)/\sigma^4 \\ &= (a^4/5)/(a^4/9) = 9/5 \end{aligned}$$

$$\text{excess kurtosis} = 9/5 - 3 = -6/5$$

7.3 Right triangular

7.3.1 pdf

For $x \in [0, a]$

$$f_{\mathrm{RT}}(x; a) = 2/a - 2x/a^2$$

7.3.2 cdf

For $x \in [0, a]$

$$F_{\mathrm{RT}}(x; a) = 2x/a - x^2/a^2$$

7.3.3 Inverse of the cdf

For $y \in [0,1]$

$$F_{\mathrm{RT}}^{-1}(y;a) = a - a\sqrt{1-y}$$

7.3.4 Moments

$$\mathrm{E}(X^k) = 2a^k/((k+1)(k+2))$$

so

$$\mathrm{E}(X) = a/3$$

$$\mathrm{E}(X^2) = a^2/6$$

$$\mathrm{E}(X^3) = a^3/10$$

$$\mathrm{E}(X^4) = a^4/15$$

7.3.5 Mean, variance, skewness, excess kurtosis

$$\mu = a/3$$

$$\sigma^2 = a^2/18$$

$$\begin{aligned}
\text{skewness} &= \mathrm{E}\left((X-\mu)^3\right)/\sigma^3 \\
&= \mathrm{E}\left(X^3 - 3X^2\mu + 3X\mu^2 - \mu^3\right)/\sigma^3 \\
&= (a^3/10 - a^3/6 + 2a^3/27)/(a^3/18^{3/2}) \\
&= 18^{3/2}/135) = 2\sqrt{2}/5
\end{aligned}$$

$$\begin{aligned}
\text{kurtosis} &= \mathrm{E}\left((X-\mu)^4\right)/\sigma^4 \\
&= \mathrm{E}\left(X^4 - 4X^3\mu + 6X^2\mu^2 - 4X\mu^3 + \mu^4\right)/\sigma^4 \\
&= (1/15 - 4/30 + 1/9 - 1/27)/(1/18)^2 \\
&= 12/5
\end{aligned}$$

$$\text{excess kurtosis} = 12/5 - 3 = -3/5$$

7.4 Double exponential

7.4.1 pdf

For $x < 0$

$$f_{\mathrm{DE}}(x;a) = a\exp(ax)/2$$

For $x > 0$

$$f_{\mathrm{DE}}(x;a) = a\exp(-ax)/2$$

7.4.2 cdf

For $x < 0$

$$F_{\mathrm{DE}}(x; a) = \exp(ax)/2$$

For $x > 0$

$$F_{\mathrm{DE}}(x; a) = 1 - \exp(-ax)/2$$

7.4.3 Inverse of the cdf

For $y < 1/2$

$$F_{\mathrm{DE}}^{-1}(y; a) = \log(2y)/a$$

For $y > 1/2$

$$F_{\mathrm{DE}}^{-1}(y; a) = -\log(2(1-y))/a$$

7.4.4 Moments

$$\mathrm{E}(X) = 0$$

$$\mathrm{E}(X^2) = 2/a^2$$

$$\mathrm{E}(X^3) = 0$$

$$\mathrm{E}(X^4) = 24/a^4$$

7.4.5 Mean, variance, skewness, excess kurtosis

$$\mu = 0$$

$$\sigma^2 = 2/a^2$$

$$\text{skewness} = 0$$

$$\begin{aligned} \text{kurtosis} &= \mathrm{E}\left((X-\mu)^4\right)/\sigma^4 \\ &= \mathrm{E}\left(X^4\right)/\sigma^4 \\ &= (24/a^4)/(4/a^4) = 6 \end{aligned}$$

$$\text{excess kurtosis} = 6 - 3 = 3$$

7.5 Exponential

7.5.1 pdf

For $x > 0$

$$f_{\mathrm{E}}(x; a) = a \exp(-ax)$$

7.5.2 cdf

For $x > 0$

$$F_{\mathrm{E}}(x; a) = 1 - \exp(-ax)$$

7.5.3 Inverse of the cdf

For $y \in [0, 1]$

$$F_{\mathrm{E}}^{-1}(y; a) = -\log(1 - y)/a$$

7.5.4 Moments

$$\mathrm{E}(X^k) = k!/a^k$$

7.5.5 Mean, variance, skewness, excess kurtosis

$$\mu = 1/a$$

$$\sigma^2 = 2/a^2 - 1/a^2 = 1/a^2$$

$$\begin{aligned}
\text{skewness} &= \mathrm{E}\left((X-\mu)^3\right)/\sigma^3 \\
&= \mathrm{E}\left(X^3 - 3X^2\mu + 3X\mu^2 - \mu^3\right)/\sigma^3 \\
&= (6/a^3 - 6/a^3 + 2/a^3)/(1/a^3) = 2
\end{aligned}$$

$$\begin{aligned}
\text{kurtosis} &= \mathrm{E}\left((X-\mu)^4\right)/\sigma^4 \\
&= \mathrm{E}\left(X^4 - 4X^3\mu + 6X^2\mu^2 - 4X\mu^3 + \mu^4\right)/\sigma^4 \\
&= (24/a^4 - 24/a^4 + 12/a^4 - 3/a^4)/(1/a^4) = 9
\end{aligned}$$

$$\text{excess kurtosis} = 9 - 3 = 6$$

Section	Distribution	Description	Parameters	Skewness	Excess kurtosis
7.1	Weibull, cv = 0.2		$\gamma = 0.1852$, $\beta = 5.796$	-0.3518	0.0036
	Weibull, cv = 0.3		$\gamma = 0.2708$, $\beta = 3.714$	-0.0261	-0.2766
	Weibull, cv = 0.4		$\gamma = 0.3557$, $\beta = 2.695$	0.2769	-0.2124
7.2	uniform	symmetric, short-tailed	$a = \sqrt{3}$	0.0000	-1.2000
7.3	right triangular	skewed, short-tailed	$a = \sqrt{18}$	0.5657	-0.6000
7.4	double exponential	symmetric, long-tailed	$a = \sqrt{2}$	0.0000	3.0000
7.5	exponential	skewed, long-tailed	$a = 1$	2.0000	6.0000

Table 1: Distributions. For definitions of γ, β, and a, see Sections 7.1 – 7.5.

				Standard allocation		Predictor sort allocation									
							Divided by $1-\rho^2$								
ρ	J	I	m	1-way anova	KW	uncorr KW	KW	1-way ranks	Van der Waerden	2-way anova	2-way ranks	Friedman	ancova	"Theoretical" 1-way	"Theoretical" 2-way
.80	5	10	1	.048	.042	.000	.081	.060	.058	.051	.052	.044	.051	.050	.050
.80	5	10	2	.049	.043	.000	.083	.061	.060	.053	.052	.045	.052	.052	.052
.80	5	10	3	.050	.044	.000	.088	.066	.065	.057	.056	.048	.057	.056	.056
.80	5	10	4	.053	.046	.000	.097	.072	.072	.065	.063	.053	.066	.064	.064
.80	5	10	5	.057	.050	.000	.110	.082	.083	.076	.073	.061	.077	.076	.076
.80	5	10	6	.062	.055	.000	.126	.097	.099	.090	.085	.072	.093	.092	.091
.80	5	10	7	.068	.060	.001	.149	.115	.119	.109	.102	.085	.114	.113	.112
.80	5	10	8	.077	.067	.001	.176	.139	.143	.132	.123	.103	.140	.140	.137
.80	5	10	9	.087	.076	.002	.208	.168	.174	.159	.147	.124	.172	.172	.168
.80	5	10	10	.098	.084	.002	.245	.202	.209	.193	.178	.149	.209	.210	.205
.80	5	10	11	.111	.096	.004	.287	.238	.251	.230	.212	.178	.252	.254	.248
.80	5	10	12	.127	.108	.005	.335	.281	.297	.277	.250	.211	.300	.304	.297
.80	5	10	13	.144	.123	.008	.385	.328	.347	.326	.293	.249	.355	.359	.350
.80	5	10	14	.163	.141	.011	.439	.380	.402	.380	.343	.290	.413	.418	.408
.80	5	10	15	.186	.159	.016	.496	.435	.459	.436	.394	.336	.475	.480	.469
.80	5	10	16	.210	.180	.023	.551	.492	.519	.498	.450	.386	.538	.543	.531
.80	5	10	17	.237	.203	.032	.607	.547	.578	.558	.505	.436	.602	.605	.593
.80	5	10	18	.264	.229	.043	.663	.606	.636	.617	.562	.490	.662	.666	.653
.80	5	10	19	.296	.256	.057	.715	.661	.693	.673	.619	.542	.720	.722	.710
.80	5	10	20	.326	.285	.074	.764	.715	.745	.726	.673	.593	.773	.774	.762
.80	5	10	21	.360	.315	.095	.808	.762	.793	.776	.724	.644	.818	.821	.809
.80	5	20	1	.050	.047	.000	.069	.058	.054	.049	.049	.045	.050	.050	.050
.80	5	20	2	.052	.048	.000	.071	.061	.058	.052	.051	.049	.053	.053	.053
.80	5	20	3	.055	.051	.000	.082	.070	.067	.063	.060	.057	.064	.063	.063
.80	5	20	4	.062	.057	.000	.102	.087	.086	.080	.075	.069	.082	.082	.081
.80	5	20	5	.071	.065	.001	.128	.113	.112	.107	.098	.089	.109	.109	.108
.80	5	20	6	.083	.075	.001	.166	.148	.151	.144	.130	.117	.147	.148	.146
.80	5	20	7	.099	.088	.002	.215	.193	.201	.192	.172	.154	.197	.198	.196
.80	5	20	8	.119	.106	.003	.275	.251	.261	.252	.225	.201	.259	.262	.259
.80	5	20	9	.142	.128	.006	.345	.318	.333	.325	.290	.256	.337	.338	.334
.80	5	20	10	.170	.153	.010	.421	.393	.415	.406	.362	.319	.421	.423	.418
.80	5	20	11	.202	.180	.017	.503	.474	.503	.491	.440	.391	.511	.513	.508
.80	5	20	12	.237	.212	.029	.588	.559	.591	.580	.524	.467	.600	.604	.598
.80	5	20	13	.278	.249	.045	.668	.642	.676	.667	.607	.546	.686	.691	.685
.80	5	20	14	.326	.292	.068	.745	.719	.753	.746	.688	.624	.765	.769	.763
.80	5	20	15	.374	.338	.102	.812	.790	.822	.817	.762	.697	.832	.835	.830
.80	5	20	16	.426	.388	.147	.865	.850	.878	.872	.824	.766	.886	.888	.884
.80	5	20	17	.481	.438	.204	.908	.895	.918	.917	.876	.825	.927	.928	.924
.80	5	20	18	.535	.491	.271	.940	.931	.949	.947	.916	.871	.955	.956	.953
.80	5	20	19	.589	.542	.348	.962	.955	.969	.968	.945	.909	.974	.974	.972
.80	5	20	20	.643	.594	.433	.977	.973	.982	.982	.966	.938	.986	.986	.985
.80	5	20	21	.692	.645	.522	.987	.984	.990	.990	.980	.961	.992	.993	.992

Table 2: Power simulation (see Section 3 for details, including column definitions), bivariate Gaussian–Weibull(.3cv), $\rho = 0.80$, $J = 5$, $I = 10, 20$. The full simulation results (covering eight bivariate predictor-response distributions, and a range of ρ, J, and I values) can be found at `www1.fpl.fs.fed.us/ps_nonpar_table2.html`.

				Standard allocation		Predictor sort allocation									
							Divided by $1-\rho^2$								
				1-way		uncorr		1-way	Van der	2-way	2-way			"Theoretical"	
ρ	J	I	m	anova	KW	KW	KW	ranks	Waerden	anova	ranks	Friedman	Ancova	1-way	2-way
.00	2	10	21	.567	.554	.552	.552	.552	.551	.517	.507	.273	.560	.562	.514
												.022			
.50	2	10	21	.557	.545	.550	.662	.662	.661	.624	.601	.346	.680	.685	.634
						.028	.054	.054				.020			
.60	2	10	21	.562	.550	.550	.742	.707	.726	.687	.661	.401	.748	.753	.702
						.021	.064		.055			.022			
.70	2	10	21	.558	.545	.555	.822	.791	.810	.772	.740	.482	.838	.841	.795
						.012	.067	.054	.058			.022			
.80	2	10	20	.518	.505	.501	.901	.877	.888	.850	.818	.574	.914	.917	.882
						.003	.079	.062	.063			.021			
.90	2	10	15	.317	.311	.191	.899	.868	.888	.819	.788	.543	.923	.924	.890
						.000	.091	.067	.080			.022			
.95	2	10	11	.185	.183	.029	.880	.880	.875	.742	.728	.476	.920	.923	.889
					.053	.000	.112	.112	.114			.021			
.99	2	10	6	.085	.086	.000	.912	.851	.870	.501	.580	.338	.961	.963	.940
					.053	.000	.284	.170	.275	.042		.021			
.00	2	20	21	.873	.853	.849	.849	.849	.861	.850	.834	.662	.868	.869	.851
												.042			
.50	2	20	20	.833	.812	.837	.906	.901	.912	.906	.886	.730	.921	.922	.908
					.047	.026	.054					.043			
.60	2	20	18	.746	.722	.760	.895	.888	.896	.888	.864	.709	.906	.906	.890
						.016	.057	.053				.041			
.70	2	20	17	.693	.665	.718	.919	.913	.920	.913	.891	.747	.931	.932	.919
						.008	.059	.054	.053			.041			
.80	2	20	14	.516	.493	.489	.900	.892	.904	.892	.860	.712	.918	.916	.901
						.002	.065	.059	.056			.041			
.90	2	20	11	.338	.322	.178	.925	.917	.925	.904	.875	.738	.940	.942	.930
						.000	.077	.069	.067			.042			
.95	2	20	8	.192	.182	.014	.916	.903	.914	.863	.840	.698	.931	.932	.919
						.000	.093	.080	.084			.041			
.99	2	20	4	.075	.075	.000	.868	.868	.861	.624	.698	.538	.906	.906	.890
						.000	.147	.147	.180	.045		.042			
.00	2	40	16	.911	.898	.897	.897	.897	.908	.905	.893	.710	.912	.912	.905
												.038			
.50	2	40	14	.816	.796	.833	.899	.899	.909	.907	.887	.715	.914	.912	.905
					.047	.025						.038			
.60	2	40	13	.755	.736	.777	.897	.895	.906	.904	.883	.710	.912	.912	.905
						.016	.054	.053				.037			
.70	2	40	12	.682	.662	.709	.910	.908	.920	.918	.894	.734	.927	.925	.919
						.008	.056	.055				.039			
.80	2	40	10	.514	.494	.482	.892	.889	.904	.899	.869	.705	.911	.912	.905
						.002	.060	.058	.055			.039			
.90	2	40	8	.342	.326	.171	.926	.921	.936	.930	.901	.756	.944	.944	.938
						.000	.068	.063	.060			.040			
.95	2	40	6	.197	.190	.010	.924	.920	.932	.914	.887	.745	.941	.943	.937
						.000	.077	.072	.069			.040			
.99	2	40	4	.101	.098	.000	.993	.991	.992	.962	.979	.909	.997	.997	.996
						.000	.109	.098	.121			.040			

Table 3.1: Some power and size results for bivariate normal data. $J = 2$, $I = 10, 20, 40$. For columns 5 – 16, the upper number in a row is power, and the lower number is size (blank if between .048 and .052).

				Standard allocation		Predictor sort allocation									
							Divided by $1-\rho^2$							"Theoretical"	
ρ	J	I	m	1-way anova	KW	uncorr KW	KW	1-way ranks	Van der Waerden	2-way anova	2-way ranks	Friedman	Ancova	1-way	2-way
.00	5	10	21	.363	.328	.320	.320	.347	.351	.350	.340	.268	.355	.359	.350
					.042	.043	.043					.043			
.50	5	10	21	.357	.322	.241	.456	.448	.459	.453	.431	.351	.470	.471	.460
					.042	.014	.054	.053				.043			
.60	5	10	21	.360	.322	.211	.538	.515	.526	.521	.493	.408	.545	.543	.531
					.042	.007	.060	.053				.044			
.70	5	10	21	.361	.324	.162	.653	.615	.630	.621	.586	.495	.654	.654	.641
					.042	.002	.067	.055				.043			
.80	5	10	21	.362	.324	.102	.819	.777	.795	.770	.735	.649	.818	.821	.809
					.042	.000	.081	.060	.059			.045			
.90	5	10	18	.266	.236	.013	.929	.897	.910	.864	.846	.765	.931	.931	.924
					.042	.000	.108	.073	.080			.043			
.95	5	10	13	.147	.128	.000	.922	.884	.902	.775	.786	.695	.921	.923	.915
					.042	.000	.148	.099	.125			.044			
.99	5	10	7	.069	.059	.000	.958	.937	.941	.505	.681	.558	.966	.967	.962
				.047	.041	.000	.367	.281	.465	.041		.045			
.00	5	20	21	.689	.655	.659	.659	.672	.688	.687	.665	.567	.694	.691	.685
					.045	.047	.047					.047			
.50	5	20	21	.689	.656	.625	.803	.800	.817	.819	.792	.702	.828	.827	.821
					.045	.014	.054	.053				.047			
.60	5	20	21	.696	.662	.609	.874	.868	.881	.882	.858	.780	.889	.888	.884
					.047	.006	.056					.046			
.70	5	20	20	.637	.606	.507	.914	.906	.918	.917	.893	.830	.927	.926	.923
					.045	.002	.062	.056				.047			
.80	5	20	17	.476	.446	.220	.914	.903	.916	.912	.885	.827	.925	.928	.924
					.047	.000	.069	.059	.055			.047			
.90	5	20	13	.274	.252	.009	.938	.924	.938	.922	.897	.841	.945	.943	.940
					.045	.000	.084	.069	.064			.046			
.95	5	20	9	.137	.127	.000	.900	.879	.897	.839	.823	.754	.903	.904	.900
					.045	.000	.096	.078	.081			.046			
.99	5	20	5	.066	.062	.000	.950	.939	.943	.710	.832	.742	.955	.956	.953
					.044	.000	.179	.148	.246	.045		.047			
.00	5	40	19	.909	.890	.893	.893	.896	.909	.910	.894	.818	.912	.908	.906
.50	5	40	17	.818	.795	.789	.907	.906	.919	.921	.903	.837	.923	.923	.922
					.047	.014	.054	.053							
.60	5	40	16	.763	.735	.707	.915	.912	.925	.927	.907	.847	.929	.931	.930
						.005	.056	.054							
.70	5	40	14	.625	.597	.492	.901	.897	.911	.911	.887	.823	.915	.915	.913
						.001	.058	.055							
.80	5	40	12	.463	.439	.206	.906	.900	.914	.913	.883	.829	.920	.919	.917
						.000	.065	.059	.054						
.90	5	40	9	.257	.241	.005	.908	.900	.914	.908	.878	.826	.921	.920	.918
						.000	.072	.065	.056						
.95	5	40	7	.154	.145	.000	.931	.923	.939	.923	.895	.852	.942	.943	.942
				.047	.047	.000	.082	.073	.067						
.99	5	40	4	.071	.069	.000	.971	.967	.972	.905	.934	.895	.977	.978	.977
					.046	.000	.111	.099	.135	.047					

Table 3.2: Some power and size results for bivariate normal data. $J = 5$, $I = 10, 20, 40$. For columns 5 – 16, the upper number in a row is power, and the lower number is size (blank if between .048 and .052).

				Standard allocation		Predictor sort allocation									
							Divided by $1-\rho^2$							"Theoretical"	
ρ	J	I	m	1-way anova	KW	uncorr KW	KW	1-way ranks	Van der Waerden	2-way anova	2-way ranks	Friedman	Ancova	1-way	2-way
.00	11	10	21	.243	.210	.212	.212	.237	.242	.242	.236	.195	.244	.245	.243
					.043	.041	.041					.042			
.50	11	10	21	.245	.210	.100	.318	.307	.314	.320	.305	.260	.326	.328	.325
					.042	.007	.056	.053				.042			
.60	11	10	21	.244	.210	.057	.393	.361	.370	.375	.354	.305	.387	.387	.384
					.041	.002	.060		.047	.047		.041	.047		
.70	11	10	21	.243	.209	.025	.503	.449	.461	.461	.435	.382	.486	.488	.484
					.042	.000	.073	.054				.043			
.80	11	10	21	.244	.211	.005	.692	.621	.641	.625	.589	.534	.668	.670	.666
					.042	.000	.084	.057	.056			.042			
.90	11	10	20	.217	.188	.000	.925	.880	.903	.851	.837	.789	.916	.918	.916
					.041	.000	.116	.070	.078			.041			
.95	11	10	15	.131	.112	.000	.949	.908	.932	.804	.832	.771	.935	.935	.932
					.043	.000	.163	.097	.143			.043			
.99	11	10	7	.062	.053	.000	.955	.922	.966	.370	.586	.479	.905	.902	.899
					.042	.000	.458	.336	.704	.040		.040			
.00	11	20	21	.522	.488	.487	.487	.503	.520	.520	.501	.446	.523	.520	.518
						.046	.046					.046			
.50	11	20	21	.529	.493	.349	.653	.647	.663	.668	.638	.584	.674	.674	.671
					.047	.007	.054					.046			
.60	11	20	21	.524	.487	.280	.741	.728	.743	.750	.717	.669	.755	.759	.756
					.045	.002	.060	.055				.047			
.70	11	20	21	.524	.485	.194	.854	.837	.854	.856	.826	.789	.865	.866	.864
					.046	.000	.062	.054				.046			
.80	11	20	19	.420	.386	.042	.909	.892	.907	.905	.876	.851	.916	.916	.915
					.047	.000	.072	.059				.046			
.90	11	20	14	.219	.199	.000	.909	.886	.904	.885	.858	.831	.911	.912	.911
					.047	.000	.088	.068	.062			.047			
.95	11	20	11	.139	.125	.000	.949	.932	.946	.906	.899	.874	.949	.952	.951
					.046	.000	.108	.081	.088			.046			
.99	11	20	6	.068	.062	.000	.984	.976	.984	.798	.922	.878	.983	.984	.983
					.047	.000	.198	.155	.356	.044		.045			
.00	11	40	21	.880	.858	.861	.861	.865	.881	.881	.864	.824	.882	.882	.882
					.046	.046	.046					.047			
.50	11	40	19	.791	.764	.694	.893	.891	.905	.906	.885	.855	.908	.910	.909
					.045	.007	.056	.055							
.60	11	40	18	.731	.700	.560	.911	.908	.920	.922	.901	.874	.924	.924	.923
						.002	.057	.055				.047			
.70	11	40	16	.598	.568	.281	.903	.898	.911	.913	.886	.861	.916	.917	.916
						.000	.059	.055				.047			
.80	11	40	14	.450	.427	.054	.928	.921	.933	.932	.906	.887	.937	.935	.935
					.047	.000	.068	.061				.047			
.90	11	40	10	.213	.200	.000	.899	.888	.902	.895	.856	.839	.905	.906	.905
						.000	.077	.068	.055						
.95	11	40	8	.142	.133	.000	.946	.938	.950	.935	.910	.899	.953	.952	.952
						.000	.089	.078	.065						
.99	11	40	4	.062	.060	.000	.930	.918	.939	.800	.852	.824	.925	.925	.924
					.047	.000	.122	.107	.168						

Table 3.3: Some power and size results for bivariate normal data. $J = 11$, $I = 10, 20, 40$. For columns 5 – 16, the upper number in a row is power, and the lower number is size (blank if between .048 and .052).

				Standard allocation		Predictor sort allocation									
							Divided by $1-\rho^2$								
				1-way		uncorr		1-way	Van der	2-way	2-way			"Theoretical"	
ρ	J	I	m	anova	KW	KW	KW	ranks	Waerden	anova	ranks	Friedman	Ancova	1-way	2-way
.00	2	10	21	.566	.555	.552	.552	.552	.553	.515	.506	.270	.558	.562	.514
					.055	.053	.053	.053				.022			
.50	2	10	21	.563	.554	.555	.667	.667	.669	.624	.607	.354	.677	.685	.634
						.031	.057	.057	.054			.023			
.60	2	10	21	.562	.554	.553	.743	.711	.731	.684	.661	.401	.742	.753	.702
					.054	.021	.062		.053		.054	.022			
.70	2	10	21	.558	.547	.559	.829	.798	.819	.772	.744	.488	.835	.841	.795
					.053	.011	.066	.053	.057			.021			
.80	2	10	20	.522	.512	.511	.905	.883	.894	.850	.822	.584	.911	.917	.882
					.054	.004	.077	.060	.062			.022			
.90	2	10	15	.322	.318	.191	.902	.872	.896	.816	.789	.549	.915	.924	.890
						.001	.091	.067	.079			.022			
.95	2	10	11	.184	.183	.030	.881	.881	.880	.742	.729	.491	.907	.923	.889
						.000	.111	.111	.112			.022	.046		
.99	2	10	6	.082	.084	.000	.913	.853	.874	.505	.584	.339	.916	.963	.940
						.000	.286	.173	.276	.043		.023	.032		
.00	2	20	21	.871	.853	.852	.852	.852	.867	.852	.838	.661	.869	.869	.851
												.042			
.50	2	20	20	.832	.813	.843	.911	.905	.918	.905	.890	.744	.921	.922	.908
						.026	.056					.043			
.60	2	20	18	.744	.724	.761	.894	.887	.900	.884	.864	.713	.904	.906	.890
						.017	.059	.055	.054			.042			
.70	2	20	17	.691	.670	.724	.921	.915	.927	.914	.892	.756	.931	.932	.919
						.008	.060	.055	.053			.042			
.80	2	20	14	.517	.500	.502	.904	.895	.910	.891	.863	.726	.916	.916	.901
						.002	.065	.060	.059			.042			
.90	2	20	11	.339	.326	.185	.929	.920	.932	.907	.880	.755	.936	.942	.930
						.000	.073	.065	.065			.041	.047		
.95	2	20	8	.192	.184	.015	.920	.908	.920	.862	.845	.710	.921	.932	.919
						.000	.092	.079	.085			.042	.046		
.99	2	20	5	.093	.091	.000	.976	.976	.971	.843	.909	.786	.976	.992	.989
						.000	.147	.147	.181			.040	.028		
.00	2	40	16	.912	.899	.902	.902	.902	.916	.906	.896	.719	.913	.912	.905
												.038			
.50	2	40	14	.819	.805	.832	.899	.899	.912	.902	.887	.716	.909	.912	.905
						.025						.040			
.60	2	40	13	.754	.739	.784	.899	.897	.913	.903	.886	.717	.911	.912	.905
						.015	.054	.053				.040			
.70	2	40	12	.677	.662	.719	.913	.911	.926	.916	.898	.744	.924	.925	.919
						.007	.055	.053				.038			
.80	2	40	10	.510	.497	.495	.897	.895	.914	.900	.875	.719	.908	.912	.905
						.002	.059	.057	.054			.038			
.90	2	40	8	.343	.332	.182	.933	.927	.945	.930	.906	.771	.941	.944	.938
						.000	.068	.062	.061			.037			
.95	2	40	6	.199	.194	.009	.931	.927	.942	.918	.895	.758	.933	.943	.937
						.000	.073	.069	.067			.039	.043		
.99	2	40	4	.101	.099	.000	.993	.992	.993	.967	.979	.912	.991	.997	.996
						.000	.109	.097	.116			.038	.024		

Table 3.4: Some power and size results for bivariate Gaussian–Weibull(.2cv) data. $J = 2$, $I = 10, 20, 40$. For columns 5 – 16, the upper number in a row is power, and the lower number is size (blank if between .048 and .052).

				Standard allocation		Predictor sort allocation									
							Divided by $1-\rho^2$							"Theoretical"	
ρ	J	I	m	1-way anova	KW	uncorr KW	KW	1-way ranks	Van der Waerden	2-way anova	2-way ranks	Friedman	Ancova	1-way	2-way
.00	5	10	21	.355	.322	.321	.321	.348	.357	.345	.339	.269	.352	.359	.350
					.043	.044	.044					.045			
.50	5	10	21	.359	.326	.248	.455	.449	.463	.452	.435	.355	.470	.471	.460
					.043	.013	.053					.043			
.60	5	10	21	.355	.323	.209	.540	.517	.534	.520	.498	.415	.540	.543	.531
					.043	.006	.060	.053				.043			
.70	5	10	21	.363	.328	.166	.657	.621	.642	.621	.591	.508	.650	.654	.641
				.047	.041	.002	.067	.054				.042			
.80	5	10	21	.360	.326	.111	.824	.783	.807	.775	.744	.661	.817	.821	.809
					.043	.000	.083	.061	.060			.044			
.90	5	10	18	.261	.234	.012	.929	.898	.917	.862	.849	.776	.923	.931	.924
					.043	.000	.105	.071	.077			.043	.045		
.95	5	10	14	.163	.143	.000	.958	.933	.946	.849	.863	.787	.947	.959	.954
				.046	.041	.000	.146	.096	.121	.046		.043	.041		
.99	5	10	8	.077	.065	.000	.986	.978	.977	.656	.834	.718	.972	.995	.993
				.047	.041	.000	.365	.280	.460	.040		.044	.020		
.00	5	20	21	.689	.663	.667	.667	.679	.701	.688	.672	.570	.693	.691	.685
					.045	.046	.046					.046			
.50	5	20	21	.692	.662	.633	.810	.808	.828	.817	.796	.711	.824	.827	.821
					.046	.014	.053					.047			
.60	5	20	21	.693	.666	.615	.873	.867	.887	.877	.858	.787	.883	.888	.884
						.006	.055					.045			
.70	5	20	20	.641	.613	.517	.919	.912	.927	.917	.900	.842	.925	.926	.923
					.046	.002	.062	.056				.045			
.80	5	20	17	.481	.455	.227	.921	.910	.927	.915	.893	.837	.926	.928	.924
					.047	.000	.069	.059	.054			.047	.047		
.90	5	20	13	.278	.260	.011	.936	.924	.940	.919	.898	.849	.934	.943	.940
					.047	.000	.083	.067	.063			.047	.045		
.95	5	20	10	.168	.157	.000	.958	.947	.960	.924	.915	.871	.950	.962	.960
					.045	.000	.096	.076	.083			.045	.040		
.99	5	20	6	.078	.071	.000	.995	.993	.993	.905	.969	.930	.984	.997	.997
				.046	.043	.000	.180	.148	.251	.046		.046	.016		
.00	5	40	19	.908	.894	.897	.897	.900	.917	.908	.898	.825	.910	.908	.906
.50	5	40	17	.823	.803	.801	.914	.914	.931	.922	.909	.849	.924	.923	.922
						.014									
.60	5	40	16	.761	.739	.720	.921	.920	.936	.930	.913	.857	.932	.931	.930
						.006	.055	.054				.047			
.70	5	40	14	.624	.600	.496	.902	.898	.918	.907	.887	.831	.911	.915	.913
					.047	.001	.060	.056							
.80	5	40	12	.468	.450	.213	.910	.904	.924	.912	.889	.840	.916	.919	.917
						.000	.062	.057				.046	.047		
.90	5	40	9	.257	.244	.005	.912	.904	.924	.907	.880	.837	.911	.920	.918
						.000	.071	.064	.056				.046		
.95	5	40	7	.155	.149	.000	.937	.930	.949	.925	.904	.867	.930	.943	.942
						.000	.082	.074	.068				.040		
.99	5	40	4	.072	.071	.000	.974	.970	.977	.916	.939	.904	.924	.978	.977
					.047	.000	.114	.102	.135	.047		.047	.014		

Table 3.5: Some power and size results for bivariate Gaussian–Weibull(.2cv) data. $J = 5$, $I = 10, 20, 40$. For columns 5 – 16, the upper number in a row is power, and the lower number is size (blank if between .048 and .052).

				Standard allocation		Predictor sort allocation									
							Divided by $1-\rho^2$								
ρ	J	I	m	1-way anova	KW	uncorr KW	KW	1-way ranks	Van der Waerden	2-way anova	2-way ranks	Friedman	Ancova	"Theoretical" 1-way	"Theoretical" 2-way
.00	11	10	21	.243	.215	.214	.214	.239	.249	.241	.238	.196	.242	.245	.243
				.047	.041	.044	.044	.053		.053	.053	.043			
.50	11	10	21	.247	.215	.099	.325	.314	.324	.321	.310	.265	.327	.328	.325
					.042	.007	.054					.041			
.60	11	10	21	.245	.215	.061	.396	.365	.380	.373	.356	.310	.383	.387	.384
					.042	.002	.061					.042			
.70	11	10	21	.243	.214	.025	.510	.456	.477	.463	.438	.391	.481	.488	.484
					.041	.000	.069					.042			
.80	11	10	21	.246	.215	.006	.705	.634	.663	.627	.601	.551	.662	.670	.666
					.043	.000	.086	.057	.054			.044			
.90	11	10	20	.226	.197	.000	.928	.887	.911	.853	.845	.804	.906	.918	.916
					.044	.000	.117	.070	.079			.042	.045		
.95	11	10	15	.128	.110	.000	.948	.909	.937	.802	.834	.780	.910	.935	.932
					.041	.000	.163	.098	.142	.047	.047	.041	.038		
.99	11	10	9	.073	.063	.000	.996	.991	.995	.653	.878	.793	.963	.996	.996
					.043	.000	.456	.333	.702	.037		.040	.011		
.00	11	20	21	.522	.493	.492	.492	.507	.535	.518	.505	.450	.521	.520	.518
					.047	.044	.044			.047		.044			
.50	11	20	21	.517	.489	.355	.662	.656	.684	.668	.647	.597	.673	.674	.671
					.045	.006			.047			.045			
.60	11	20	21	.521	.491	.289	.749	.737	.763	.750	.725	.683	.756	.759	.756
					.046	.002	.060	.055				.046			
.70	11	20	21	.522	.493	.201	.860	.845	.869	.856	.832	.801	.863	.866	.864
					.046	.000	.066	.057							
.80	11	20	19	.423	.395	.045	.913	.896	.917	.903	.879	.861	.912	.916	.915
					.045	.000	.072	.058				.045			
.90	11	20	15	.250	.232	.000	.951	.936	.953	.931	.915	.900	.942	.953	.952
					.046	.000	.088	.068	.063			.046	.043		
.95	11	20	11	.138	.125	.000	.954	.937	.955	.909	.903	.886	.931	.952	.951
					.046	.000	.108	.082	.088			.047	.036		
.99	11	20	6	.066	.061	.000	.985	.978	.987	.813	.926	.888	.904	.984	.983
					.045	.000	.194	.151	.355	.043	.047	.046	.008		
.00	11	40	21	.878	.863	.866	.866	.869	.892	.880	.869	.826	.881	.882	.882
.50	11	40	19	.788	.767	.701	.900	.898	.920	.909	.894	.865	.911	.910	.909
						.006	.053								
.60	11	40	18	.728	.705	.573	.916	.913	.931	.921	.905	.883	.923	.924	.923
					.047	.002	.055								
.70	11	40	16	.600	.575	.294	.910	.905	.925	.915	.894	.874	.916	.917	.916
					.047	.000	.061	.057							
.80	11	40	14	.450	.433	.058	.929	.923	.939	.930	.907	.895	.930	.935	.935
						.000	.068	.062							
.90	11	40	11	.259	.246	.000	.958	.952	.967	.954	.933	.928	.955	.962	.961
					.047	.000	.078	.068	.054			.045	.042		
.95	11	40	8	.139	.132	.000	.952	.944	.959	.937	.917	.912	.933	.952	.952
						.000	.090	.078	.065			.046	.035		
.99	11	40	5	.073	.071	.000	.998	.997	.998	.983	.991	.988	.979	.998	.998
						.000	.120	.104	.163	.046		.047	.006		

Table 3.6: Some power and size results for bivariate Gaussian–Weibull(.2cv) data. $J = 11$, $I = 10, 20, 40$. For columns 5 – 16, the upper number in a row is power, and the lower number is size (blank if between .048 and .052).

				Standard allocation		Predictor sort allocation									
							Divided by $1-\rho^2$								
				1-way		uncorr		1-way	Van der	2-way	2-way			"Theoretical"	
ρ	J	I	m	anova	KW	KW	KW	ranks	Waerden	anova	ranks	Friedman	Ancova	1-way	2-way
.00	2	10	21	.561	.542	.538	.538	.538	.542	.512	.495	.260	.557	.562	.514
					.054	.053	.053	.053				.021			
.50	2	10	21	.559	.537	.531	.643	.643	.650	.617	.584	.338	.672	.685	.634
					.054	.029	.055	.055	.053	.053	.054	.024			
.60	2	10	21	.559	.537	.539	.730	.695	.721	.688	.649	.397	.749	.753	.702
						.020	.064		.054		.053	.022			
.70	2	10	21	.558	.539	.542	.815	.786	.810	.773	.731	.476	.837	.841	.795
						.011	.066		.057			.022			
.80	2	10	20	.518	.496	.487	.895	.869	.886	.853	.808	.572	.913	.917	.882
					.055	.004	.078	.060	.063		.053	.022			
.90	2	10	15	.311	.301	.176	.892	.860	.886	.824	.776	.544	.921	.924	.890
						.000	.090	.066	.078			.020			
.95	2	10	11	.185	.182	.027	.874	.874	.873	.749	.715	.478	.914	.923	.889
					.054	.000	.112	.112	.110			.022			
.99	2	10	6	.085	.085	.000	.907	.843	.868	.515	.571	.332	.954	.963	.940
				.053	.056	.000	.288	.174	.274	.042		.020	.047		
.00	2	20	21	.868	.838	.836	.836	.836	.855	.848	.821	.643	.865	.869	.851
												.043			
.50	2	20	20	.835	.804	.831	.903	.896	.912	.908	.882	.724	.923	.922	.908
						.025	.053					.041			
.60	2	20	18	.742	.704	.746	.884	.877	.892	.888	.854	.700	.905	.906	.890
						.016	.058	.053				.041			
.70	2	20	17	.694	.658	.700	.911	.905	.921	.915	.882	.744	.932	.932	.919
						.009	.061	.056	.053			.041			
.80	2	20	14	.515	.482	.469	.892	.883	.901	.894	.850	.711	.916	.916	.901
						.002	.063	.057	.055			.041			
.90	2	20	11	.333	.309	.173	.919	.910	.927	.911	.869	.744	.940	.942	.930
						.000	.076	.067	.066			.040			
.95	2	20	8	.189	.177	.012	.908	.895	.911	.871	.830	.699	.928	.932	.919
						.000	.095	.081	.087			.042			
.99	2	20	5	.094	.090	.000	.972	.972	.968	.868	.897	.775	.989	.992	.989
						.000	.147	.147	.179	.047		.041	.047		
.00	2	40	16	.913	.889	.888	.888	.888	.905	.903	.879	.696	.909	.912	.905
												.040			
.50	2	40	14	.821	.787	.818	.889	.889	.907	.906	.877	.700	.913	.912	.905
						.026	.054	.054				.039			
.60	2	40	13	.754	.720	.759	.886	.884	.903	.903	.870	.700	.911	.912	.905
						.017	.056	.054				.039			
.70	2	40	12	.679	.642	.692	.904	.902	.920	.918	.885	.726	.926	.925	.919
						.007	.056	.055				.040			
.80	2	40	10	.509	.477	.465	.885	.881	.905	.900	.859	.705	.911	.912	.905
						.002	.058	.056	.053			.038			
.90	2	40	8	.335	.314	.162	.922	.916	.937	.932	.893	.757	.944	.944	.938
						.000	.066	.060	.058			.040			
.95	2	40	6	.196	.183	.008	.920	.916	.935	.923	.880	.747	.942	.943	.937
					.047	.000	.074	.070	.067			.037	.047		
.99	2	40	4	.103	.099	.000	.992	.990	.992	.976	.974	.907	.996	.997	.996
						.000	.106	.095	.115			.039	.044		

Table 3.7: Some power and size results for bivariate Gaussian–Weibull(.3cv) data. $J = 2$, $I = 10, 20, 40$. For columns 5 – 16, the upper number in a row is power, and the lower number is size (blank if between .048 and .052).

				Standard allocation		Predictor sort allocation									
							Divided by $1-\rho^2$								
ρ	J	I	m	1-way anova	KW	uncorr KW	KW	1-way ranks	Van der Waerden	2-way anova	2-way ranks	Friedman	Ancova	"Theoretical" 1-way	"Theoretical" 2-way
.00	5	10	21	.362	.314	.312	.312	.338	.349	.347	.330	.262	.356	.359	.350
					.042	.043	.043					.043			
.50	5	10	21	.358	.313	.233	.441	.433	.449	.452	.418	.345	.468	.471	.460
					.041	.015	.056	.054	.053			.044			
.60	5	10	21	.357	.312	.197	.524	.500	.522	.521	.480	.396	.541	.543	.531
					.041	.006	.062	.054				.043			
.70	5	10	21	.354	.309	.147	.636	.598	.623	.613	.567	.485	.646	.654	.641
					.043	.002	.067	.055				.044			
.80	5	10	21	.360	.315	.095	.808	.762	.793	.776	.724	.644	.818	.821	.809
					.042	.000	.081	.060	.058			.044			
.90	5	10	18	.261	.226	.010	.919	.884	.908	.870	.832	.764	.928	.931	.924
					.044	.000	.105	.071	.077			.043			
.95	5	10	13	.146	.123	.000	.919	.879	.902	.797	.782	.697	.922	.923	.915
					.040	.000	.146	.098	.123			.044			
.99	5	10	7	.069	.059	.000	.954	.932	.941	.542	.670	.552	.956	.967	.962
					.041	.000	.366	.284	.462	.044		.042	.046		
.00	5	20	21	.693	.646	.640	.640	.653	.683	.683	.646	.549	.690	.691	.685
					.047										
.50	5	20	21	.694	.644	.606	.791	.789	.817	.821	.780	.696	.826	.827	.821
					.047	.014	.053								
.60	5	20	21	.689	.640	.588	.861	.855	.880	.881	.843	.770	.888	.888	.884
					.046	.006	.056	.053				.046			
.70	5	20	20	.641	.592	.482	.904	.896	.917	.916	.882	.825	.923	.926	.923
					.046	.001	.061	.055							
.80	5	20	17	.481	.438	.204	.908	.895	.918	.917	.876	.825	.927	.928	.924
					.047	.000	.069	.058	.054			.045			
.90	5	20	13	.277	.249	.008	.927	.912	.934	.922	.882	.838	.941	.943	.940
					.045	.000	.083	.068	.065			.046			
.95	5	20	9	.137	.124	.000	.890	.869	.896	.854	.810	.754	.902	.904	.900
				.047	.045	.000	.100	.080	.086			.047			
.99	5	20	5	.070	.065	.000	.945	.932	.945	.768	.824	.742	.944	.956	.953
						.000	.180	.149	.248	.046		.045	.046		
.00	5	40	19	.909	.878	.880	.880	.883	.907	.906	.882	.804	.908	.908	.906
						.047	.047								
.50	5	40	17	.821	.780	.775	.899	.898	.922	.922	.892	.829	.924	.923	.922
						.015	.055	.055							
.60	5	40	16	.759	.720	.691	.909	.907	.928	.930	.900	.843	.932	.931	.930
				.047	.047	.006	.055	.054							
.70	5	40	14	.622	.577	.463	.893	.888	.913	.913	.876	.821	.917	.915	.913
						.001	.057	.054				.047			
.80	5	40	12	.465	.428	.189	.897	.891	.915	.914	.873	.826	.919	.919	.917
						.000	.063	.058	.054						
.90	5	40	9	.252	.230	.004	.899	.891	.918	.911	.863	.825	.919	.920	.918
						.000	.073	.067	.057						
.95	5	40	7	.156	.143	.000	.925	.917	.940	.927	.888	.853	.940	.943	.942
						.000	.082	.073	.067						
.99	5	40	4	.072	.068	.000	.967	.963	.973	.931	.926	.891	.972	.978	.977
						.000	.113	.101	.133				.043		

Table 3.8: Some power and size results for bivariate Gaussian–Weibull(.3cv) data. $J = 5$, $I = 10, 20, 40$. For columns 5 – 16, the upper number in a row is power, and the lower number is size (blank if between .048 and .052).

				Standard allocation		Predictor sort allocation									
							Divided by $1-\rho^2$								
				1-way		uncorr		1-way	Van der	2-way	2-way			"Theoretical"	
ρ	J	I	m	anova	KW	KW	KW	ranks	Waerden	anova	ranks	Friedman	Ancova	1-way	2-way
.00	11	10	21	.243	.203	.204	.204	.230	.242	.242	.228	.188	.245	.245	.243
					.040	.042	.042					.043			
.50	11	10	21	.243	.205	.091	.310	.300	.312	.321	.295	.252	.328	.328	.325
					.042	.007	.054					.043			
.60	11	10	21	.245	.205	.053	.378	.348	.365	.374	.342	.298	.387	.387	.384
					.042	.002	.060		.047			.043			
.70	11	10	21	.246	.205	.023	.491	.439	.465	.465	.423	.380	.487	.488	.484
					.043	.000	.072	.055				.042			
.80	11	10	21	.243	.204	.004	.676	.605	.641	.630	.572	.529	.666	.670	.666
					.040	.000	.087	.058	.056			.041			
.90	11	10	20	.222	.185	.000	.917	.871	.904	.861	.826	.788	.916	.918	.916
					.041	.000	.117	.071	.078			.042			
.95	11	10	15	.133	.109	.000	.937	.896	.930	.824	.816	.763	.929	.935	.932
					.042	.000	.165	.100	.145			.042			
.99	11	10	8	.067	.055	.000	.982	.965	.984	.564	.735	.631	.965	.976	.975
					.040	.000	.460	.338	.708	.042		.042	.044		
.00	11	20	21	.516	.462	.472	.472	.487	.521	.521	.484	.431	.524	.520	.518
				.047	.044							.047			
.50	11	20	21	.520	.468	.328	.633	.627	.661	.666	.618	.571	.670	.674	.671
					.047	.007						.045			
.60	11	20	21	.517	.465	.259	.723	.710	.745	.749	.697	.659	.756	.759	.756
					.045	.002	.059	.054				.045			
.70	11	20	21	.520	.469	.178	.841	.824	.857	.858	.809	.782	.866	.866	.864
					.046	.000	.065	.056				.047			
.80	11	20	19	.417	.373	.036	.900	.881	.906	.904	.862	.842	.915	.916	.915
					.046	.000	.072	.059				.047			
.90	11	20	14	.214	.188	.000	.900	.876	.906	.890	.841	.829	.910	.912	.911
					.044	.000	.091	.070	.062			.046			
.95	11	20	11	.140	.124	.000	.946	.927	.949	.918	.888	.873	.949	.952	.951
					.046	.000	.107	.081	.088			.046			
.99	11	20	6	.069	.062	.000	.982	.974	.985	.854	.912	.875	.978	.984	.983
					.047	.000	.196	.153	.354	.046		.044	.040		
.00	11	40	21	.885	.849	.848	.848	.853	.885	.885	.851	.807	.885	.882	.882
.50	11	40	19	.787	.742	.666	.881	.879	.908	.909	.873	.843	.910	.910	.909
						.007	.053								
.60	11	40	18	.729	.683	.537	.900	.896	.922	.923	.888	.868	.925	.924	.923
						.002	.058	.056							
.70	11	40	16	.594	.549	.258	.893	.886	.913	.914	.873	.855	.915	.917	.916
						.000	.061	.057				.047			
.80	11	40	14	.449	.409	.048	.916	.908	.931	.930	.891	.884	.935	.935	.935
						.000	.068	.061				.047			
.90	11	40	10	.212	.192	.000	.887	.875	.903	.897	.842	.836	.903	.906	.905
						.000	.076	.067	.054						
.95	11	40	8	.137	.127	.000	.939	.929	.951	.938	.898	.897	.948	.952	.952
					.047	.000	.087	.076	.066				.047		
.99	11	40	4	.063	.060	.000	.918	.907	.939	.839	.834	.820	.903	.925	.924
						.000	.122	.105	.163				.040		

Table 3.9: Some power and size results for bivariate Gaussian–Weibull(.3cv) data. $J = 11$, $I = 10, 20, 40$. For columns 5 – 16, the upper number in a row is power, and the lower number is size (blank if between .048 and .052).

				Standard allocation		Predictor sort allocation									
							Divided by $1-\rho^2$								
				1-way		uncorr		1-way	Van der	2-way	2-way			"Theoretical"	
ρ	J	I	m	anova	KW	KW	KW	ranks	Waerden	anova	ranks	Friedman	Ancova	1-way	2-way
.00	2	10	21	.561	.542	.542	.542	.542	.549	.513	.497	.261	.558	.562	.514
						.053	.053	.053	.053		.053	.022			
.50	2	10	21	.558	.537	.540	.656	.656	.663	.622	.590	.342	.678	.685	.634
						.028	.054	.054				.022			
.60	2	10	21	.556	.536	.540	.734	.700	.728	.682	.649	.395	.743	.753	.702
						.021	.066	.054	.056		.053	.023			
.70	2	10	21	.556	.541	.546	.821	.790	.818	.774	.734	.480	.839	.841	.795
					.054	.012	.065		.056			.022			
.80	2	10	20	.514	.498	.494	.898	.874	.892	.853	.813	.580	.910	.917	.882
						.004	.076	.059	.061			.022			
.90	2	10	15	.316	.306	.176	.897	.864	.894	.824	.781	.548	.915	.924	.890
					.053	.000	.091	.067	.080			.021	.047		
.95	2	10	11	.182	.180	.027	.879	.879	.881	.752	.721	.490	.908	.923	.889
						.000	.112	.112	.112			.022	.047		
.99	2	10	6	.083	.085	.000	.908	.847	.872	.520	.576	.338	.919	.963	.940
					.054	.000	.283	.172	.274	.043		.021	.037		
.00	2	20	21	.870	.843	.844	.844	.844	.867	.854	.828	.649	.871	.869	.851
												.041			
.50	2	20	20	.835	.805	.830	.902	.895	.915	.904	.879	.729	.920	.922	.908
				.047	.047	.026	.055		.053			.042			
.60	2	20	18	.745	.709	.746	.886	.879	.898	.885	.855	.700	.903	.906	.890
						.015	.057	.053				.041			
.70	2	20	17	.691	.656	.706	.912	.907	.924	.914	.883	.745	.931	.932	.919
				.053		.009	.061	.057	.054			.042			
.80	2	20	14	.519	.487	.473	.893	.884	.908	.891	.851	.715	.912	.916	.901
						.002	.064	.058	.056			.042			
.90	2	20	11	.335	.312	.172	.921	.912	.930	.905	.868	.751	.936	.942	.930
						.000	.079	.070	.068			.042			
.95	2	20	8	.190	.177	.012	.913	.900	.923	.873	.835	.709	.924	.932	.919
						.000	.092	.079	.083			.041	.044		
.99	2	20	5	.096	.091	.000	.973	.973	.971	.864	.898	.780	.974	.992	.989
				.053		.000	.151	.151	.182	.047		.044	.033		
.00	2	40	16	.915	.893	.893	.893	.893	.914	.908	.886	.703	.914	.912	.905
												.038			
.50	2	40	14	.817	.786	.818	.888	.888	.910	.903	.876	.709	.909	.912	.905
						.027	.053	.053				.038			
.60	2	40	13	.756	.724	.766	.891	.889	.912	.902	.874	.709	.910	.912	.905
						.016	.055	.053				.038	.047		
.70	2	40	12	.682	.649	.692	.903	.901	.925	.917	.886	.732	.925	.925	.919
						.007	.055	.053				.037			
.80	2	40	10	.513	.482	.468	.886	.883	.912	.900	.863	.714	.909	.912	.905
						.002	.058	.056	.054			.039			
.90	2	40	8	.340	.320	.164	.921	.916	.942	.929	.892	.765	.939	.944	.938
						.000	.067	.061	.060			.038	.047		
.95	2	40	6	.202	.188	.008	.921	.917	.941	.920	.882	.756	.935	.943	.937
						.000	.075	.070	.069			.038	.044		
.99	2	40	4	.104	.100	.000	.991	.989	.993	.972	.975	.910	.990	.997	.996
						.000	.109	.097	.119			.038	.030		

Table 3.10: Some power and size results for bivariate Gaussian–Weibull(.4cv) data. $J = 2$, $I = 10, 20, 40$. For columns 5 – 16, the upper number in a row is power, and the lower number is size (blank if between .048 and .052).

				Standard allocation		Predictor sort allocation									
							Divided by $1-\rho^2$								
ρ	J	I	m	1-way anova	KW	uncorr KW	KW	1-way ranks	Van der Waerden	2-way anova	2-way ranks	Friedman	Ancova	"Theoretical" 1-way	"Theoretical" 2-way
.00	5	10	21	.360	.318	.311	.311	.337	.352	.347	.330	.261	.354	.359	.350
					.042	.046	.046	.054	.053		.054	.045			
.50	5	10	21	.357	.313	.233	.442	.434	.457	.449	.420	.347	.467	.471	.460
					.042	.014	.053					.043			
.60	5	10	21	.359	.314	.198	.522	.500	.526	.518	.480	.404	.539	.543	.531
					.042	.006	.058					.043			
.70	5	10	21	.362	.319	.154	.640	.600	.634	.619	.574	.496	.647	.654	.641
					.043	.002	.068	.055				.043			
.80	5	10	21	.359	.313	.096	.809	.764	.800	.772	.724	.656	.809	.821	.809
					.044	.000	.078	.057	.057		.047	.043	.046		
.90	5	10	18	.260	.227	.010	.919	.887	.913	.869	.835	.771	.921	.931	.924
				.047	.041	.000	.107	.073	.080			.044	.046		
.95	5	10	13	.144	.125	.000	.917	.878	.910	.797	.782	.705	.904	.923	.915
					.042	.000	.150	.099	.125			.043	.045		
.99	5	10	8	.076	.066	.000	.985	.975	.978	.693	.820	.708	.972	.995	.993
				.047	.041	.000	.365	.281	.465	.042		.044	.028		
.00	5	20	21	.691	.646	.644	.644	.656	.693	.683	.651	.548	.688	.691	.685
					.045	.046	.046					.044			
.50	5	20	21	.691	.646	.613	.794	.791	.825	.819	.782	.700	.826	.827	.821
					.047	.013	.053								
.60	5	20	21	.692	.646	.590	.859	.853	.884	.877	.841	.776	.884	.888	.884
					.045	.006	.057	.054							
.70	5	20	20	.638	.594	.492	.910	.901	.927	.919	.888	.837	.927	.926	.923
						.002	.061	.055				.046			
.80	5	20	17	.475	.434	.207	.909	.897	.924	.914	.878	.834	.923	.928	.924
					.046	.000	.068	.058	.054			.046			
.90	5	20	13	.277	.252	.007	.930	.916	.944	.922	.888	.848	.935	.943	.940
					.047	.000	.079	.064	.063			.045	.046		
.95	5	20	10	.167	.150	.000	.951	.939	.960	.926	.903	.866	.951	.962	.960
					.047	.000	.099	.079	.085			.045	.043		
.99	5	20	6	.083	.074	.000	.993	.991	.992	.921	.962	.924	.982	.997	.997
					.046	.000	.180	.151	.248	.046		.047	.024		
.00	5	40	19	.910	.881	.881	.881	.885	.916	.907	.882	.808	.909	.908	.906
					.047										
.50	5	40	17	.817	.780	.776	.901	.900	.928	.919	.894	.834	.921	.923	.922
				.046	.046	.014									
.60	5	40	16	.760	.721	.695	.910	.907	.936	.929	.900	.850	.931	.931	.930
						.006	.057	.055							
.70	5	40	14	.618	.577	.470	.893	.889	.919	.910	.877	.825	.913	.915	.913
						.001	.058	.055							
.80	5	40	12	.463	.427	.193	.896	.889	.924	.911	.873	.834	.915	.919	.917
						.000	.065	.061							
.90	5	40	9	.253	.234	.004	.902	.893	.927	.911	.866	.835	.915	.920	.918
					.047	.000	.070	.063	.055				.045		
.95	5	40	7	.150	.139	.000	.925	.917	.948	.927	.888	.863	.931	.943	.942
				.047	.044	.000	.080	.071	.066				.042		
.99	5	40	4	.069	.067	.000	.968	.964	.978	.927	.929	.903	.931	.978	.977
					.047	.000	.113	.100	.136				.018		

Table 3.11: Some power and size results for bivariate Gaussian–Weibull(.4cv) data. $J = 5$, $I = 10, 20, 40$. For columns 5 – 16, the upper number in a row is power, and the lower number is size (blank if between .048 and .052).

				Standard allocation		Predictor sort allocation									
							Divided by $1-\rho^2$								
				1-way		uncorr		1-way	Van der	2-way	2-way			"Theoretical"	
ρ	J	I	m	anova	KW	KW	KW	ranks	Waerden	anova	ranks	Friedman	Ancova	1-way	2-way
.00	11	10	21	.245	.204	.204	.204	.229	.246	.241	.228	.190	.243	.245	.243
					.042	.042	.042					.043			
.50	11	10	21	.243	.203	.092	.308	.298	.317	.316	.293	.252	.322	.328	.325
					.042	.006	.054					.040			
.60	11	10	21	.246	.204	.055	.382	.351	.376	.373	.344	.302	.385	.387	.384
					.040	.002	.061					.042			
.70	11	10	21	.245	.204	.023	.493	.441	.473	.462	.424	.384	.483	.488	.484
					.043	.000	.070	.053				.042			
.80	11	10	21	.244	.205	.004	.681	.607	.657	.628	.579	.544	.662	.670	.666
					.042	.000	.087	.056	.055			.042			
.90	11	10	20	.218	.183	.000	.920	.871	.913	.860	.828	.798	.908	.918	.916
					.041	.000	.117	.072	.081			.042	.047		
.95	11	10	15	.129	.108	.000	.943	.903	.939	.823	.817	.775	.915	.935	.932
					.041	.000	.165	.101	.143			.043	.043		
.99	11	10	9	.072	.059	.000	.995	.990	.995	.700	.866	.784	.967	.996	.996
					.041	.000	.456	.336	.704	.040		.041	.021		
.00	11	20	21	.521	.472	.467	.467	.483	.530	.515	.481	.428	.518	.520	.518
					.047	.046	.046					.046			
.50	11	20	21	.519	.472	.336	.640	.634	.682	.669	.625	.584	.674	.674	.671
					.046	.007	.056	.054				.047			
.60	11	20	21	.522	.475	.266	.731	.717	.763	.749	.704	.671	.756	.759	.756
					.047	.002	.057					.045			
.70	11	20	21	.517	.468	.178	.842	.826	.867	.856	.811	.790	.862	.866	.864
					.047	.000	.062	.054				.043	.047		
.80	11	20	19	.415	.374	.038	.901	.884	.918	.905	.865	.853	.913	.916	.915
					.044	.000	.074	.059				.045			
.90	11	20	15	.251	.221	.000	.942	.927	.954	.936	.904	.900	.945	.953	.952
					.046	.000	.087	.067	.059			.047	.044		
.95	11	20	11	.138	.125	.000	.943	.925	.955	.914	.887	.880	.934	.952	.951
					.045	.000	.108	.081	.088			.045	.037		
.99	11	20	6	.068	.062	.000	.982	.975	.988	.850	.915	.886	.918	.984	.983
					.045	.000	.197	.153	.356	.046		.046	.015		
.00	11	40	21	.884	.849	.850	.850	.855	.896	.885	.854	.809	.886	.882	.882
.50	11	40	19	.786	.745	.672	.885	.883	.920	.910	.878	.852	.911	.910	.909
				.047	.047	.007						.047			
.60	11	40	18	.731	.686	.539	.902	.898	.930	.919	.890	.872	.921	.924	.923
						.002	.057	.055							
.70	11	40	16	.594	.547	.266	.894	.888	.924	.912	.874	.863	.914	.917	.916
						.000	.062	.059							
.80	11	40	14	.451	.415	.051	.916	.909	.940	.927	.892	.890	.929	.935	.935
					.047	.000	.070	.063	.054						
.90	11	40	11	.264	.240	.000	.949	.942	.966	.954	.921	.928	.955	.962	.961
						.000	.078	.069	.055				.044		
.95	11	40	8	.137	.126	.000	.943	.934	.961	.939	.903	.910	.937	.952	.952
						.000	.087	.075	.065				.038		
.99	11	40	5	.074	.070	.000	.996	.995	.998	.985	.988	.987	.982	.998	.998
						.000	.119	.103	.164	.046	.047	.047	.010		

Table 3.12: Some power and size results for bivariate Gaussian–Weibull(.4cv) data. $J = 11$, $I = 10, 20, 40$. For columns 5 – 16, the upper number in a row is power, and the lower number is size (blank if between .048 and .052).

				Standard allocation		Predictor sort allocation									
							Divided by $1-\rho^2$								
				1-way		uncorr		1-way	Van der	2-way	2-way			"Theoretical"	
ρ	J	I	m	anova	KW	KW	KW	ranks	Waerden	anova	ranks	Friedman	Ancova	1-way	2-way
.00	2	10	21	.544	.508	.508	.508	.508	.551	.497	.463	.239	.543	.562	.514
				.054	.055					.054	.053	.021			
.50	2	10	21	.545	.505	.503	.620	.620	.666	.612	.560	.328	.661	.685	.634
						.028	.055	.055		.053		.022			
.60	2	10	21	.546	.506	.503	.704	.667	.739	.677	.617	.385	.731	.753	.702
					.054	.020	.063		.054			.021			
.70	2	10	21	.545	.507	.506	.789	.756	.826	.765	.698	.479	.819	.841	.795
						.011	.065		.054			.020	.047		
.80	2	10	21	.549	.510	.504	.904	.881	.927	.879	.815	.620	.920	.941	.911
				.053	.053	.004	.075	.058	.062			.021	.047		
.90	2	10	16	.341	.324	.197	.918	.892	.949	.875	.811	.637	.916	.953	.927
				.053	.053	.001	.091	.068	.079			.022	.042		
.95	2	10	13	.233	.227	.046	.953	.953	.977	.903	.846	.680	.937	.982	.968
					.053	.000	.112	.112	.114	.047		.022	.036		
.99	2	10	9	.128	.128	.000	.997	.992	.998	.948	.926	.774	.944	1.000	1.000
						.000	.285	.173	.276	.046		.021	.015		
.00	2	20	21	.872	.812	.813	.813	.813	.882	.855	.797	.607	.874	.869	.851
												.039			
.50	2	20	20	.834	.770	.798	.880	.873	.930	.907	.852	.718	.922	.922	.908
						.025	.054					.042			
.60	2	20	19	.792	.728	.769	.897	.891	.943	.915	.865	.751	.930	.934	.921
						.016	.057					.040			
.70	2	20	17	.687	.628	.664	.893	.886	.944	.906	.851	.755	.921	.932	.919
						.008	.061	.056	.054			.042			
.80	2	20	15	.568	.520	.537	.919	.912	.964	.926	.878	.808	.937	.949	.938
						.002	.062	.057	.055			.040	.045		
.90	2	20	11	.329	.305	.155	.918	.909	.968	.902	.856	.804	.906	.942	.930
						.000	.076	.067	.066			.042	.041		
.95	2	20	9	.227	.215	.020	.962	.956	.990	.943	.904	.866	.930	.977	.970
					.047	.000	.092	.078	.085			.043	.031		
.99	2	20	5	.094	.090	.000	.978	.978	.995	.937	.906	.857	.760	.992	.989
						.000	.148	.148	.179			.043	.008		
.00	2	40	16	.911	.868	.870	.870	.870	.943	.908	.862	.670	.915	.912	.905
												.038			
.50	2	40	14	.817	.764	.798	.873	.873	.949	.901	.860	.718	.909	.912	.905
						.023						.039			
.60	2	40	13	.754	.700	.746	.878	.875	.953	.901	.858	.740	.907	.912	.905
						.017	.053					.037			
.70	2	40	12	.682	.635	.678	.894	.892	.964	.912	.872	.788	.918	.925	.919
						.007	.055	.054				.037			
.80	2	40	11	.596	.555	.585	.933	.931	.984	.944	.912	.859	.946	.957	.953
						.002	.055	.053		.047	.047	.037	.045		
.90	2	40	8	.337	.317	.159	.923	.916	.985	.919	.885	.856	.909	.944	.938
						.000	.066	.060	.059			.038	.039		
.95	2	40	6	.192	.185	.006	.927	.923	.989	.913	.883	.863	.872	.943	.937
						.000	.074	.070	.070			.041	.028		
.99	2	40	4	.099	.098	.000	.994	.993	1.000	.989	.977	.964	.841	.997	.996
						.000	.106	.095	.117			.039	.003		

Table 3.13: Some power and size results for bivariate Gaussian–uniform data. $J = 2$, $I = 10, 20, 40$. For columns 5 – 16, the upper number in a row is power, and the lower number is size (blank if between .048 and .052).

				Standard allocation		Predictor sort allocation									
							Divided by $1-\rho^2$							"Theoretical"	
ρ	J	I	m	1-way anova	KW	uncorr KW	KW	1-way ranks	Van der Waerden	2-way anova	2-way ranks	Friedman	Ancova	1-way	2-way
.00	5	10	21	.352	.295	.291	.291	.317	.397	.337	.312	.248	.345	.359	.350
					.043	.043	.043					.043			
.50	5	10	21	.352	.296	.217	.422	.415	.518	.441	.401	.357	.450	.471	.460
					.042	.015	.056	.054	.053	.054	.053	.046	.053		
.60	5	10	21	.352	.296	.180	.505	.482	.600	.511	.462	.431	.524	.543	.531
					.041	.007	.060	.053				.044			
.70	5	10	21	.352	.297	.133	.621	.584	.712	.609	.551	.539	.624	.654	.641
					.043	.002	.069	.056	.054			.044			
.80	5	10	21	.354	.300	.077	.801	.754	.872	.772	.708	.708	.783	.821	.809
					.041	.000	.081	.059	.059			.045	.047		
.90	5	10	19	.288	.242	.009	.947	.919	.979	.916	.863	.876	.912	.956	.951
					.042	.000	.105	.071	.076	.047		.042	.038		
.95	5	10	15	.182	.153	.000	.975	.959	.993	.935	.896	.903	.913	.981	.978
				.047	.040	.000	.146	.098	.119			.043	.028		
.99	5	10	9	.085	.072	.000	.997	.995	1.000	.949	.928	.909	.767	.999	.999
					.040	.000	.370	.283	.466	.047		.045	.005		
.00	5	20	21	.686	.619	.620	.620	.632	.778	.682	.625	.521	.687	.691	.685
				.047	.044	.047	.047					.045			
.50	5	20	21	.689	.619	.575	.771	.768	.894	.810	.753	.713	.817	.827	.821
					.046	.014	.053					.045			
.60	5	20	21	.688	.620	.559	.844	.838	.942	.872	.822	.806	.878	.888	.884
						.006	.058	.054				.047			
.70	5	20	20	.635	.571	.457	.894	.885	.966	.911	.865	.871	.913	.926	.923
					.046	.002	.063	.057	.053						
.80	5	20	17	.471	.419	.188	.903	.890	.974	.906	.862	.891	.906	.928	.924
					.045	.000	.068	.057	.055			.046	.044		
.90	5	20	13	.272	.246	.007	.930	.916	.988	.916	.879	.923	.894	.943	.940
					.046	.000	.082	.067	.064				.035		
.95	5	20	10	.164	.149	.000	.956	.946	.996	.934	.905	.944	.872	.962	.960
					.045	.000	.099	.079	.084			.046	.022		
.99	5	20	6	.079	.072	.000	.996	.994	1.000	.980	.967	.977	.632	.997	.997
					.046	.000	.180	.149	.247			.046	.001		
.00	5	40	19	.910	.867	.864	.864	.868	.966	.905	.866	.785	.908	.908	.906
					.047										
.50	5	40	17	.821	.768	.757	.888	.887	.977	.918	.880	.859	.918	.923	.922
					.047	.015	.055	.054							
.60	5	40	16	.762	.709	.678	.903	.900	.982	.924	.889	.891	.925	.931	.930
					.047	.006	.055	.054							
.70	5	40	14	.619	.569	.452	.887	.883	.980	.903	.865	.895	.902	.915	.913
						.002	.056	.054				.047	.047		
.80	5	40	12	.460	.423	.188	.899	.892	.986	.905	.871	.922	.898	.919	.917
					.047	.000	.064	.059	.053				.044		
.90	5	40	9	.253	.235	.003	.906	.897	.992	.896	.867	.944	.864	.920	.918
						.000	.071	.064	.056			.046	.033		
.95	5	40	7	.156	.149	.000	.937	.930	.998	.918	.897	.964	.828	.943	.942
						.000	.081	.071	.067				.018		
.99	5	40	4	.073	.070	.000	.976	.973	1.000	.954	.940	.980	.374	.978	.977
						.000	.111	.098	.134			.047	.000		

Table 3.14: Some power and size results for bivariate Gaussian–uniform data. $J = 5$, $I = 10, 20, 40$. For columns 5 – 16, the upper number in a row is power, and the lower number is size (blank if between .048 and .052).

				Standard allocation		Predictor sort allocation									
							Divided by $1-\rho^2$							"Theoretical"	
ρ	J	I	m	1-way anova	KW	uncorr KW	KW	1-way ranks	Van der Waerden	2-way anova	2-way ranks	Friedman	Ancova	1-way	2-way
.00	11	10	21	.240	.200	.200	.200	.226	.320	.241	.223	.185	.242	.245	.243
					.041	.043	.043					.042			
.50	11	10	21	.240	.197	.089	.308	.297	.428	.316	.292	.283	.321	.328	.325
					.040	.006	.057	.053				.044			
.60	11	10	21	.243	.200	.048	.373	.342	.495	.366	.334	.345	.369	.387	.384
					.043	.002	.061		.047			.041			
.70	11	10	21	.242	.200	.020	.488	.435	.618	.459	.417	.457	.462	.488	.484
					.042	.000	.069					.041	.047		
.80	11	10	21	.245	.200	.003	.681	.605	.811	.622	.568	.647	.619	.670	.666
					.040	.000	.086	.056	.055			.040	.044		
.90	11	10	21	.242	.199	.000	.946	.910	.988	.908	.863	.919	.887	.947	.945
					.042	.000	.114	.068	.078	.046	.045	.042	.032		
.95	11	10	16	.141	.116	.000	.970	.945	.996	.912	.879	.925	.842	.966	.964
					.041	.000	.162	.097	.142	.046	.047	.042	.021		
.99	11	10	9	.074	.061	.000	.997	.993	1.000	.910	.890	.892	.447	.996	.996
					.043	.000	.453	.331	.701		.047	.040	.001		
.00	11	20	21	.518	.460	.460	.460	.474	.686	.514	.472	.420	.517	.520	.518
					.046	.047	.047					.046			
.50	11	20	21	.520	.462	.316	.622	.616	.833	.655	.605	.623	.659	.674	.671
					.046	.007	.053					.047			
.60	11	20	21	.523	.464	.255	.717	.705	.897	.740	.688	.737	.742	.759	.756
					.047	.002	.058	.053	.047			.047			
.70	11	20	21	.519	.460	.172	.838	.821	.962	.848	.799	.867	.846	.866	.864
					.047	.000	.066	.057		.053		.047			
.80	11	20	18	.381	.336	.020	.859	.837	.972	.851	.806	.901	.838	.875	.874
					.046	.000	.071	.059				.046	.044		
.90	11	20	13	.184	.165	.000	.853	.820	.979	.807	.770	.901	.746	.851	.849
					.045	.000	.087	.067	.061	.047	.047	.045	.030		
.95	11	20	10	.120	.109	.000	.901	.872	.992	.840	.812	.930	.669	.888	.887
					.047	.000	.106	.079	.086	.047		.046	.014		
.99	11	20	6	.072	.065	.000	.988	.983	1.000	.946	.930	.972	.212	.984	.983
				.053		.000	.198	.155	.356			.046	.000		
.00	11	40	21	.885	.841	.841	.841	.846	.976	.883	.844	.798	.884	.882	.882
												.047			
.50	11	40	19	.787	.736	.659	.879	.878	.986	.904	.870	.891	.906	.910	.909
					.047	.007			.047				.047		
.60	11	40	18	.727	.677	.529	.899	.895	.990	.916	.883	.925	.916	.924	.923
						.001	.056	.054							
.70	11	40	15	.523	.481	.188	.848	.840	.982	.855	.819	.908	.851	.867	.866
						.000	.061	.057							
.80	11	40	13	.382	.353	.025	.867	.857	.989	.859	.827	.940	.845	.882	.882
				.053		.000	.067	.060			.047	.047	.042		
.90	11	40	9	.171	.160	.000	.815	.796	.984	.772	.747	.923	.693	.807	.806
						.000	.080	.070	.057				.029		
.95	11	40	7	.111	.105	.000	.861	.844	.994	.810	.790	.952	.593	.850	.849
						.000	.092	.079	.068				.011		
.99	11	40	4	.063	.061	.000	.940	.930	1.000	.876	.864	.972	.053	.925	.924
					.047	.000	.121	.104	.164	.047			.000		

Table 3.15: Some power and size results for bivariate Gaussian–uniform data. $J = 11$, $I = 10, 20, 40$. For columns 5 – 16, the upper number in a row is power, and the lower number is size (blank if between .048 and .052).

				Standard allocation		Predictor sort allocation									
							Divided by $1-\rho^2$								
				1-way		uncorr		1-way	Van der	2-way	2-way			"Theoretical"	
ρ	J	I	m	anova	KW	KW	KW	ranks	Waerden	anova	ranks	Friedman	Ancova	1-way	2-way
.00	2	10	21	.552	.549	.551	.551	.551	.573	.505	.504	.269	.550	.562	.514
						.053	.053	.053				.022			
.50	2	10	21	.553	.551	.549	.661	.661	.686	.614	.599	.352	.666	.685	.634
						.029	.054	.054				.022			
.60	2	10	21	.553	.547	.549	.738	.705	.751	.675	.654	.408	.728	.753	.702
						.020	.062		.053			.021	.047		
.70	2	10	21	.552	.548	.554	.819	.789	.835	.762	.734	.492	.818	.841	.795
					.053	.010	.063		.055			.020	.047		
.80	2	10	21	.554	.550	.567	.922	.902	.931	.876	.844	.629	.919	.941	.911
					.053	.004	.075	.059	.062			.022	.045		
.90	2	10	16	.348	.357	.256	.934	.913	.950	.866	.838	.629	.917	.953	.927
					.053	.000	.091	.068	.080	.046		.021	.037		
.95	2	10	13	.238	.252	.082	.962	.962	.977	.886	.866	.667	.931	.982	.968
					.053	.000	.110	.110	.110	.047		.021	.030		
.99	2	10	9	.135	.145	.002	.998	.994	.998	.901	.933	.746	.935	1.000	1.000
					.053	.000	.281	.165	.270	.042		.021	.010		
.00	2	20	21	.872	.850	.846	.846	.846	.884	.851	.832	.653	.867	.869	.851
												.040			
.50	2	20	20	.833	.809	.840	.906	.901	.935	.903	.883	.747	.918	.922	.908
						.026	.055					.042			
.60	2	20	19	.789	.771	.815	.921	.916	.947	.914	.893	.779	.928	.934	.921
						.017	.059	.054	.053			.042			
.70	2	20	17	.691	.677	.728	.921	.915	.950	.905	.887	.778	.920	.932	.919
						.007	.058	.053		.047		.041	.045		
.80	2	20	15	.575	.574	.611	.941	.936	.967	.922	.906	.817	.932	.949	.938
						.002	.065	.059	.058			.042	.045		
.90	2	20	11	.335	.344	.220	.939	.932	.970	.898	.884	.805	.902	.942	.930
						.000	.078	.069	.068	.047		.040	.036		
.95	2	20	9	.231	.243	.045	.973	.968	.989	.937	.930	.864	.925	.977	.970
						.000	.095	.081	.086			.042	.027		
.99	2	20	5	.093	.097	.000	.985	.985	.995	.909	.928	.850	.722	.992	.989
						.000	.145	.145	.181			.041	.004		
.00	2	40	16	.916	.908	.905	.905	.905	.947	.904	.898	.726	.912	.912	.905
				.047	.046							.039			
.50	2	40	14	.818	.816	.846	.908	.908	.953	.900	.896	.759	.907	.912	.905
						.026						.039			
.60	2	40	13	.754	.758	.804	.911	.909	.956	.897	.896	.771	.904	.912	.905
						.016	.056	.055				.038			
.70	2	40	12	.677	.687	.746	.927	.924	.968	.910	.909	.808	.916	.925	.919
						.008	.054	.053				.037	.046		
.80	2	40	11	.598	.613	.671	.956	.954	.985	.941	.938	.870	.944	.957	.953
						.002	.059	.058	.053			.039	.044		
.90	2	40	8	.334	.356	.229	.950	.946	.986	.921	.923	.857	.906	.944	.938
						.000	.065	.060	.058			.037	.035		
.95	2	40	6	.195	.209	.015	.951	.948	.988	.913	.917	.860	.863	.943	.937
						.000	.075	.071	.069			.038	.024		
.99	2	40	4	.101	.109	.000	.997	.997	1.000	.985	.987	.962	.803	.997	.996
						.000	.108	.097	.118			.038	.002		

Table 3.16: Some power and size results for bivariate Gaussian–right triangular data. $J = 2$, $I = 10, 20, 40$. For columns 5 – 16, the upper number in a row is power, and the lower number is size (blank if between .048 and .052).

				Standard allocation		Predictor sort allocation									
							Divided by $1-\rho^2$								
ρ	J	I	m	1-way anova	KW	uncorr KW	KW	1-way ranks	Van der Waerden	2-way anova	2-way ranks	Friedman	Ancova	"Theoretical" 1-way	"Theoretical" 2-way
.00	5	10	21	.355	.339	.341	.341	.369	.422	.346	.359	.285	.353	.359	.350
				.047	.041	.044	.044					.043			
.50	5	10	21	.354	.339	.262	.481	.473	.543	.446	.453	.390	.457	.471	.460
					.042	.014	.055					.042			
.60	5	10	21	.354	.337	.226	.560	.537	.618	.509	.514	.459	.522	.543	.531
				.047	.041	.006	.061	.054				.045			
.70	5	10	21	.354	.338	.177	.679	.642	.728	.608	.605	.561	.620	.654	.641
					.044	.002	.068	.055	.053			.043	.046		
.80	5	10	21	.353	.337	.117	.841	.802	.878	.763	.752	.719	.771	.821	.809
				.047	.041	.000	.077	.057	.057			.044	.040		
.90	5	10	19	.289	.276	.022	.964	.945	.980	.907	.899	.878	.906	.956	.951
					.043	.000	.107	.072	.076			.044	.032		
.95	5	10	15	.182	.175	.000	.984	.973	.993	.921	.922	.897	.901	.981	.978
					.043	.000	.148	.098	.123	.047		.044	.018		
.99	5	10	9	.082	.076	.000	.998	.997	.999	.888	.949	.903	.681	.999	.999
				.045	.038	.000	.368	.285	.461	.044		.043	.002		
.00	5	20	21	.688	.682	.685	.685	.696	.791	.684	.689	.589	.690	.691	.685
					.046	.045	.045					.046			
.50	5	20	21	.695	.690	.657	.833	.830	.905	.816	.816	.761	.820	.827	.821
					.046	.015	.055	.054							
.60	5	20	21	.690	.685	.648	.892	.887	.946	.874	.872	.838	.879	.888	.884
					.047	.006	.055					.045	.047		
.70	5	20	20	.640	.639	.556	.930	.923	.971	.912	.906	.888	.913	.926	.923
						.001	.061	.055				.046	.047		
.80	5	20	17	.475	.479	.267	.938	.929	.977	.904	.906	.904	.902	.928	.924
					.046	.000	.069	.058	.055				.042		
.90	5	20	13	.278	.287	.015	.958	.948	.989	.915	.919	.925	.886	.943	.940
						.000	.083	.068	.066				.029		
.95	5	20	10	.166	.173	.000	.974	.967	.995	.928	.936	.942	.850	.962	.960
					.045	.000	.101	.080	.087				.015		
.99	5	20	6	.081	.080	.000	.998	.997	1.000	.964	.980	.975	.520	.997	.997
					.046	.000	.179	.149	.246			.047	.000		
.00	5	40	19	.909	.912	.913	.913	.915	.969	.908	.913	.847	.910	.908	.906
					.047						.053				
.50	5	40	17	.819	.827	.833	.929	.928	.980	.916	.923	.895	.917	.923	.922
					.047	.013	.053	.053							
.60	5	40	16	.761	.775	.765	.939	.938	.984	.923	.929	.915	.923	.931	.930
						.005	.053						.047		
.70	5	40	14	.618	.639	.564	.928	.925	.981	.901	.910	.911	.899	.915	.913
						.001	.060	.057							
.80	5	40	12	.465	.492	.272	.936	.931	.985	.902	.915	.930	.891	.919	.917
					.047	.000	.062	.057					.040		
.90	5	40	9	.254	.272	.009	.941	.935	.991	.894	.912	.944	.852	.920	.918
						.000	.071	.064	.058				.029		
.95	5	40	7	.152	.163	.000	.964	.959	.996	.919	.936	.960	.804	.943	.942
						.000	.082	.073	.066				.012		
.99	5	40	4	.072	.074	.000	.989	.987	1.000	.948	.968	.978	.270	.978	.977
					.047	.000	.113	.101	.136			.047	.000		

Table 3.17: Some power and size results for bivariate Gaussian–right triangular data. $J = 5$, $I = 10, 20, 40$. For columns 5 – 16, the upper number in a row is power, and the lower number is size (blank if between .048 and .052).

				Standard allocation		Predictor sort allocation									
							Divided by $1-\rho^2$								
ρ	J	I	m	1-way anova	KW	uncorr KW	KW	1-way ranks	Van der Waerden	2-way anova	2-way ranks	Friedman	Ancova	"Theoretical" 1-way	"Theoretical" 2-way
.00	11	10	21	.240	.231	.232	.232	.258	.333	.242	.256	.213	.242	.245	.243
					.042	.041	.041					.041			
.50	11	10	21	.244	.234	.113	.354	.342	.437	.321	.337	.311	.322	.328	.325
					.042	.007	.054					.043			
.60	11	10	21	.243	.232	.071	.430	.398	.508	.366	.388	.379	.366	.387	.384
					.040	.001	.062					.042			
.70	11	10	21	.245	.236	.032	.550	.498	.630	.451	.473	.485	.451	.488	.484
					.042	.000	.072	.054				.041	.045		
.80	11	10	21	.243	.232	.007	.746	.680	.816	.620	.640	.666	.605	.670	.666
					.042	.000	.087	.058	.055			.042	.038		
.90	11	10	21	.242	.235	.000	.968	.944	.987	.898	.904	.923	.870	.947	.945
					.042	.000	.119	.072	.080			.042	.025		
.95	11	10	16	.146	.137	.000	.985	.969	.996	.898	.921	.928	.797	.966	.964
					.043	.000	.163	.096	.142	.047	.046	.041	.010		
.99	11	10	9	.071	.065	.000	.999	.997	1.000	.819	.925	.900	.265	.996	.996
					.042	.000	.459	.336	.709	.042		.042	.000		
.00	11	20	21	.522	.536	.535	.535	.551	.705	.520	.548	.490	.522	.520	.518
					.047	.045	.045					.046			
.50	11	20	21	.517	.529	.408	.707	.702	.842	.664	.690	.685	.666	.674	.671
					.047	.007	.055	.053							
.60	11	20	21	.518	.531	.335	.787	.774	.902	.738	.759	.775	.737	.759	.756
					.045	.002	.058	.053				.047			
.70	11	20	21	.515	.527	.249	.892	.880	.964	.847	.861	.889	.841	.866	.864
						.000	.064	.055				.046	.045		
.80	11	20	18	.371	.384	.039	.909	.892	.975	.851	.867	.912	.832	.875	.874
					.047	.000	.072	.058				.043	.037		
.90	11	20	13	.189	.196	.000	.905	.884	.978	.810	.841	.907	.720	.851	.849
					.046	.000	.088	.068	.061			.045	.023		
.95	11	20	10	.118	.121	.000	.941	.922	.993	.835	.870	.929	.603	.888	.887
					.046	.000	.108	.081	.089			.046	.007		
.99	11	20	6	.068	.066	.000	.994	.992	1.000	.913	.960	.971	.098	.984	.983
					.046	.000	.198	.155	.356			.046	.000		
.00	11	40	21	.879	.894	.895	.895	.898	.974	.880	.897	.861	.881	.882	.882
.50	11	40	19	.788	.813	.760	.927	.926	.986	.902	.920	.925	.903	.910	.909
						.007	.054	.053							
.60	11	40	17	.666	.696	.557	.910	.906	.981	.873	.896	.918	.871	.882	.882
						.001	.058	.055							
.70	11	40	15	.521	.554	.277	.902	.896	.982	.851	.879	.923	.844	.867	.866
					.045	.000	.062	.057					.044		
.80	11	40	12	.315	.342	.024	.859	.848	.972	.777	.819	.902	.746	.806	.805
						.000	.066	.059		.047		.047	.037		
.90	11	40	9	.170	.184	.000	.877	.863	.982	.774	.822	.922	.662	.807	.806
						.000	.079	.068	.055				.021		
.95	11	40	7	.112	.120	.000	.913	.901	.994	.809	.856	.950	.529	.850	.849
						.000	.088	.076	.066			.047	.006		
.99	11	40	4	.063	.063	.000	.969	.963	.999	.873	.921	.973	.019	.925	.924
						.000	.122	.106	.168				.000		

Table 3.18: Some power and size results for bivariate Gaussian–right triangular data. $J = 11$, $I = 10, 20, 40$. For columns 5 – 16, the upper number in a row is power, and the lower number is size (blank if between .048 and .052).

				Standard allocation		Predictor sort allocation									
							Divided by $1-\rho^2$								
				1-way		uncorr		1-way	Van der	2-way	2-way			"Theoretical"	
ρ	J	I	m	anova	KW	KW	KW	ranks	Waerden	anova	ranks	Friedman	Ancova	1-way	2-way
.00	2	10	21	.591	.657	.659	.659	.659	.624	.549	.611	.358	.593	.562	.514
				.046		.053	.053	.053			.053	.023	.047		
.50	2	10	21	.596	.662	.670	.766	.766	.732	.645	.711	.433	.700	.685	.634
				.047		.029	.056	.056			.053	.023			
.60	2	10	21	.593	.657	.677	.832	.807	.787	.701	.763	.480	.759	.753	.702
				.047	.053	.020	.064		.054			.023			
.70	2	10	21	.594	.660	.684	.895	.875	.855	.765	.830	.550	.832	.841	.795
				.046	.053	.010	.064		.054	.046		.020	.047		
.80	2	10	21	.595	.661	.701	.963	.951	.936	.855	.914	.667	.923	.941	.911
				.047	.054	.004	.076	.060	.061	.044		.020			
.90	2	10	16	.393	.454	.421	.970	.957	.949	.814	.912	.654	.923	.953	.927
				.047		.001	.091	.068	.080	.040		.020	.044		
.95	2	10	12	.244	.289	.141	.968	.968	.949	.736	.887	.615	.909	.961	.937
				.047	.053	.000	.109	.109	.111	.038		.022	.041		
.99	2	10	8	.126	.153	.008	.995	.989	.984	.629	.910	.626	.917	.999	.998
				.047	.054	.000	.287	.174	.275	.031		.021	.018		
.00	2	20	20	.831	.901	.904	.904	.904	.875	.815	.893	.741	.831	.833	.813
										.047		.042	.047		
.50	2	20	18	.752	.840	.867	.925	.920	.896	.832	.907	.749	.854	.856	.837
						.025	.054			.047		.041			
.60	2	20	17	.705	.798	.841	.939	.935	.908	.841	.919	.760	.866	.869	.851
				.047		.016	.056					.040			
.70	2	20	15	.597	.699	.749	.928	.923	.897	.814	.902	.737	.845	.856	.836
						.009	.061	.056	.054			.042			
.80	2	20	13	.477	.575	.607	.939	.933	.908	.812	.912	.735	.851	.869	.851
					.047	.002	.061	.056	.055	.047		.040	.047		
.90	2	20	10	.300	.374	.281	.952	.946	.926	.796	.919	.746	.856	.889	.872
						.000	.077	.069	.066	.046		.040	.045		
.95	2	20	8	.204	.251	.065	.973	.968	.953	.786	.940	.783	.869	.932	.919
						.000	.091	.079	.084	.044		.043	.038		
.99	2	20	5	.099	.115	.000	.995	.995	.985	.687	.972	.860	.829	.992	.989
						.000	.149	.149	.178	.037		.042	.011		
.00	2	40	14	.822	.914	.915	.915	.915	.880	.812	.908	.747	.822	.819	.809
												.039			
.50	2	40	13	.754	.868	.899	.945	.945	.916	.848	.936	.775	.859	.864	.856
						.025						.038			
.60	2	40	12	.683	.811	.858	.942	.940	.912	.839	.933	.762	.852	.859	.850
						.016	.054	.053				.039			
.70	2	40	11	.604	.736	.804	.950	.948	.923	.850	.939	.772	.862	.871	.863
						.008	.057	.055	.053			.039			
.80	2	40	9	.432	.562	.592	.932	.930	.901	.802	.916	.723	.821	.838	.828
					.047	.002	.059	.057	.054			.039			
.90	2	40	7	.274	.361	.237	.948	.943	.920	.800	.927	.734	.823	.860	.851
						.000	.066	.059	.058			.038	.044		
.95	2	40	6	.205	.268	.059	.985	.984	.972	.861	.974	.841	.887	.943	.937
						.000	.071	.067	.066	.047		.038	.034		
.99	2	40	3	.072	.086	.000	.956	.950	.920	.589	.905	.707	.511	.879	.871
						.000	.109	.098	.121	.043	.053	.039	.008		

Table 3.19: Some power and size results for bivariate Gaussian–double exponential data. $J = 2$, $I = 10, 20, 40$. For columns 5 – 16, the upper number in a row is power, and the lower number is size (blank if between .048 and .052).

				Standard allocation		Predictor sort allocation									
							Divided by $1-\rho^2$							"Theoretical"	
ρ	J	I	m	1-way anova	KW	uncorr KW	KW	1-way ranks	Van der Waerden	2-way anova	2-way ranks	Friedman	Ancova	1-way	2-way
.00	5	10	21	.378	.459	.458	.458	.487	.444	.372	.478	.384	.379	.359	.350
				.044	.039	.045	.045			.047		.044	.047		
.50	5	10	21	.377	.458	.392	.611	.603	.555	.464	.586	.468	.482	.471	.460
				.045	.042	.013	.056	.054		.047		.044			
.60	5	10	21	.381	.463	.355	.692	.673	.624	.521	.655	.521	.549	.543	.531
				.045	.041	.006	.061	.054		.047		.044			
.70	5	10	21	.384	.466	.317	.801	.773	.730	.608	.750	.609	.651	.654	.641
				.046	.043	.002	.067	.054		.047		.044			
.80	5	10	21	.383	.464	.259	.922	.899	.868	.738	.875	.750	.800	.821	.809
				.046	.042	.000	.082	.061	.060	.045		.044	.047		
.90	5	10	17	.251	.308	.045	.965	.949	.926	.740	.918	.800	.848	.895	.886
				.046	.042	.000	.111	.076	.081	.044		.044	.044		
.95	5	10	13	.151	.180	.001	.977	.962	.946	.651	.918	.791	.825	.923	.915
				.044	.040	.000	.144	.097	.123	.034		.043	.032		
.99	5	10	8	.077	.082	.000	.997	.995	.988	.438	.938	.808	.734	.995	.993
				.045	.041	.000	.364	.282	.460	.022		.043	.006		
.00	5	20	21	.696	.824	.829	.829	.837	.785	.694	.830	.739	.700	.691	.685
					.047							.046			
.50	5	20	21	.693	.824	.821	.927	.926	.890	.812	.920	.841	.822	.827	.821
				.045	.045	.015	.055	.054			.053	.047			
.60	5	20	19	.593	.730	.706	.916	.911	.870	.781	.904	.805	.794	.804	.798
				.047	.047	.006	.057	.054				.046			
.70	5	20	18	.544	.684	.621	.944	.939	.910	.823	.932	.845	.839	.852	.847
						.001	.061	.055		.047		.046	.047		
.80	5	20	15	.384	.503	.302	.941	.932	.899	.782	.920	.818	.807	.835	.830
				.047	.047	.000	.070	.060	.056						
.90	5	20	11	.204	.270	.015	.940	.926	.893	.716	.903	.785	.759	.821	.816
				.047	.046	.000	.080	.065	.064	.046	.047	.046	.040		
.95	5	20	9	.141	.183	.000	.976	.971	.951	.733	.951	.858	.791	.904	.900
					.046	.000	.099	.079	.084	.044		.046	.028		
.99	5	20	5	.067	.074	.000	.992	.990	.976	.479	.958	.867	.478	.956	.953
				.046	.044	.000	.176	.145	.246	.028		.046	.003		
.00	5	40	17	.822	.933	.931	.931	.933	.895	.819	.932	.876	.821	.820	.818
												.047			
.50	5	40	15	.695	.850	.856	.941	.941	.905	.818	.939	.868	.824	.829	.826
						.014									
.60	5	40	14	.630	.790	.778	.943	.942	.905	.819	.938	.859	.824	.833	.831
						.006	.054								
.70	5	40	12	.469	.630	.544	.921	.918	.874	.766	.909	.810	.774	.786	.783
					.047	.001	.061	.057	.053						
.80	5	40	11	.395	.541	.360	.957	.953	.923	.825	.946	.863	.833	.854	.852
						.000	.064	.060	.053	.047			.046		
.90	5	40	8	.205	.285	.013	.945	.940	.906	.756	.924	.818	.756	.823	.821
						.000	.071	.064	.057				.038		
.95	5	40	6	.118	.152	.000	.947	.941	.908	.699	.920	.807	.671	.821	.818
					.046	.000	.081	.072	.068	.047			.024		
.99	5	40	4	.071	.080	.000	.998	.997	.991	.703	.993	.960	.520	.978	.977
					.047	.000	.112	.101	.135	.036			.001		

Table 3.20: Some power and size results for bivariate Gaussian–double exponential data. $J = 5$, $I = 10, 20, 40$. For columns 5 – 16, the upper number in a row is power, and the lower number is size (blank if between .048 and .052).

				Standard allocation		Predictor sort allocation									
							Divided by $1-\rho^2$							"Theoretical"	
ρ	J	I	m	1-way anova	KW	uncorr KW	KW	1-way ranks	Van der Waerden	2-way anova	2-way ranks	Friedman	Ancova	1-way	2-way
.00	11	10	21	.253	.327	.328	.328	.359	.314	.249	.355	.296	.251	.245	.243
				.046	.041	.042	.042					.042			
.50	11	10	21	.249	.322	.193	.477	.465	.405	.323	.458	.373	.332	.328	.325
					.042	.006	.056					.042			
.60	11	10	21	.253	.327	.138	.568	.536	.470	.372	.524	.424	.391	.387	.384
					.042	.002	.062					.045			
.70	11	10	21	.252	.325	.081	.699	.651	.584	.455	.634	.518	.488	.488	.484
				.047	.043	.000	.071	.053		.046		.043			
.80	11	10	21	.252	.328	.031	.863	.817	.758	.584	.793	.673	.639	.670	.666
					.042	.000	.088	.059	.056	.045		.043	.046		
.90	11	10	18	.184	.233	.001	.959	.931	.898	.646	.902	.797	.758	.832	.828
				.047	.042	.000	.118	.072	.077	.042		.042	.038		
.95	11	10	14	.119	.141	.000	.979	.961	.944	.569	.922	.817	.739	.887	.883
					.042	.000	.162	.098	.141	.034		.042	.024		
.99	11	10	8	.065	.065	.000	.997	.995	.992	.273	.916	.792	.437	.976	.975
					.041	.000	.457	.336	.703	.019		.043	.002		
.00	11	20	21	.529	.698	.694	.694	.707	.634	.523	.706	.646	.526	.520	.518
					.044	.046	.046					.045			
.50	11	20	21	.527	.698	.607	.852	.849	.780	.665	.841	.767	.672	.674	.671
				.047	.046	.007	.054					.046			
.60	11	20	21	.528	.696	.554	.911	.905	.854	.742	.900	.833	.752	.759	.756
					.046	.002	.058	.053				.047			
.70	11	20	19	.427	.583	.319	.920	.910	.857	.741	.902	.831	.754	.767	.765
					.044	.000	.063	.055				.047			
.80	11	20	17	.336	.468	.091	.953	.943	.901	.772	.932	.863	.793	.823	.821
					.046	.000	.073	.059	.053	.047		.046	.045		
.90	11	20	13	.191	.259	.000	.968	.957	.924	.742	.943	.871	.773	.851	.849
					.047	.000	.088	.068	.061	.044		.045	.035		
.95	11	20	10	.122	.155	.000	.980	.972	.950	.695	.957	.893	.724	.888	.887
					.047	.000	.107	.081	.086	.042		.046	.020		
.99	11	20	5	.063	.064	.000	.982	.975	.964	.312	.915	.811	.151	.876	.874
						.000	.198	.156	.353	.026		.047	.000		
.00	11	40	19	.788	.926	.927	.927	.929	.881	.786	.929	.897	.786	.788	.787
.50	11	40	16	.597	.787	.730	.913	.912	.853	.741	.907	.852	.745	.749	.748
						.007	.055	.054							
.60	11	40	15	.528	.720	.586	.919	.917	.861	.743	.910	.848	.747	.760	.758
				.053		.002	.056	.053				.047			
.70	11	40	14	.451	.635	.391	.945	.941	.894	.780	.933	.874	.785	.799	.798
					.047	.000	.061	.057				.047			
.80	11	40	12	.318	.467	.083	.950	.945	.901	.768	.934	.866	.772	.806	.805
					.047	.000	.067	.061					.045		
.90	11	40	9	.173	.241	.000	.953	.947	.902	.739	.928	.855	.720	.807	.806
						.000	.079	.070	.054	.047			.033		
.95	11	40	7	.112	.144	.000	.970	.965	.933	.729	.948	.880	.654	.850	.849
						.000	.091	.079	.067	.046		.047	.015		
.99	11	40	4	.062	.068	.000	.991	.989	.977	.538	.975	.929	.143	.925	.924
						.000	.123	.106	.167	.035			.000		

Table 3.21: Some power and size results for bivariate Gaussian–double exponential data. $J = 11$, $I = 10, 20, 40$. For columns 5 – 16, the upper number in a row is power, and the lower number is size (blank if between .048 and .052).

				Standard allocation		Predictor sort allocation									
							Divided by $1-\rho^2$								
				1-way		uncorr		1-way	Van der	2-way	2-way			"Theoretical"	
ρ	J	I	m	anova	KW	KW	KW	ranks	Waerden	anova	ranks	Friedman	Ancova	1-way	2-way
.00	2	10	21	.621	.745	.743	.743	.743	.725	.579	.701	.430	.618	.562	.514
				.044						.041		.022	.042		
.50	2	10	21	.617	.739	.755	.835	.835	.822	.658	.778	.512	.699	.685	.634
				.042		.028	.053	.053		.040		.021	.042		
.60	2	10	21	.619	.742	.759	.884	.865	.865	.700	.820	.564	.740	.753	.702
				.043		.020	.063		.053	.039		.022	.042		
.70	2	10	21	.617	.743	.779	.933	.920	.923	.761	.874	.629	.802	.841	.795
				.043	.053	.011	.064		.054	.038		.020	.037		
.80	2	10	20	.581	.713	.761	.968	.960	.962	.810	.917	.700	.843	.917	.882
				.044	.053	.004	.077	.060	.062	.038		.021	.036		
.90	2	10	15	.383	.520	.523	.978	.970	.979	.769	.919	.685	.788	.924	.890
				.045	.054	.000	.089	.067	.077	.033		.022	.023		
.95	2	10	11	.228	.339	.208	.978	.978	.982	.695	.903	.646	.660	.923	.889
				.043		.000	.111	.111	.110	.030		.021	.013		
.99	2	10	7	.112	.172	.009	.997	.993	.996	.597	.922	.659	.393	.994	.988
				.043	.053	.000	.287	.169	.275	.024		.022	.001		
.00	2	20	18	.759	.912	.909	.909	.909	.913	.737	.897	.755	.756	.745	.722
				.046						.047	.053	.042	.047		
.50	2	20	16	.659	.850	.872	.927	.922	.930	.724	.905	.774	.744	.761	.738
				.046	.047	.025	.056			.046		.042	.046		
.60	2	20	15	.608	.814	.849	.937	.932	.941	.728	.912	.785	.746	.769	.747
				.045		.017	.057			.043		.041	.042		
.70	2	20	14	.553	.765	.817	.952	.949	.958	.745	.925	.812	.755	.801	.779
				.047		.008	.060	.056	.054	.045		.040	.041		
.80	2	20	12	.433	.654	.714	.961	.957	.970	.732	.931	.824	.724	.806	.785
				.046		.003	.066	.060	.057	.043		.041	.034		
.90	2	20	9	.264	.439	.392	.971	.967	.981	.696	.934	.824	.626	.807	.786
						.000	.077	.068	.068	.043		.042	.021		
.95	2	20	7	.169	.291	.108	.981	.978	.991	.675	.945	.842	.499	.842	.822
				.044		.000	.094	.081	.086	.039		.042	.008		
.99	2	20	4	.077	.115	.000	.991	.991	.997	.539	.949	.844	.093	.906	.890
				.046		.000	.148	.148	.178	.029		.043	.000		
.00	2	40	12	.694	.920	.919	.919	.919	.938	.681	.915	.762	.692	.681	.670
												.038			
.50	2	40	11	.611	.874	.901	.942	.942	.961	.700	.934	.810	.706	.722	.712
						.025	.053	.053				.039	.047		
.60	2	40	10	.533	.816	.852	.935	.934	.958	.665	.922	.798	.668	.700	.689
						.018	.058	.056	.055		.053	.039			
.70	2	40	9	.445	.732	.793	.941	.939	.965	.655	.924	.810	.643	.696	.685
						.008	.057	.055				.038	.045		
.80	2	40	8	.360	.639	.698	.958	.956	.979	.674	.940	.844	.638	.731	.720
						.002	.060	.058	.055			.040	.035		
.90	2	40	6	.211	.404	.322	.963	.960	.986	.634	.938	.846	.509	.717	.706
						.000	.068	.063	.061	.047		.040	.019		
.95	2	40	5	.153	.289	.077	.986	.985	.997	.680	.968	.899	.416	.808	.798
						.000	.073	.070	.067	.044		.038	.006		
.99	2	40	3	.074	.112	.000	.996	.995	.999	.611	.982	.930	.035	.879	.871
						.000	.106	.095	.118	.038		.039	.000		

Table 3.22: Some power and size results for bivariate Gaussian–exponential data. $J = 2$, $I = 10, 20, 40$. For columns 5 – 16, the upper number in a row is power, and the lower number is size (blank if between .048 and .052).

				Standard allocation		Predictor sort allocation									
							Divided by $1-\rho^2$								
ρ	J	I	m	1-way anova	KW	uncorr KW	KW	1-way ranks	Van der Waerden	2-way anova	2-way ranks	Friedman	Ancova	"Theoretical" 1-way	"Theoretical" 2-way
.00	5	10	21	.399	.611	.608	.608	.636	.650	.381	.624	.513	.391	.359	.350
				.043	.041	.044	.044			.042		.042	.043		
.50	5	10	21	.388	.604	.564	.763	.757	.777	.471	.734	.640	.483	.471	.460
				.041	.043	.013	.055	.053		.043		.046	.043		
.60	5	10	21	.394	.608	.550	.840	.826	.844	.528	.796	.710	.536	.543	.531
				.044	.043	.007	.061	.055	.053	.042		.044	.041		
.70	5	10	21	.395	.608	.516	.917	.900	.916	.605	.870	.795	.604	.654	.641
				.043	.041	.002	.068	.054		.039		.043	.034		
.80	5	10	19	.323	.523	.350	.954	.938	.955	.630	.903	.836	.604	.722	.710
				.042	.041	.000	.084	.061	.061	.038		.044	.027		
.90	5	10	15	.204	.352	.072	.980	.969	.984	.613	.933	.870	.504	.787	.775
				.045	.044	.000	.105	.073	.077	.031		.043	.011		
.95	5	10	11	.116	.199	.001	.985	.976	.990	.504	.923	.850	.276	.785	.773
				.042	.040	.000	.145	.096	.122	.028		.044	.003		
.99	5	10	7	.067	.096	.000	.999	.999	1.000	.377	.964	.894	.031	.967	.962
				.044	.040	.000	.363	.279	.459	.015		.042	.000		
.00	5	20	20	.657	.908	.914	.914	.920	.935	.654	.916	.848	.660	.640	.634
				.045	.045					.046			.046		
.50	5	20	18	.556	.849	.843	.936	.935	.954	.653	.925	.880	.655	.673	.666
				.046	.046	.014	.053			.046		.046	.043		
.60	5	20	16	.448	.751	.729	.925	.921	.949	.601	.907	.861	.595	.627	.621
						.007	.059	.056		.045		.046	.041		
.70	5	20	15	.392	.696	.640	.950	.945	.967	.628	.930	.896	.603	.671	.665
				.047	.047	.002	.062	.055		.045		.045	.037		
.80	5	20	13	.291	.564	.400	.964	.958	.980	.618	.939	.914	.553	.691	.685
				.045	.047	.000	.067	.058	.054	.042		.046	.026		
.90	5	20	9	.142	.286	.019	.950	.940	.980	.487	.904	.875	.291	.606	.599
				.046	.047	.000	.083	.067	.064	.041		.045	.009		
.95	5	20	7	.096	.178	.000	.973	.966	.993	.471	.931	.902	.133	.651	.645
				.047	.047	.000	.098	.079	.086	.038			.001		
.99	5	20	4	.056	.077	.000	.993	.990	.999	.307	.950	.913	.000	.752	.746
				.044	.044	.000	.178	.146	.249	.023		.045	.000		
.00	5	40	14	.625	.938	.937	.937	.939	.965	.628	.937	.881	.630	.620	.617
				.047		.047	.047			.046			.046		
.50	5	40	12	.475	.841	.840	.933	.933	.971	.578	.927	.890	.573	.597	.594
						.013							.045		
.60	5	40	11	.396	.762	.745	.932	.930	.971	.551	.918	.893	.538	.581	.578
					.047	.006	.055	.053		.046			.042		
.70	5	40	10	.333	.674	.611	.939	.936	.977	.546	.923	.910	.510	.589	.586
						.002	.059	.056		.046		.047	.035		
.80	5	40	9	.265	.567	.406	.966	.964	.991	.581	.950	.945	.488	.645	.642
				.047	.047	.000	.065	.059	.053	.047			.025		
.90	5	40	7	.155	.349	.039	.978	.975	.997	.576	.963	.963	.327	.677	.674
						.000	.073	.066	.057	.046			.008		
.95	5	40	5	.093	.173	.000	.968	.964	.997	.465	.938	.942	.083	.605	.602
						.000	.084	.074	.068	.044			.001		
.99	5	40	3	.056	.076	.000	.992	.991	1.000	.381	.972	.970	.000	.706	.703
				.047	.046	.000	.114	.102	.138	.033			.000		

Table 3.23: Some power and size results for bivariate Gaussian–exponential data. $J = 5$, $I = 10, 20, 40$. For columns 5 – 16, the upper number in a row is power, and the lower number is size (blank if between .048 and .052).

				Standard allocation		Predictor sort allocation									
							Divided by $1-\rho^2$							"Theoretical"	
ρ	J	I	m	1-way anova	KW	uncorr KW	KW	1-way ranks	Van der Waerden	2-way anova	2-way ranks	Friedman	Ancova	1-way	2-way
.00	11	10	21	.255	.498	.499	.499	.534	.587	.252	.529	.457	.253	.245	.243
					.042	.042	.042			.046		.042	.046		
.50	11	10	21	.258	.501	.374	.682	.671	.729	.320	.653	.605	.320	.328	.325
				.047	.042	.008	.055			.044	.053	.044	.044		
.60	11	10	21	.259	.501	.309	.773	.749	.804	.366	.727	.688	.358	.387	.384
					.044	.002	.061			.043		.042	.039		
.70	11	10	21	.253	.497	.231	.882	.855	.901	.442	.825	.796	.413	.488	.484
				.046	.041	.000	.070	.053		.042		.042	.033		
.80	11	10	20	.234	.459	.099	.953	.931	.963	.517	.901	.882	.438	.615	.610
					.044	.000	.086	.057	.056	.038		.042	.020		
.90	11	10	15	.132	.258	.001	.973	.955	.984	.446	.911	.889	.242	.630	.625
					.042	.000	.118	.070	.077	.030		.041	.006		
.95	11	10	11	.086	.144	.000	.981	.965	.992	.332	.904	.875	.058	.627	.622
				.046	.042	.000	.163	.097	.139	.023	.046	.041	.000		
.99	11	10	7	.058	.071	.000	1.000	.999	1.000	.195	.966	.926	.000	.902	.899
				.045	.040	.000	.457	.334	.704	.009		.040	.000		
.00	11	20	21	.529	.888	.887	.887	.894	.936	.528	.893	.851	.531	.520	.518
				.046	.045							.046			
.50	11	20	19	.431	.808	.756	.925	.923	.960	.546	.914	.901	.539	.560	.558
					.045	.007	.054			.046		.046	.043		
.60	11	20	18	.386	.757	.651	.944	.940	.973	.554	.929	.924	.535	.583	.581
						.002	.058	.054		.045		.046	.039		
.70	11	20	16	.298	.642	.402	.946	.939	.975	.524	.924	.926	.476	.571	.569
					.046	.000	.064	.054		.044		.047	.032		
.80	11	20	13	.187	.437	.074	.938	.926	.976	.449	.898	.913	.330	.520	.518
				.046	.045	.000	.076	.062	.054	.045		.046	.020		
.90	11	20	10	.118	.250	.000	.962	.952	.992	.424	.922	.938	.153	.553	.551
					.046	.000	.089	.069	.061	.041		.044	.003		
.95	11	20	7	.075	.128	.000	.947	.931	.991	.307	.878	.902	.015	.481	.479
				.047	.046	.000	.107	.081	.086	.036		.045	.000		
.99	11	20	4	.055	.065	.000	.986	.980	1.000	.164	.922	.921	.000	.584	.582
					.047	.000	.197	.153	.354	.016	.046	.044	.000		
.00	11	40	15	.529	.923	.920	.920	.923	.972	.526	.922	.889	.527	.521	.520
					.047	.047	.047			.047		.047	.047		
.50	11	40	13	.383	.809	.764	.927	.926	.978	.492	.918	.919	.481	.511	.510
						.007	.054	.053					.047		
.60	11	40	12	.325	.730	.616	.928	.925	.981	.475	.915	.927	.449	.503	.502
				.047	.045	.002	.057	.054		.047			.039		
.70	11	40	11	.267	.641	.393	.943	.939	.988	.476	.927	.949	.413	.522	.520
						.000	.062	.057					.033		
.80	11	40	9	.175	.432	.064	.929	.923	.985	.416	.898	.940	.281	.474	.473
						.000	.068	.061		.047			.018		
.90	11	40	7	.113	.247	.000	.953	.946	.996	.411	.920	.965	.111	.504	.503
						.000	.079	.070	.055	.046			.003		
.95	11	40	5	.072	.124	.000	.932	.924	.996	.315	.880	.946	.006	.437	.436
						.000	.088	.078	.065	.042			.000		
.99	11	40	3	.053	.063	.000	.979	.975	1.000	.231	.941	.973	.000	.534	.533
					.047	.000	.123	.106	.170	.029			.000		

Table 3.24: Some power and size results for bivariate Gaussian–exponential data. $J = 11$, $I = 10, 20, 40$. For columns 5 – 16, the upper number in a row is power, and the lower number is size (blank if between .048 and .052).

J	Order Statistic	I	Theoretical Coverage $\rho = 0.0$	Simulation Coverage $\rho = 0.0$	$\rho = 0.7$	$\rho = 0.8$	$\rho = 0.9$
2	1	28	0.7622	0.764	0.777	0.792	0.810
	2	53	0.7500	0.752	0.765	0.775	0.793
	3	78	0.7543	0.752	0.770	0.777	0.796
3	1	28	0.7622	0.761	0.786	0.798	0.831
	2	53	0.7500	0.748	0.772	0.787	0.806
	3	78	0.7543	0.752	0.775	0.787	0.814
5	1	28	0.7622	0.765	0.794	0.811	0.845
	2	53	0.7500	0.748	0.775	0.790	0.826
	3	78	0.7543	0.756	0.772	0.794	0.824

Table 4: For standard I's, coverages of nonparametric one-sided, lower nominal 75% confidence bounds on fifth percentiles after a predictor sort allocation. Each simulation estimate was based on 40,000 trials. J is the number of treatments. I is the number of observations for each treatment. ρ is the correlation between the predictor used in the predictor sort allocation and the response. In the simulation that produced this table, the predictor and response had a bivariate normal distribution.

OS	J	ρ .7 I	.7 Coverage	.8 I	.8 Coverage	.9 I	.9 Coverage
1	2	26	.752	25	.752	24	.757
		25	.741	24	.738	23	.740
	3	26	.764	25	.763	23	.759
		25	.745	24	.749	22	.746
	5	25	.754	24	.761	22	.759
		24	.739	23	.743	21	.745
2	2	52	.755	51	.757	50	.762
		51	.748	50	.747	49	.749
	3	51	.752	50	.755	48	.756
		50	.741	49	.745	47	.742
	5	51	.757	49	.753	47	.758
		50	.744	48	.740	46	.745
3	2	76	.755	75	.752	73	.753
		75	.743	74	.744	72	.742
	3	75	.750	74	.754	72	.756
		74	.743	73	.743	71	.747
	5	75	.753	73	.752	71	.762
		74	.746	72	.742	70	.749

Table 5: For nonstandard I's, coverages of nonparametric one-sided, lower confidence bounds on fifth percentiles after a predictor sort allocation. Each simulation estimate was based on a minimum of 80,000 trials. OS is the order statistic that was used as the confidence bound. J is the number of treatments. I is the number of observations for each treatment. ρ is the correlation between the predictor used in the predictor sort allocation and the response. In the simulation that produced this table, the predictor and response had a bivariate normal distribution.

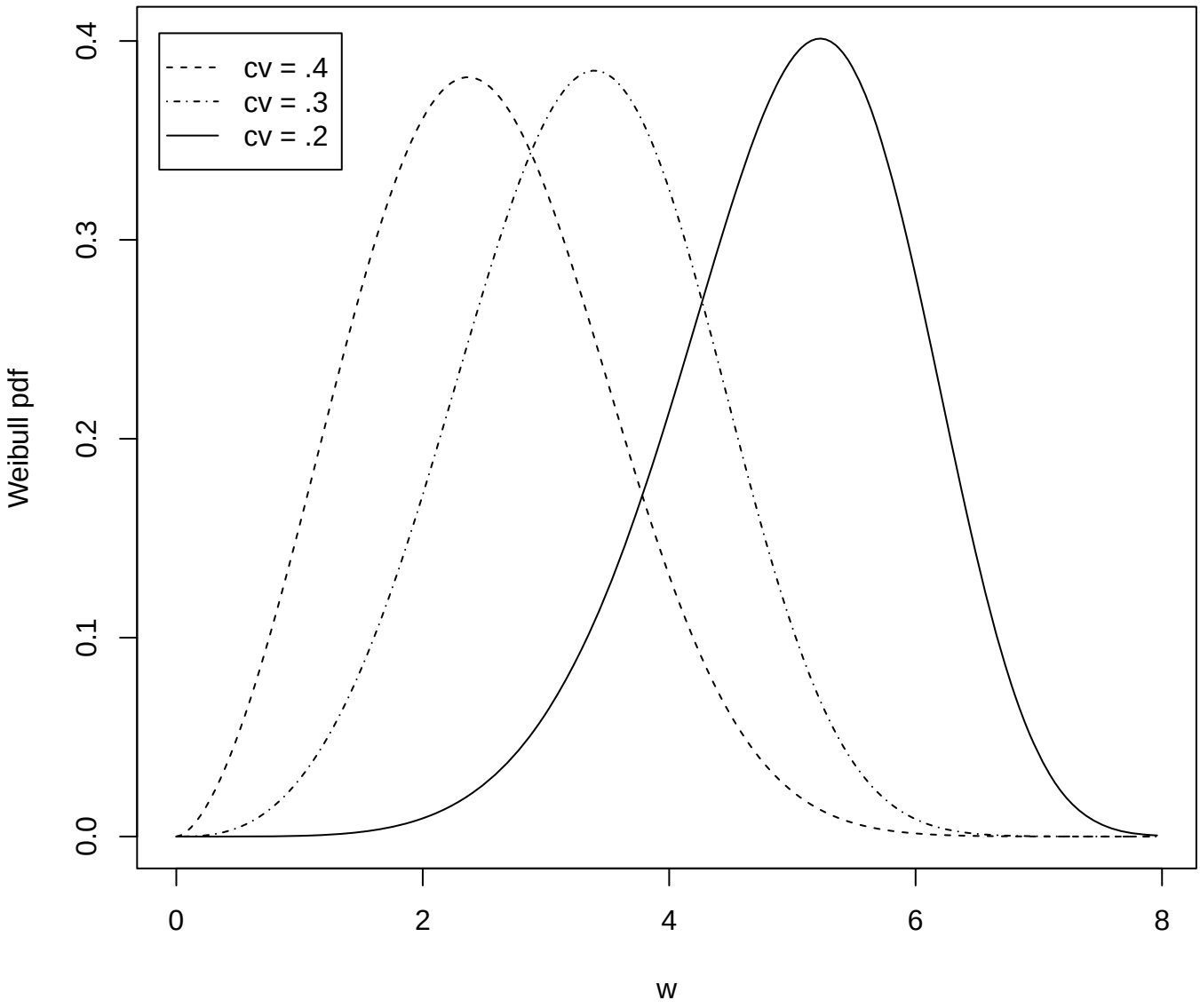

Figure 1: Probability density functions of the three Weibull distributions

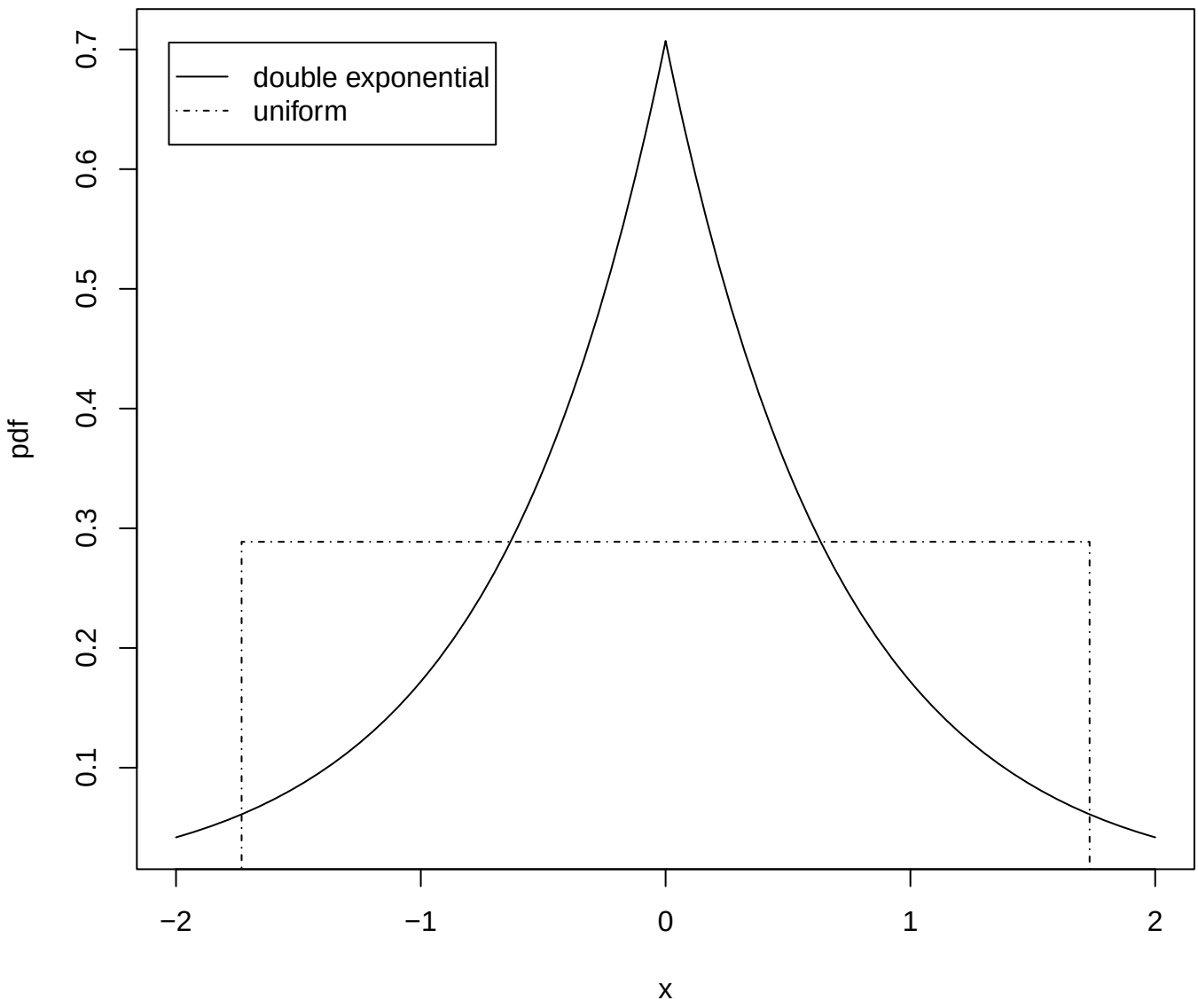

Figure 2: Probability density functions of the double exponential and uniform distributions

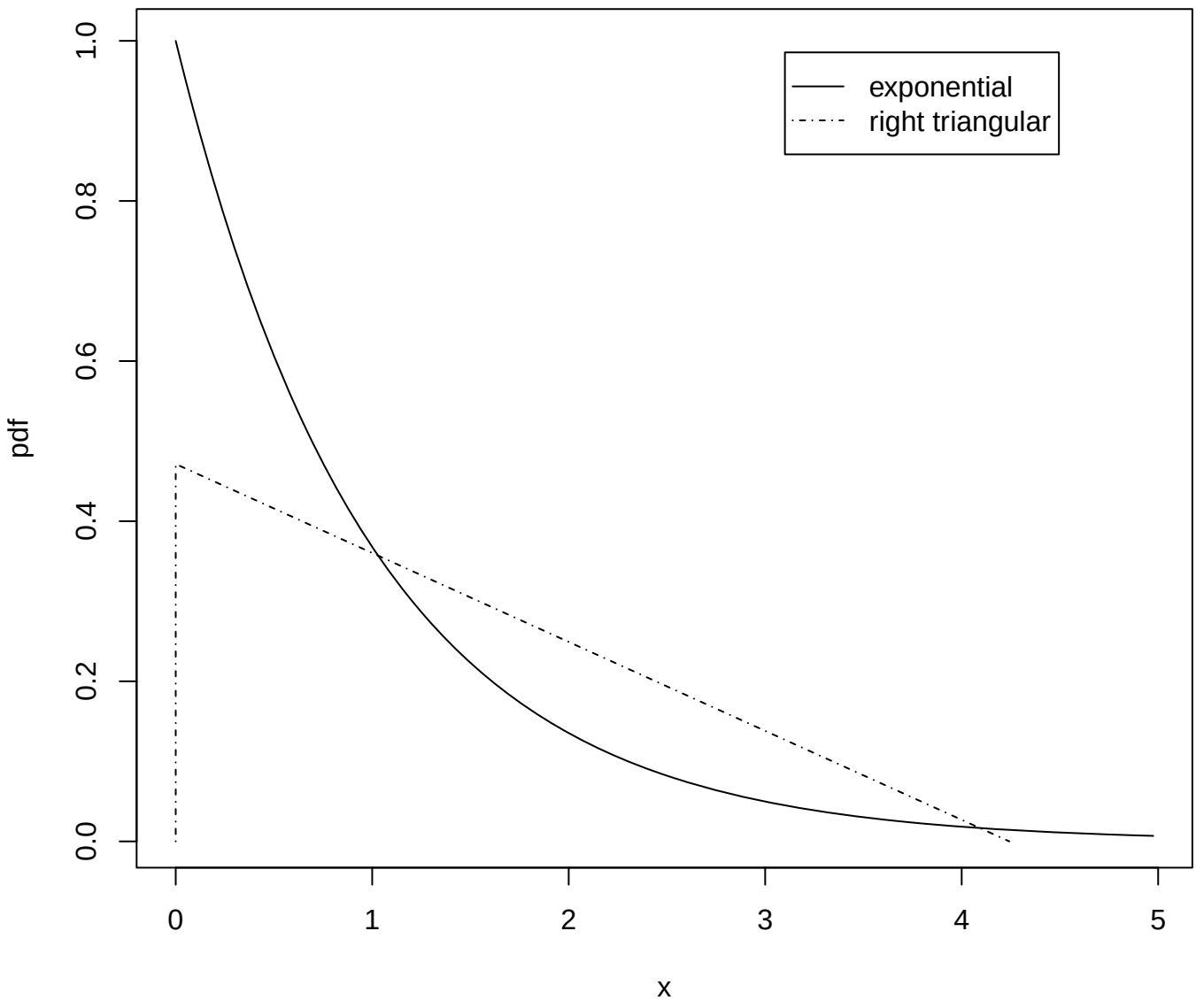

Figure 3: Probability density functions of the exponential and right triangular distributions

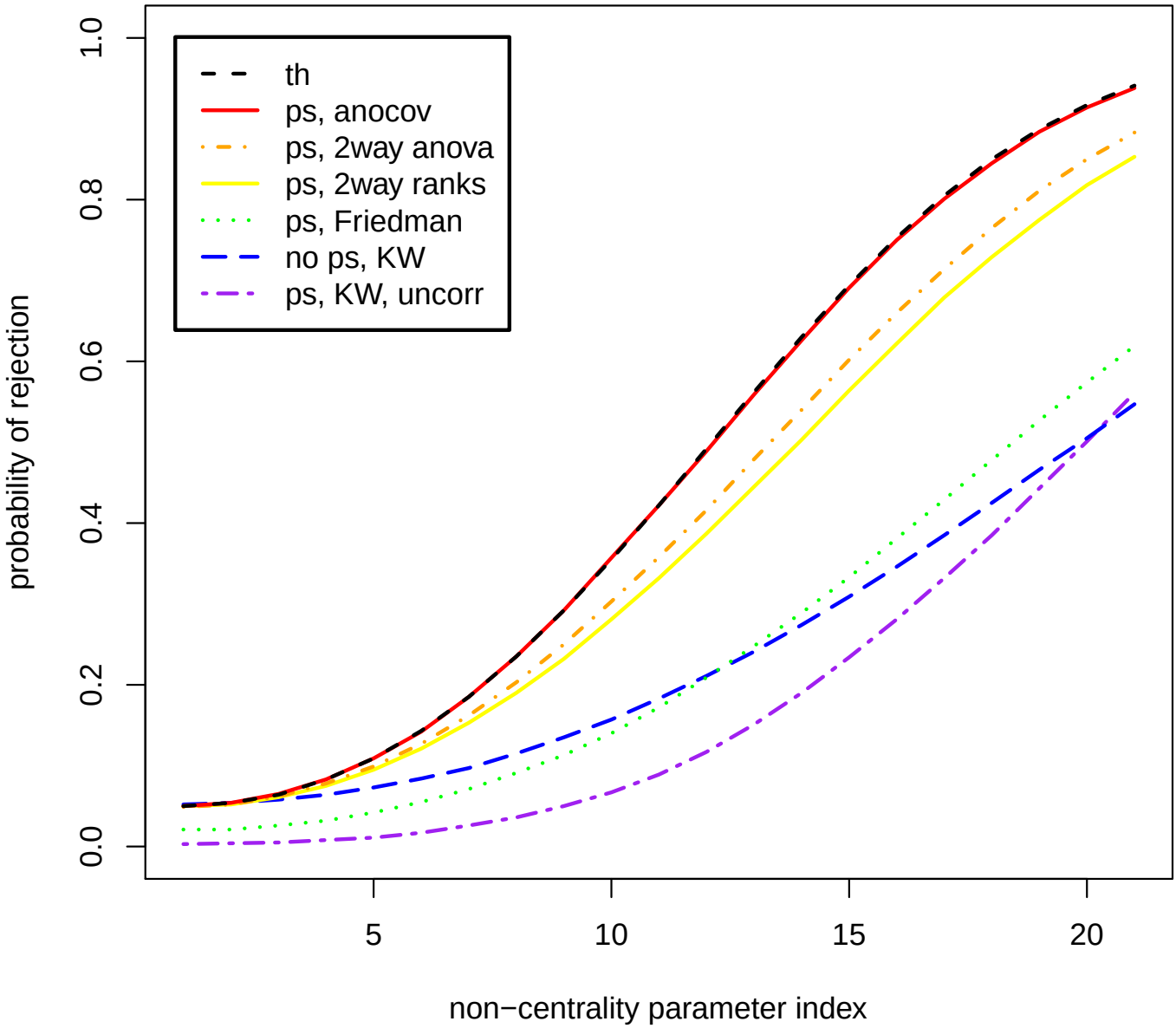

Figure 4: Power plot, $\rho = 0.8$, bivariate normal, $J = 2$, $I = 10$

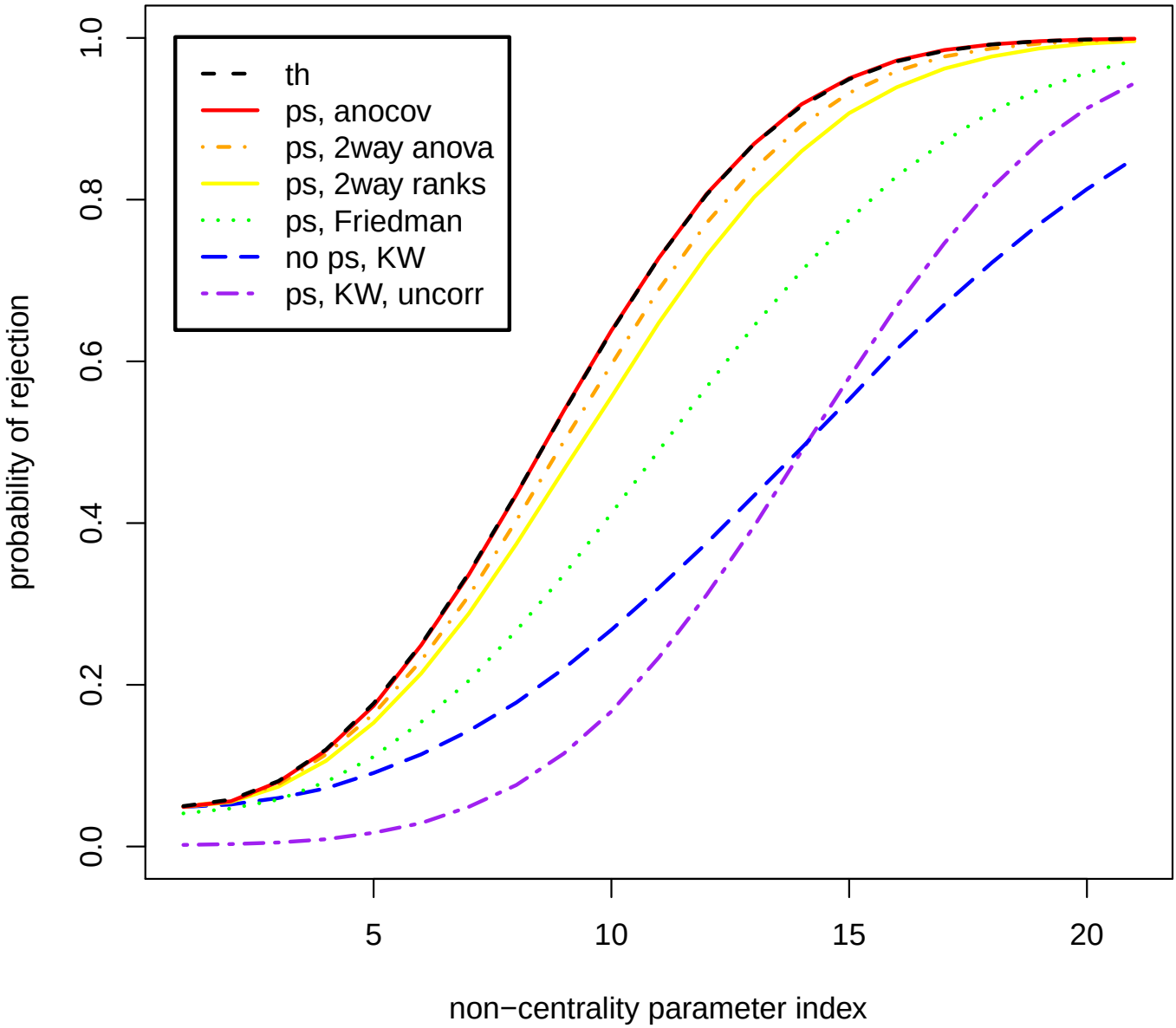

Figure 5: Power plot, $\rho = 0.8$, bivariate normal, $J = 2$, $I = 20$

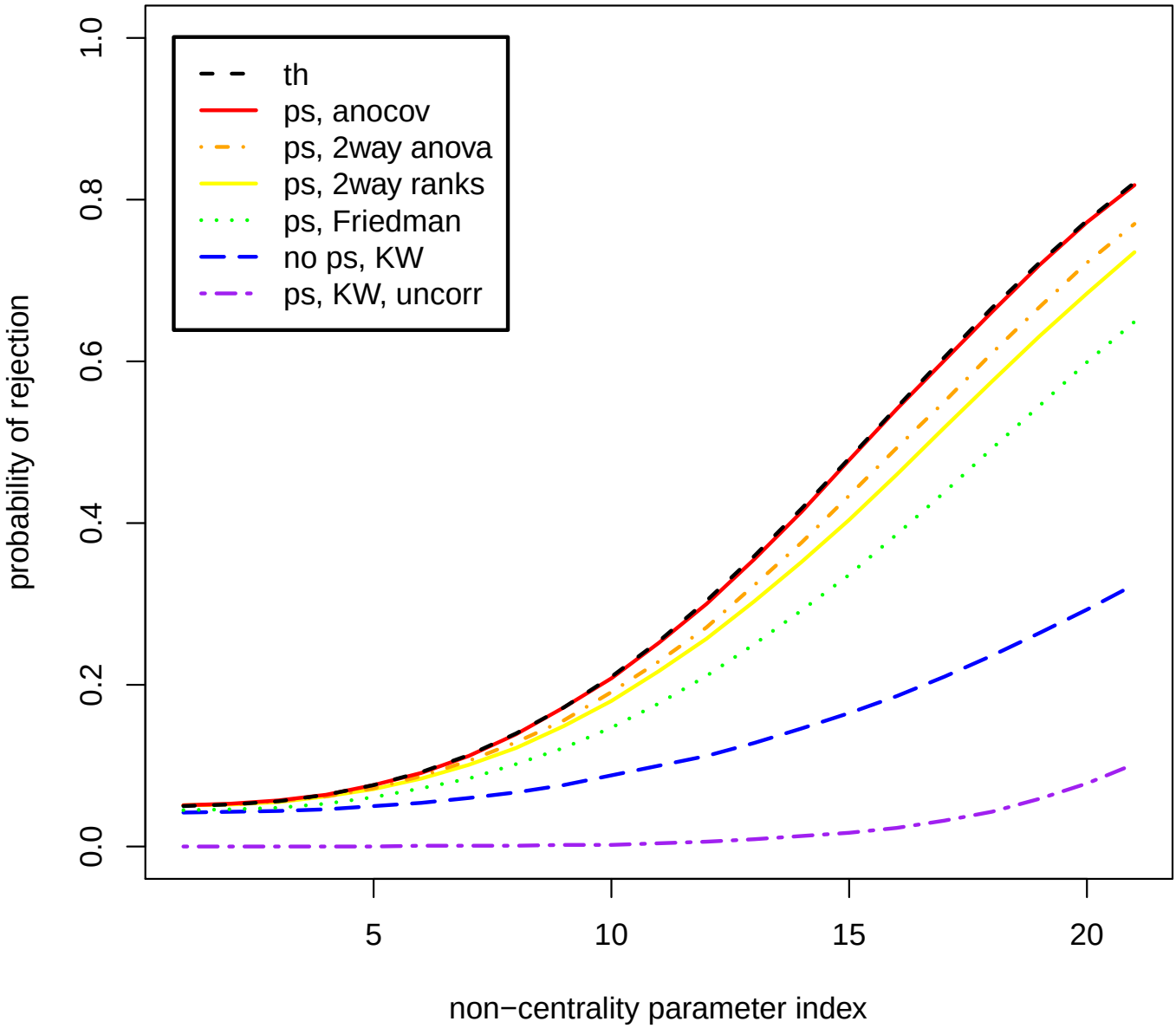

Figure 6: Power plot, $\rho = 0.8$, bivariate normal, $J = 5$, $I = 10$

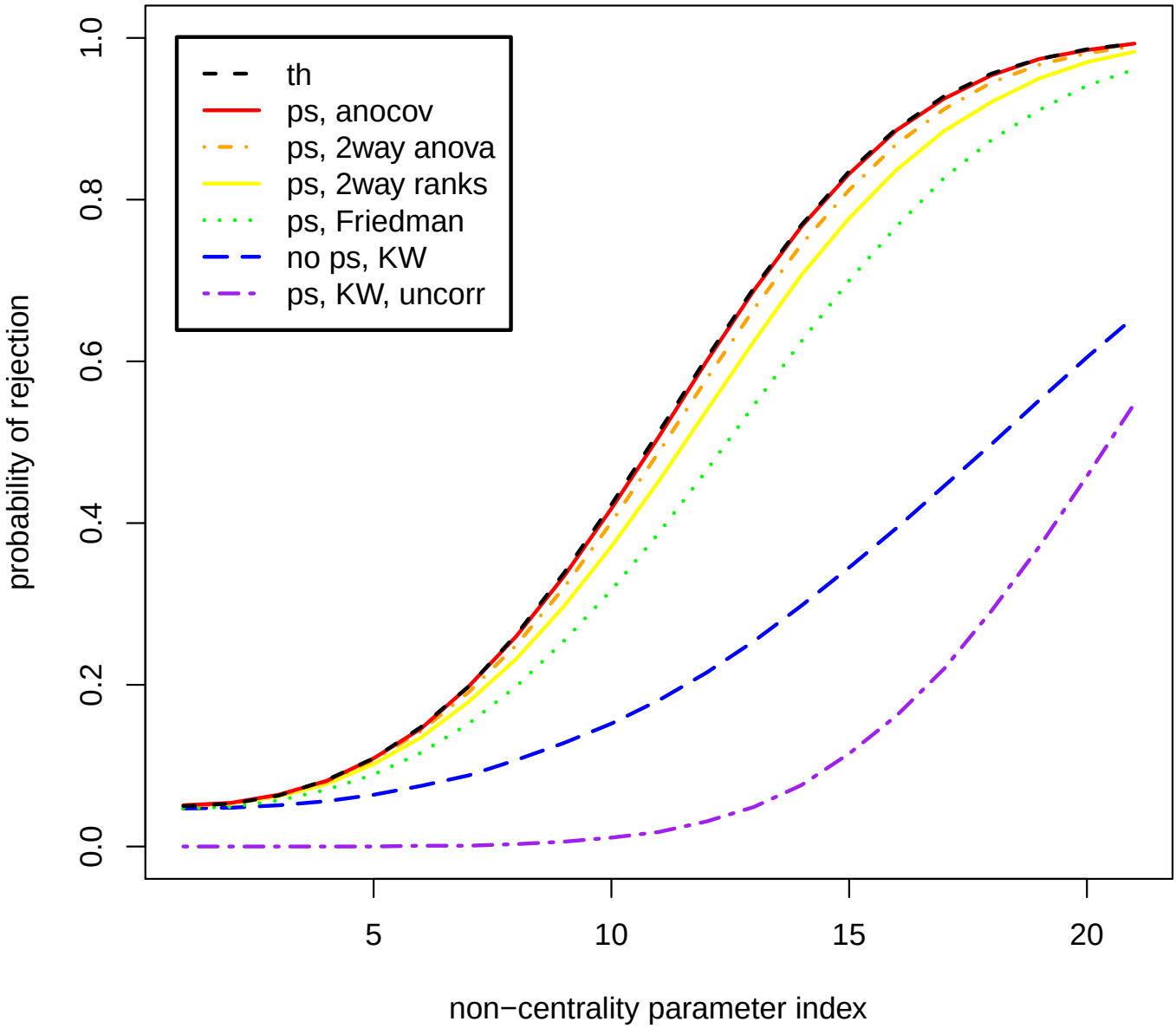

Figure 7: Power plot, $\rho = 0.8$, bivariate normal, $J = 5$, $I = 20$

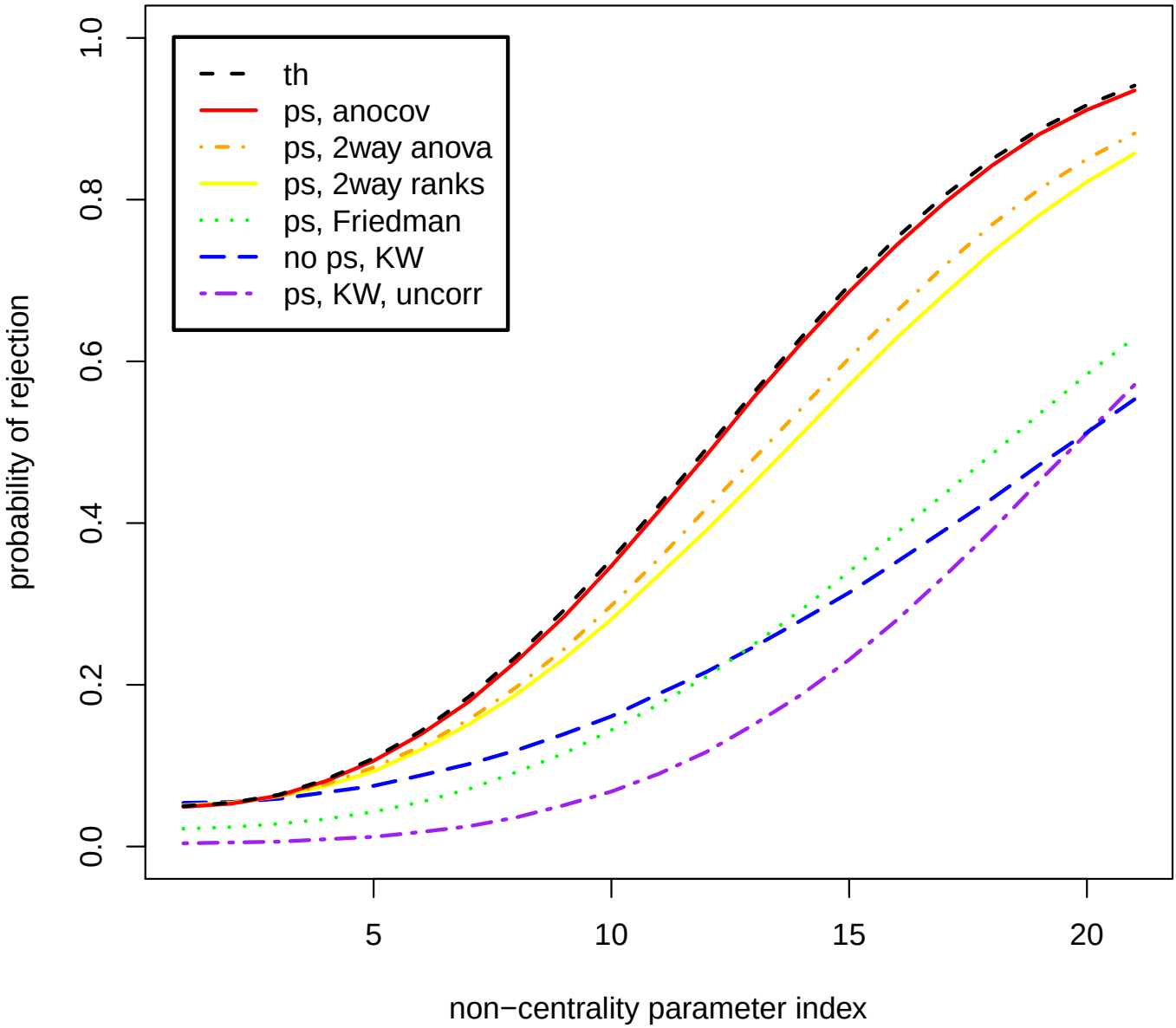

Figure 8: Power plot, $\rho = 0.8$, bivariate Gaussian–Weibull (.2 CV), $J = 2$, $I = 10$

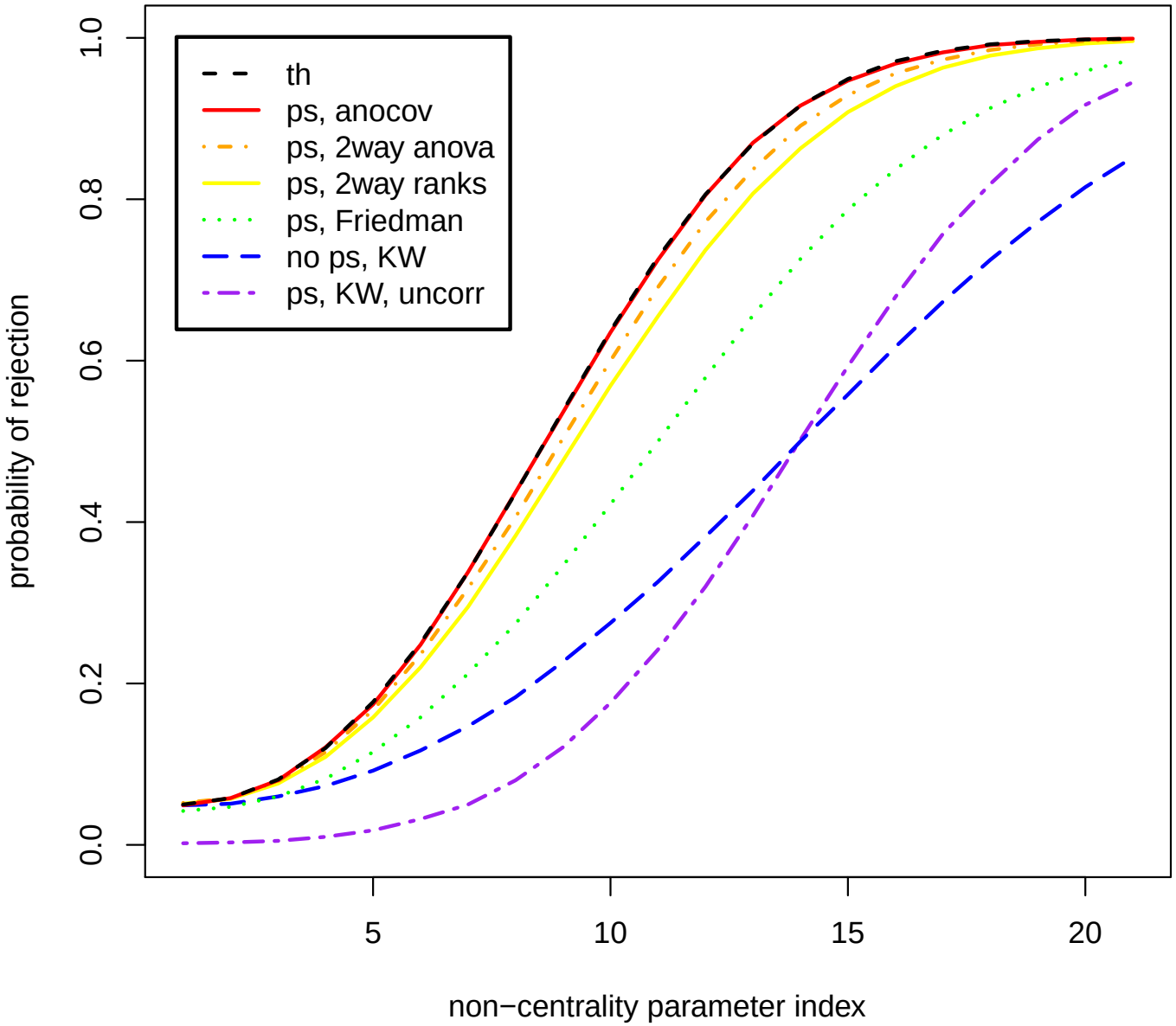

Figure 9: Power plot, $\rho = 0.8$, bivariate Gaussian–Weibull (.2 CV), $J = 2$, $I = 20$

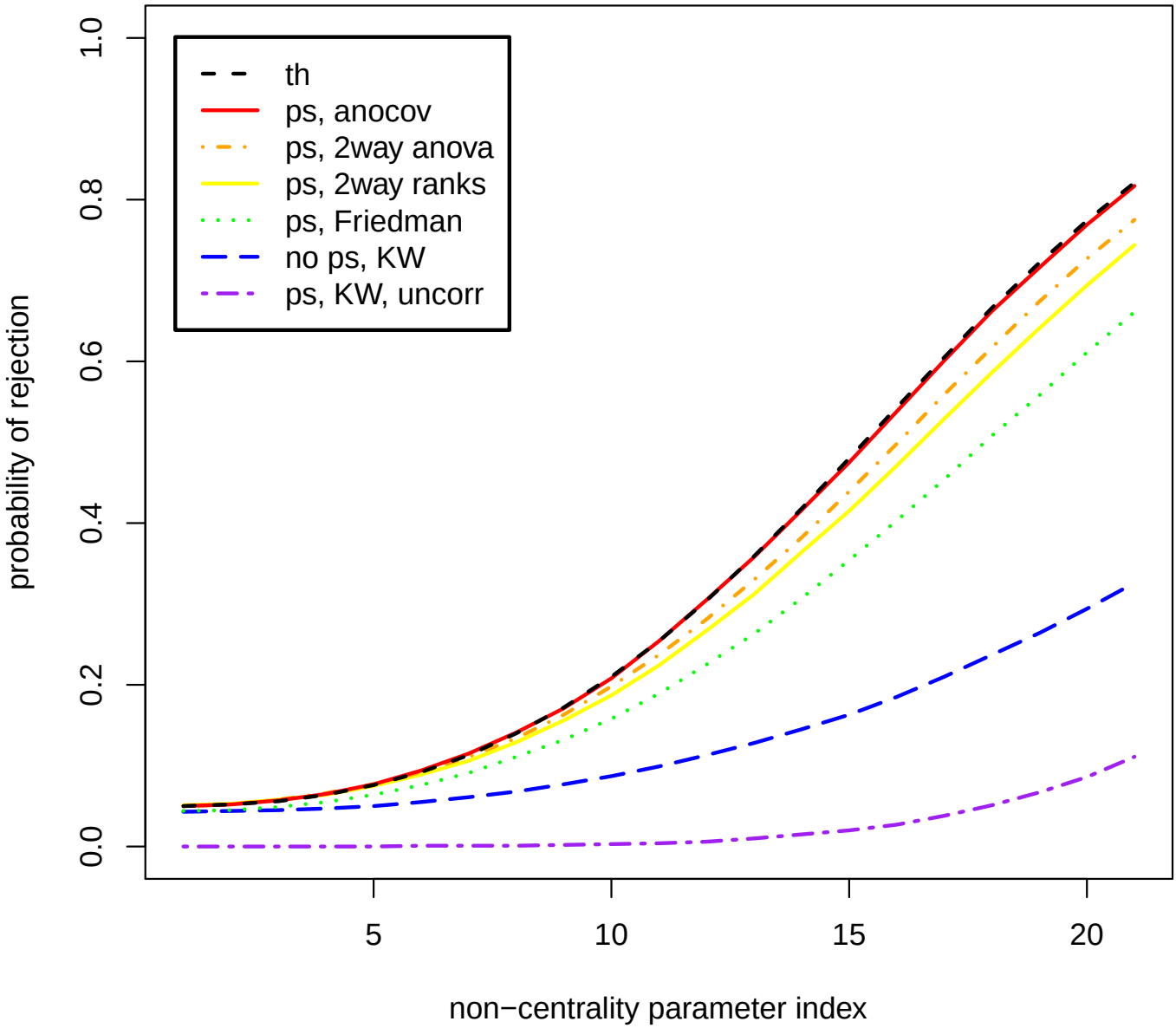

Figure 10: Power plot, $\rho = 0.8$, bivariate Gaussian–Weibull (.2 CV), $J = 5$, $I = 10$

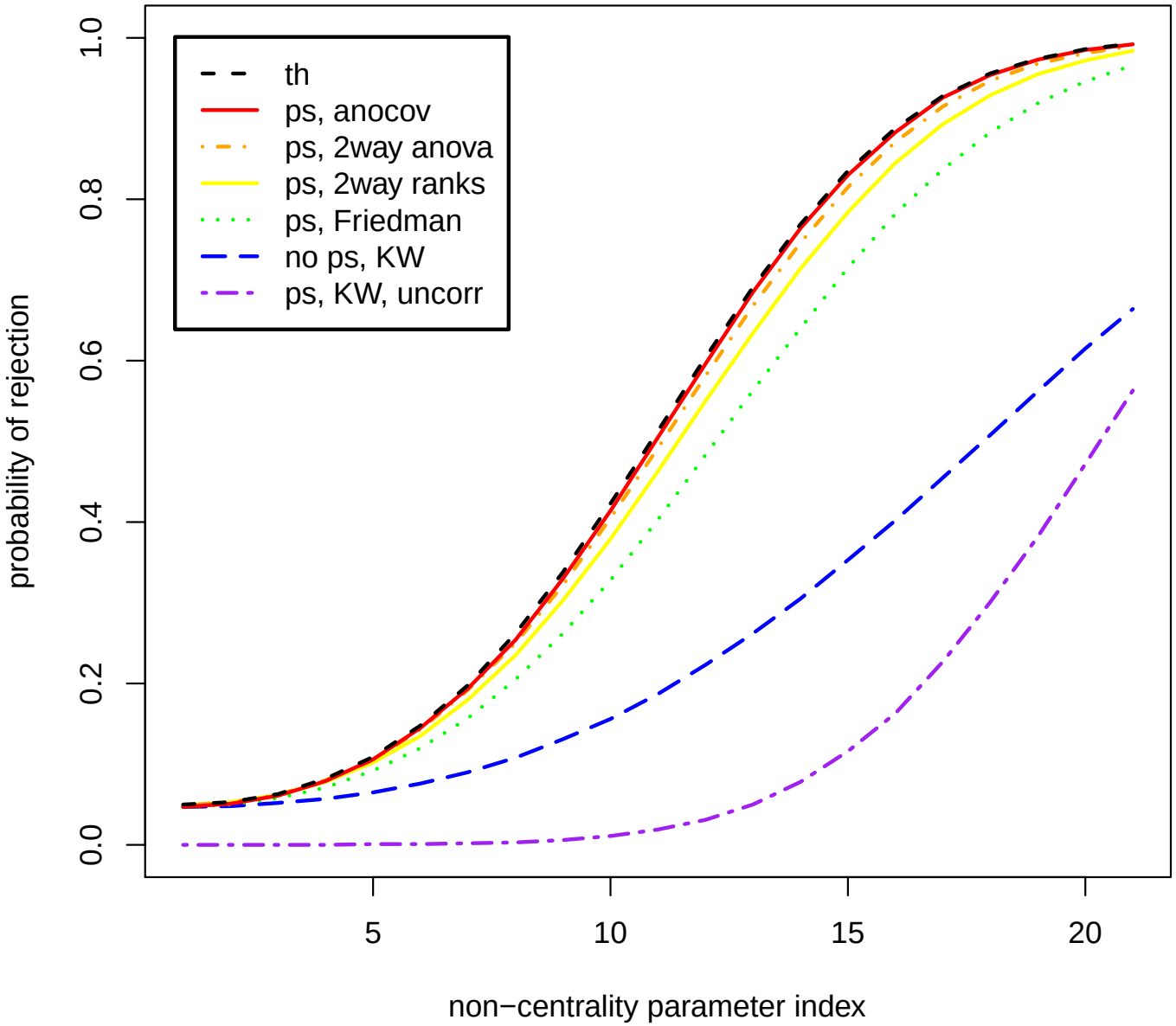

Figure 11: Power plot, $\rho = 0.8$, bivariate Gaussian–Weibull (.2 CV), $J = 5$, $I = 20$

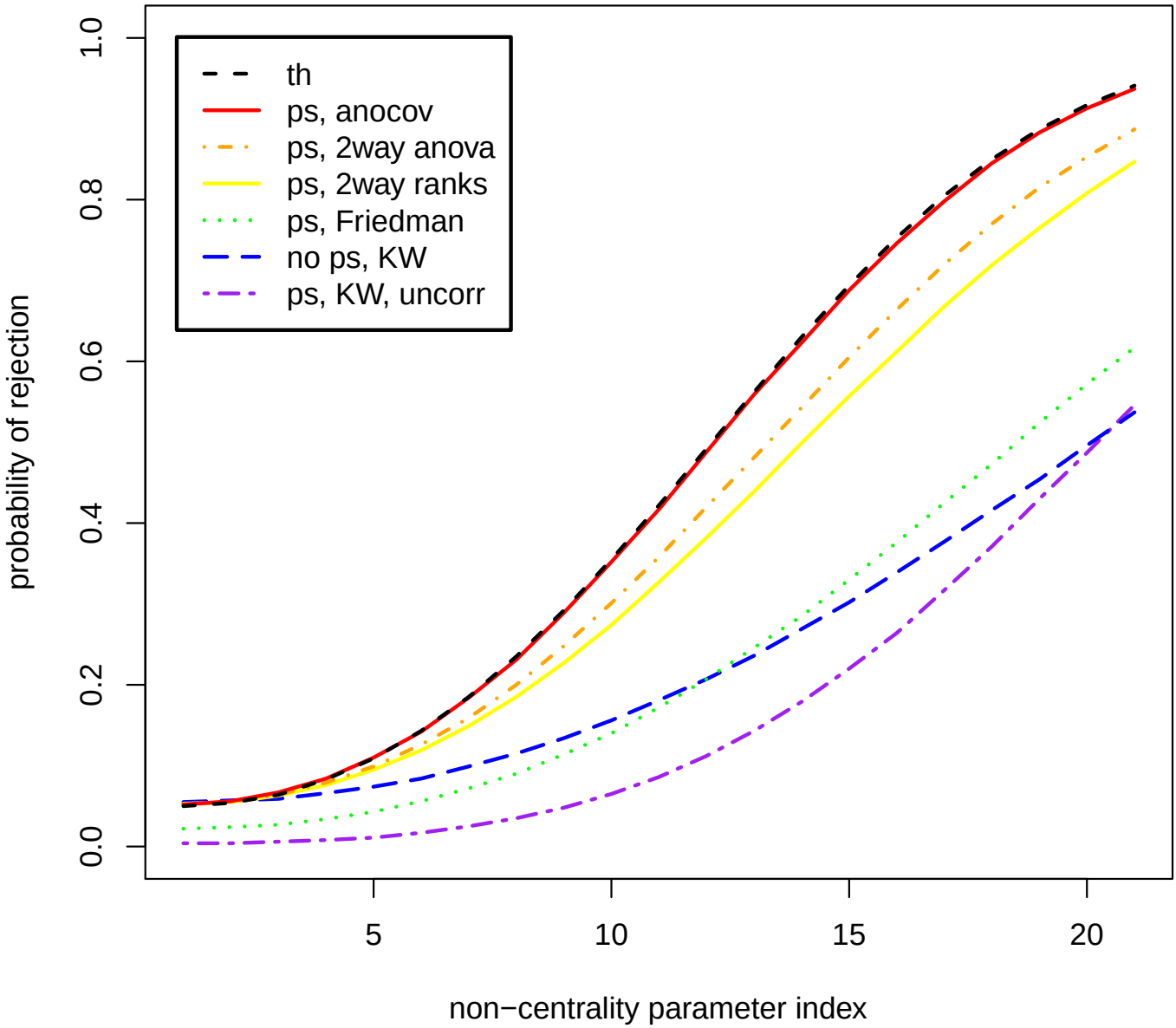

Figure 12: Power plot, $\rho = 0.8$, bivariate Gaussian–Weibull (.3 CV), $J = 2$, $I = 10$

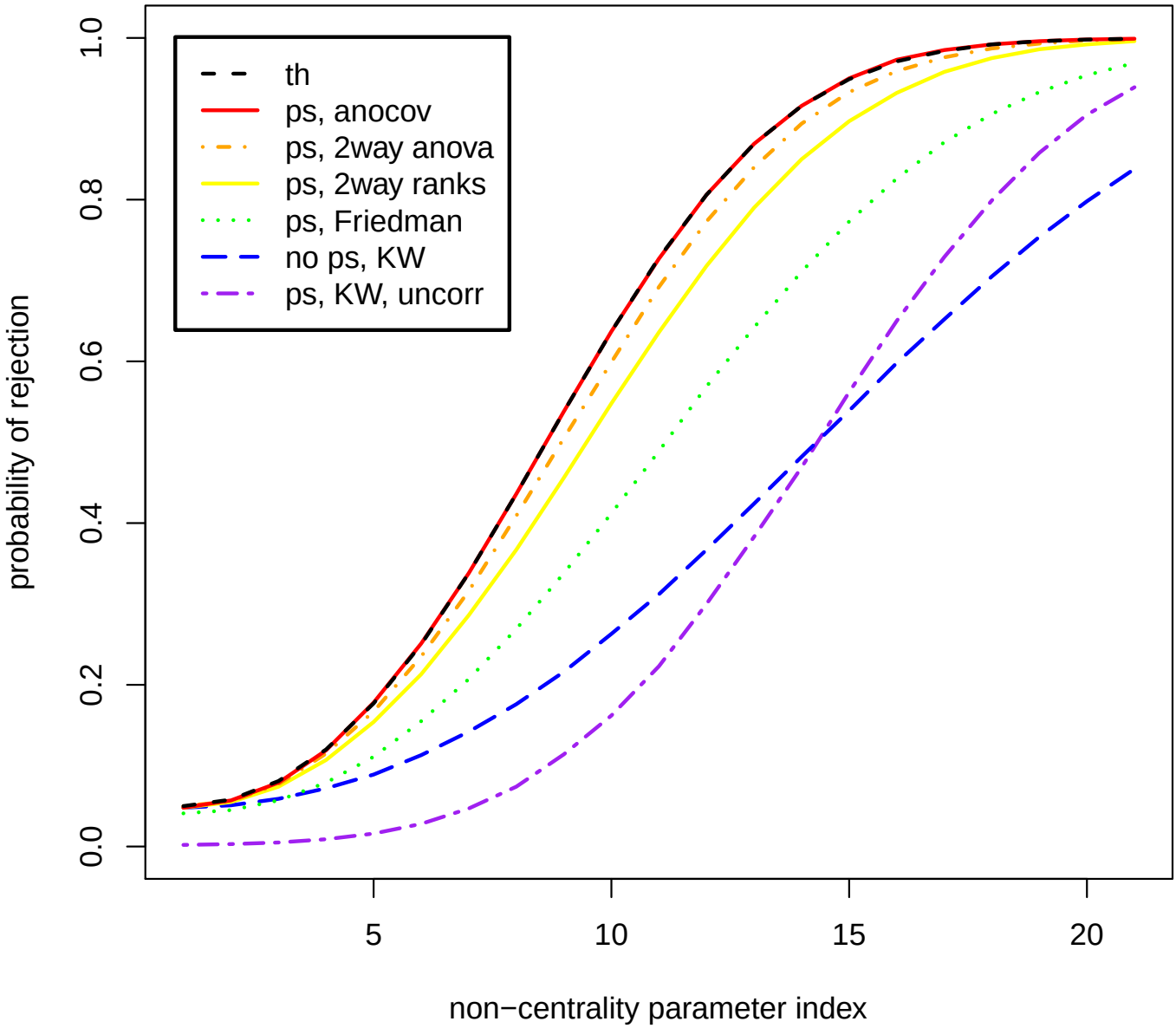

Figure 13: Power plot, $\rho = 0.8$, bivariate Gaussian–Weibull (.3 CV), $J = 2$, $I = 20$

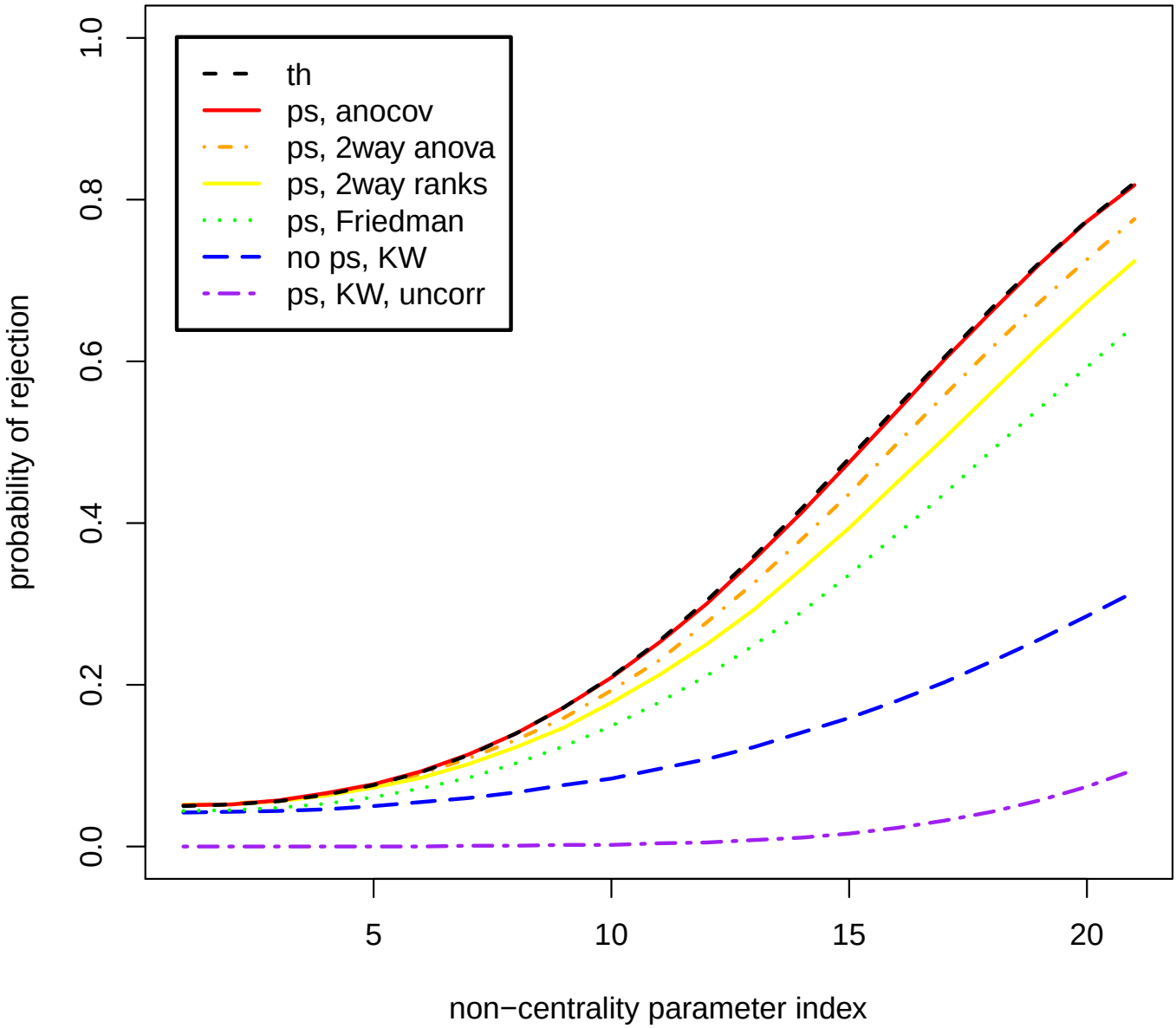

Figure 14: Power plot, $\rho = 0.8$, bivariate Gaussian–Weibull (.3 CV), $J = 5$, $I = 10$

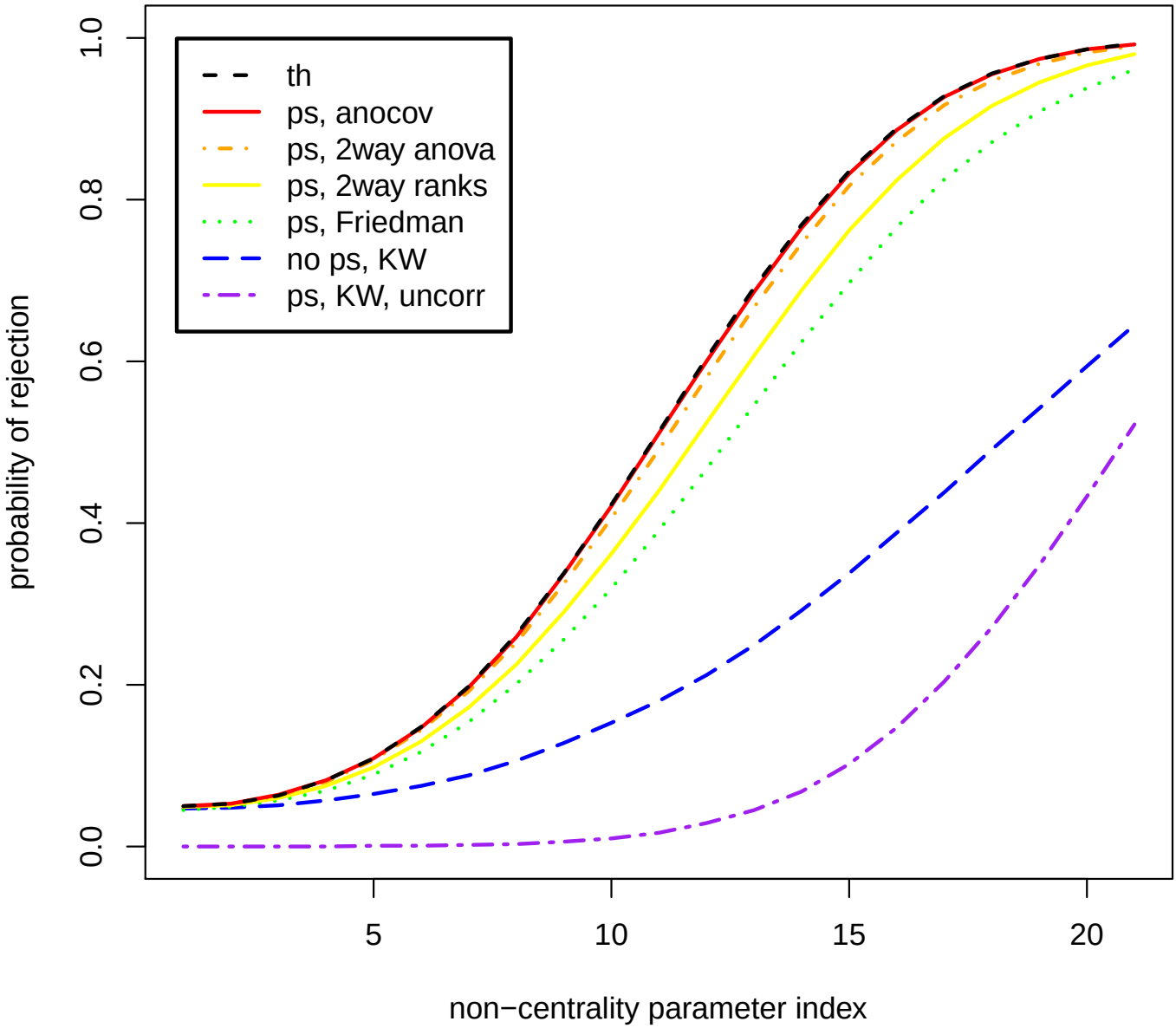

Figure 15: Power plot, $\rho = 0.8$, bivariate Gaussian–Weibull (.3 CV), $J = 5$, $I = 20$

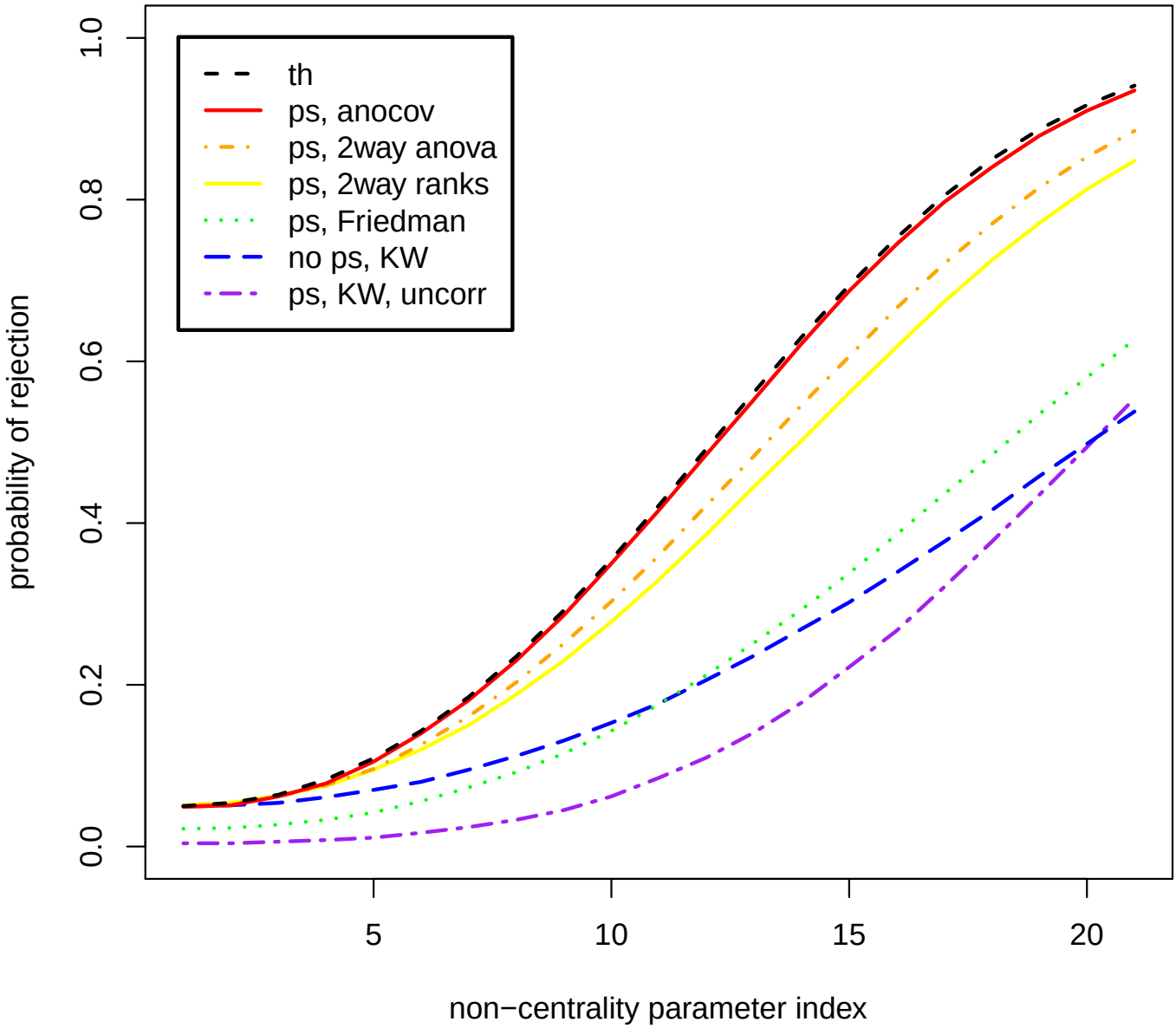

Figure 16: Power plot, $\rho = 0.8$, bivariate Gaussian–Weibull (.4 CV), $J = 2$, $I = 10$

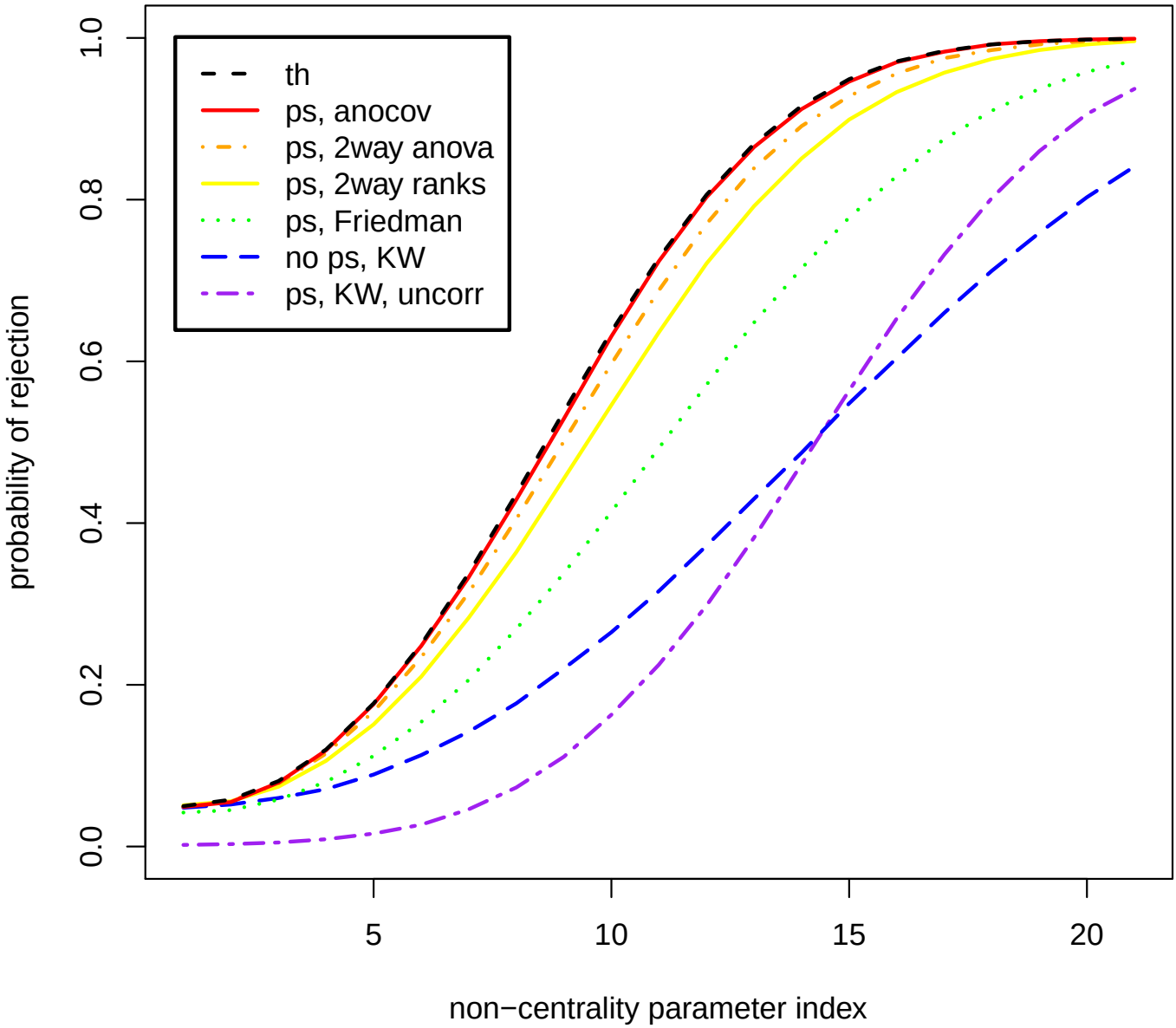

Figure 17: Power plot, $\rho = 0.8$, bivariate Gaussian–Weibull (.4 CV), $J = 2$, $I = 20$

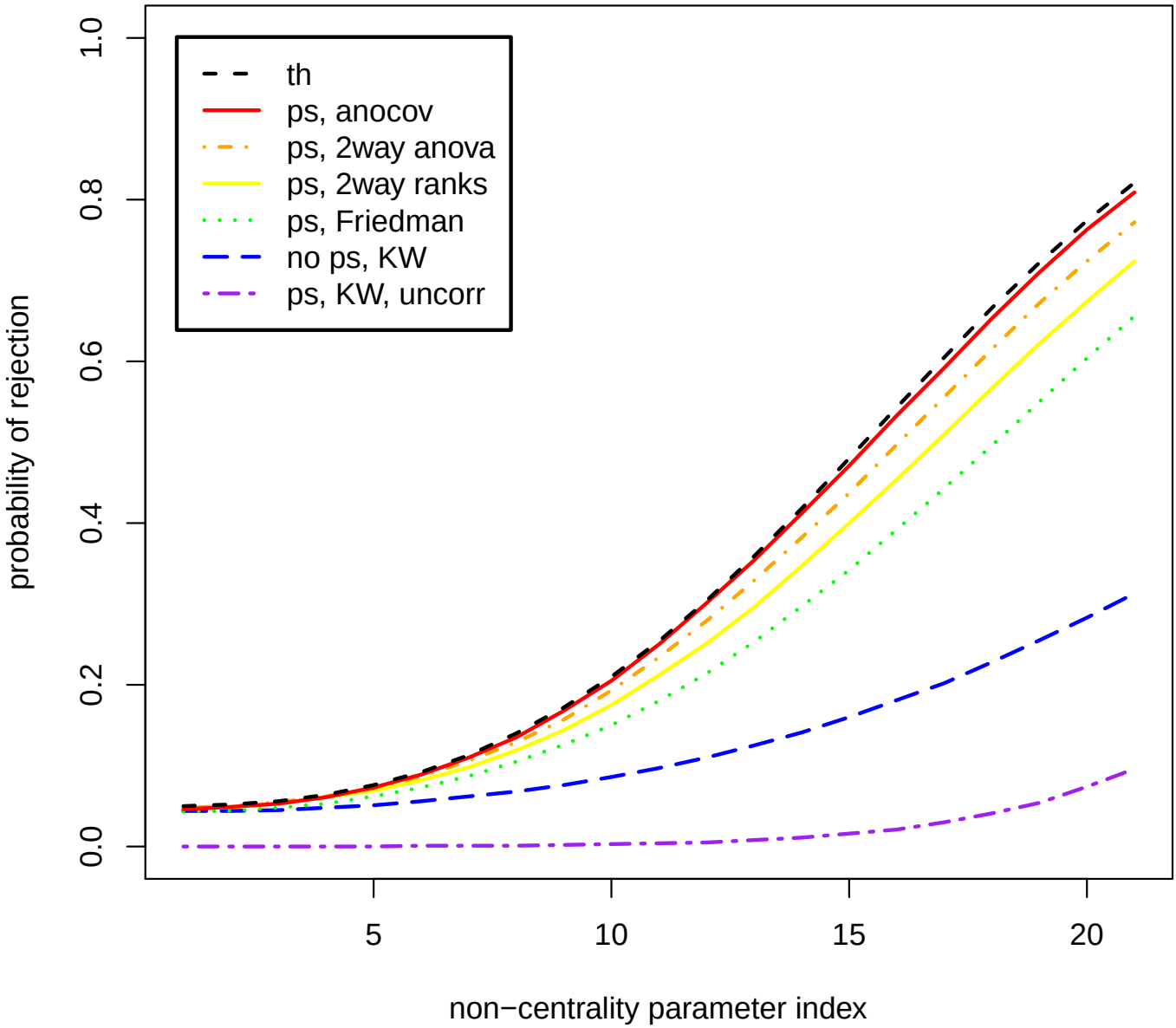

Figure 18: Power plot, $\rho = 0.8$, bivariate Gaussian–Weibull (.4 CV), $J = 5$, $I = 10$

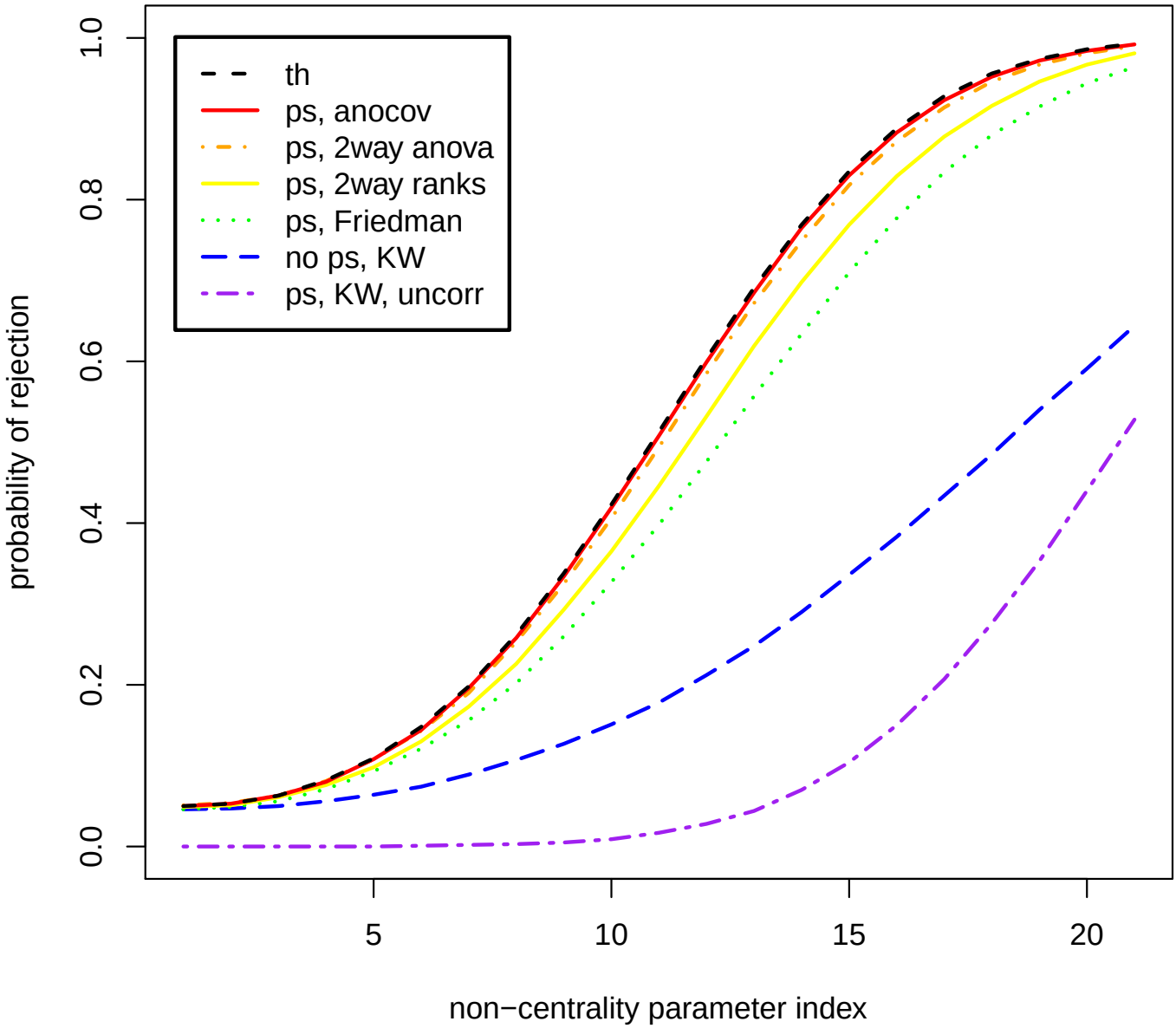

Figure 19: Power plot, $\rho = 0.8$, bivariate Gaussian–Weibull (.4 CV), $J = 5$, $I = 20$

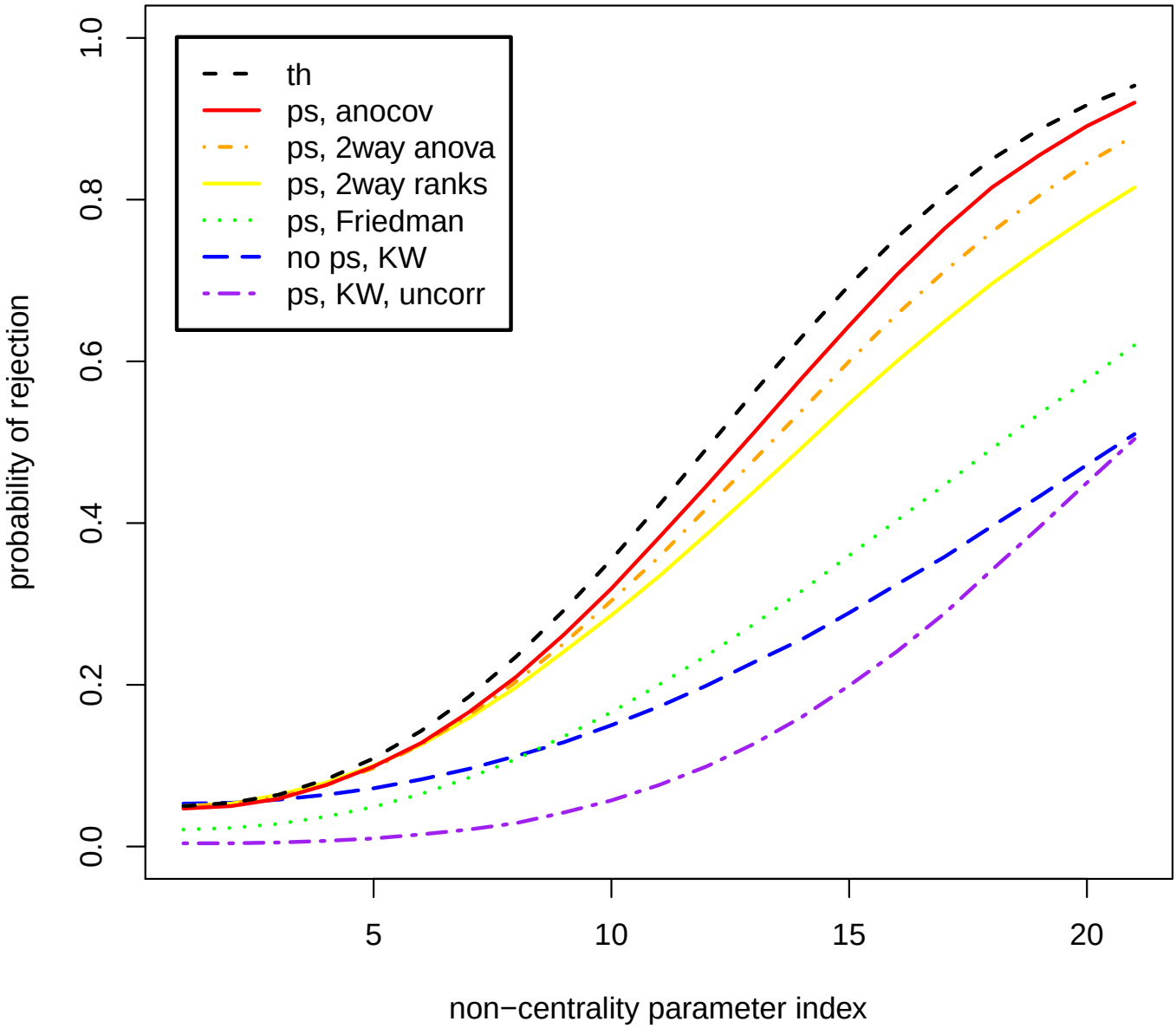

Figure 20: Power plot, $\rho = 0.8$, bivariate Gaussian–uniform, $J = 2$, $I = 10$

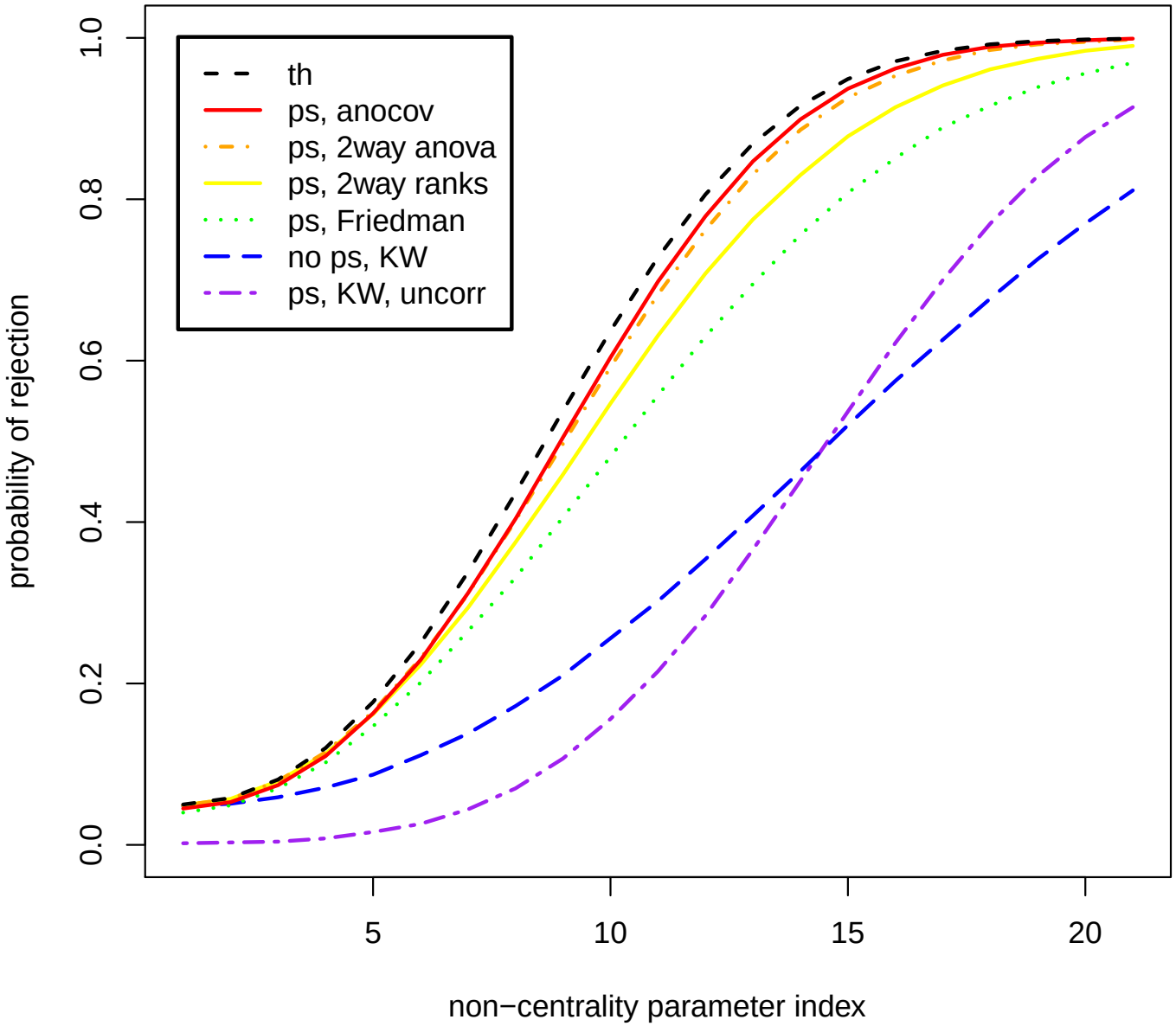

Figure 21: Power plot, $\rho = 0.8$, bivariate Gaussian–uniform, $J = 2$, $I = 20$

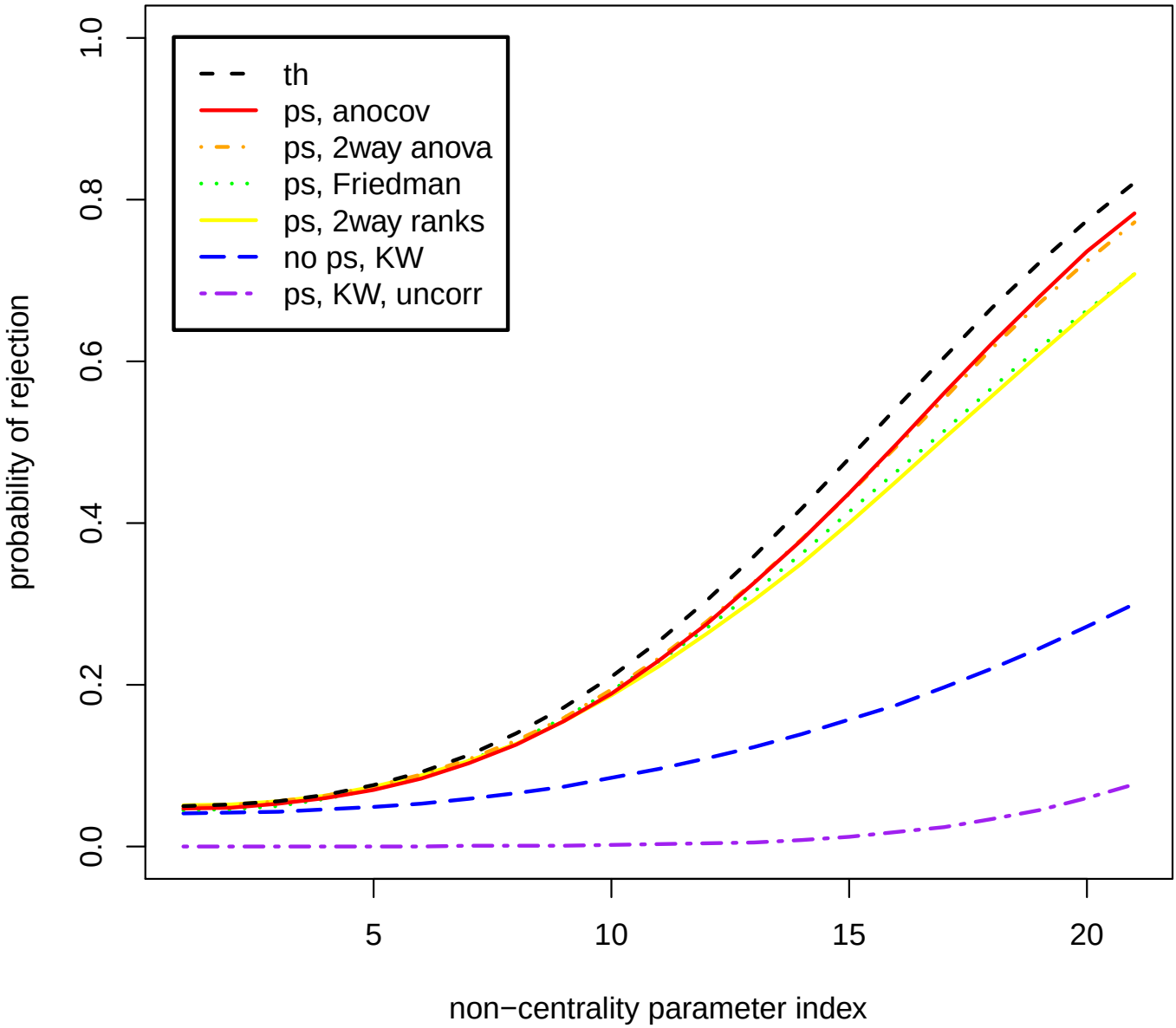

Figure 22: Power plot, $\rho = 0.8$, bivariate Gaussian–uniform, $J = 5$, $I = 10$

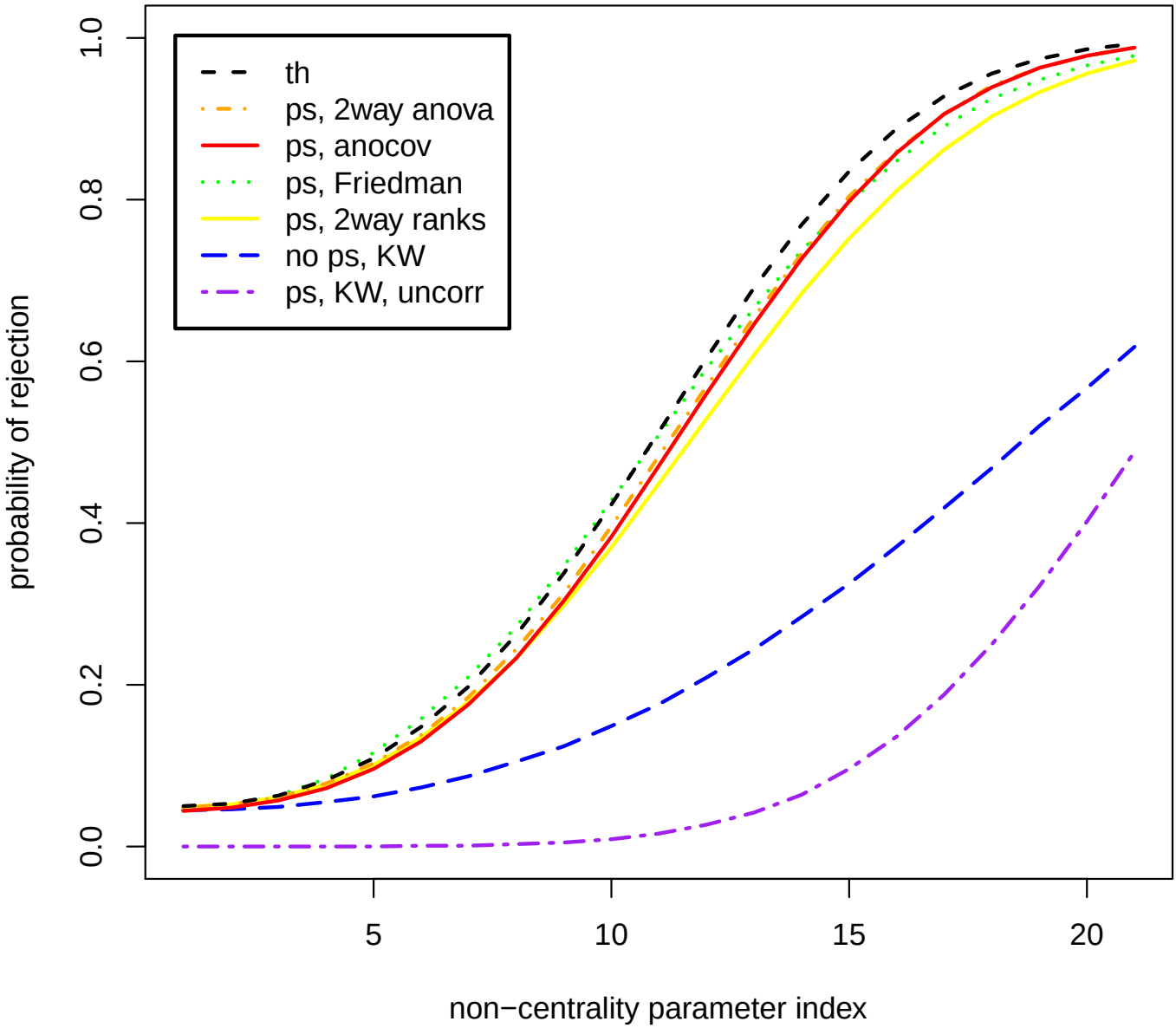

Figure 23: Power plot, $\rho = 0.8$, bivariate Gaussian–uniform, $J = 5$, $I = 20$

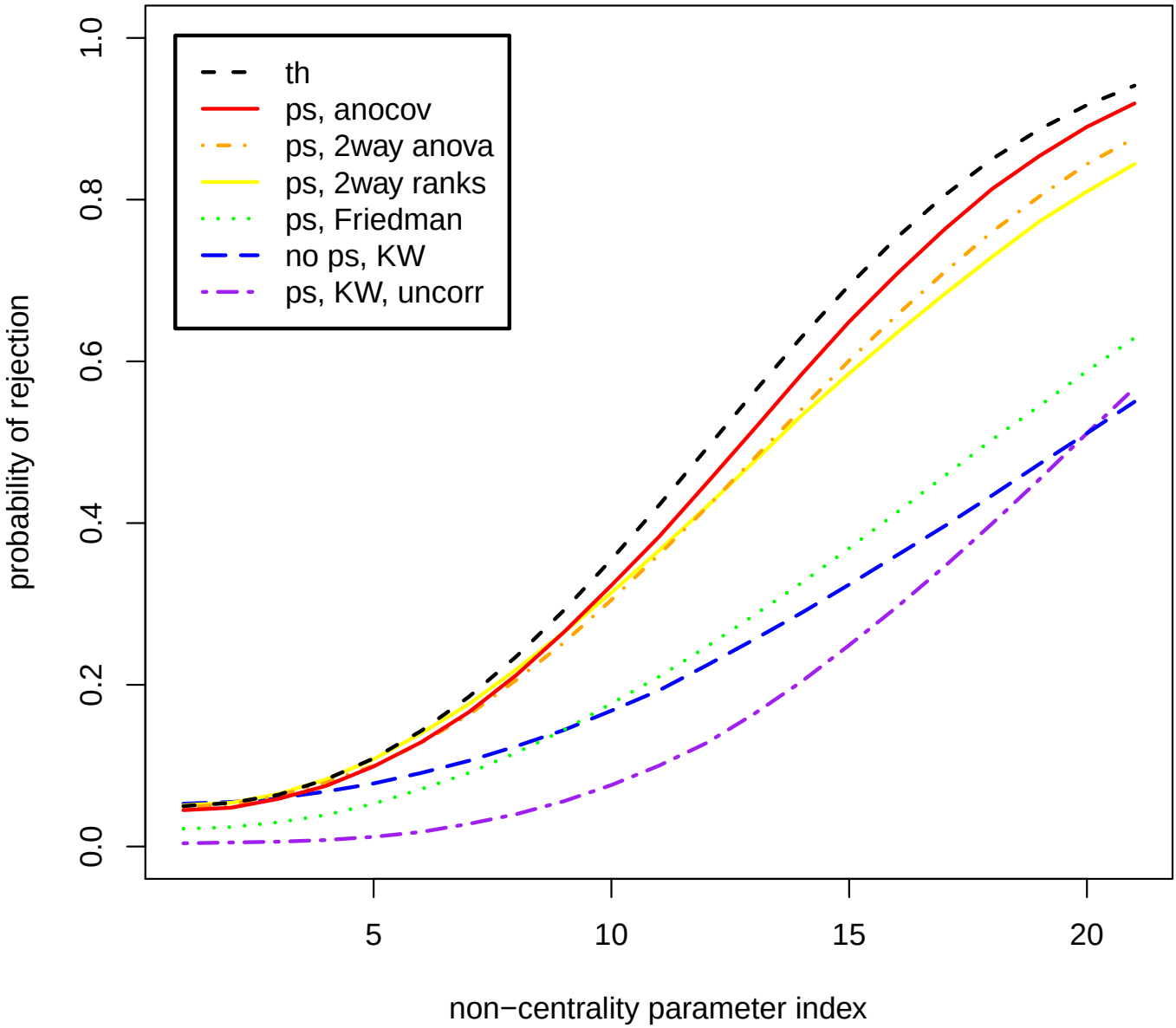

Figure 24: Power plot, $\rho = 0.8$, bivariate Gaussian–right triangular, $J = 2$, $I = 10$

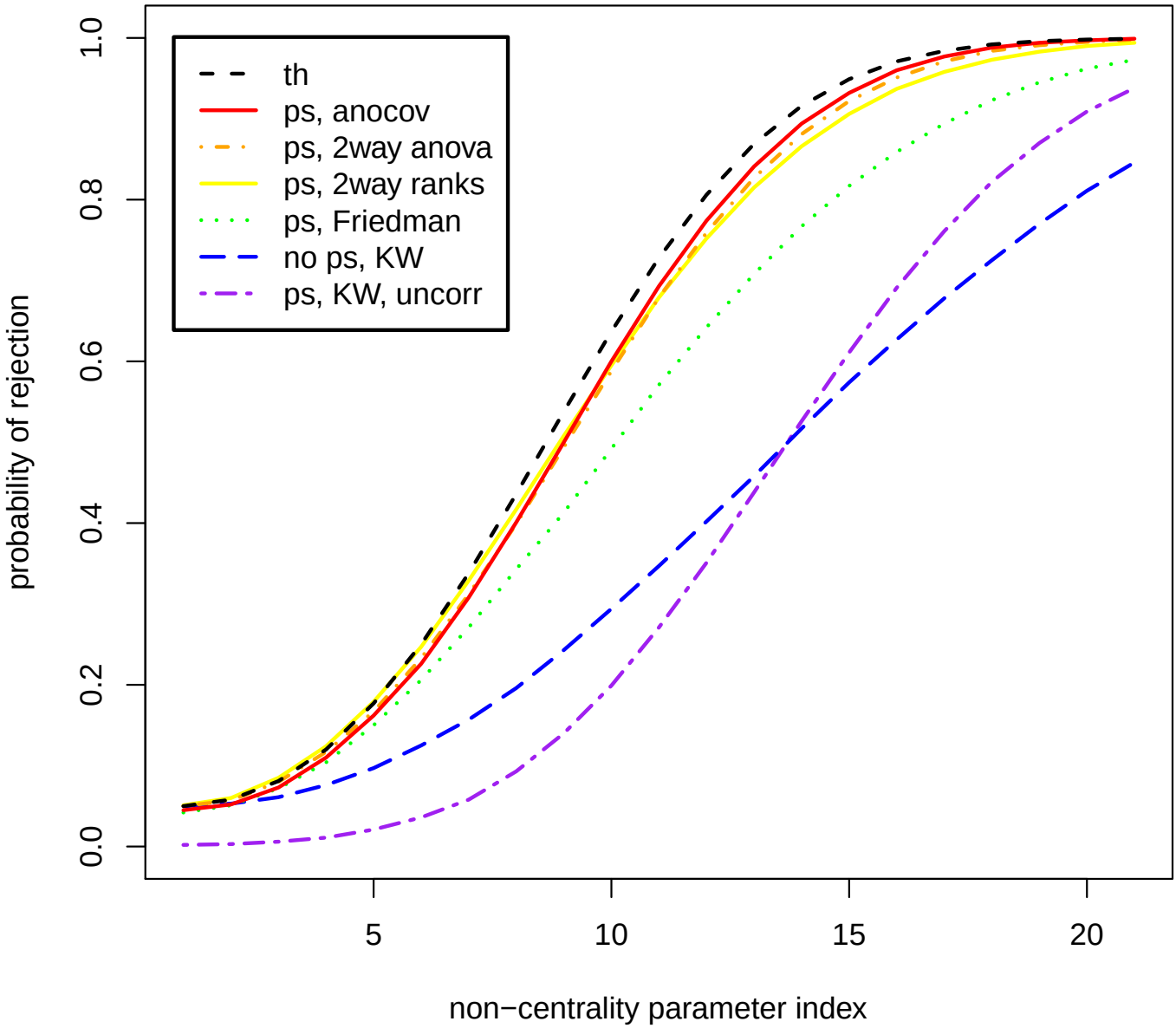

Figure 25: Power plot, $\rho = 0.8$, bivariate Gaussian–right triangular, $J = 2$, $I = 20$

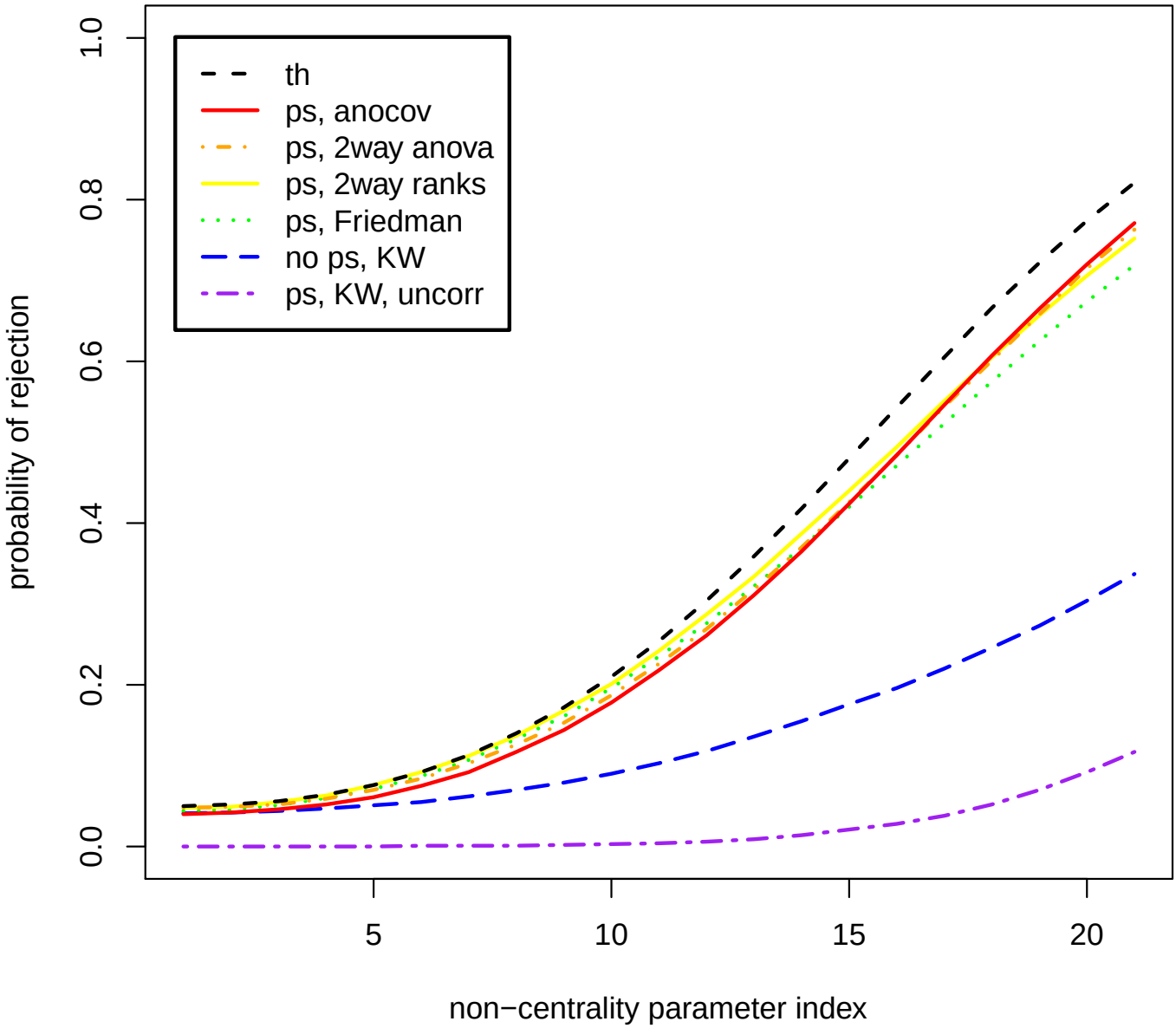

Figure 26: Power plot, $\rho = 0.8$, bivariate Gaussian–right triangular, $J = 5$, $I = 10$

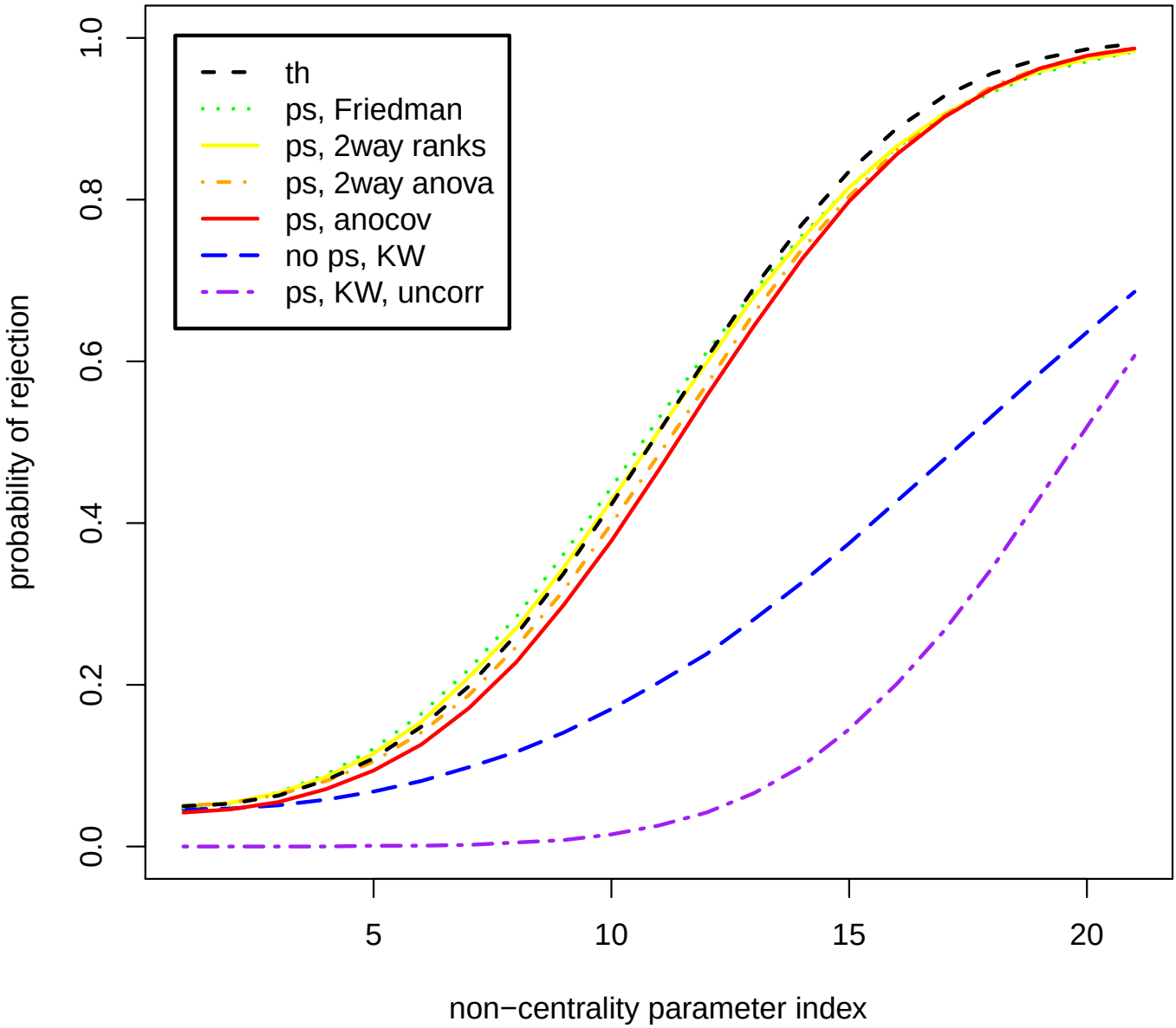

Figure 27: Power plot, $\rho = 0.8$, bivariate Gaussian–right triangular, $J = 5$, $I = 20$

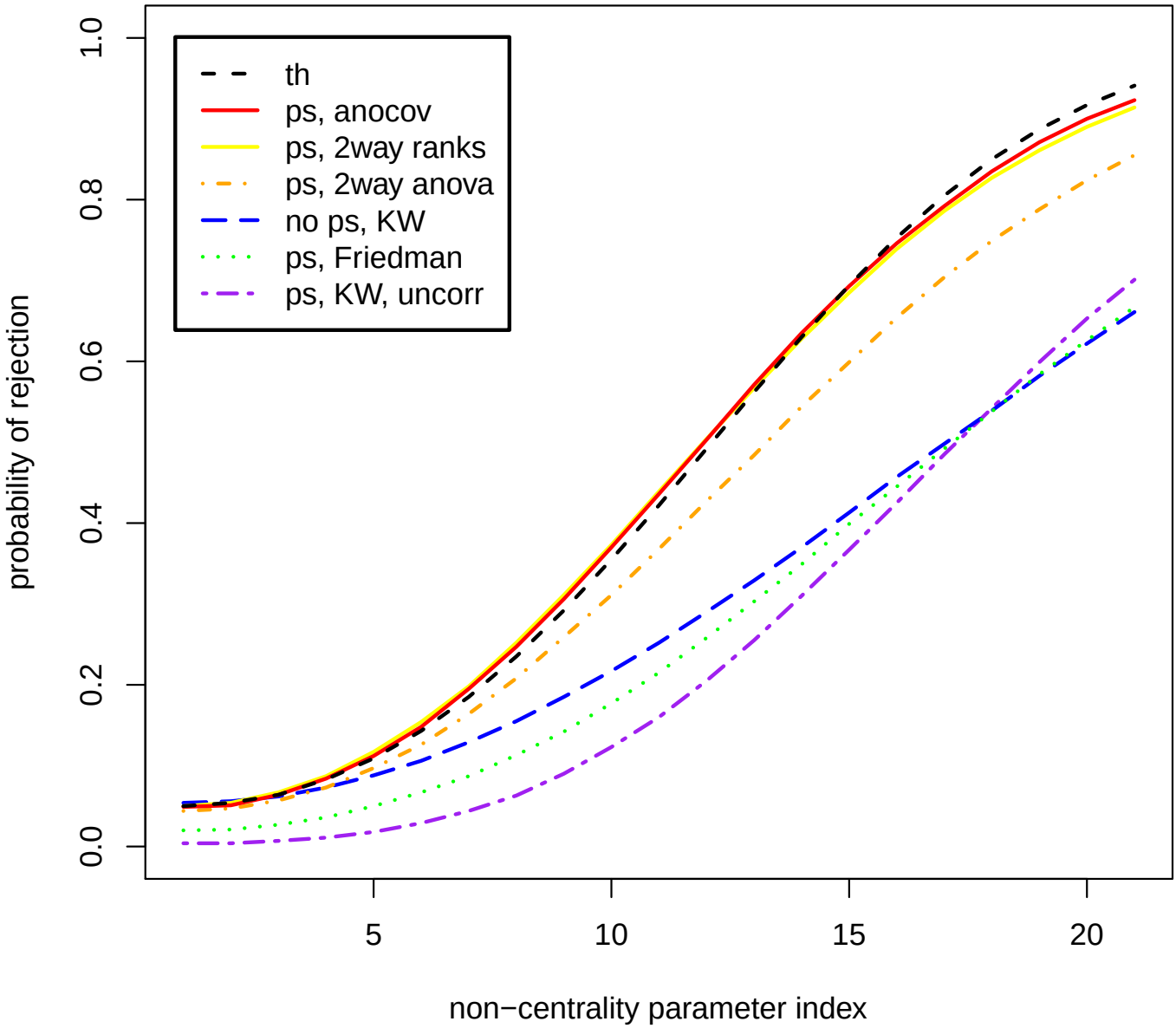

Figure 28: Power plot, $\rho = 0.8$, bivariate Gaussian–double exponential, $J = 2$, $I = 10$

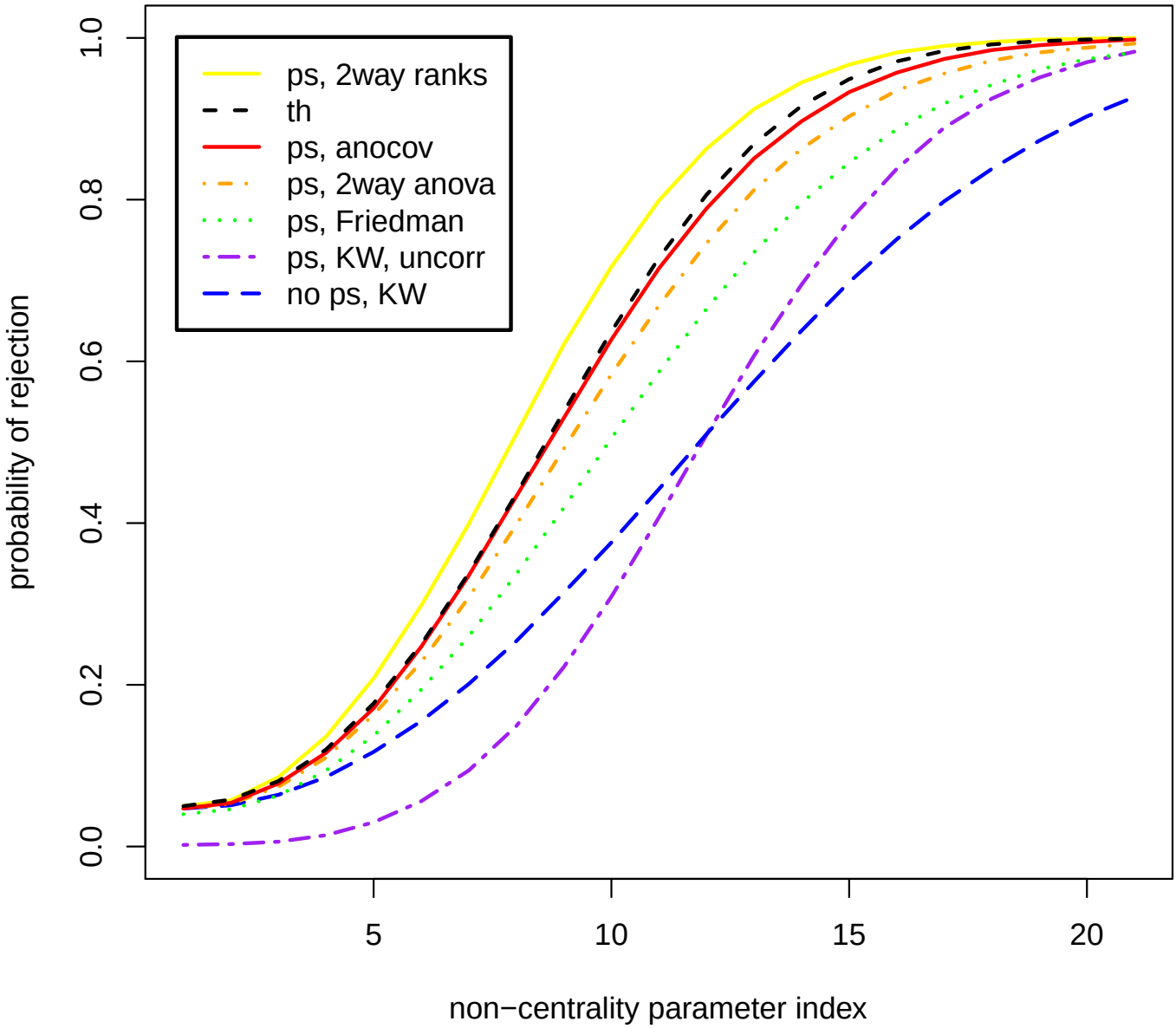

Figure 29: Power plot, $\rho = 0.8$, bivariate Gaussian–double exponential, $J = 2$, $I = 20$

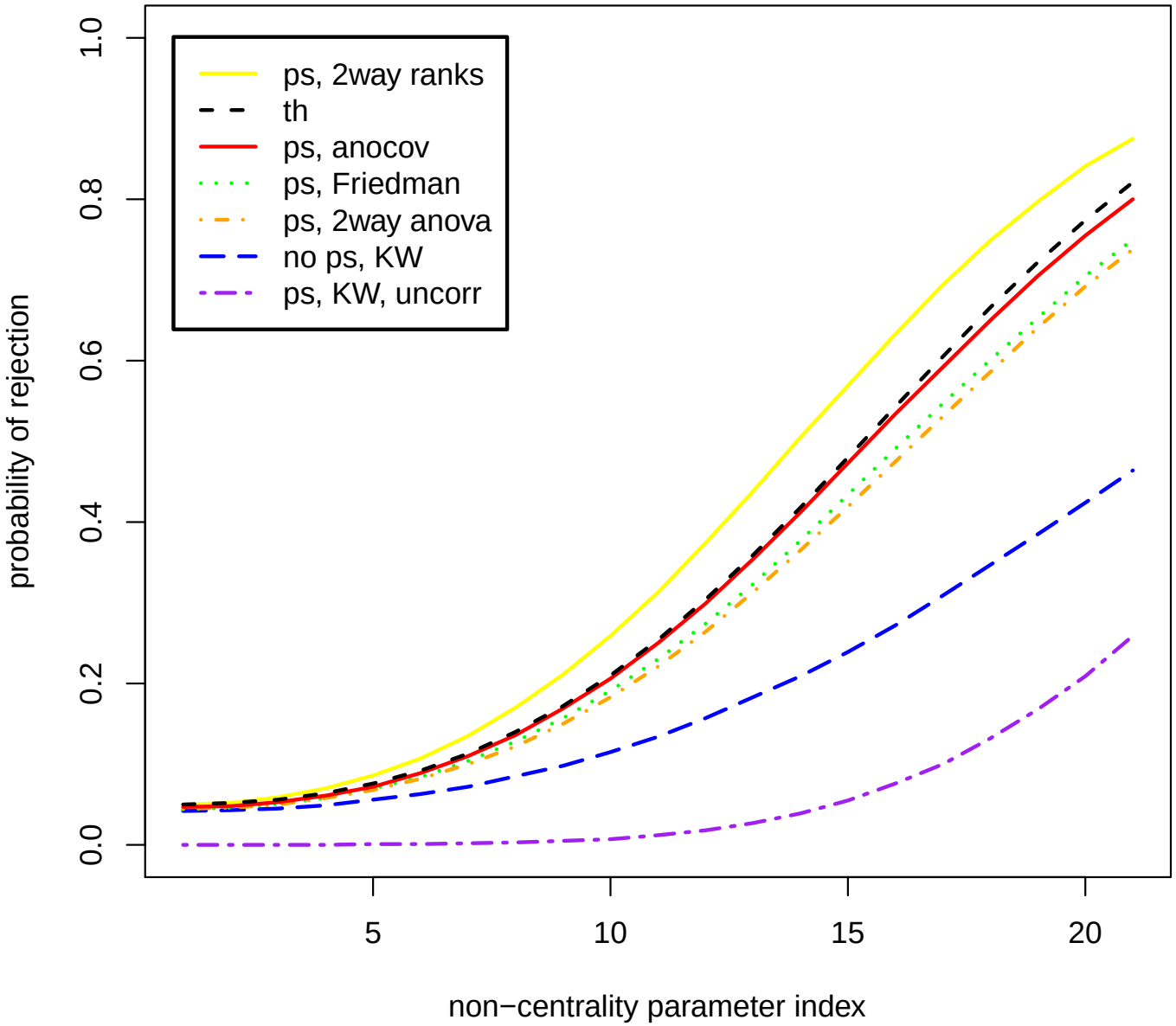

Figure 30: Power plot, $\rho = 0.8$, bivariate Gaussian–double exponential, $J = 5$, $I = 10$

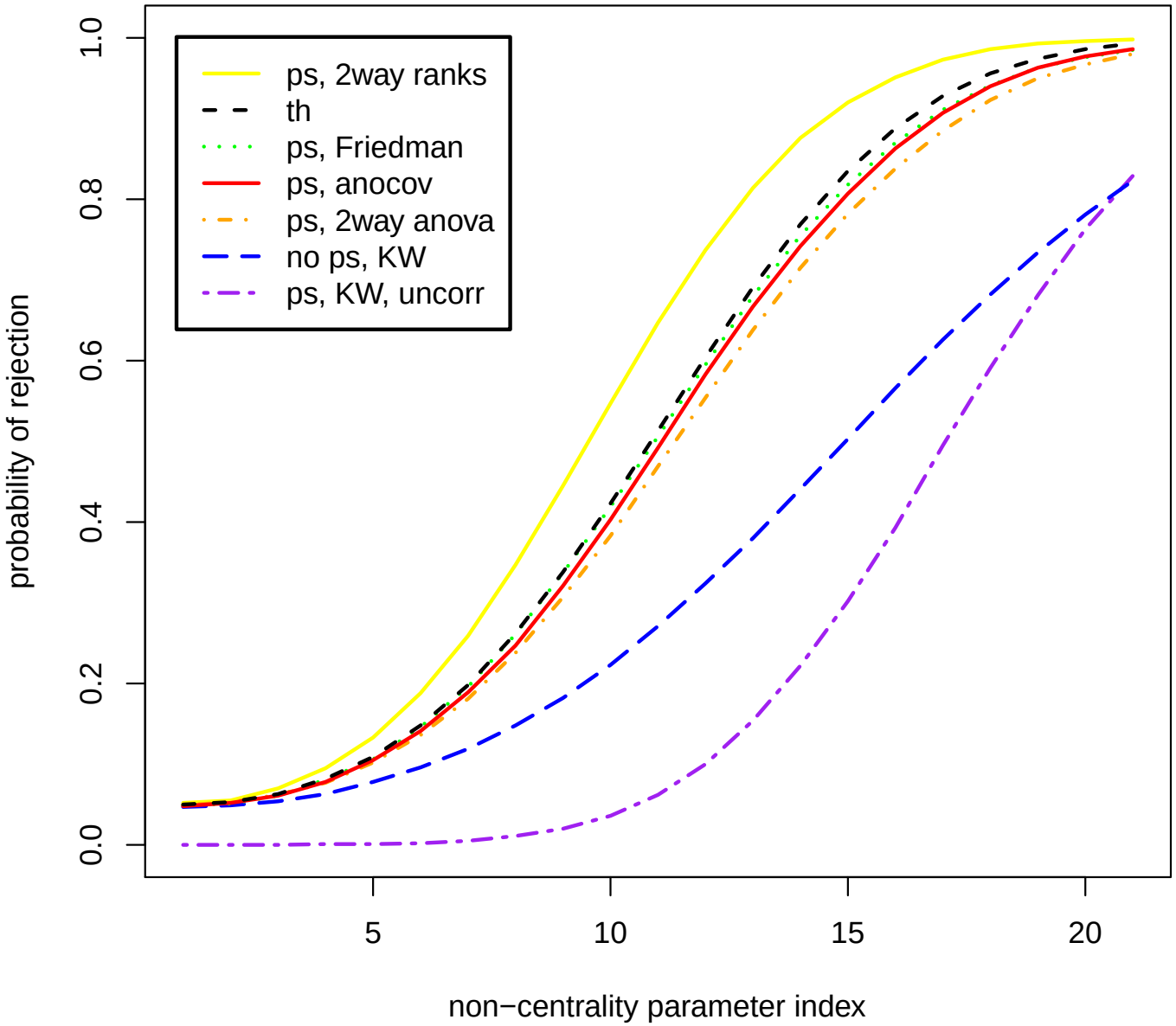

Figure 31: Power plot, $\rho = 0.8$, bivariate Gaussian–double exponential, $J = 5$, $I = 20$

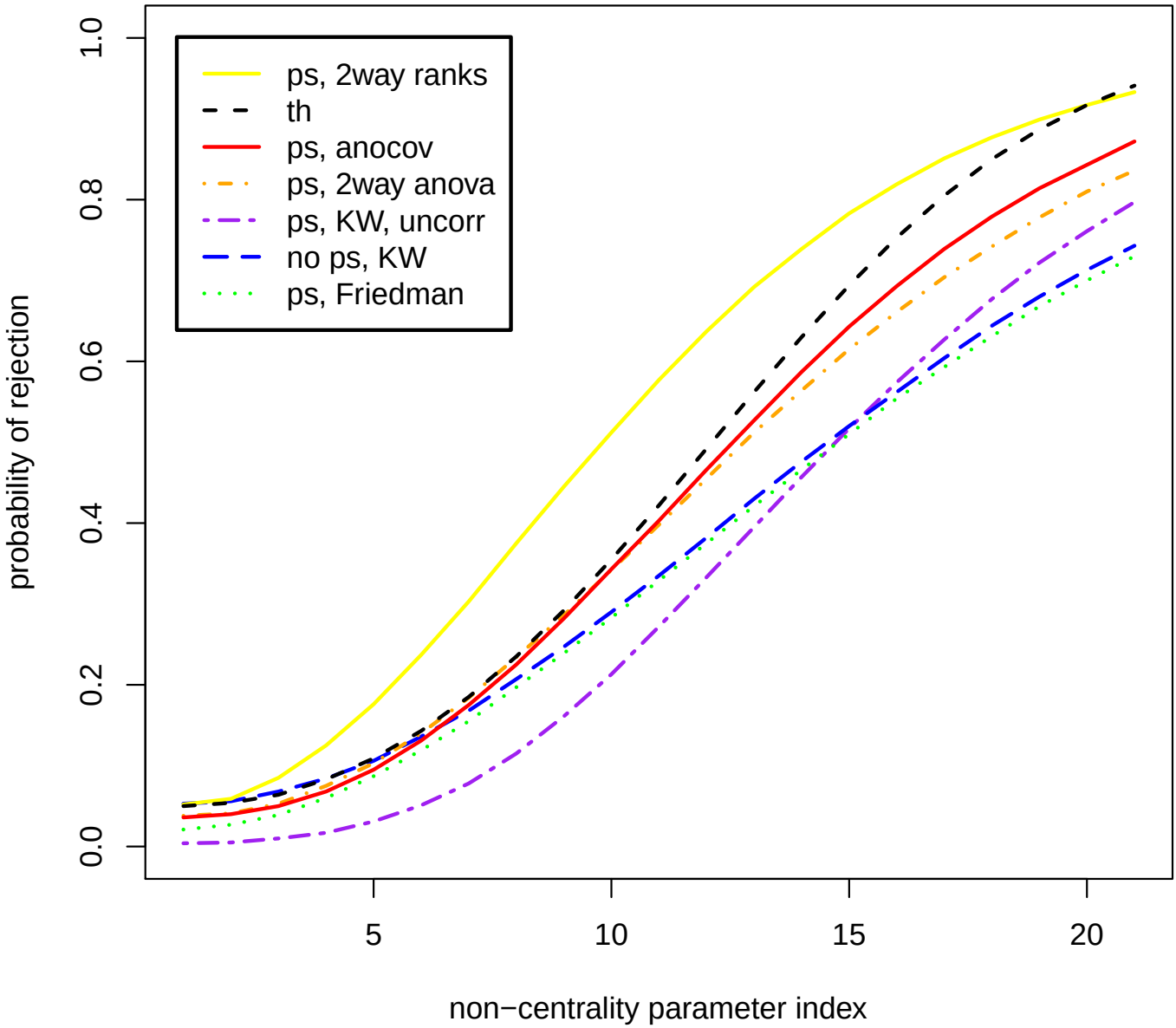

Figure 32: Power plot, $\rho = 0.8$, bivariate Gaussian–exponential, $J = 2$, $I = 10$

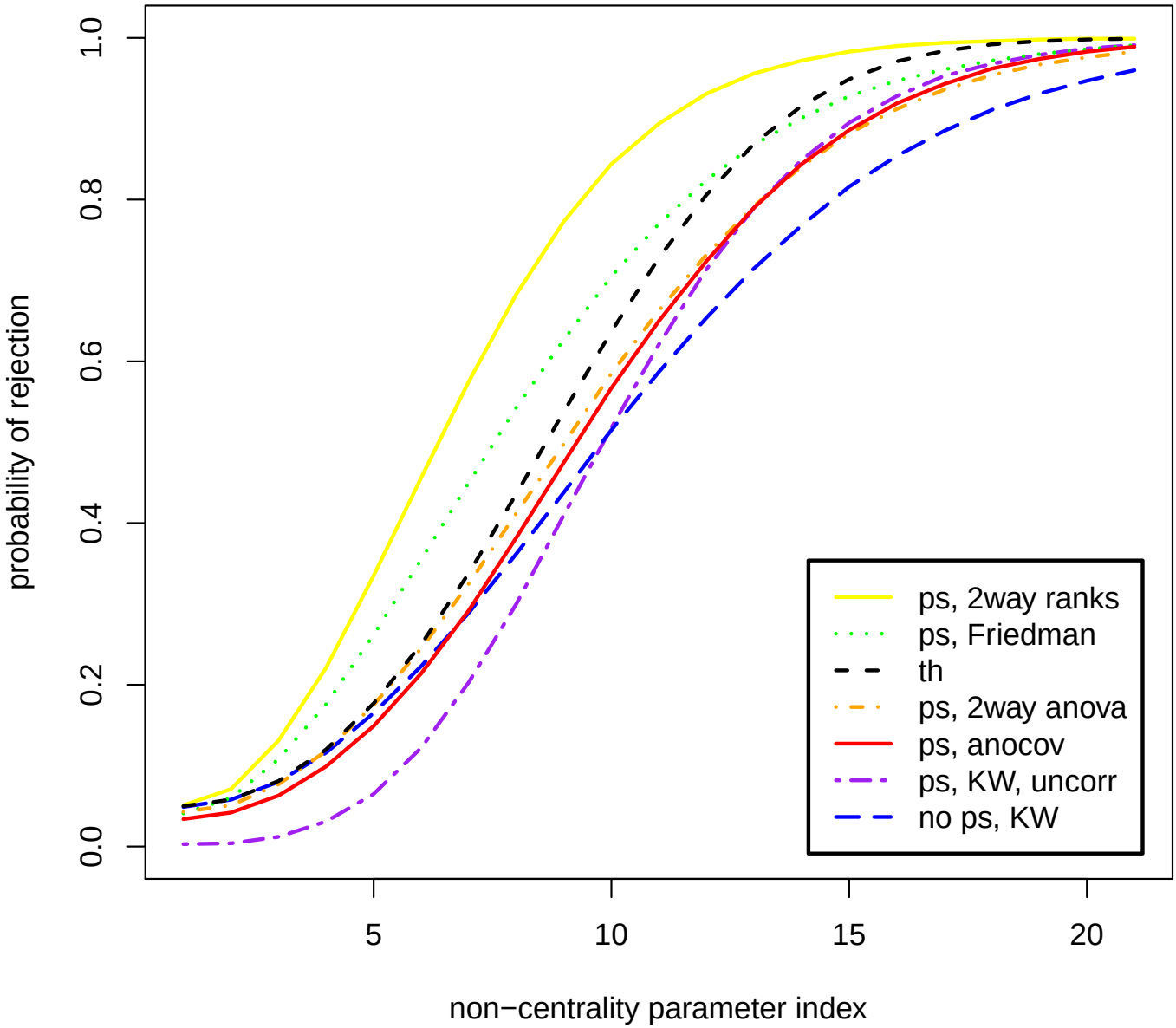

Figure 33: Power plot, $\rho = 0.8$, bivariate Gaussian–exponential, $J = 2$, $I = 20$

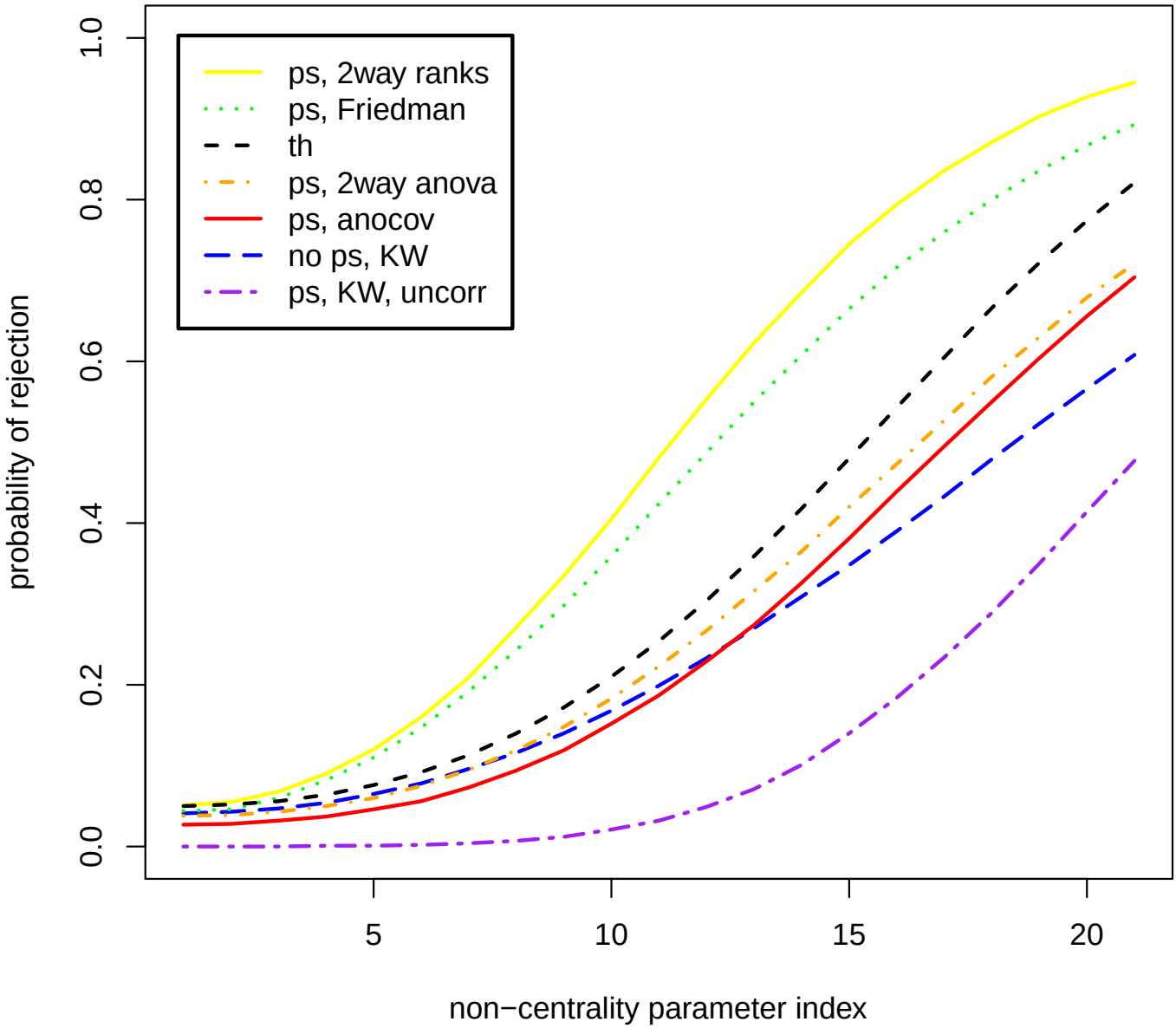

Figure 34: Power plot, $\rho = 0.8$, bivariate Gaussian–exponential, $J = 5$, $I = 10$

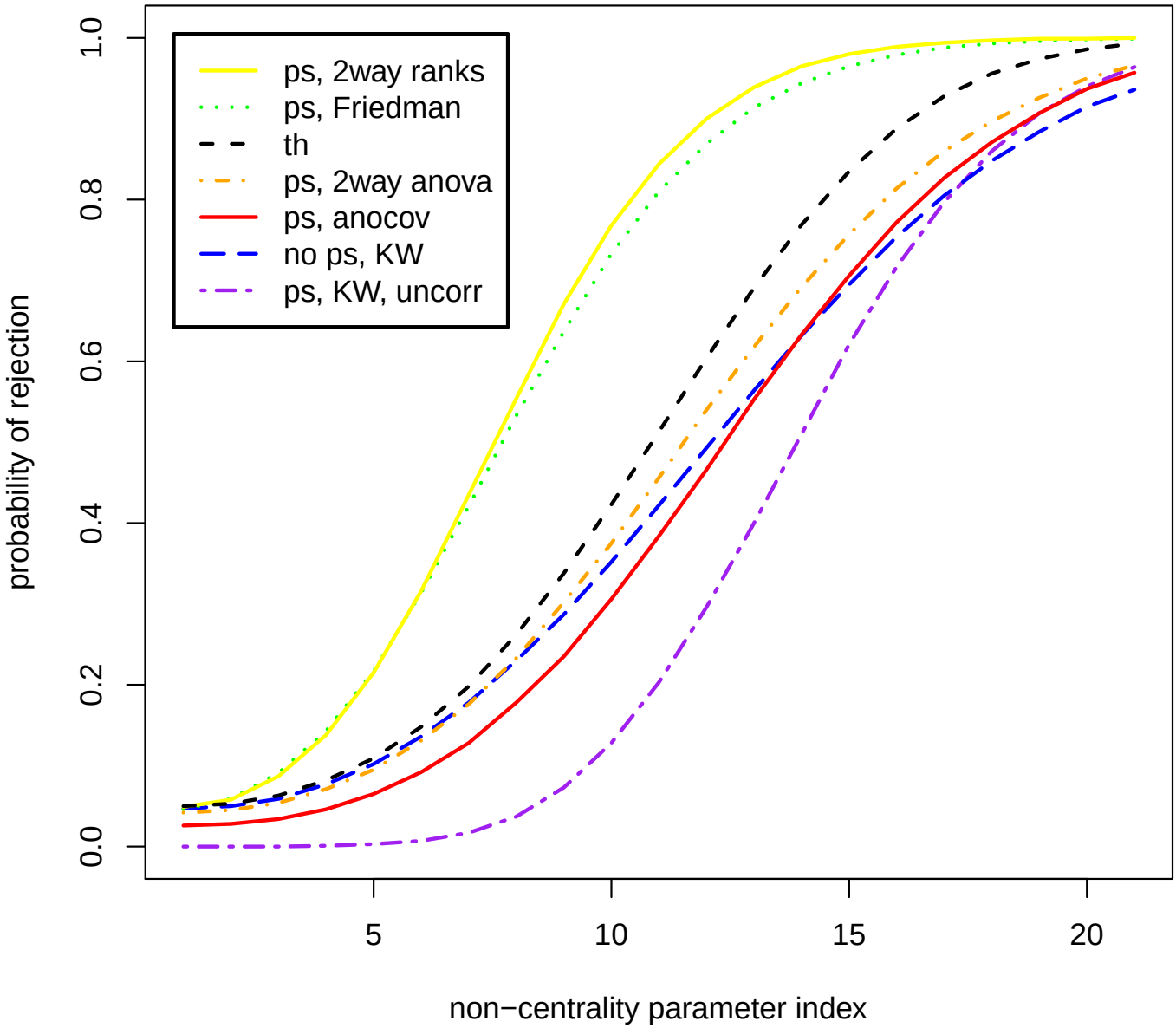

Figure 35: Power plot, $\rho = 0.8$, bivariate Gaussian–exponential, $J = 5$, $I = 20$