Developing Inventory Projection Models Using Empirical Net Forest Growth and Growing-Stock Density Relationships Across U.S. Regions and Species Groups

Prakash Nepal
Peter J. Ince
Kenneth E. Skog
Sun J. Chang
Abstract

This paper describes a set of empirical net forest growth models based on forest growing-stock density relationships for three U.S. regions (North, South, and West) and two species groups (softwoods and hardwoods) at the regional aggregate level. The growth models accurately predict historical U.S. timber inventory trends when we incorporate historical timber harvests. The models also project future timber inventory trends when linked to a model of regional timber harvest, forest product markets, and trade, specifically the U.S. Forest Products Module (USFPM) within the Global Forest Products Model (GFPM). The market model also takes into account the timber supply and market impacts of projected trends in U.S. timber inventory, and results show the sensitivity of U.S. regional timber inventory projections to alternative timber market scenarios. Given the parsimonious nature of the model and its simplicity, the developed net forest growth models can be very useful in providing projections of growing-stock inventory trends across U.S. regions and species groups for alternative U.S. and global timber market scenarios.

Keywords: Forest Inventory and Analysis (FIA); Forest and Rangeland Resource Planning Act (RPA); growing-stock density; growing-stock inventory projection; net forest growth; nonlinear regression; prediction error

Acknowledgments

We acknowledge the support of the USDA Forest Service for funding this project, and especially Dr. Linda Heath of the Forest Service who helped to plan and arrange support for the project. We also acknowledge the guidance provided by Dr. Joseph Buongiorno of UW-Madison in helping design this study. The authors thank three reviewers whose comments helped improve this paper.

Contents

Introduction.........................................................1
Theoretical Framework.........................................3
Methods............................................................3
Data......................................................................3
Model....................................................................6
Model Validation and Application.........................7
Results and Discussion........................................10
Net Forest Growth and Growing-Stock Density Relationship........................................10
Model Comparison and Application.....................12
Conclusions.........................................................16
Literature Cited.....................................................19
Developing Inventory Projection Models Using Empirical Net Forest Growth and Growing-Stock Density Relationships Across U.S. Regions and Species Groups

Prakash Nepal, Post-Doctoral Research Associate
Louisiana State University Agricultural Center
Baton Rouge, Louisiana

Peter J. Ince, Research Forester
Kenneth E. Skog, Supervisory Research Forester
Forest Products Laboratory, Madison, Wisconsin

Sun J. Chang, Professor
Louisiana State University Agricultural Center
Baton Rouge, Louisiana

Introduction

Forest sector models have been widely used in projecting forest product market trends, timber harvest levels, and overall forest resource conditions and trends. Such models differ in their geographic scope (e.g., regional, national, global), economic scope (e.g., partial market equilibrium, general equilibrium), and temporal scope (e.g., static, dynamic). One important aspect of such models is the link between timber supply and forest products demand (Turner and others 2006). Among variables that govern long-run shifts of timber supply, available timber growing-stock inventory is often considered to be the most important based on the logic that supply of timber is proportional to physical volume of growing stock available in the forest, which is influenced over time by forest growth and harvest levels. Therefore, a reliable projection of growing-stock inventory is a critical aspect of all kinds of forest sector models. Growing-stock inventory is projected by adding growth and subtracting harvest quantity to the initial growing stock. Harvest quantity is an element related to the timber market and is generally determined by modeling equilibrium timber market demand and supply, whereas forest growth rate is a biological concept and generally modeled as some function of biological or silvicultural measures related to forest resources (e.g., growing-stock density).

The USDA Forest Service, as mandated by the Forest and Rangeland Renewable Resources Planning Act of 1974 (RPA), is responsible for assessing nationwide timber demand, supply, and conditions of forest resources. For several decades, forest sector models have played a key role in producing long-range RPA projections of U.S. forest resources, conditions, and trends (Adams and Haynes 2007). The USDA Forest Service has used a combination of forest sector models, including the Timber Assessment Market Model (TAMM) (Adams and Haynes 1996); North American Pulp and Paper Model (NAPAP) (Zhang and others 1996; Ince 1994); and Aggregate Timberland Assessment System (ATLAS) (Mills and Kincaid 1992). Recently, the USDA Forest Service designed a new forest sector modeling framework for the 2010 RPA assessment. This new modeling system consists of a group of submodels, collectively known as the U.S. Forest Assessment System (USFAS). As a part of this system of models, the U.S. Forest Products Module (USFPM) (Ince and others 2011) was developed at the USDA Forest Products Laboratory (FPL). USFPM is a dynamic partial market equilibrium model of the U.S. forest product sector that operates within the Global Forest Products Model (GFPM) (Buongiorno and others 2003). The USFPM/GFPM produces long-range projections of U.S. forest products markets, forest product trade, and regional U.S. timber markets (Ince and others 2011; Buongiorno and others 2003).

Some of these forest sector models project growing-stock inventory based on very complex growth models that require more effort to gather and analyze input data (e.g., the ATLAS model, or the forest dynamics module of USFAS), whereas others have only limited capability to project U.S. forest inventory (e.g., USFPM, GFPM). Growth in ATLAS is modeled as a function of relative density change as defined by the ratio of the existing inventory volume per acre and the corresponding base-yield table volume per acre (Mills and Kincaid 1992). The requirement of the base-yield tables and specification of relative density change parameters makes it complex to use because such parameters and base-yield tables are not available for every species in the United States. For RPA purposes, ATLAS has now been replaced by a more complex set of probabilistic forest
transition models developed at the USDA Southern Research Station to simulate the dynamics of U.S. forest growth and changes in forest inventory for tens of thousands of forest sample plots within each region. Such plot transition models have been developed for the U.S. North and South regions, and the southern regional plot transition model was used also in the recently completed Southern Forest Futures Project (Wear and others 2011). Such a plot-level forest transition model runs stochastic simulations of harvest and management responses for all forest plots, but one complex aspect of such a stochastic simulation model is that it may typically produce different results each time the model is run, requiring many model runs to derive a representative sample of results (Wear and others 2011). Another example of this approach to modeling forest growth presented by Abt and others (2000) used Forest Inventory and Analysis (FIA) plot-level data to estimate forest growth per acre by species groups, physiographic regions, and management types in the U.S. South. Growth was modeled as a function of state, owner, stand age, and interaction between owner and stand age. One problem with using such a model is that plot age data are not always available.

At the opposite end of the complexity spectrum, GFPM estimates change in forest growth using a single nonlinear functional relationship between net forest growth and growing-stock density applied to all countries (Turner and others 2006). This functional relationship was derived from cross-sectional analysis of data on aggregate forest growth and forest growing-stock density of 180 countries, and the GFPM used the same elasticity of growth rate with respect to growing-stock density across all countries (as estimated in Turner and others 2006). Note that although the GFPM employs the same elasticity of growth rate with growing-stock density across all countries, the base year growth rate and growing-stock density in a given country may be different and thus may change differently over time as determined by the elasticity of growth rate with growing-stock density. For example, a typical projection of the growth rate of growing-stock inventory in Russia remains quite different from that in Germany because of different initial growth rates and growing-stock densities in those countries.

Nevertheless, this global approach is more generalized and may not precisely account for U.S. regional or species group differences in the relationship between forest growth and stocking density (if the same elasticity relationship that is applied uniformly to all countries and all timber species is also applied to U.S. subregions and species groups). Such an approach to modeling forest growth is of course less complex, based on global data that are readily available and may be appropriate for analysis of global forest trends in aggregate. However, in our earlier attempts to predict U.S. regional growth and timber inventory, we found that the global elasticity relationship employed in the GFPM did not accurately represent the dynamics of observed forest growth and growing-stock density relationships among species groups and regions in the United States. In this sense, the current versions of the GFPM and USFPM have limited capability to project U.S. regional growing-stock inventory with acceptable accuracy (and for RPA applications, the much more complex stochastic plot transition model of the USFAS was employed to produce U.S. forest inventory projections, but the model simulations were only indirectly related to the USFPM/GFPM timber market and timber harvest projections).

Therefore, we needed tools to precisely project U.S. timber inventory trends or changes in forest carbon stocks in direct relation to USFPM/GFPM timber market scenarios and had a compelling need to develop basic forest growth models that are simple to apply, yet credible enough to reliably project growing-stock inventory across U.S. regions and species groups in relation to U.S. and global timber market scenarios. Accordingly, this study had two purposes. The first purpose was to develop empirical net forest growth and forest growing-stock density relationships for three U.S. regions (North, South, and West) and two species groups (softwoods and hardwoods) that would allow USFPM to accurately predict U.S. timber inventory trends when including projected aggregate timber harvest, forest product markets, and trade at the U.S. regional and national level. Note that the USFPM/GFPM forest products market model projects production, consumption, and trade of timber and forest products markets at the national level in all other countries (but production and timber supply at a regional aggregate level for the U.S. only). Therefore, one of the goals of this study was to develop regional timber growth models that can make accurate projections of timber inventory by U.S. regions and species groups so that these projections can be consistently applied in conjunction with a broader-scale market projection model such as USFPM/GFPM.

While detailed stand-level growth models can be developed as functions of several variables related to forest ecology such as relative density (e.g., stand density index), stand age, and site quality and they might provide more precise projections of timber inventory at the stand level, such detailed models are generally not necessary in the context of more generalized national or global level timber market models. This is because market parameters such as timber harvest, stumpage price, lumber demand, and supply are generally not analyzed at a stand level but only at regional and/or national aggregate levels.

Our secondary purpose in this study was to develop long-term inventory projections that would be suitable for broadly modeling forest carbon and timber market implications of different carbon offset and timber market scenarios at U.S. regional and national levels (an objective we plan to pursue in subsequent research). This paper focuses primarily on methods used to estimate our regional timber growth and inventory models in relation to growing-stock density and also the linkage of U.S. timber growth and inventory models to USFPM/GFPM.
Developing Inventory Projection Models Using Empirical Net Forest Growth and Growing-Stock Density Relationships

In this paper, we use regional forest growth data that are based on the U.S. timber inventory concept of “net annual growth” as defined in Smith and others (2009), specifically “The average annual net increase in volume of trees during the period between inventories. Components include the increment in net volume of trees at the beginning of the specific year surviving to its end, plus the net volume of trees reaching the minimum size class during the year, minus the volume of trees that died during the year and minus the net volume of trees that became cull trees during the year.” This measure of growth is thus net of tree mortality, and is therefore akin to the concept of net stand growth (growth net of mortality, but not net of harvest).

The U.S. regional net annual growth rate models developed in this study were based on the theoretical relationship between net stand growth rate (measured by timber “net annual growth” as a percentage of inventory volume) and growing-stock density. This theoretical framework was chosen because the county-level forest net annual growth and growing-stock density data used in this study showed a similar empirical relationship as one would expect between the net annual growth rate and growing-stock density (i.e., a declining relationship, similar to the implicit relationship shown in Fig. 1). Specifically, for a given species group and region, the counties with higher growing-stock density had generally lower growth rates per unit of inventory and vice versa. In addition, based on the empirical data (Table 1), we found unique growth/density relationships for softwoods and hardwoods and for each region. For example, softwoods in the U.S. South are observed to have the most productive growth rate (annual growth rate as percentage of inventory) among all regions and species groups (Table 1).

Methods

Data

This study used cross-sectional data of net annual forest growth and growing-stock density by county obtained from the FIA database of the USDA Forest Service (Miles 2011). County level cross-sectional data were used because cross-sectional data aggregated at U.S. state level (50 observations) or time series data aggregated at U.S. regional level (9 observations, 1953, 1963, 1970, 1977, 1987, 1992, 1997,
2002, 2007) did not show expected relationships between net annual forest growth and growing-stock density, probably due to relatively small sample size. The growth equations derived from cross-sectional analysis of county-level data represent regional aggregate level relationships, based on an aggregate region-wide data (the counties within each region). Parenthetically, the aggregate growth and growing-stock density relationship estimated at the regional level do not necessarily apply to any individual county, but will reflect the range of growth and growing-stock density data observed at the county level.

The EVALIDator tool (USDA 2011) was used to retrieve net annual growth and growing-stock density data according to two species groups (softwoods and hardwoods) in various counties in three U.S. regions (Fig. 2). Figure 3 shows the empirical relationships between net growth in growing stock (%/y) and growing-stock density (m$^3$/ha/y) by U.S. region and species group. The cross-sectional county-level net annual growth data were not available for five states in the Western region (Alaska, California, Hawaii, Oregon, and Washington). Thus, the empirical relationship analyzed in this study for the U.S. West does not include those five states, but we made adjustments to our model for the West so that it would accurately predict growth for the entire region (including the Pacific Coast states). For the projections in this study, we calibrated the estimated growth parameters for the West so that the model would project timber growth and inventory for the entire West quite similar to the region-wide net annual growth as reported for 2006–2007 in the Forest Resources report from the Forest Service (Smith and others 2009). This data calibration was a final step in our model estimation procedure and was necessary because our origi-
Figure 3—Empirical relationship between net growth in growing stock (%/y) and growing stock density (m³/ha) for (a) softwoods in the U.S. North, (b) hardwoods in the U.S. North, (c) softwoods in the U.S. South, (d) hardwoods in the U.S. South, (e) softwoods in the U.S. West, and (f) hardwoods in the U.S. West.
nal estimates of growth parameters for the West were based on county-level data that did not include Pacific Coast states. We know those states have generally much lower average timber growth rates than the region as a whole (as a percentage of timber inventory volume) partly because of relatively high timber inventory volumes within the Pacific Coast states.

Table 1 presents the descriptive statistics of the county-level data used in the study (excluding the Pacific Coast states). The number of observations (number of counties with timber data) ranged from as low as 273 for softwoods in the West to as high as 1,013 for hardwoods in the South. The softwood forest inventory in the U.S. South was characterized by the largest average net annual growth as percentage of inventory (5.91%/y), followed by softwoods in the North (3.97%/y). Similarly, the largest average growth in hardwood forest inventory was observed in the West (3.76%/y), followed by the South (3.10%/y). The lowest softwood inventory growth rate was observed in the West (2.50%/y), and the lowest hardwood inventory growth rate was observed in the North (3.01%/y). The lowest average net growth rate was also accompanied by the largest growing-stock density. For example, hardwood forest inventory in the North and softwood forest inventory in the West had the largest average growing-stock density (98.31 and 88.34 m³/ha, respectively) (Table 1).

According to Smith and others (2009), net annual growth of softwoods in the entire U.S. West in 2006 was 0.173 billion m³, whereas net annual growth of hardwoods in the entire U.S. West was 0.022 billion m³. Also, total volume of softwood growing-stock in the entire U.S. West in 2007 was 10.049 billion m³, whereas total volume of hardwood growing-stock in the entire U.S. West was 1.152 billion m³ (Smith and others 2009). These data include all Pacific Coast states, including Alaska and Hawaii, as well as Oregon, Washington, and California. On the basis of these data, net annual growth rates as percentages of inventories in the entire West were 1.72% for softwoods and 1.91% for hardwoods. These growth rates are clearly lower than the Table 1 growth rates for the West obtained as mean values from county data that excluded the Pacific Coast states, which explains why we must make adjustments to growth parameters that we estimate for the West.

Softwood and hardwood growth and stocking density data used in this study pertain to total timberland area in all ownership categories across the three U.S. regions. Therefore, the growth models developed in this study are regional models of softwoods and hardwoods that include all ownership categories. Furthermore, stocking densities for both species groups (hardwood and softwood) were computed on the basis of total timberland area, so growing-stock densities are naturally the highest for species that are regionally dominant on the landscape, specifically hardwoods in the East (North and South) and softwoods in the West (Table 1). In addition, softwood timber in the South has by far the highest mean growth rate as a percentage of inventory, largely because of widespread plantations and productive management of Southern Pines with relatively short timber rotations. Mean growth rate for softwood is much lower in the West (at 1.72% of growing stock) because a large share of forest land is in public ownership with relatively passive timber management practices and long timber rotations that contribute to high stocking density and slower growth rates for softwood in the West.

**Model**

The study used nonlinear least squares regression models to estimate empirical relationships between net annual growth and growing-stock density across U.S. regions for the two species groups, hardwoods and softwoods. The regression model was represented by the following functional form (Turner and others 2006):

\[
G_i = (\alpha + \alpha_1P_i)(S_i)\gamma + u_i
\]

where \(G_i\) is net annual growth as percentage of growing-stock inventory (%/y) in county \(i\), \(S_i\) is growing-stock density (m³/ha) (growing-stock inventory divided by timberland area in county \(i\); \(P_i\) is proportion of area in plantations in county \(i\); \(\alpha\) and \(\gamma\) are parameters to be estimated; and \(u_i\) is an error term. Such a functional form is consistent with theoretical relationships described in the literature between net forest growth rate and growing-stock density (Turner and others 2006; Oliver and Larson 1996; Smith and others 1996). For a given species group and region, the counties with lower stocking density generally have higher growth rates per unit of inventory and vice versa. Thus, \(\gamma\) should be negative. In addition, counties that have more timberland areas under plantations will generally have higher percentage net forest growth. Thus, \(\alpha_1\) should be positive. The FIA reporting of data does not include timberland area under plantations. However, it reports stand origin indicating whether the stand was naturally or artificially regenerated (trees were planted). Although artificial regeneration does not necessarily mean that a stand is an intensively managed tree plantation, it can be used as a proxy indicator for the higher productivity of plantations because artificially regenerated stands certainly include managed plantations, and artificial regeneration is also likely to enhance forest growth.

We used the proportion of timberland area that is artificially regenerated as a proxy for \(P_i\) in Equation (1). Such a proxy variable was applied for the softwoods in the South only because the proportion of the timber resource that is artificially regenerated and intensively managed in the South is relatively high compared with other regions. Although there may be some degree of uncertainty about the FIA stand origin data because of a perceived difficulty in determining the origin of older stands on survey plots, the data indicate that planted timberland area is 15% of the timberland area in the Pacific Coast region. In the West as a whole (both Rocky Mountain and Pacific Coast) planted timberland area is only 8% of total timberland area, and in the North the planted
timberland area is about 4% of total timberland area (Smith and others 2009). By contrast, about 22% of the total timberland area is artificially regenerated in the South (Miles 2011; Smith and others 2009).

Furthermore, most artificially regenerated stands in the South are planted pines, and much of the pine forest area in the South consists of managed pine plantations (Wear and others 2011). By contrast, managed plantations are a much smaller share of the timber resource in the West, even for species that are most commonly planted, such as Douglas-fir. Planted stands account for only 13% of the growing-stock inventory of Douglas-fir timber in the West, whereas planted stands account for 46% of the inventory of longleaf-slash pine and 43% of the loblolly–shortleaf pine inventory in the South (Smith and others 2009). Therefore, we did not include $P_i$ as an additional variable in our inventory growth models for the North and West or for hardwoods in the South, because in those cases the proportion of the total resource that is artificially regenerated and intensively managed region-wide was thought to be relatively small compared with southern softwoods. However, if county-level data were to become available for the Pacific Coast region, we would consider extending our analysis by including $P_i$ as a variable for the softwood inventory growth model in the West because the Pacific Coast has the next largest area of planted timberland area after the South.

The model parameters were estimated by nonlinear least squares method (Marquardt’s method, SAS 1999). Because the functional form used in this study represents a constant elasticity functional form, the estimates of $\gamma_i$ provide a direct measure of elasticity. It tells us the percentage change in net growth for a 1% change in stocking density. The plausibility of each model was evaluated based on the conformity of the estimated parameters with expected signs, the statistical significance of the overall model, and individual parameters. The validity of estimated parameters for meaningful inferences was determined using Hougaard’s measure of skewness (Ratkowsky 1990; Hougaard 1985). According to Ratkowsky (1990), when the parameter estimates from nonlinear regression models are “close-to-linear,” then the least-squares estimators of the parameters are close to being unbiased, are normally distributed, and have minimum variance, thus allowing their standard errors and confidence intervals to be safely used for inferences. Hougaard’s measure of skewness ($g_{1i}$) can be used to assess the extent to which a parameter exhibits close-to-linear behavior (Ratkowsky 1990; Hougaard 1985). The value of $|g_{1i}| < 0.1$ indicates that parameter estimates are very close-to-linear in behavior, and $0.1 < |g_{1i}| < 0.25$ indicates that parameter estimates are reasonably close-to-linear (Ratkowsky 1990).

**Model Validation and Application**

One goal of the growth models developed in this study is to provide a reliable basis for timber inventory projection across U.S. regions and species groups. Therefore, it was important to evaluate and understand the reliability of these models in projecting inventory. The reliability of such models was assessed by comparing historically observed growing-stock inventory with predicted inventory using the developed growth models and historical harvest data (Figs. 4, 5). The predictive accuracy of the model was evaluated based on the estimated root mean square error as follows (Greene 2003):

$$\text{RMSE} = \sqrt{\frac{1}{n^0} \sum_{i} (y_i - \hat{y}_i)^2}$$  \hspace{1cm} (2)

where, $n^0$ is the number of periods being predicted, $y_i$ is the individual observation for a particular year, and $\hat{y}_i$ is the predicted value for a particular year. The statistical understanding of the prediction error (PE) over the predicted periods was estimated by taking percentage of root mean squared error against the mean of the observed historical values over the predicted period as follows:

$$\text{PE} = \frac{\text{RMSE}}{\overline{y}} \times 100$$ \hspace{1cm} (3)

For this comparison, we used nine different historical years (1953, 1963, 1970, 1977, 1987, 1992, 1997, 2002, and 2007) for which U.S. inventory, growth, harvest, and area data were available as reported in various USDA Forest Service Forest Resources reports. The historically observed growing-stock inventory was projected by subtracting the harvest quantity and adding the growth predicted by the developed models (using regional historical growing-stock density) with two separate approaches. The first approach was to use historical inventory and growing-stock density data at each time period separately. In other words, we used 1953 actual inventories, harvest quantities, and growing-stock densities to project 1963 inventory, and we used 1963 actual inventories, harvest quantities, and stocking densities to project 1970 inventory, and so forth. Another approach was to predict timber inventory for all historical years using only the initial inventory in 1953 and the historical harvest from 1953 to 2007. Results of these two inventory projection approaches are shown in Figures 4 and 5 in comparison with actual historical inventory data.

The developed growth models were applied also in making a long-range growing-stock inventory projection for three U.S. regions under three alternative 2010 RPA scenarios and using timber harvest projections from USFPM/GFPM (the model used to produce timber harvest projections for the 2010 RPA). The RPA scenarios were based on various assumptions about global economic growth, global wood energy consumption, population changes, and climate changes and were expected to have different effects on future U.S. forest resources conditions and trends (Ince and others 2011). These scenarios were termed A1B, A2, and B2 in line with similar scenarios developed by the Intergovernmental Panel on Climate Change (IPCC) (Nakicenovic and Swart 2000). The A1B scenario represented the highest global economic growth and wood energy consumption coupled...
Figure 4—Comparison of observed historical growing stock inventory and predicted inventory (using predictions for each time period separately) for (a) softwoods in the U.S. North, (b) hardwoods in the U.S. North, (c) softwoods in the U.S. South, (d) hardwoods in the U.S. South, (e) softwoods in the U.S. West, (f) hardwoods in the U.S. West, (g) softwoods, U.S., (h) hardwoods, U.S., and (i) softwood and hardwoods combined, United States.
Figure 5—Comparison of observed historical growing stock inventory and predicted inventory, (using only 1953 inventory and 1953–2007 harvest data) for (a) softwoods in the U.S. North, (b) hardwoods in the U.S. North, (c) softwoods in the U.S. South, (d) hardwoods in the U.S. South, (e) softwoods in the U.S. West, (f) hardwoods in the U.S. West, (g) softwoods, U.S., (h) hardwoods, U.S., and (i) softwood and hardwoods combined, U.S.
Table 2—Summary of the three alternative RPA scenarios

<table>
<thead>
<tr>
<th>Summary</th>
<th>A1B</th>
<th>A2</th>
<th>B2</th>
</tr>
</thead>
<tbody>
<tr>
<td>General description</td>
<td>Globalization, economic convergence</td>
<td>Heterogenic regionalism, less trade</td>
<td>Localized solutions, slow change</td>
</tr>
<tr>
<td>Social development themes</td>
<td>Economic growth, new technologies, capacity</td>
<td>Self-reliance, preservation of local identities</td>
<td>Sustainable development, diversified technology</td>
</tr>
<tr>
<td>Global real GDP growth (2010–2060)</td>
<td>High (6.2×)</td>
<td>Medium (3.2×)</td>
<td>Medium (3.5×)</td>
</tr>
<tr>
<td>Global population growth (2010–2060)</td>
<td>Medium (1.3×)</td>
<td>High (1.7×)</td>
<td>Medium (1.4×)</td>
</tr>
<tr>
<td>U.S. GDP growth (2006–2060)</td>
<td>Medium (3.3×)</td>
<td>Low (2.6×)</td>
<td>Low (2.2×)</td>
</tr>
<tr>
<td>U.S. population growth (2006–2060)</td>
<td>Medium (1.5×)</td>
<td>High (1.7×)</td>
<td>Medium (1.3×)</td>
</tr>
<tr>
<td>Global expansion of primary biomass energy production (2000–2060)</td>
<td>High (5.9×)</td>
<td>Medium (3.1×)</td>
<td>Medium (3.2×)</td>
</tr>
</tbody>
</table>

*Ince and others (2011).}

with slowing population growth, whereas the A2 and B2 scenarios were associated with considerably lower economic growth and lower wood energy consumption. The A2 scenario assumed higher global population growth coupled with lower economic growth compared with other scenarios, and the B2 scenario assumed lowest population growth with mid-level global economic growth. Table 2 summarizes various assumptions of the selected scenarios.

We made timber inventory projections for these scenarios by adding to the base year (2006) inventory the net forest growth (predicted by the empirical growth models developed in this study) and subtracting annual harvest quantities by regions based on USFPM projected timber harvest levels (from 2006 to 2060). In addition, the regional U.S. timber supply equations in USFPM were shifted over time by our projected regional timber inventories (using an inventory elasticity of 1.0 for the supply equations). The Gauss–Seidel technique of iterative solutions (Jeffreys and Jeffreys 2000) was used to derive convergent equilibrium solutions for each scenario by linking our spreadsheet model of regional timber growth and inventory to USFPM/GFPM. The spreadsheet model used USFPM projections of timber harvest by region, and in turn the spreadsheet model provided projections of growing-stock inventory to USFPM, where they were used to shift regional timber supply curves. The models were run iteratively until a reasonably stable convergent equilibrium was obtained (usually less than 12 iterations). In a separate analysis, we compared inventory projections with projections reported in other recent literature related to U.S. forest resources.

**Results and Discussion**

**Net Forest Growth and Growing-Stock Density Relationship**

Table 3 presents estimates of model parameters and related statistics. All regression models were statistically significant at 1% or better significance level ($F \leq 0.0001$). The models also fulfilled an a priori expectation on the signs of the estimated parameters, a negative sign on the growing-stock density coefficient ($\gamma$) and a positive sign on the coefficient associated with fraction of area under artificial regeneration in the U.S. South ($a_i$). Individual parameter estimates were strongly significant at 1% or better significance level ($t \leq 0.0001$) for all regions and species. The coefficient of determination ($R^2$), interpreted as the proportion of variation in dependent variable explained by variation in independent variable, has meaning only in case of least squares regression applied to a linear equation with a constant term (Greene 2003). This usual interpretation of $R^2$ does not apply in the case of nonlinear regression without a constant term, as used in this study. Therefore, we did not use $R^2$ as a basis of evaluating our models. However, an analogous measure of $R^2$ called Pseudo-$R^2$ (Gujarati 2003) was calculated based on residual and total corrected sums of squares and is reported in Table 3. The absolute value of Hougaard’s measure of skewness for all parameter estimates ranged from less than 0.01 to 0.25 for all regions and species groups, indicating that the parameter estimates were close-to-linear and that their standard errors and confidence intervals could be safely used for inferences.

Figure 3 presents observed and predicted relationships between net annual growth rates and growing-stock inventory for the three U.S. regions and two species groups. Results indicated a higher percentage net growth rate for lower growing-stock densities and a lower percentage net growth rate for higher stocking densities in all regions and species. However, the decline in net growth rate with increasing growing-stock density differed among regions and species groups. For example, the net growth rate declined strongly with higher growing-stock density for hardwoods in the North ($\gamma = -0.47$). A gamma value of $-0.47$ indicated that
doubling of growing-stock density would decrease the net growth rate by almost half (–47%). A similar response to increase in stocking density was observed for softwoods in the West (\(\gamma = –0.41\)). In contrast, softwoods in the North and hardwoods in the West had relatively smaller declines in net growth rates in response to higher stocking density (\(\gamma = –0.28\) and –0.20, respectively). The observed response of net growth to stocking density was markedly lower in the South for both softwoods and hardwoods, relative to the North and West. With a doubling of stocking density, the hardwood forest in the South showed a decline of only 16% in net annual growth, and similarly only 15% for softwoods. Furthermore, southern softwood stands showed significantly different rates of growth for naturally and artificially regenerated stands. For a given stocking density, net annual growth as a percentage of inventory was significantly higher (five times) in artificially regenerated softwood stands relative to naturally regenerated softwood stands in the South. Our result that growth was five times higher in artificially regenerated stands for softwoods in the South is consistent with estimates provided in other studies. For example, Ryan and others (2010) reported that planted pines with improved seedlings, competition control, and fertilization grow four times faster than naturally regenerated second-growth pine stands in the U.S. South.

Overall, our results showed a relatively smaller decline in percentage net growth for an increase in growing-stock density for all U.S. regions and species groups relative to estimates reported by Turner and others (2006), who investigated a similar relationship at the global level using a singular nonlinear functional relationship. Their results indicated that doubling of growing-stock density would decrease net forest growth by 81%. Results from this study did not indicate as strong a decline in U.S. forest stock growth with increasing density as reported globally by Turner and others (2006). The differences in results between this study and those of Turner and others (2006) can be explained by the

### Table 3—Parameter estimates of net forest growth (%/y) and growing stock density (m³/ha/y) relationships by three U.S. regions and softwood and hardwood species groups

<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimate</th>
<th>Approximately 95% confidence limits</th>
<th>Hougaard’s measure of skewness ((g_1))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Lower</td>
<td>Upper</td>
</tr>
<tr>
<td>North, softwoods</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\alpha)</td>
<td>6.1323 (0.2314)***</td>
<td>5.6779</td>
<td>6.5867</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>(-0.2809 (0.0191)***)</td>
<td>(-0.3200)</td>
<td>(-0.2400)</td>
</tr>
<tr>
<td>(F) value</td>
<td>351.1100***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pseudo-(R^2)</td>
<td>0.2100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>North, hardwoods</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\alpha)</td>
<td>23.2566 (1.4047)***</td>
<td>20.5001</td>
<td>26.0132</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>(-0.4648 (0.0152)***)</td>
<td>(-0.4986)</td>
<td>(-0.4390)</td>
</tr>
<tr>
<td>(F) value</td>
<td>1119.4900***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pseudo-(R^2)</td>
<td>0.2300</td>
<td></td>
<td></td>
</tr>
<tr>
<td>South, softwoods</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\alpha)</td>
<td>5.6224 (0.3274)***</td>
<td>4.9800</td>
<td>6.2649</td>
</tr>
<tr>
<td>(\alpha_1)</td>
<td>22.8506 (2.8764)***</td>
<td>17.2060</td>
<td>28.4952</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>(-0.1453 (0.0220)***)</td>
<td>(-0.1885)</td>
<td>(-0.1021)</td>
</tr>
<tr>
<td>(F) value</td>
<td>1028.2600***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pseudo-(R^2)</td>
<td>0.1600</td>
<td></td>
<td></td>
</tr>
<tr>
<td>South, hardwoods</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\alpha)</td>
<td>5.3815 (0.3932)***</td>
<td>5.0599</td>
<td>6.6031</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>(-0.1584 (0.0171)***)</td>
<td>(-0.1920)</td>
<td>(-0.1248)</td>
</tr>
<tr>
<td>(F) value</td>
<td>1659.4100***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pseudo-(R^2)</td>
<td>0.0500</td>
<td></td>
<td></td>
</tr>
<tr>
<td>West, softwoods</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\alpha)</td>
<td>9.3639 (0.5017)***</td>
<td>8.3763</td>
<td>10.3516</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>(-0.4097 (0.0265)***)</td>
<td>(-0.4619)</td>
<td>(-0.3574)</td>
</tr>
<tr>
<td>(F) value</td>
<td>175.1500***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pseudo-(R^2)</td>
<td>0.4500</td>
<td></td>
<td></td>
</tr>
<tr>
<td>West, hardwoods</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\alpha)</td>
<td>5.7107 (0.4185)***</td>
<td>4.8876</td>
<td>6.5337</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>(-0.1992 (0.0309)***)</td>
<td>(-0.2600)</td>
<td>(-0.1384)</td>
</tr>
<tr>
<td>(F) value</td>
<td>101.6400***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pseudo-(R^2)</td>
<td>0.1000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

---

*Numbers in parentheses indicate approximate standard error; ***, statistically significant at 1% significance level.

†, parameters are very close-to-linear; ‡, parameters are reasonably close-to-linear.

The model includes proportion of artificial regenerated area as additional variable.
geographic coverage of the analyses and by the fact that we used different data sets. The single functional relationship for the world developed by Turner and others (2006) was derived from cross-sectional analysis of FAO global data on the net growth rate and forest stocking density among 180 countries.

This study was designed to account for regional and species group differences by developing separate empirical relationships based on cross-sectional analysis of FIA county-level data for each region, as opposed to global data by Turner and others (2006). Therefore, it was logical to expect differences between our estimated regional growth models and the generalized global formula developed by Turner and others (2006). Also, preliminary research conducted at FPL indicated that the global relationship employed in the GFPM did not accurately fit forest growth and growing-stock density data for any of the U.S. regions or species groups. However, employing such relationships specific to U.S. regions and species groups as developed in this study showed a much improved fit, indicating that the general theory of a growth-to-growing-stock density relationship is valid, and growth models developed in this study more accurately represented growth dynamics for the U.S. regions and species groups.

**Model Comparison and Application**

Figures 4 and 5 compare historically observed and predicted growing-stock inventory for U.S. regions and species groups using two approaches to predict inventory (predictions made at each time period separately versus using only 1953 inventory and 1953–2007 harvest data). The comparison shows that the region- and species-specific net annual growth and growing-stock density models developed in this study were successful in predicting growing-stock inventory trends and inventory levels very close to historically observed inventory when predictions were made at each timber period separately. The overall differences between predicted and actual historical observations, as measured by the percentage of root mean square error over the mean of the historically observed inventories (Eq. (2)), ranged from 3% to 6% for hardwoods and 4% to 7% for softwoods for individual regions (using each time period prediction approach). The prediction showed a 38% increase in total U.S. growing-stock inventory during 1963–2007, compared with a 41% increase in actual historical trend during the same period. However, when predictions were made using only 1953 inventory and 1953–2007 harvest data, the overall average prediction error for individual region and species group increased and ranged from 12% for hardwoods in the North to as much 35% for hardwoods in the South (Fig. 5). The predictions for the U.S. aggregate inventory (Figs. 5g–i), however, remained very close to the actual historical observations.

To further validate our regional inventory projections across species groups, we compared them with corresponding inventories projected for the 2005 RPA base scenario. The 2005 RPA base scenario was developed based on assumptions about the overall future U.S. economy and biological and market conditions affecting U.S. timber supply on both public and private lands (Haynes and others 2007). The 2005 RPA base scenario projected rising trends in U.S. softwoods, hardwoods, and total inventories, which we reconciled with trends in our projections. Specifically, we applied the same harvest levels and timberland area assumptions employed in the 2005 RPA assessment to our spreadsheet model and compared the resulting inventory projections. Figures 6a–d show that our original estimates of growth model parameters (Table 3) yield inventory projections for the North and for softwood in the South quite close to 2005 RPA projections when using the same projected timber harvest and land area assumptions. However, our original estimates of growth model parameters yield large over-projections of timber inventories in the West and hardwood timber inventory in the South. The inaccuracy of our original estimates of growth parameters for the West can be attributed as explained earlier to our lack of county data for the Pacific Coast states, which have generally much lower net annual growth as a percentage of growing-stock inventory than the West as a whole (Smith and others 2009). Therefore, we made calibrating adjustments to our growth parameters for the West (Table 4), and as a result our timber inventory projections for the West come much closer to the 2005 RPA projections, as shown in Figures 7a–d. We also made some additional minor adjustments to growth parameters for other regions to bring our inventory projections more closely in line with the 2005 RPA projections, particularly for hardwood inventory in the South.

The most likely reason for our over-projection of hardwood inventory in the South relative to the 2005 RPA was that our model did not take into account large-scale future conversions of hardwood forest types to planted pines, as projected by Haynes and others (2007). Using ATLAS to model forest growth, Haynes and others (2007) projected a big decline in the area of upland hardwoods in the South because of conversions to planted pine, resulting in less region-wide hardwood growth and smaller hardwood inventories. In ATLAS, when hardwood area was converted or removed from the timberland base in the 2005 RPA projections, area was removed across a range of age classes and not just older, slower growing stands (John Mills, personal communication, USDA Forest Service, Pacific Northwest Research Station, October 18, 2011). Our broad region-wide growth model (based on cross-sectional county-level data of just one year) cannot capture effects of such forest cover changes or management conversions occurring at different time periods except by making exogenous adjustments to growth parameters. Without such adjustments, our model produces over-projections of hardwood inventories in the South.

In general, our region-wide growth model parameters can be further adjusted to reflect other assumptions about future forest resource conditions and trends: for example, timberland area changes from forest type conversion, increased forest management intensity, increased plantation area, and
so forth. As shown in Table 4, we actually made modest adjustments to the alpha (α) or gamma (γ) parameters (or both) for every region and species group to produce the inventory projections shown in Figures 7a–d, which matched quite closely to the 2005 RPA inventory projections. The necessary adjustments were very small for the North and also for softwood in the South, but they were larger for the West and for hardwood in the South. In almost all cases (except for hardwoods in the West) the adjusted parameter values were still within the 95% confidence limits of our original estimates (Table 4).

Figure 8 presents historical and projected U.S. growing-stock inventories for three alternative 2010 RPA scenarios for the period between 1953 and 2060, based on iterative convergent solutions between the USFPM/GFPM model and the U.S. regional forest growth and inventory models that we developed in this study (without calibration to 2005 RPA base scenarios). Specifically, the projected regional growing-stock inventories took into account our originally
estimated forest growth and stocking density relationships (Table 3) and also the projected regional timber harvest levels from USFPM, and in turn the USFPM regional timber supply equations were shifted by the projected regional inventories for hardwoods and softwoods. Consistent with the assumed economic growth, energy consumption level, and population growth, the projections (Fig. 8) indicated higher inventory levels over the entire projection period for both hardwoods and softwoods in A2 and B2 scenarios, which featured modest expansion of timber harvest and represented modest growth in the economy, population, and wood energy consumption. In contrast, forest inventory levels were lower and declining in the A1B scenario, which represented a scenario with the largest economic growth and wood energy consumption and highest timber harvest levels. Relative to 2010, softwoods, hardwoods, and total inventory increased by 56%, 76%, and 65%, respectively, by 2060 in the B2 scenario. Similarly, increases in softwoods,
Developing Inventory Projection Models Using Empirical Net Forest Growth and Growing-Stock Density Relationships

Figure 8—Historical and projected U.S. growing stock inventories (billion cubic meters), 1952–2060, by three alternative 2010 RPA scenarios, (a) softwoods, (b) hardwoods, and (c) total, based on USFPM harvest projections for 2010 RPA scenarios linked to our regional growth and inventory models without any adjustments except for the West (Table 3).
hardwoods, and total inventory for the A2 scenario were 45%, 53%, and 48%, respectively. Projected total inventories in these two scenarios were fairly close to the historical trend in total inventory from 1953 to 2007, during which total inventory increased by 51% (Smith and others 2009). Consistent with higher global economic growth and high wood energy consumption inducing by far the largest projected timber removals in the A1B scenario, U.S. softwood, hardwood, and total inventory levels increased only by 30%, 11%, and 22%, respectively, in the A1B scenario by 2060 relative to 2010.

Finally, we used the adjusted parameters of growth equations (Fig. 4 or 5) for both individual regions and the U.S. aggregate. Because of better performance in predicting historical inventory, we plan to apply our unadjusted growth equations in future studies.

### Conclusions

This study presents a parsimonious model of regional net forest growth that is linked to a regional and global forest product market model to project future U.S. timber inventory by region and species group (hardwood and softwood). For each region and species group, the model requires data for only one independent variable and growing-stock density but allows for the inclusion of plantation area as an additional variable (for softwoods in the South). Data and models support theoretical relationships between net forest growth and stocking density. By estimating separate regional growth models according to species group, this study was successful in taking into account distinguishing relationships between net forest growth and stocking density across U.S. regions and species groups. The developed growth models can be easily linked via a spreadsheet model to USFPM/GFPM or other forest sector market models.

Results showed a different response in net forest growth to changes in growing-stock density across U.S. regions and species groups. Inventory projections for the 2010 RPA scenarios showed contrasting dynamics between the A1B scenario and other scenarios because of large differences related to the adjusted model parameters for each region and species group.
Figure 9—Historical and projected U.S. growing stock inventories (billion cubic meters), 1952–1960, by three alternative 2010 RPA scenarios for (a) softwoods, (b) hardwoods, and (c) total, based on our growth models adjusted to match 2005 RPA base scenario inventory projection (Table 4).
Figure 10—Comparison of observed historical growing stock inventory and predicted inventory (using only 1953 inventory and 1953–2007 harvest data) after adjusting estimated growth parameters to match 2005 RPA baseline projection (see Table 4) for (a) softwoods in the U.S. North, (b) hardwoods in the U.S. North, (c) softwoods in the U.S. South, (d) hardwoods in the U.S. South, (e) softwoods in the U.S. West, (f) hardwoods in the U.S. West, (g) softwoods, U.S., (h) hardwoods, U.S., and (i) softwood and hardwoods combined, U.S.
in projected timber harvest. The RPA scenario resulting in highest removals (A1B) was accompanied by lowest projected inventory levels, and the scenario resulting in lowest removals (B2) resulted in highest projected inventory levels. The A1B scenario, which represented the highest economic growth coupled with high demand for wood energy, induced largest removals, leading to a decline in softwood, hardwood, and total U.S. growing-stock inventory (Fig. 8). In contrast, with modest economic growth and modest removals, inventories were projected to increase over the entire projection period in the B2 scenario.

Prior to the development of our growth models, USFPM/GFPM could not be used to accurately simulate regional impacts of climate change policies on forest carbon storage. However, analysis of climate change policies is important for predicting trends in future forest resource conditions and trends. This study is one step toward enabling the USFPM/GFPM to provide such analysis. Earlier unpublished work at the Forest Products Laboratory had shown a strong linear relationship between growing-stock inventory and forest carbon stocks. This suggests that the developed inventory growth models can equip USFPM with forest carbon stock accounting capability by projecting growing-stock inventory. Such a capability in forest carbon accounting will allow USFPM/GFPM to simulate the impact of national and global climate change policies (e.g., forest carbon offset policies) on U.S. timber markets and trade and conversely to analyze how market and trade scenarios affect forest carbon stocks. By enhancing USFPM/GFPM’s capability to model changes in the U.S. forest sector carbon inventory in response to changing global timber markets, wood energy demands, forest growth, and forest carbon offset credits, this study can contribute to a more meaningful analysis of U.S. and global forest carbon offset strategies. Using USFPM/GFPM and our growth models, we plan to evaluate how forest sector carbon offset credits will influence U.S. forest carbon sequestration and forest products markets, focusing initially on the strategy to pay forest landowners for reducing or avoiding timber harvests.

**Literature Cited**


