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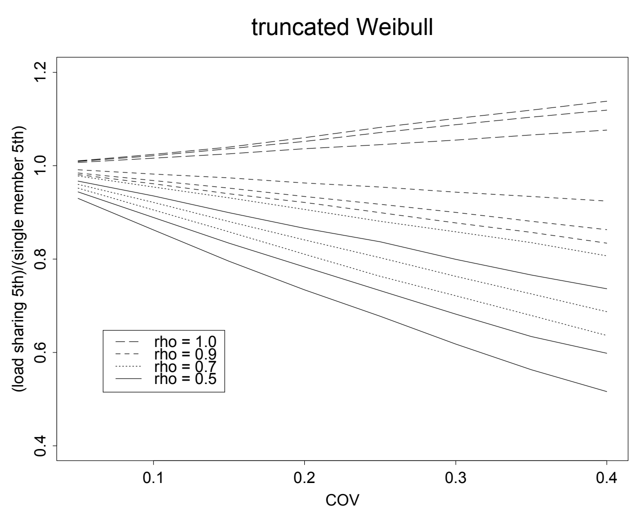
Repetitive Member Factors for the Allowable Properties of Wood Products

Steve P. Verrill
David E. Kretschmann



Abstract

We identify confusion in the literature about the definition and calculation of repetitive member factors. This confusion casts some doubt on the validity of the 1.15 repetitive member factor permitted in ASTM D 245 and ASTM D 1990.

Keywords: repetitive members, assembly, reliability, redundancy, load sharing, composite action, residual capacity

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Repetitive Member Factors for the Allowable Properties of Wood Products

Steve P. Verrill, Mathematical Statistician
David E. Kretschmann, Research General Engineer
USDA Forest Products Laboratory, Madison, Wisconsin

1 Introduction

It is generally accepted that there should be an upward repetitive member allowable property adjustment. ASTM D 245 (ASTM 2007a) and ASTM D 1990 (ASTM 2007b) specify a 1.15 factor for allowable bending stress. This factor is also listed in table 1 of ASTM D 6555 (ASTM 2007c). In this report we identify sources of confusion regarding appropriate repetitive member factors. This confusion casts some doubt on the validity of the 1.15 value.

The two main sources of confusion that we discuss are associated with the fact that a full repetitive member factor can be modeled as a combination of subfactors. ASTM D 6555 identifies three subfactors — load sharing,[1] composite action,[2] and residual capacity.[3]

The first confusion is associated with the proper calculation of the load sharing subfactor. The second confusion is associated with the question of whether the third subfactor, residual capacity, is to be included in the "full" repetitive member factor.

In Sections 2 and 3 we describe these two main sources of confusion in some detail. In Section 4 we discuss additional sources of ambiguity. In Appendix A we discuss a simple simulation that focuses on the load sharing factor.

2 Confusion about the method of calculating a repetitive member adjustment

In section X2.8 of ASTM D 6555, the repetitive member adjustment is determined experimentally by finding an estimate of the fifth percentile of the strength distribution of an assembly (actually a one-sided lower confidence bound on the fifth percentile) where an assembly's strength is taken to be the load at first member failure, and dividing that by an estimate of the fifth percentile of individual member strength (again, actually a one-sided lower confidence bound on the fifth percentile). In the standard's example, this ratio is 1.11. However, in section 8.4.1 of ASTM D 6555, the repetitive member adjustment is defined *differently.* Rather than the ratio of the fifth percentile of an assembly's strength distribution to the fifth percentile of an individual member's strength distribution, it is defined as the ratio of the fifth percentile of an assembly's strength distribution to the fifth percentile of the *minimum* of k individual strengths (where an assembly is composed of k members). The fifth percentile of this "minimum of k" strength distribution will be lower than the fifth percentile of the strength distribution of a single member, and the calculated repetitive

[1]In ASTM D 6555, section 3.1.4, "load sharing" is defined as "distribution of load among adjacent, parallel members in proportion to relative member stiffness."

[2]In ASTM D 6555, section 3.1.1, "composite action" is defined as "interaction of two or more connected wood members that increases the effective section properties over that determined for the individual members."

[3]In ASTM D 6555, section 3.1.8, "residual capacity" is defined as "ratio of the maximum assembly capacity to the assembly capacity at first failure of an individual member or connection."

member factor will be larger. Either of the standard's definitions is permissible *if* subsequently applied correctly in calculating allowable properties, but the ratio described in section 8.4.1 of ASTM D 6555 cannot be applied to the fifth percentile of an individual member (as is suggested in section 8.5.1 of ASTM D 6555). It would have to be applied to the fifth percentile of the distribution of the minimum strength of k members. Otherwise, we would be overestimating assembly strength.

To make this point clear, note that in the load sharing simulations described in Appendix A, for strength, stiffness correlations below 1.0, we have

$$y_{\text{sngl}} > y_{\text{assem}} > y_{\text{mink}} \tag{1}$$

where

$$\begin{aligned} y_{\text{sngl}} &= q\text{th percentile of the single member strength distribution} \\ y_{\text{assem}} &= q\text{th percentile of the load sharing assembly strength distribution} \\ y_{\text{mink}} &= q\text{th percentile of the distribution of the minimum strength of } k \text{ single members} \end{aligned}$$

where the assembly is composed of k members and $q = 1, 5, 10$.

To obtain a correct y_{assem} value, we could do (**ratio** R_{ADS} **definition**)

$$y_{\text{assem}} = (y_{\text{assem}}/y_{\text{sngl}}) \times y_{\text{sngl}} \equiv R_{\text{ADS}} \times y_{\text{sngl}}$$

as is suggested by the example in section X2.8 of ASTM D 6555, or we could do (**ratio** R_{ADM} **definition**)

$$y_{\text{assem}} = (y_{\text{assem}}/y_{\text{mink}}) \times y_{\text{mink}} \equiv R_{\text{ADM}} \times y_{\text{mink}}$$

but

$$y_{\text{assem}} \neq (y_{\text{assem}}/y_{\text{mink}}) \times y_{\text{sngl}}$$

In fact, since $y_{\text{sngl}} > y_{\text{mink}}$, $(y_{\text{assem}}/y_{\text{mink}}) \times y_{\text{sngl}}$ can be a serious overestimate of y_{assem}. Unfortunately, repetitive member factors reported in the literature are sometimes of the $y_{\text{assem}}/y_{\text{mink}}$ variety, and it is common practice to apply these factors to y_{sngl} values (as suggested in section 8.5.1 of ASTM D 6555).

We give some examples from the literature below.

In a load sharing simulation in which assemblies contained five members, Zahn (1970, table 4) reports load sharing fractional increases of 0.128, 0.096, 0.125, 0.082, and 0.125. However, these fractional increases are not calculated as

$$(y_{\text{assem}} - y_{\text{sngl}})/y_{\text{sngl}} = R_{\text{ADS}} - 1$$

nor as

$$(y_{\text{assem}} - y_{\text{mink}})/y_{\text{mink}} = R_{\text{ADM}} - 1$$

Instead they are calculated as

$$(y_{\text{assem}} - y_{\text{mink}})/y_{\text{sngl}}$$

From Zahn's table, we can calculate the "correct" R_{ADS} $(y_{\text{assem}}/y_{\text{sngl}})$ load sharing factors: 0.978, 1.077, 0.891, 0.681, and 0.929. Thus, in four of the five cases that Zahn simulated, the assembly was weaker (at the fifth percentile) than an individual member. (This does not take into account composite action.)

Zahn also references an experiment conducted by Atherton and Corder (1965) at Oregon State. In this experiment, "floors" constructed of 14 beams and 1/2-inch sheathing grade plywood were loaded to failure. The experimentally determined mean load capacity of five such floors was compared to the mean load capacity of five unsheathed "floors." The load capacity of an unsheathed "floor" was taken to be the minimum of the strengths of the 14 beams that had been assigned to it. Thus the "increase in load capacity" reported in Zahn's table 5 (taken from the Oregon State report) has a R_{ADM} nature. That is, it represents the ratio of the strength of a load-sharing assembly to that of a weakest link assembly rather than to the strength of an individual member.

Wolfe and LaBissoniere (1991, table 3) report load sharing increases that range from 9% to 47%. However, these are neither R_{ADS} nor R_{ADM} values. They constructed three roof assemblies, each containing eight trusses and a gable end. They defined assembly failure as the point at which maximum load was achieved. This point did not correspond to first member failure. (They discuss the failure of trusses within the roof assembly prior to the failure of the assembly.)

Their repetitive member factor was the ratio of the assembly load at failure to the smallest maximum load on a member of the assembly that failed in the course of the assembly failure. This is not a ratio of fifth percentile estimates. Also, the denominator is closer to a minimum of k strength value than to an individual member strength value.

Because Wolfe and LaBissoniere also tested individual trusses to failure, it is possible to calculate R_{ADS} ratios (although they are at *mean* population values rather than at fifth percentiles). From their table 1, for their three assemblies we obtain R_{ADS} values of 0.84, 1.00, and 1.02[4] These ratios do not support a repetitive member load sharing increase (assuming that we are beginning from a single member allowable property base).

3 Confusion about which literature values are relevant for the ASTM standards

ASTM D 6555 notes that repetitive member factors should account for both load sharing and composite action. However, the failure criterion in ASTM D 6555 is *currently* a serviceability criterion, not a safety criterion. That is, assembly failure is defined to be first member failure, and *residual capacity cannot be considered.* Unfortunately, the repetitive member factors in the literature that significantly exceed 1 are, for the most part, based on multiple member failure criteria. In those cases in which the criterion is first member failure, the evidence for a repetitive member factor above 1 is at best mixed.

In Appendix A, we describe simple load sharing simulations that define assembly failure as first member failure, do not include composite action, and assume perfectly rigid sheathing (so that loads are distributed in proportion to member stiffnesses). We find that under these conditions, R_{ADS} ratios fall below 1.

Rosowsky and Ellingwood (1992) performed load sharing simulations that included duration of load effects, but did not include partial composite action. Their simulations yielded repetitive member factors 1.11, 1.33, and 1.53 for three species of wood. However, their definition of assembly failure was the failure of any two members for systems with fewer than eight members and the failure of two adjacent members for systems with eight or more members. If one considers figure 6 of their paper, one sees that for 10-member assemblies of Douglas Fir-Larch and Southern Pine, equal loads lead to higher probabilities of failure for assemblies than for single members even if we define assembly failure as the failure of any two members.

[4] 0.84 equals the mean of 3280, 4210, ..., 2980 divided by the mean of 4530, 4360, 4540, and 4270. 1.00 equals the mean of 4290, 5280, ..., 4750 divided by the mean of 4460, 5780, 5880, and 5510. 1.02 equals the mean of 4070, 4970, ..., 2240 divided by the mean of 5080, 4890, 4000, and 6900. See their table 1.

Bulleit and Liu (1995) performed simulations that were based on a version of McCutcheon's (1984) beam-spring analog model. Both duration of load and partial composite action were included in their simulations. They defined assembly failure variously as the failure of any one member, of any two members, of any three members, or of any four members. They report three cases (see their table 9) in which assembly failure was defined to be first member failure. The system factors in these three cases were 1.04, 1.09, and 1.24. Bulleit and Liu also found cases in which system factors were less than 1. See their table 6.

Wheat *et al.* (1986) created 15 full-scale test floors (three replications of five distinct test floors). For each of these floors they reported both the ultimate load and the load at first member failure. We can compare the mean loads at first member failure with the population mean strengths of individual members. For the single member means, we used No. 2, 15% moisture content in-grade values (Green and Evans, 1987).

The resulting strength ratios were 0.60, 0.62, 0.79, 0.79, and 0.58. (Note that these are ratios at the mean, not at the fifth percentile.) See their tables 1 and 5 for the data needed to calculate these values. Given that Wheat *et al.* used No. 2 and better boards, these system factors are probably overestimates.

In a thorough and insightful analytical study, Rosowsky and Yu (2004) made use of an extension (Yu, 2003) of McCutcheon's (1984) beam-spring analog model to evaluate a portfolio of wall systems. In this paper they considered four ratios related to our $R_{\mathrm{ADS}} = y_{\mathrm{assem}}/y_{\mathrm{sngl}}$ and $R_{\mathrm{ADM}} = y_{\mathrm{assem}}/y_{\mathrm{mink}}$:

$$K_{\mathrm{PY}} \approx (\text{fifth percentile of ultimate system strength})/\ y_{\mathrm{assem}} \tag{2}$$

where PY denotes "post yield";

$$K_{\mathrm{LS}} \approx y_{\mathrm{assem}}/y_{\mathrm{mink}} = R_{\mathrm{ADM}} \tag{3}$$

where LS denotes "load sharing";

$$K_{\mathrm{NMEM}} \approx y_{\mathrm{mink}}/y_{\mathrm{sngl}} \tag{4}$$

where NMEM denotes "number of members"; and

$$K_{\mathrm{PCA}} \equiv (\text{fifth percentile of T-beam strength})/y_{\mathrm{sngl}} \tag{5}$$

where PCA denotes "partial composite action".

Equations 2, 3, and 4 are only approximations because our y's do not include partial composite action.

Rosowsky and Yu calculated their raw repetitive member factor as

$$K_{\mathrm{PY}} \times K_{\mathrm{LS}} \times K_{\mathrm{NMEM}} \times K_{\mathrm{PCA}}$$

They found that, by far, the largest contributor to a repetitive member factor greater than 1 was K_{PY}. That is, residual capacity is the greatest contributor. However, D 6555 does not currently permit one to include residual capacity in the calculation of a repetitive member factor. When K_{PY} is set to 1 as it must be if we define assembly failure as first member failure, then Rosowsky and Yu's work yields repetitive member factors (that include partial composite action) that fall below 1. (See, for example, their table 3. Also see Rosowsky *et al.*, 2005.) This is driven by the small value of K_{NMEM}, essentially the ratio of the the fifth percentile of the minimum of the strengths of k members to the fifth percentile of the strength distribution of a single member (where there are k members in the assembly).

Note that our $R_{\text{ADS}} = y_{\text{assem}}/y_{\text{sngl}}$ is approximately equal to Rosowsky and Yu's (2004) $K_{\text{LS}} \times K_{\text{NMEM}}$. They found (see their figure 8), as we have, that this product decreases with increasing modulus of rupture coefficient of variation (COV).

We note that there *are* studies that calculate the repetitive member factor as done in section X2.8 of ASTM D 6555 (fifth percentile tolerance limit for the assembly strength distribution to the fifth percentile tolerance limit of an individual member strength distribution) and obtain values greater than 1. See, for example, table 4 of Bohnhoff *et al.* (1991). (Note that ratios of fifth percentile estimates can differ significantly from ratios of tolerance limits — compare the 1.27 and 1.40 values in the last column of Bohnhoff *et al.*'s table 4. Further note that although their ratio of fifth percentile estimates is above 1, their ratio of means is below 1.)

Using finite element methods, Folz and Foschi (1989) obtained an average system factor of 1.38 for systems in which assembly failure was first member failure. (However, they also obtained system factors that lay near or considerably below 1 in specific cases. See their table 7.) They did not consider duration of load effects.

Thus, we are not claiming that there are no cases in which the repetitive member factor should be greater than 1. We are saying, however, that there is confusion in the literature about the definition of a repetitive member factor and that it is by no means clear (at least if we restrict ourselves to adjustments due to load sharing and partial composite action, and a definition of assembly failure as first member failure) that the factor should, in general, be greater than 1 when it is defined as $y_{\text{assem}}/y_{\text{sngl}}$, the ratio of the fifth percentile of the assembly strength distribution to the fifth percentile of the individual member distribution (as it is commonly *applied* but not always calculated).

4 Additional sources of ambiguity

Even if we settle on an R_{ADS} (rather than an R_{ADM}) definition of the repetitive member factor, and we agree, for the current purposes of ASTM D 6555, to include only load sharing and composite action in this factor, we can obtain a wide range of R_{ADS} values. This point has been made previously by (at the least) Folz and Foschi (1989), Bulleit and Liu (1995), Rosowsky and Yu (2004), and comments in the standard itself. For example:

- As the number of members in an assembly increases, the repetitive member factor decreases. This is in accord with intuition and our simulations, and has been previously observed by Folz and Foschi (1989), Rosowsky and Ellingwood (1991), Bulleit and Liu (1995), and Rosowsky and Yu (2004).

- As the correlation between strength and stiffness increases — as load sharing becomes more effective — the repetitive member factor increases. This is in accord with intuition and our simulations, and has been previously observed by Folz and Foschi (1989) and Bulleit and Liu (1995).

- As the modulus of rupture COV increases, R_{ADS} values decrease. In fact, for large COVs, load sharing assembly strengths can be *much* less than single member strengths. (It is true that R_{ADM} values do increase with COV. That is, the advantage of load sharing assemblies over weakest link assemblies (but not over single members) increases with increasing COV.)

  Intuitions about the effect of an increase in the coefficient of variation of MOR have been, in some cases, incorrect. (See, for example, Verrill and Kretschmann, 2009.) For a one member failure criterion, a decline in the repetitive member factor with an increase in MOR COV has been observed by Folz and Foschi (1989), Bulleit and Liu (1995), and Rosowsky and Yu (2004). It is also apparent in our simulations. For a two adjacent members failure criterion, a decline in the repetitive member factor with an increase in MOR COV has been observed by Rosowsky and Ellingwood (1991).

  In contrast, for various failure criteria that require more than one member failure, Bulleit and Liu (1995) report that the repetitive member factor increases as modulus of rupture COV increases.

- Folz and Foschi (1989) and Bulleit and Liu (1995) have identified a number of other variables (for example, sheathing thickness and fastener stiffness) that can have a significant effect on the repetitive member factor.

5 Summary and conclusions

We have identified confusion in the literature about the proper choices for ASTM repetitive member factors. It appears that at least some of the justification for an 1.15 ASTM repetitive member factor may be based on this confusion. First, engineers might have R_{ADM} values in mind rather than the appropriate R_{ADS} values. Second, they might have R_{ADS} values in mind that are appropriate for a multiple member failure criterion, but not for a first member failure criterion. Finally, we have reminded readers of the fact that there are additional variables (such as number of members in the assembly and COV of the modulus of rupture) that can have a large effect on the repetitive member factor.

Our work suggests that authors must be quite careful when they report the results of repetitive member studies, and standards bodies must be even more careful when they interpret these results. For example, answers to the following questions must be clear:

1. Which of the subfactors — load sharing, partial composite action, residual capacity — does a reported system factor include?

2. What is the definition of assembly failure? Is it first member failure as specified in ASTM D 6555, or the failure of two members, or the failure of two adjacent members, or maximum assembly load capacity, or ...?

3. What definition of repetitive member factor is used? If the repetitive factor is calculated as a ratio of strengths:

   - Is it calculated as $R_{\mathrm{ADS}} = y_{\mathrm{assem}}/y_{\mathrm{sngl}}$ or $R_{\mathrm{ADM}} = y_{\mathrm{assem}}/y_{\mathrm{mink}}$?
   - If the R_{ADM} definition is used, how is the factor to be applied?
   - At what percentile (e.g., 50th, 5th, ...) is the ratio calculated? ASTM D 6555 specifies the fifth percentile, but some data sets only permit it to be calculated at the mean.
   - Is it a ratio of fifth percentiles or a ratio of tolerance bounds? ASTM D 6555 specifies the fifth percentile in Section 8.4.1.2, but uses tolerance bounds in the example in Section X2.8.

4. How many members are in the assembly?

5. What distributions of single member strength are considered?

6. What is the COV of the individual member strength distributions?

7. What is the correlation between strength and stiffness?

The correct repetitive member factor can depend heavily upon the answers to these questions.

Taken together, these dependencies suggest that a single repetitive member factor is unlikely to be appropriate. This point has been made previously by Folz and Foschi (1989), Bulleit and Liu (1995), Rosowsky and Yu (2004), and in ASTM D 6555 itself.

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7 Appendix A — The simulations

We worked with "truncated normal", "truncated lognormal", and "truncated Weibull" individual member distributions. We were attempting to model the selection of No. 2 boards. For a given correlation between modulus of elasticity (MOE) and modulus of rupture (MOR), we generated 100,000 pairs from the appropriate bivariate normal distribution. In the lognormal and Weibull cases one member of each pair had to be further transformed. See Appendix B for details. We then identified the 60,000 boards with the largest MOEs (the top 60% of the MOEs) and treated them as MSR (machine stress rated) No. 2 and better. We treated the 40,000 of these 60,000 with the lowest MOEs (the bottom two-thirds of the No. 2 and better) as No. 2 boards.

We considered assemblies of 3, 7, and 11 members. We modeled load sharing by assuming an infinitely stiff diaphragm, which ensured that all members were displaced equally so that loads were proportional to member stiffnesses. We did not include composite action in our model, and we defined assembly failure to be first member failure (so we did not consider residual capacity[5]).

In our simulations we calculated the ratio of the qth quantile ($q = 0.01, 0.05, 0.10$) of the strength distribution of a load sharing assembly to the qth quantile of the single member strength distribution (an R_{ADS} ratio) and the ratio of the qth quantile of the strength distribution of a load sharing assembly to the qth quantile of the distribution of the minimum of k member strengths (an R_{ADM} ratio)

The R_{ADS} values appear in column 7 of our tables. The R_{ADM} values appear in column 8 of the tables. In Figures 1–9 we plot R_{ADS} values versus COV. Figures 1–3 correspond to $q = 0.01, 0.05, 0.10$ for a "truncated normal" distribution. Figures 4–6 correspond to $q = 0.01, 0.05, 0.10$ for a "truncated lognormal" distribution. Figures 7–9 correspond to $q = 0.01, 0.05, 0.10$ for a "truncated Weibull" distribution. In each figure, there are three lines that correspond to each rho (correlation between strength and stiffness) value. For rho values less than 1.0, the lowest of the

[5]One reviewer, while accepting the results of the paper, argued that defining assembly failure as "first member failure" is not appropriate for tightly coupled assemblies such as laminates. We concur. However, for floors and roofs, we believe that first member failure is a reasonable assembly failure criterion. In any event, the current ASTM D 6555 does not permit a repetitive member adjustment based on residual capacity.

the three lines in a triplet corresponds to the 11-member assembly, the middle line corresponds to the 7-member assembly, and the highest line corresponds to the 3-member assembly. For rho equal to 1.0, this ordering is reversed. (In the in-grade data (Green and Evans, 1987), MOR/MOE correlations generally ranged from 0.60 to 0.75.)

Our simulations indicate that the ratios depend upon the assumed distribution ("truncated normal," "truncated lognormal," "truncated Weibull") of the individual members, the number of members (3,7,11) in the assembly, the correlation between the MOR and the MOE, the q value, and the COV of the individual member strength distribution.

In particular, the simulations demonstrated the following:

- The R_{ADM} values are (as we would expect) always above 1. That is, load sharing assemblies deserve an upward adjustment from weakest link assemblies.
- The R_{ADS} values are below 1 for strength, stiffness correlations below 1. That is, load sharing assemblies do *not* deserve an upward adjustment from single member values. In fact their allowable properties should be adjusted *downward* from the allowable properties of single members. (We emphasize that we are not considering composite action in our simulations. However, Rosowsky and Yu's (2004) work suggests that this conclusion holds even if we do include composite action.)
- The downward adjustment should be greater for assemblies with more members.
- As we would expect, the downward adjustment is less as the correlation between strength and stiffness increases — as load sharing becomes more effective. In fact, given a perfect correlation between strength and stiffness, the R_{ADS} values are greater than 1.
- As material variability increases (as COV increases), R_{ADS} values decrease. In fact, for large COVs, load sharing assembly strengths can be *much* less than single member strengths. (It is true that R_{ADM} values do increase with COV. That is, the advantage of load sharing assemblies over weakest link assemblies (but not over single members) increases with increasing COV.)

The FORTRAN computer programs that we used in our simulations can be found at `http://www1.fpl.fs.fed.us/RepMem_sim.html`.

8 Appendix B — Correlations and the generation of the strengths and stiffnesses

The MOEs were all normally distributed. The MORs were distributed as normal, lognormal, or Weibull random variables. In all cases the medians of the distributions were equal to 100.

8.1 Normal strengths

The MOE of an MOE,MOR pair was generated as

$$\text{MOE} = 100 + 100 \times \text{COV} \times \epsilon_1$$

where ϵ_1 represents a generated N(0,1) random variable. The correlated MOR value was generated as

$$\text{MOR} = 100 + 100 \times \text{COV} \times (\rho \times \epsilon_1 + \sqrt{1-\rho^2} \times \epsilon_2)$$

where ϵ_1 and ϵ_2 are "independent" generated N(0,1)'s. The correlation between the MOE and MOR values is then ρ.

8.2 Lognormal strengths

The MOE of an MOE,MOR pair was generated as

$$\text{MOE} = 100 + 100 \times \text{COV} \times \epsilon_1$$

where ϵ_1 represents a generated N(0,1) random variable. The correlated MOR value was generated as

$$\text{MOR} = 100 \times \exp\left(\sigma \times \left(\rho \times \epsilon_1 + \sqrt{1-\rho^2} \times \epsilon_2\right)\right)$$

where ϵ_1 and ϵ_2 are "independent" generated N(0,1)'s, and

$$\sigma = \sqrt{\log(1 + \text{COV}^2)}$$

This ensures that the coefficient of variation of the MOR will be COV. However, the correlation between the two elements of an MOE,MOR pair will not be exactly equal to ρ. The actual correlation is presented in Table 10.

8.3 Weibull strengths

The MOE of an MOE,MOR pair was generated as

$$\text{MOE} = 100 + 100 \times \text{COV} \times \epsilon_1$$

where ϵ_1 represents a generated N(0,1) random variable. The correlated MOR values was generated via three steps. First a N(0,1) correlated to the MOE was generated:

$$z = \rho \times \epsilon_1 + \sqrt{1-\rho^2} \times \epsilon_2$$

where ϵ_1 and ϵ_2 are "independent" generated N(0,1)'s. Second, the N(0,1) random variable was transformed into a Uniform(0,1) random variable:

$$u = \Phi(z)$$

where Φ denotes the N(0,1) distribution function. Finally, the Uniform(0,1) random variable was transformed into a Weibull random variable:

$$w = (-\log(1-u))^{1/\beta}/\gamma$$

where γ is the scale parameter and β is the shape parameter. β was determined by

$$\text{COV} = \sqrt{2\beta\Gamma(2/\beta) - \Gamma^2(1/\beta)}/\Gamma(1/\beta)$$

where Γ denotes the gamma function. This ensures that the coefficient of variation of the Weibull will be COV. The γ value was then determined via

$$\gamma = \log(2)^{1/\beta}/100$$

which ensures that the median of the Weibull will be 100.

The correlation between the two elements of an MOE,MOR pair will not be exactly equal to ρ. The actual correlation is presented in Table 10.

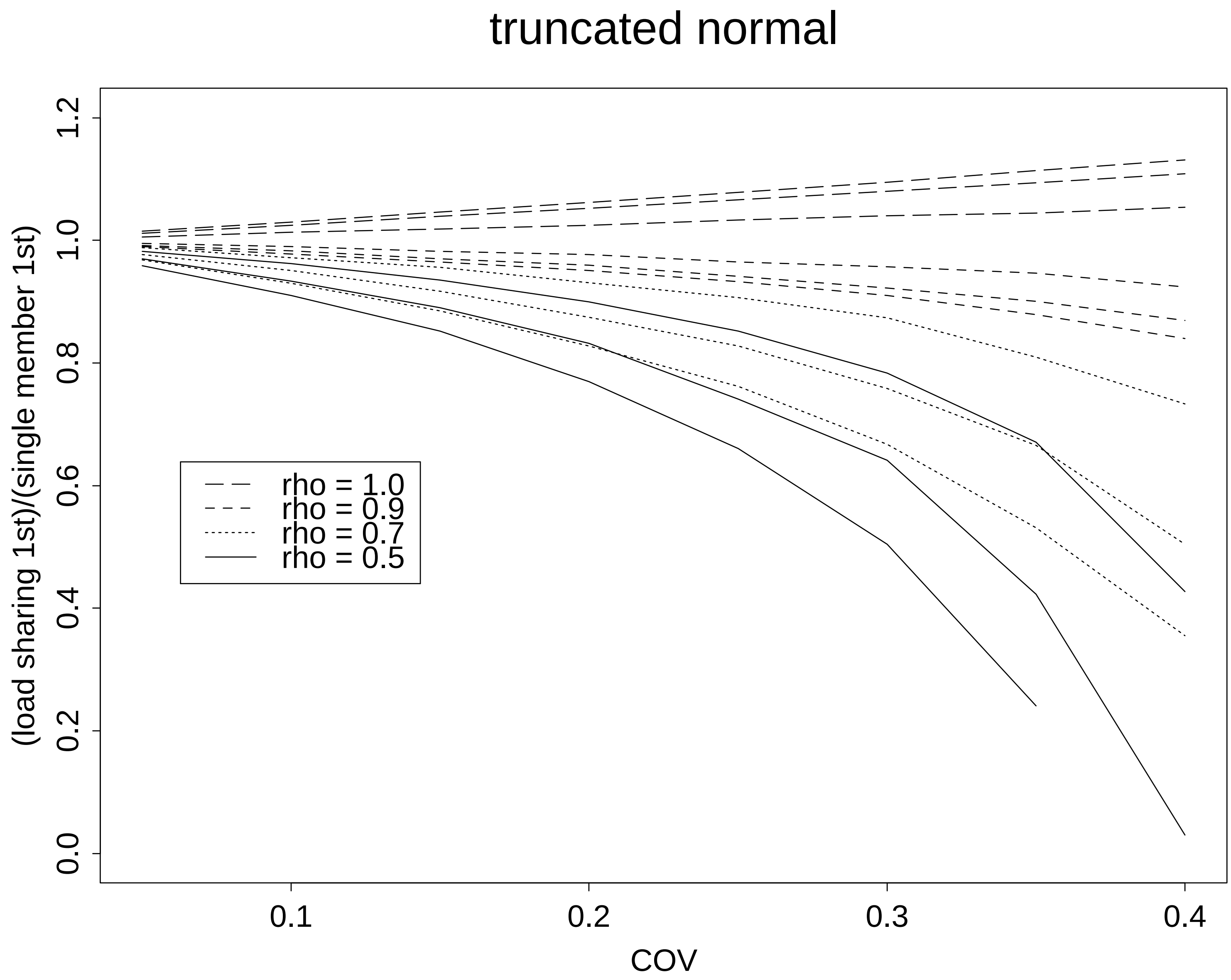

Figure 1: "Truncated normal" distribution. Ratios of first percentile estimates. Based on 100,000 simulation trials. In this figure, there are three lines that correspond to each rho (correlation between strength and stiffness) value. For rho values less than 1.0, the lowest of the three lines in a triplet corresponds to the 11-member assembly, the middle line corresponds to the 7-member assembly, and the highest line corresponds to the 3-member assembly. For rho equal to 1.0, this ordering is reversed.

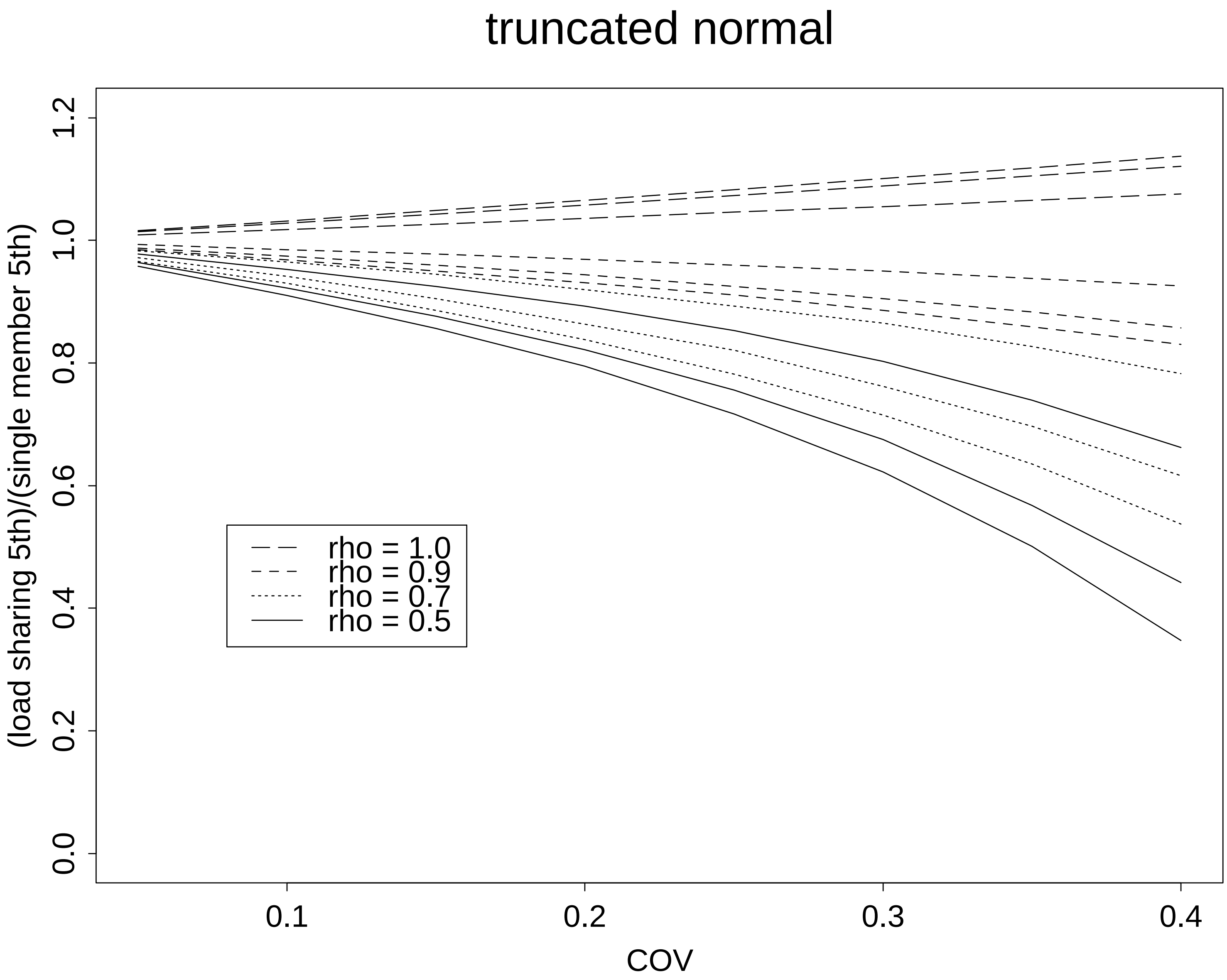

Figure 2: "Truncated normal" distribution. Ratios of fifth percentile estimates. Based on 100,000 simulation trials. In this figure, there are three lines that correspond to each rho (correlation between strength and stiffness) value. For rho values less than 1.0, the lowest of the three lines in a triplet corresponds to the 11-member assembly, the middle line corresponds to the 7-member assembly, and the highest line corresponds to the 3-member assembly. For rho equal to 1.0, this ordering is reversed.

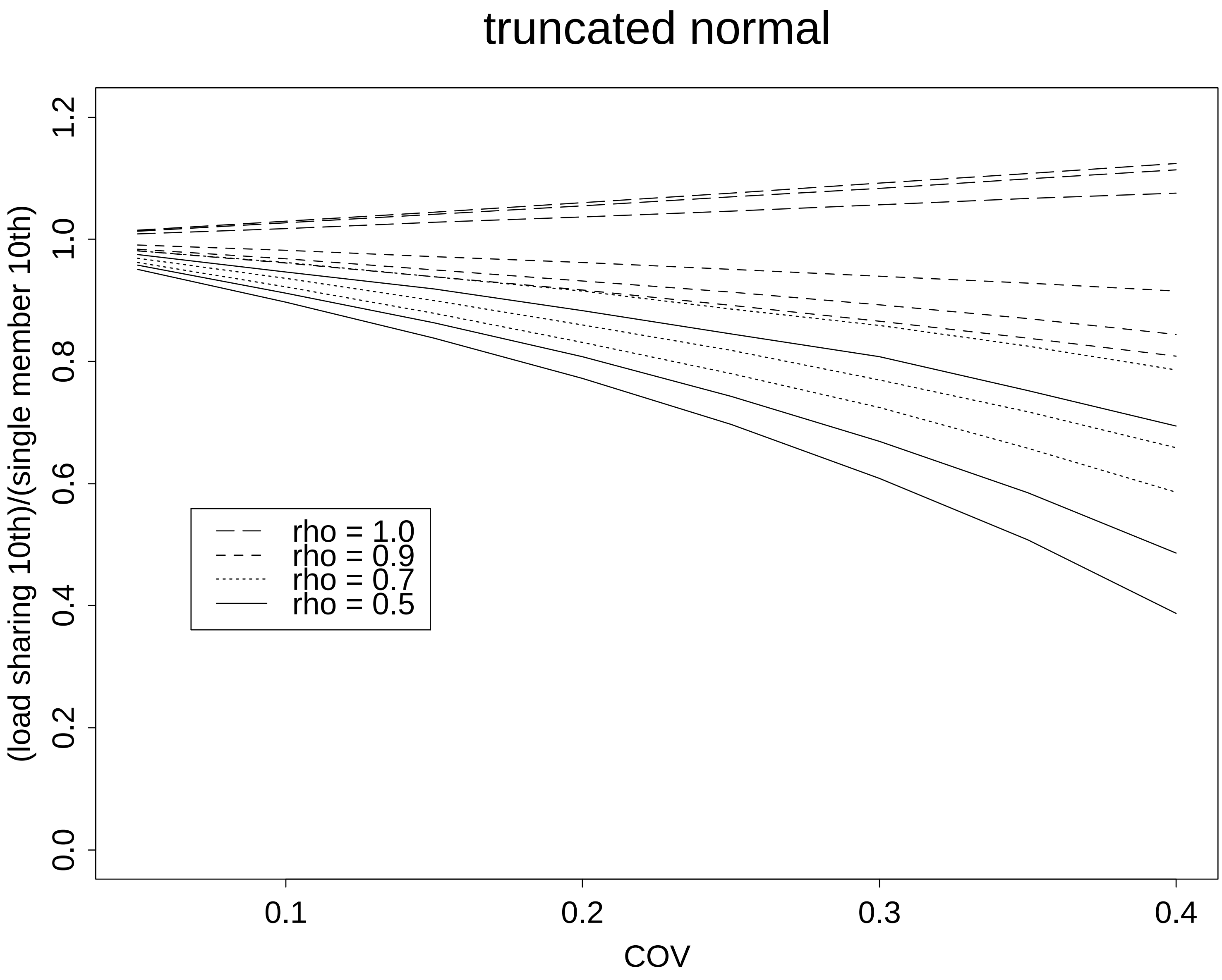

Figure 3: "Truncated normal" distribution. Ratios of tenth percentile estimates. Based on 100,000 simulation trials. In this figure, there are three lines that correspond to each rho (correlation between strength and stiffness) value. For rho values less than 1.0, the lowest of the three lines in a triplet corresponds to the 11-member assembly, the middle line corresponds to the 7-member assembly, and the highest line corresponds to the 3-member assembly. For rho equal to 1.0, this ordering is reversed.

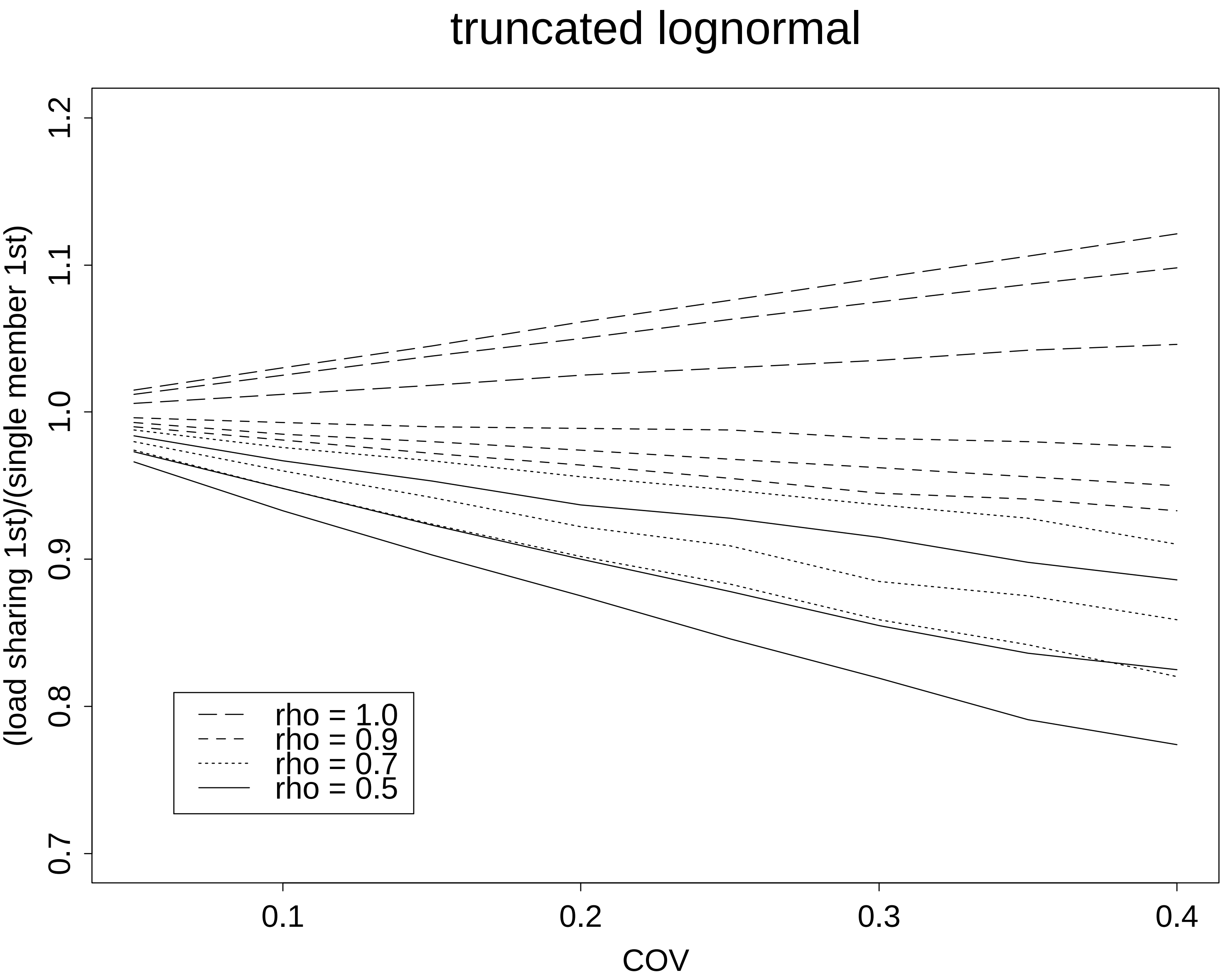

Figure 4: “Truncated lognormal” distribution. Ratios of first percentile estimates. Based on 100,000 simulation trials. In this figure, there are three lines that correspond to each rho (correlation between ln(strength) and stiffness) value. For rho values less than 1.0, the lowest of the three lines in a triplet corresponds to the 11-member assembly, the middle line corresponds to the 7-member assembly, and the highest line corresponds to the 3-member assembly. For rho equal to 1.0, this ordering is reversed.

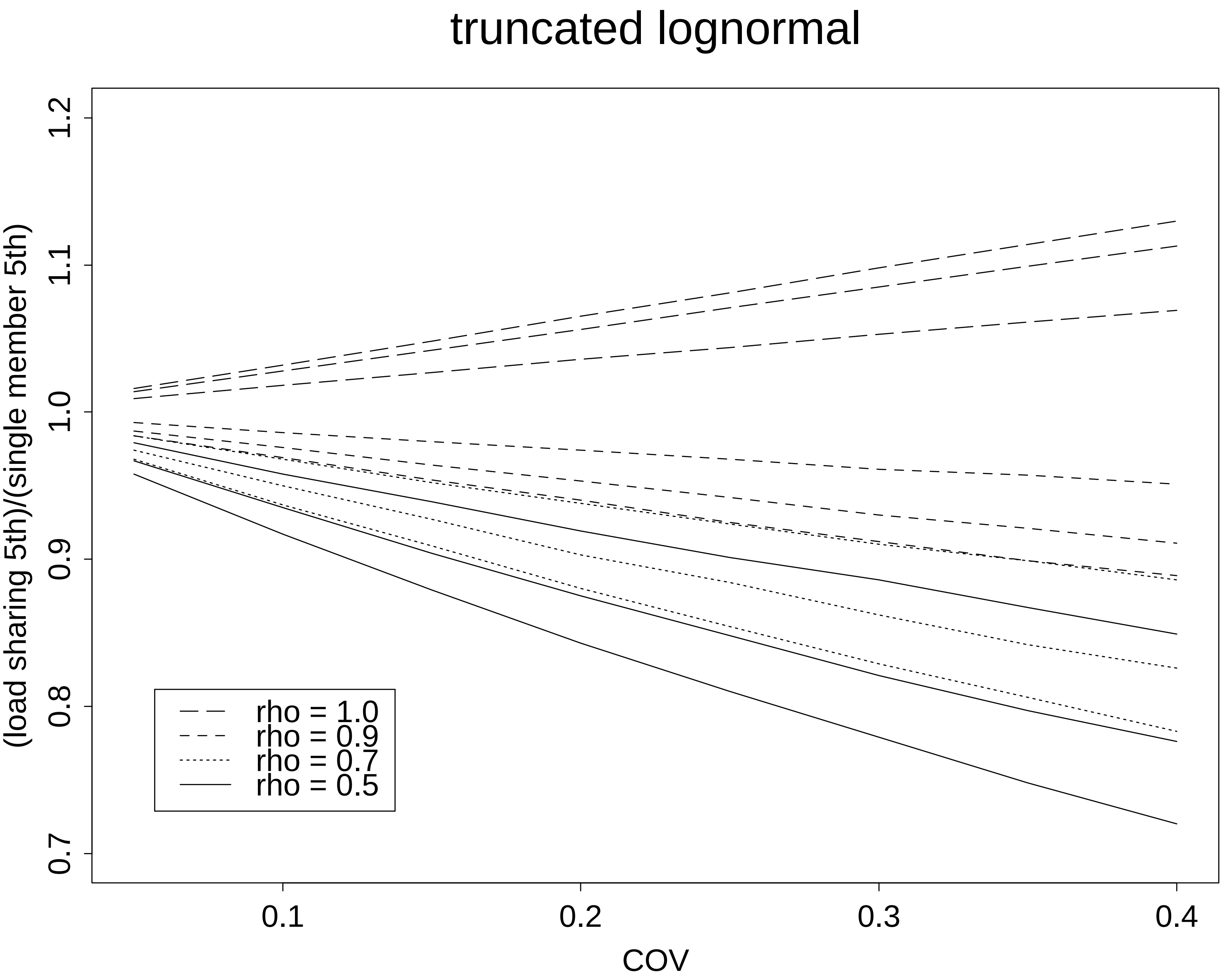

Figure 5: "Truncated lognormal" distribution. Ratios of fifth percentile estimates. Based on 100,000 simulation trials. In this figure, there are three lines that correspond to each rho (correlation between ln(strength) and stiffness) value. For rho values less than 1.0, the lowest of the three lines in a triplet corresponds to the 11-member assembly, the middle line corresponds to the 7-member assembly, and the highest line corresponds to the 3-member assembly. For rho equal to 1.0, this ordering is reversed.

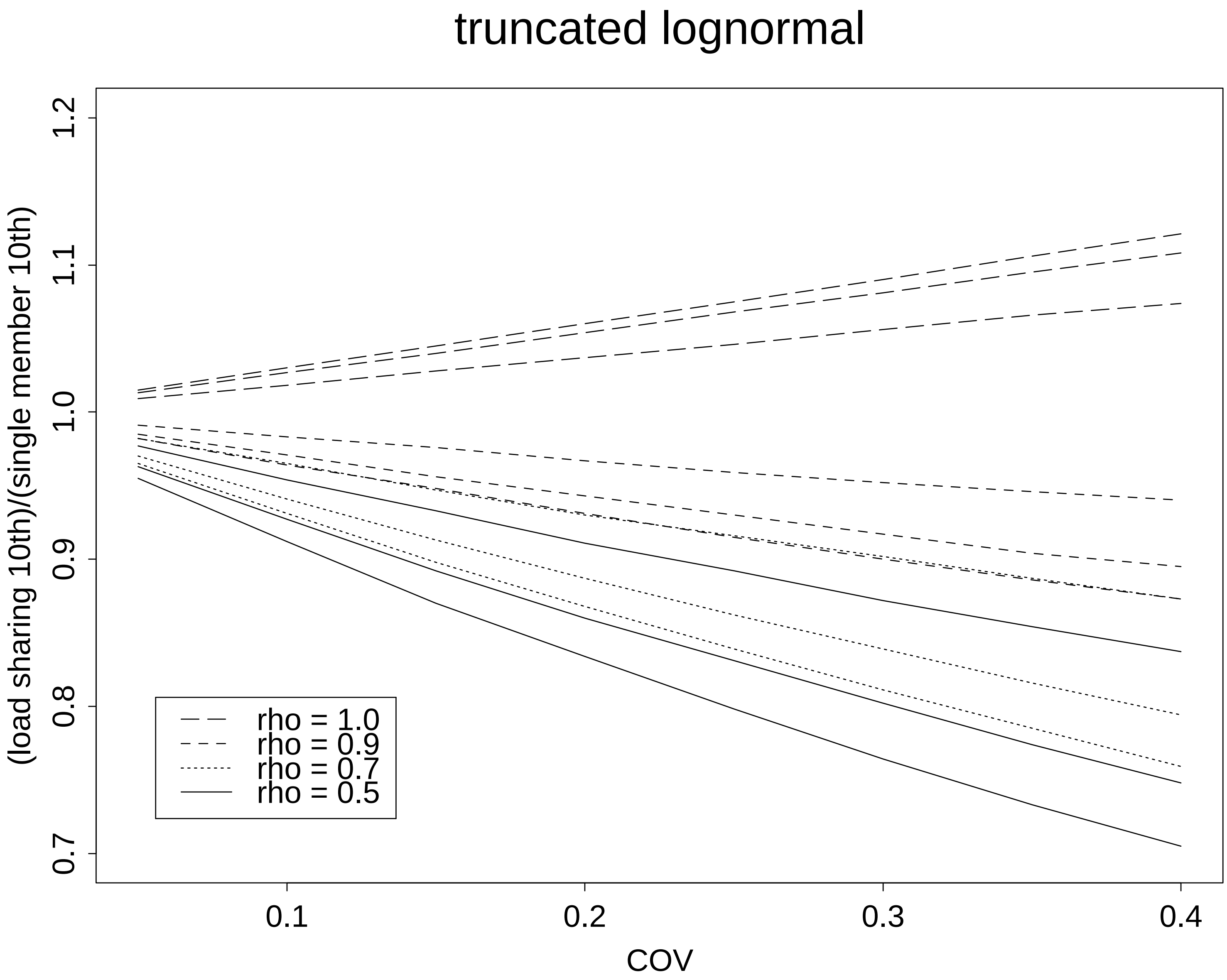

Figure 6: "Truncated lognormal" distribution. Ratios of tenth percentile estimates. Based on 100,000 simulation trials. In this figure, there are three lines that correspond to each rho (correlation between ln(strength) and stiffness) value. For rho values less than 1.0, the lowest of the three lines in a triplet corresponds to the 11-member assembly, the middle line corresponds to the 7-member assembly, and the highest line corresponds to the 3-member assembly. For rho equal to 1.0, this ordering is reversed.

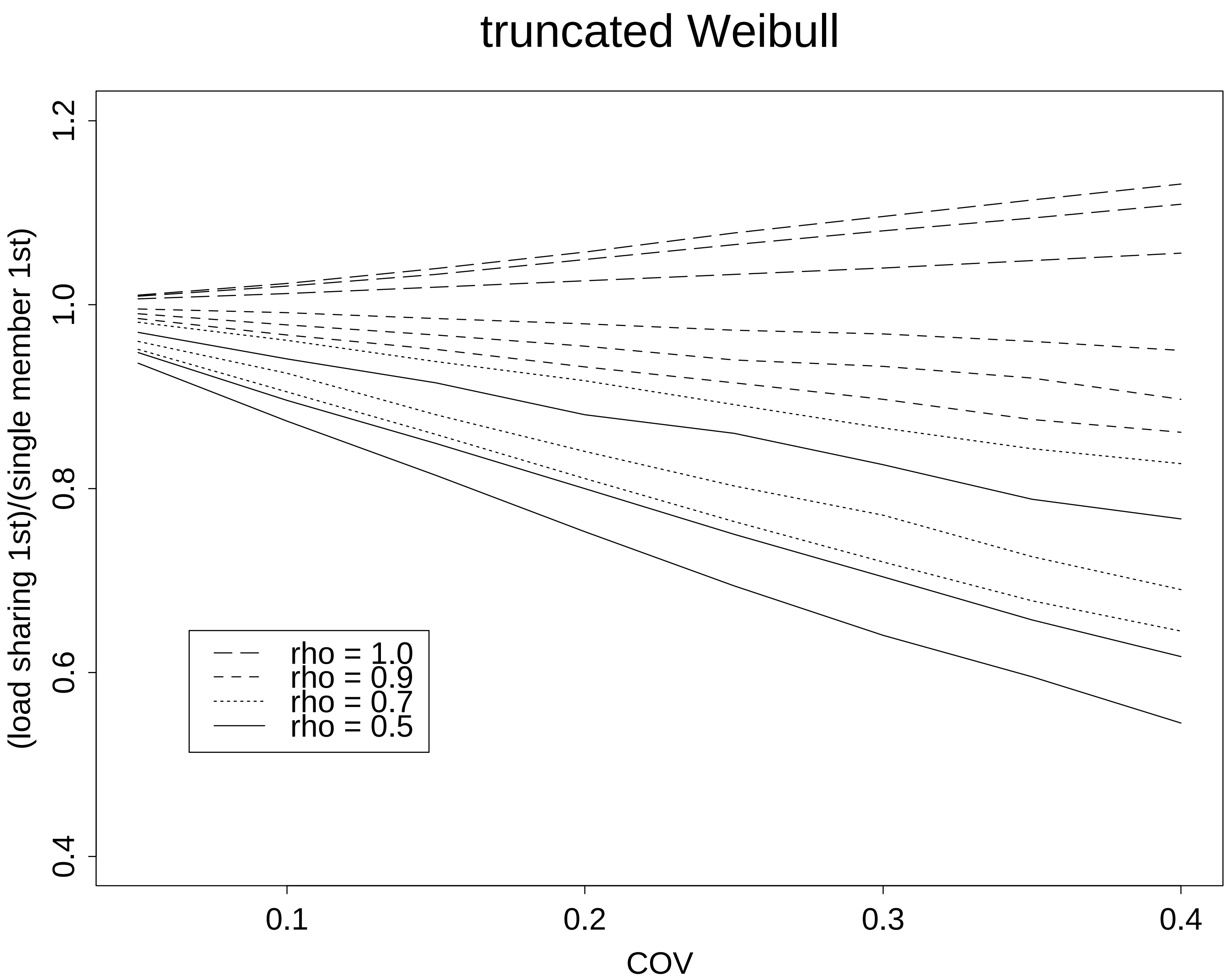

Figure 7: "Truncated Weibull" distribution. Ratios of first percentile estimates. Based on 100,000 simulation trials. In this figure, there are three lines that correspond to each rho value. For rho values less than 1.0, the lowest of the three lines in a triplet corresponds to the 11-member assembly, the middle line corresponds to the 7-member assembly, and the highest line corresponds to the 3-member assembly. For rho equal to 1.0, this ordering is reversed.

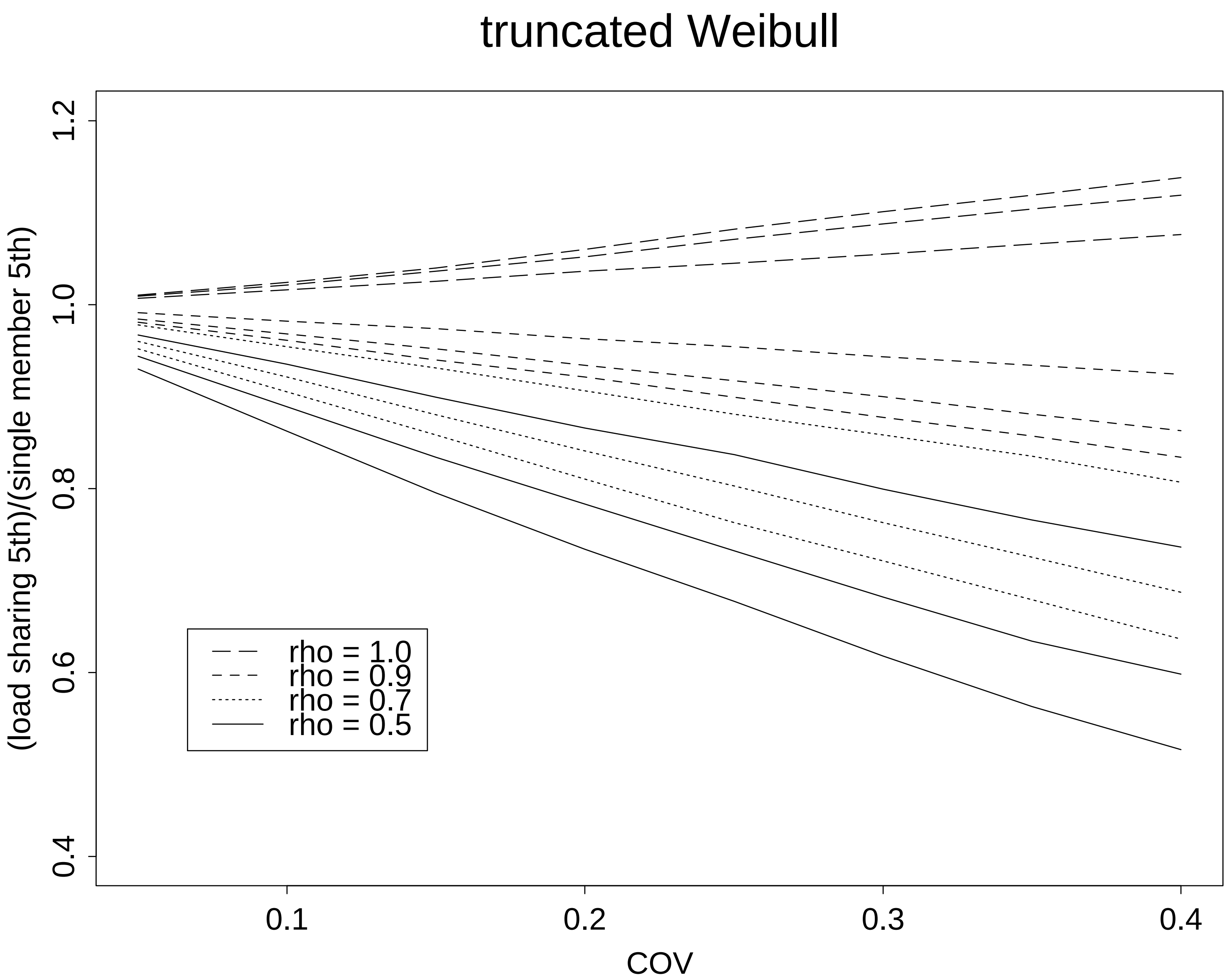

Figure 8: "Truncated Weibull" distribution. Ratios of fifth percentile estimates. Based on 100,000 simulation trials. In this figure, there are three lines that correspond to each rho value. For rho values less than 1.0, the lowest of the three lines in a triplet corresponds to the 11-member assembly, the middle line corresponds to the 7-member assembly, and the highest line corresponds to the 3-member assembly. For rho equal to 1.0, this ordering is reversed.

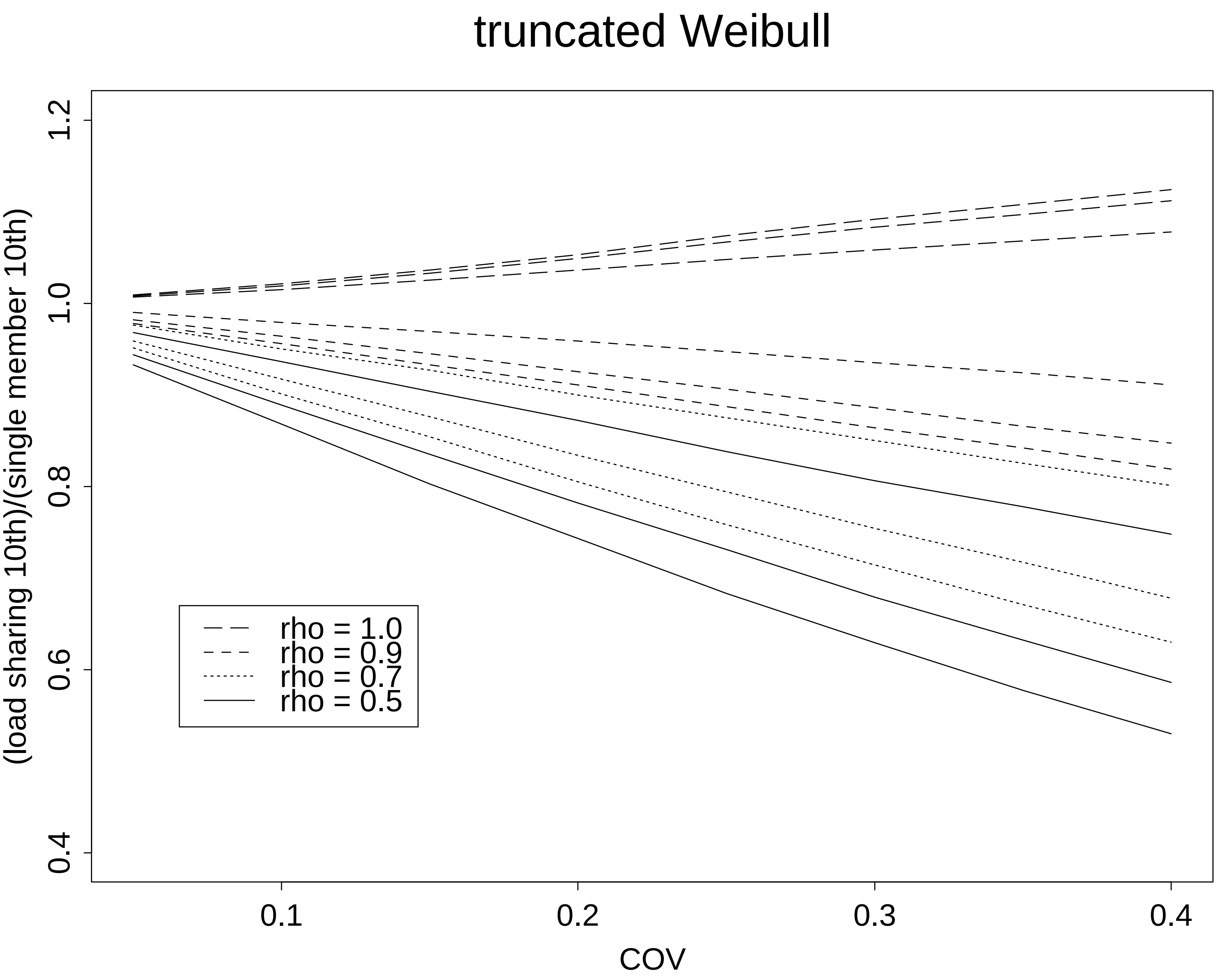

Figure 9: "Truncated Weibull" distribution. Ratios of tenth percentile estimates. Based on 100,000 simulation trials. In this figure, there are three lines that correspond to each rho value. For rho values less than 1.0, the lowest of the three lines in a triplet corresponds to the 11-member assembly, the middle line corresponds to the 7-member assembly, and the highest line corresponds to the 3-member assembly. For rho equal to 1.0, this ordering is reversed.

ρ	Number of members	COV	$y_{\text{sngl}} =$ Single member	$y_{\text{assem}} =$ Load sharing	$y_{\text{mink}} =$ Weakest link	$R_{\text{ADS}} =$ $y_{\text{assem}}/y_{\text{sngl}}$	$R_{\text{ADM}} =$ $y_{\text{assem}}/y_{\text{mink}}$
		0.05	90.5	89.0	88.9	0.982	1.001
		0.10	81.1	78.0	77.7	0.962	1.004
		0.15	71.6	67.0	66.6	0.935	1.005
		0.20	62.2	55.9	55.5	0.900	1.008
	3	0.25	52.7	44.9	44.3	0.852	1.014
		0.30	43.3	33.9	33.2	0.784	1.022
		0.35	33.8	22.7	22.1	0.671	1.028
		0.40	24.4	10.4	10.9	0.427	0.950
		0.05	90.4	87.6	87.6	0.970	1.001
		0.10	80.7	75.3	75.1	0.934	1.003
		0.15	71.1	63.2	62.7	0.890	1.009
		0.20	61.4	51.1	50.2	0.832	1.017
0.500	7	0.25	51.8	38.3	37.8	0.741	1.015
		0.30	42.1	27.0	25.3	0.641	1.065
		0.35	32.5	13.7	12.9	0.423	1.065
		0.40	22.8	0.7	0.5	0.030	1.486
		0.05	90.4	86.7	86.7	0.959	1.000
		0.10	80.8	73.5	73.4	0.910	1.002
		0.15	71.2	60.7	60.0	0.852	1.011
		0.20	61.6	47.4	46.7	0.770	1.015
	11	0.25	52.0	34.3	33.4	0.660	1.028
		0.30	42.4	21.4	20.1	0.504	1.065
		0.35	32.8	7.9	6.7	0.241	1.175
		0.40	23.2	—	—	—	—
		0.05	92.2	91.1	90.7	0.988	1.005
		0.10	84.4	82.1	81.4	0.972	1.009
		0.15	76.6	73.3	72.1	0.956	1.017
		0.20	68.9	64.1	62.7	0.931	1.022
	3	0.25	61.1	55.4	53.4	0.907	1.037
		0.30	53.3	46.6	44.1	0.874	1.055
		0.35	45.5	36.8	34.8	0.810	1.059
		0.40	37.7	27.6	25.5	0.733	1.084
		0.05	92.2	90.1	89.8	0.977	1.004
		0.10	84.4	80.3	79.6	0.951	1.009
		0.15	76.6	70.3	69.4	0.917	1.013
		0.20	68.9	60.2	59.2	0.875	1.018
0.700	7	0.25	61.1	50.6	49.0	0.828	1.033
		0.30	53.3	40.4	38.8	0.758	1.041
		0.35	45.5	30.3	28.6	0.666	1.061
		0.40	37.7	19.0	18.4	0.504	1.036
		0.05	92.2	89.3	89.1	0.968	1.002
		0.10	84.5	78.6	78.2	0.930	1.005
		0.15	76.7	67.9	67.3	0.885	1.008
		0.20	69.0	57.1	56.4	0.828	1.012
	11	0.25	61.2	46.6	45.5	0.762	1.024
		0.30	53.4	35.6	34.6	0.667	1.029
		0.35	45.7	24.3	23.7	0.531	1.023
		0.40	37.9	13.5	12.8	0.355	1.051

Table 1: First percentile strengths and ratios of first percentile strengths for a truncated normal distribution. Based on 100,000 trials.

ρ	Number of members	COV	$y_{\text{sngl}} =$ Single member	$y_{\text{assem}} =$ Load sharing	$y_{\text{mink}} =$ Weakest link	$R_{\text{ADS}} = y_{\text{assem}}/y_{\text{sngl}}$	$R_{\text{ADM}} = y_{\text{assem}}/y_{\text{mink}}$
		0.05	95.3	94.9	94.3	0.995	1.006
		0.10	90.7	89.8	88.7	0.990	1.013
		0.15	86.0	84.5	83.0	0.982	1.018
		0.20	81.3	79.4	77.3	0.977	1.028
	3	0.25	76.7	74.0	71.6	0.965	1.033
		0.30	72.0	68.9	66.0	0.957	1.045
		0.35	67.3	63.7	60.3	0.947	1.057
		0.40	62.7	57.9	54.6	0.924	1.061
		0.05	95.3	94.5	93.7	0.992	1.009
		0.10	90.6	89.0	87.4	0.983	1.019
		0.15	85.9	83.3	81.0	0.970	1.028
		0.20	81.2	77.9	74.7	0.960	1.043
0.900	7	0.25	76.5	72.0	68.4	0.941	1.052
		0.30	71.8	66.2	62.1	0.922	1.066
		0.35	67.1	60.4	55.7	0.901	1.084
		0.40	62.4	54.2	49.4	0.869	1.097
		0.05	95.3	94.4	93.4	0.990	1.011
		0.10	90.7	88.7	86.7	0.978	1.023
		0.15	86.0	83.0	80.1	0.965	1.037
		0.20	81.4	77.4	73.4	0.951	1.054
	11	0.25	76.7	71.6	66.8	0.933	1.072
		0.30	72.1	65.5	60.1	0.910	1.090
		0.35	67.4	59.2	53.5	0.879	1.107
		0.40	62.8	52.7	46.9	0.840	1.126
		0.05	98.8	99.4	98.7	1.006	1.007
		0.10	97.6	98.8	97.5	1.013	1.013
		0.15	96.3	98.1	96.2	1.019	1.020
		0.20	95.1	97.5	95.0	1.025	1.027
	3	0.25	93.9	97.0	93.7	1.033	1.035
		0.30	92.7	96.4	92.5	1.040	1.042
		0.35	91.5	95.6	91.2	1.045	1.048
		0.40	90.2	95.1	90.0	1.054	1.058
		0.05	98.8	100.0	98.8	1.012	1.013
		0.10	97.6	100.1	97.5	1.025	1.026
		0.15	96.4	100.1	96.3	1.039	1.040
		0.20	95.2	100.2	95.0	1.052	1.054
1.000	7	0.25	94.0	100.2	93.8	1.066	1.068
		0.30	92.8	100.2	92.6	1.080	1.083
		0.35	91.6	100.3	91.3	1.094	1.098
		0.40	90.4	100.2	90.1	1.109	1.113
		0.05	98.8	100.3	98.7	1.015	1.015
		0.10	97.6	100.5	97.5	1.030	1.031
		0.15	96.3	100.8	96.2	1.046	1.048
		0.20	95.1	101.0	94.9	1.062	1.064
	11	0.25	93.9	101.2	93.7	1.078	1.081
		0.30	92.7	101.5	92.4	1.095	1.098
		0.35	91.4	101.9	91.1	1.114	1.118
		0.40	90.2	102.1	89.9	1.131	1.136

Table 1 continued: First percentile strengths and ratios of first percentile strengths for a truncated normal distribution. Based on 100,000 trials.

ρ	Number of members	COV	$y_{\text{sngl}} =$ Single member	$y_{\text{assem}} =$ Load sharing	$y_{\text{mink}} =$ Weakest link	$R_{\text{ADS}} = y_{\text{assem}}/y_{\text{sngl}}$	$R_{\text{ADM}} = y_{\text{assem}}/y_{\text{mink}}$
0.500	3	0.05	93.5	91.4	91.4	0.978	1.001
		0.10	86.9	82.8	82.7	0.953	1.001
		0.15	80.4	74.4	74.1	0.925	1.004
		0.20	73.8	65.9	65.4	0.893	1.007
		0.25	67.3	57.4	56.8	0.853	1.010
		0.30	60.7	48.7	48.1	0.803	1.013
		0.35	54.2	40.0	39.5	0.739	1.014
		0.40	47.6	31.5	30.8	0.662	1.022
	7	0.05	93.4	90.0	89.9	0.964	1.000
		0.10	86.7	80.0	79.9	0.922	1.002
		0.15	80.1	70.2	69.8	0.876	1.005
		0.20	73.5	60.4	59.8	0.822	1.011
		0.25	66.9	50.5	49.7	0.756	1.016
		0.30	60.2	40.6	39.7	0.675	1.024
		0.35	53.6	30.5	29.6	0.568	1.029
		0.40	47.0	20.8	19.6	0.442	1.061
	11	0.05	93.5	89.5	89.4	0.958	1.001
		0.10	86.9	79.1	78.9	0.910	1.002
		0.15	80.4	68.8	68.3	0.856	1.008
		0.20	73.8	58.7	57.8	0.795	1.016
		0.25	67.3	48.2	47.2	0.717	1.022
		0.30	60.7	37.8	36.6	0.622	1.031
		0.35	54.2	27.2	26.1	0.501	1.042
		0.40	47.6	16.5	15.5	0.347	1.065
0.700	3	0.05	94.8	93.2	93.0	0.983	1.002
		0.10	89.6	86.5	86.1	0.965	1.004
		0.15	84.4	79.8	79.1	0.945	1.008
		0.20	79.2	72.9	72.2	0.920	1.010
		0.25	74.0	66.1	65.2	0.893	1.014
		0.30	68.8	59.5	58.3	0.865	1.022
		0.35	63.7	52.7	51.3	0.827	1.026
		0.40	58.5	45.8	44.4	0.783	1.032
	7	0.05	94.8	92.1	91.8	0.972	1.004
		0.10	89.6	84.3	83.5	0.941	1.010
		0.15	84.4	76.3	75.3	0.905	1.014
		0.20	79.1	68.3	67.0	0.863	1.019
		0.25	73.9	60.7	58.8	0.821	1.032
		0.30	68.7	52.3	50.6	0.762	1.035
		0.35	63.5	44.3	42.3	0.697	1.046
		0.40	58.3	35.9	34.1	0.616	1.054
	11	0.05	94.7	91.5	91.2	0.966	1.004
		0.10	89.5	83.2	82.3	0.930	1.010
		0.15	84.2	74.6	73.5	0.886	1.016
		0.20	78.9	66.2	64.6	0.838	1.024
		0.25	73.7	57.6	55.8	0.782	1.032
		0.30	68.4	48.9	46.9	0.715	1.042
		0.35	63.1	40.1	38.1	0.635	1.053
		0.40	57.9	31.1	29.3	0.537	1.062

Table 2: Fifth percentile strengths and ratios of fifth percentile strengths for a truncated normal distribution. Based on 100,000 trials.

ρ	Number of members	COV	$y_{\text{sngl}}=$ Single member	$y_{\text{assem}}=$ Load sharing	$y_{\text{mink}}=$ Weakest link	$R_{\text{ADS}}=$ $y_{\text{assem}}/y_{\text{sngl}}$	$R_{\text{ADM}}=$ $y_{\text{assem}}/y_{\text{mink}}$
0.900	3	0.05	96.9	96.3	95.8	0.993	1.005
		0.10	93.9	92.5	91.6	0.985	1.010
		0.15	90.8	88.9	87.4	0.978	1.017
		0.20	87.8	85.0	83.2	0.969	1.023
		0.25	84.7	81.4	79.0	0.960	1.030
		0.30	81.7	77.6	74.8	0.950	1.038
		0.35	78.6	73.8	70.5	0.938	1.046
		0.40	75.6	70.0	66.3	0.926	1.055
	7	0.05	97.0	95.7	95.0	0.987	1.008
		0.10	93.9	91.5	89.9	0.974	1.017
		0.15	90.9	87.2	84.9	0.960	1.027
		0.20	87.8	82.9	79.9	0.944	1.037
		0.25	84.8	78.5	74.9	0.925	1.048
		0.30	81.8	74.0	69.8	0.905	1.059
		0.35	78.7	69.5	64.8	0.883	1.073
		0.40	75.7	64.8	59.8	0.857	1.085
	11	0.05	96.9	95.5	94.6	0.985	1.009
		0.10	93.9	90.9	89.2	0.968	1.020
		0.15	90.8	86.3	83.8	0.950	1.030
		0.20	87.8	81.7	78.4	0.931	1.043
		0.25	84.7	77.1	73.0	0.911	1.057
		0.30	81.6	72.3	67.5	0.886	1.071
		0.35	78.6	67.5	62.1	0.859	1.087
		0.40	75.5	62.7	56.7	0.830	1.105
1.000	3	0.05	99.0	99.8	98.8	1.009	1.010
		0.10	98.0	99.7	97.7	1.018	1.021
		0.15	97.0	99.5	96.5	1.026	1.032
		0.20	96.0	99.4	95.3	1.036	1.043
		0.25	95.0	99.3	94.1	1.046	1.055
		0.30	94.0	99.1	93.0	1.055	1.066
		0.35	93.0	99.0	91.8	1.065	1.079
		0.40	92.0	98.9	90.6	1.076	1.092
	7	0.05	99.0	100.4	98.8	1.014	1.016
		0.10	98.0	100.7	97.5	1.028	1.033
		0.15	96.9	101.1	96.3	1.043	1.050
		0.20	95.9	101.5	95.1	1.058	1.068
		0.25	94.9	101.9	93.8	1.073	1.086
		0.30	93.9	102.3	92.6	1.089	1.104
		0.35	92.9	102.6	91.3	1.105	1.123
		0.40	91.8	103.0	90.1	1.121	1.143
	11	0.05	99.0	100.6	98.8	1.016	1.018
		0.10	98.0	101.1	97.5	1.032	1.037
		0.15	97.0	101.7	96.3	1.049	1.057
		0.20	96.0	102.2	95.0	1.065	1.076
		0.25	95.0	102.9	93.8	1.083	1.097
		0.30	94.0	103.4	92.5	1.101	1.118
		0.35	92.9	103.9	91.3	1.118	1.139
		0.40	91.9	104.5	90.0	1.137	1.161

Table 2 continued: Fifth percentile strengths and ratios of fifth percentile strengths for a truncated normal distribution. Based on 100,000 trials.

ρ	Number of members	COV	$y_{\text{sngl}} =$ Single member	$y_{\text{assem}} =$ Load sharing	$y_{\text{mink}} =$ Weakest link	$R_{\text{ADS}} = y_{\text{assem}}/y_{\text{sngl}}$	$R_{\text{ADM}} = y_{\text{assem}}/y_{\text{mink}}$
0.500	3	0.05	95.0	92.6	92.7	0.975	1.000
		0.10	90.1	85.3	85.3	0.947	0.999
		0.15	85.1	78.2	78.0	0.919	1.003
		0.20	80.2	70.8	70.7	0.883	1.002
		0.25	75.2	63.6	63.3	0.845	1.004
		0.30	70.3	56.7	56.0	0.808	1.013
		0.35	65.3	49.1	48.7	0.752	1.009
		0.40	60.4	41.9	41.3	0.694	1.013
	7	0.05	95.1	91.2	91.1	0.958	1.001
		0.10	90.3	82.3	82.2	0.912	1.002
		0.15	85.4	73.7	73.3	0.863	1.005
		0.20	80.5	65.0	64.4	0.808	1.010
		0.25	75.6	56.2	55.5	0.743	1.013
		0.30	70.8	47.3	46.6	0.669	1.016
		0.35	65.9	38.5	37.7	0.585	1.022
		0.40	61.0	29.6	28.8	0.486	1.029
	11	0.05	95.1	90.4	90.3	0.951	1.001
		0.10	90.1	80.8	80.7	0.897	1.002
		0.15	85.2	71.4	71.0	0.838	1.005
		0.20	80.3	61.9	61.3	0.772	1.009
		0.25	75.3	52.5	51.7	0.697	1.015
		0.30	70.4	42.8	42.0	0.608	1.019
		0.35	65.5	33.3	32.4	0.508	1.028
		0.40	60.5	23.4	22.7	0.387	1.032
0.700	3	0.05	96.1	94.3	94.0	0.981	1.002
		0.10	92.1	88.6	88.1	0.962	1.006
		0.15	88.2	82.8	82.1	0.939	1.009
		0.20	84.3	77.2	76.2	0.915	1.013
		0.25	80.4	71.2	70.2	0.886	1.014
		0.30	76.4	65.7	64.3	0.859	1.022
		0.35	72.5	59.8	58.3	0.825	1.026
		0.40	68.6	53.9	52.4	0.786	1.029
	7	0.05	96.1	93.1	92.9	0.969	1.002
		0.10	92.2	86.3	85.8	0.936	1.005
		0.15	88.3	79.5	78.7	0.900	1.009
		0.20	84.4	72.6	71.6	0.860	1.013
		0.25	80.5	65.8	64.5	0.818	1.020
		0.30	76.6	59.0	57.5	0.770	1.026
		0.35	72.7	52.2	50.4	0.718	1.036
		0.40	68.8	45.4	43.3	0.659	1.049
	11	0.05	96.2	92.6	92.4	0.962	1.002
		0.10	92.4	85.1	84.7	0.922	1.005
		0.15	88.5	77.8	77.1	0.879	1.010
		0.20	84.7	70.4	69.4	0.831	1.014
		0.25	80.9	63.1	61.8	0.780	1.021
		0.30	77.1	55.8	54.2	0.725	1.031
		0.35	73.2	48.2	46.5	0.658	1.036
		0.40	69.4	40.7	38.9	0.586	1.046

Table 3: Tenth percentile strengths and ratios of tenth percentile strengths for a truncated normal distribution. Based on 100,000 trials.

ρ	Number of members	COV	y_{sngl} = Single member	y_{assem} = Load sharing	y_{mink} = Weakest link	$R_{\text{ADS}} = y_{\text{assem}}/y_{\text{sngl}}$	$R_{\text{ADM}} = y_{\text{assem}}/y_{\text{mink}}$
0.900	3	0.05	97.9	97.0	96.5	0.991	1.005
		0.10	95.7	94.0	93.0	0.982	1.011
		0.15	93.6	91.0	89.6	0.972	1.016
		0.20	91.4	88.0	86.1	0.962	1.022
		0.25	89.3	85.0	82.6	0.951	1.029
		0.30	87.2	81.9	79.1	0.940	1.036
		0.35	85.0	78.9	75.6	0.928	1.044
		0.40	82.9	75.9	72.2	0.915	1.052
	7	0.05	97.8	96.3	95.7	0.984	1.007
		0.10	95.7	92.6	91.3	0.968	1.014
		0.15	93.5	88.9	87.0	0.950	1.022
		0.20	91.4	85.2	82.7	0.932	1.031
		0.25	89.2	81.6	78.3	0.914	1.042
		0.30	87.1	77.8	74.0	0.893	1.052
		0.35	84.9	73.9	69.6	0.870	1.062
		0.40	82.8	69.9	65.3	0.844	1.071
	11	0.05	97.9	96.0	95.2	0.981	1.008
		0.10	95.8	92.0	90.4	0.961	1.017
		0.15	93.7	88.0	85.7	0.939	1.027
		0.20	91.6	84.0	80.9	0.917	1.039
		0.25	89.5	79.8	76.1	0.892	1.049
		0.30	87.3	75.7	71.3	0.866	1.061
		0.35	85.2	71.5	66.5	0.838	1.074
		0.40	83.1	67.2	61.7	0.809	1.089
1.000	3	0.05	99.2	100.2	98.9	1.009	1.013
		0.10	98.5	100.3	97.8	1.018	1.026
		0.15	97.7	100.5	96.7	1.028	1.039
		0.20	97.0	100.6	95.6	1.037	1.052
		0.25	96.2	100.7	94.5	1.046	1.066
		0.30	95.5	100.9	93.4	1.057	1.081
		0.35	94.7	101.1	92.3	1.067	1.095
		0.40	94.0	101.2	91.2	1.076	1.109
	7	0.05	99.3	100.6	98.8	1.013	1.018
		0.10	98.5	101.2	97.7	1.027	1.036
		0.15	97.8	101.8	96.5	1.041	1.055
		0.20	97.0	102.4	95.3	1.055	1.074
		0.25	96.3	103.0	94.1	1.070	1.094
		0.30	95.5	103.5	93.0	1.084	1.113
		0.35	94.8	104.1	91.8	1.099	1.134
		0.40	94.0	104.7	90.6	1.114	1.156
	11	0.05	99.3	100.7	98.8	1.015	1.019
		0.10	98.6	101.5	97.7	1.030	1.039
		0.15	97.8	102.2	96.5	1.045	1.060
		0.20	97.1	103.0	95.3	1.060	1.080
		0.25	96.4	103.7	94.1	1.076	1.102
		0.30	95.7	104.5	93.0	1.092	1.124
		0.35	94.9	105.2	91.8	1.108	1.146
		0.40	94.2	105.9	90.6	1.124	1.169

Table 3 continued: Tenth percentile strengths and ratios of tenth percentile strengths for a truncated normal distribution. Based on 100,000 trials.

ρ	Number of members	COV	$y_{\text{sngl}} =$ Single member	$y_{\text{assem}} =$ Load sharing	$y_{\text{mink}} =$ Weakest link	$R_{\text{ADS}} = y_{\text{assem}}/y_{\text{sngl}}$	$R_{\text{ADM}} = y_{\text{assem}}/y_{\text{mink}}$
0.500	3	0.05	90.9	89.4	89.4	0.984	1.000
		0.10	82.6	79.9	80.0	0.967	0.999
		0.15	75.1	71.6	71.6	0.953	0.999
		0.20	68.4	64.1	64.2	0.937	0.998
		0.25	62.4	57.9	57.7	0.928	1.004
		0.30	57.0	52.2	51.9	0.915	1.005
		0.35	52.1	46.8	46.8	0.898	1.001
		0.40	47.8	42.4	42.3	0.886	1.002
	7	0.05	90.8	88.4	88.4	0.973	1.000
		0.10	82.6	78.3	78.2	0.948	1.001
		0.15	75.1	69.3	69.3	0.923	1.001
		0.20	68.4	61.6	61.4	0.900	1.002
		0.25	62.3	54.7	54.6	0.878	1.002
		0.30	56.9	48.6	48.6	0.855	1.001
		0.35	52.1	43.5	43.3	0.836	1.004
		0.40	47.7	39.3	38.8	0.825	1.015
	11	0.05	90.9	87.8	87.9	0.966	0.999
		0.10	82.7	77.1	77.4	0.933	0.997
		0.15	75.2	67.9	68.1	0.903	0.997
		0.20	68.5	60.0	60.1	0.875	0.999
		0.25	62.5	52.8	53.1	0.846	0.996
		0.30	57.1	46.8	47.0	0.819	0.996
		0.35	52.3	41.3	41.7	0.791	0.992
		0.40	47.9	37.1	37.1	0.774	1.001
0.700	3	0.05	92.6	91.5	91.2	0.988	1.003
		0.10	85.8	83.7	83.3	0.976	1.005
		0.15	79.5	76.9	76.0	0.967	1.011
		0.20	73.7	70.5	69.5	0.956	1.014
		0.25	68.4	64.8	63.6	0.947	1.019
		0.30	63.6	59.6	58.3	0.937	1.022
		0.35	59.2	55.0	53.6	0.928	1.026
		0.40	55.2	50.2	49.3	0.910	1.020
	7	0.05	92.6	90.8	90.3	0.980	1.005
		0.10	85.8	82.4	81.7	0.960	1.009
		0.15	79.6	74.9	73.9	0.942	1.015
		0.20	73.8	68.1	66.9	0.922	1.018
		0.25	68.6	62.4	60.6	0.909	1.028
		0.30	63.8	56.5	55.1	0.885	1.025
		0.35	59.4	52.0	50.1	0.875	1.037
		0.40	55.4	47.6	45.7	0.859	1.041
	11	0.05	92.5	90.1	89.7	0.974	1.005
		0.10	85.6	81.2	80.4	0.948	1.009
		0.15	79.2	73.2	72.2	0.924	1.014
		0.20	73.4	66.2	64.9	0.902	1.020
		0.25	68.1	60.1	58.4	0.883	1.029
		0.30	63.3	54.3	52.7	0.859	1.031
		0.35	58.8	49.5	47.6	0.842	1.040
		0.40	54.8	44.9	43.1	0.820	1.042

Table 4: First percentile strengths and ratios of first percentile strengths for a truncated lognormal distribution. Based on 100,000 trials.

ρ	Number of members	COV	y_{sngl} = Single member	y_{assem} = Load sharing	y_{mink} = Weakest link	$R_{ADS} = y_{assem}/y_{sngl}$	$R_{ADM} = y_{assem}/y_{mink}$
0.900	3	0.05	95.3	95.0	94.5	0.996	1.006
		0.10	90.9	90.3	89.2	0.993	1.012
		0.15	86.7	85.9	84.3	0.990	1.018
		0.20	82.8	81.8	79.8	0.989	1.026
		0.25	79.0	78.1	75.5	0.988	1.034
		0.30	75.5	74.2	71.5	0.982	1.037
		0.35	72.3	70.8	67.8	0.980	1.044
		0.40	69.2	67.6	64.4	0.976	1.049
	7	0.05	95.4	94.7	93.8	0.993	1.009
		0.10	91.0	89.6	88.0	0.985	1.018
		0.15	86.8	85.1	82.7	0.980	1.029
		0.20	82.8	80.6	77.7	0.974	1.038
		0.25	79.1	76.6	73.0	0.968	1.049
		0.30	75.6	72.7	68.7	0.962	1.058
		0.35	72.4	69.2	64.8	0.956	1.068
		0.40	69.3	65.9	61.2	0.950	1.077
	11	0.05	95.5	94.6	93.8	0.990	1.008
		0.10	91.2	89.5	88.0	0.981	1.017
		0.15	87.2	84.7	82.5	0.972	1.027
		0.20	83.3	80.3	77.5	0.964	1.036
		0.25	79.7	76.1	72.8	0.955	1.045
		0.30	76.3	72.1	68.5	0.945	1.052
		0.35	73.1	68.8	64.6	0.941	1.065
		0.40	70.1	65.4	60.9	0.933	1.074
1.000	3	0.05	98.8	99.4	98.8	1.006	1.007
		0.10	97.6	98.8	97.5	1.012	1.013
		0.15	96.4	98.2	96.3	1.018	1.019
		0.20	95.3	97.6	95.2	1.025	1.026
		0.25	94.2	97.0	94.0	1.030	1.031
		0.30	93.1	96.4	92.9	1.035	1.037
		0.35	92.1	95.9	91.8	1.042	1.044
		0.40	91.1	95.2	90.8	1.046	1.049
	7	0.05	98.8	100.0	98.7	1.012	1.013
		0.10	97.6	100.0	97.5	1.025	1.026
		0.15	96.4	100.0	96.3	1.038	1.039
		0.20	95.2	100.0	95.1	1.050	1.052
		0.25	94.1	100.0	93.9	1.063	1.065
		0.30	93.0	100.0	92.8	1.075	1.078
		0.35	92.0	100.0	91.7	1.087	1.091
		0.40	91.0	99.9	90.7	1.098	1.102
	11	0.05	98.8	100.2	98.7	1.015	1.015
		0.10	97.6	100.5	97.5	1.030	1.031
		0.15	96.4	100.7	96.2	1.045	1.047
		0.20	95.2	101.0	95.0	1.061	1.063
		0.25	94.1	101.3	93.9	1.076	1.079
		0.30	93.0	101.5	92.7	1.091	1.094
		0.35	92.0	101.7	91.6	1.106	1.109
		0.40	90.9	101.9	90.6	1.121	1.125

Table 4 continued: First percentile strengths and ratios of first percentile strengths for a truncated lognormal distribution. Based on 100,000 trials.

ρ	Number of members	COV	$y_{\text{sngl}} =$ Single member	$y_{\text{assem}} =$ Load sharing	$y_{\text{mink}} =$ Weakest link	$R_{\text{ADS}} = y_{\text{assem}}/y_{\text{sngl}}$	$R_{\text{ADM}} = y_{\text{assem}}/y_{\text{mink}}$
0.500	3	0.05	93.7	91.7	91.7	0.979	1.000
		0.10	87.8	84.1	84.1	0.958	1.000
		0.15	82.3	77.2	77.2	0.939	1.000
		0.20	77.2	70.9	70.9	0.919	1.000
		0.25	72.5	65.3	65.2	0.901	1.001
		0.30	68.1	60.4	60.1	0.886	1.004
		0.35	64.1	55.6	55.5	0.867	1.002
		0.40	60.4	51.3	51.3	0.849	1.001
	7	0.05	93.7	90.5	90.4	0.967	1.002
		0.10	87.7	82.0	81.7	0.935	1.004
		0.15	82.2	74.3	73.9	0.904	1.006
		0.20	77.1	67.5	66.9	0.875	1.008
		0.25	72.4	61.4	60.7	0.848	1.011
		0.30	68.0	55.9	55.2	0.821	1.013
		0.35	64.0	51.0	50.2	0.797	1.016
		0.40	60.3	46.8	45.8	0.776	1.022
	11	0.05	93.7	89.7	89.7	0.958	1.001
		0.10	87.8	80.5	80.4	0.917	1.001
		0.15	82.3	72.4	72.2	0.879	1.002
		0.20	77.2	65.1	64.9	0.843	1.003
		0.25	72.5	58.8	58.4	0.810	1.006
		0.30	68.2	53.1	52.7	0.779	1.008
		0.35	64.2	48.0	47.6	0.748	1.009
		0.40	60.5	43.6	43.1	0.720	1.010
0.700	3	0.05	95.0	93.5	93.2	0.984	1.002
		0.10	90.3	87.4	87.0	0.968	1.005
		0.15	85.8	81.8	81.2	0.952	1.008
		0.20	81.7	76.6	75.8	0.938	1.011
		0.25	77.7	71.8	70.8	0.924	1.014
		0.30	74.0	67.4	66.3	0.910	1.017
		0.35	70.6	63.5	62.1	0.899	1.022
		0.40	67.4	59.7	58.3	0.886	1.024
	7	0.05	94.9	92.5	92.2	0.974	1.003
		0.10	90.1	85.6	85.1	0.950	1.006
		0.15	85.6	79.3	78.5	0.927	1.010
		0.20	81.3	73.4	72.6	0.903	1.012
		0.25	77.3	68.3	67.1	0.884	1.018
		0.30	73.6	63.4	62.2	0.862	1.020
		0.35	70.1	59.1	57.7	0.842	1.024
		0.40	66.9	55.3	53.6	0.826	1.031
	11	0.05	94.8	91.8	91.6	0.968	1.003
		0.10	90.0	84.3	83.9	0.937	1.006
		0.15	85.4	77.6	76.9	0.909	1.010
		0.20	81.1	71.4	70.5	0.880	1.012
		0.25	77.1	65.8	64.8	0.854	1.016
		0.30	73.3	60.7	59.6	0.829	1.019
		0.35	69.8	56.2	54.9	0.806	1.024
		0.40	66.5	52.1	50.7	0.783	1.027

Table 5: Fifth percentile strengths and ratios of fifth percentile strengths for a truncated lognormal distribution. Based on 100,000 trials.

ρ	Number of members	COV	$y_{\text{sngl}} =$ Single member	$y_{\text{assem}} =$ Load sharing	$y_{\text{mink}} =$ Weakest link	$R_{\text{ADS}} = y_{\text{assem}}/y_{\text{sngl}}$	$R_{\text{ADM}} = y_{\text{assem}}/y_{\text{mink}}$
0.900	3	0.05	97.0	96.3	95.9	0.993	1.005
		0.10	94.1	92.9	92.0	0.986	1.010
		0.15	91.3	89.5	88.2	0.980	1.015
		0.20	88.7	86.4	84.7	0.974	1.020
		0.25	86.1	83.4	81.3	0.968	1.026
		0.30	83.7	80.5	78.1	0.961	1.030
		0.35	81.4	77.8	75.1	0.957	1.036
		0.40	79.2	75.3	72.3	0.951	1.041
	7	0.05	97.0	95.8	95.1	0.987	1.007
		0.10	94.1	91.9	90.5	0.976	1.015
		0.15	91.4	88.1	86.1	0.964	1.023
		0.20	88.7	84.5	82.0	0.953	1.031
		0.25	86.2	81.2	78.1	0.942	1.040
		0.30	83.7	77.9	74.5	0.930	1.046
		0.35	81.4	75.0	71.1	0.921	1.056
		0.40	79.2	72.2	67.9	0.911	1.063
	11	0.05	97.0	95.5	94.7	0.984	1.008
		0.10	94.1	91.1	89.7	0.969	1.017
		0.15	91.3	87.1	84.9	0.954	1.025
		0.20	88.6	83.3	80.5	0.940	1.034
		0.25	86.0	79.6	76.4	0.925	1.042
		0.30	83.6	76.2	72.5	0.912	1.051
		0.35	81.2	73.1	68.9	0.899	1.060
		0.40	79.0	70.2	65.6	0.889	1.070
1.000	3	0.05	99.0	99.9	98.8	1.009	1.011
		0.10	98.0	99.7	97.7	1.018	1.021
		0.15	97.0	99.6	96.5	1.027	1.032
		0.20	96.1	99.5	95.4	1.036	1.043
		0.25	95.2	99.4	94.3	1.044	1.053
		0.30	94.3	99.2	93.3	1.053	1.064
		0.35	93.4	99.1	92.3	1.061	1.074
		0.40	92.5	98.9	91.3	1.069	1.084
	7	0.05	99.0	100.4	98.8	1.014	1.016
		0.10	98.0	100.7	97.6	1.028	1.032
		0.15	97.0	101.1	96.4	1.042	1.049
		0.20	96.0	101.4	95.2	1.056	1.065
		0.25	95.1	101.8	94.1	1.071	1.082
		0.30	94.2	102.2	93.0	1.085	1.099
		0.35	93.3	102.5	91.9	1.099	1.115
		0.40	92.4	102.9	90.9	1.113	1.132
	11	0.05	99.0	100.6	98.8	1.016	1.018
		0.10	98.0	101.1	97.5	1.032	1.037
		0.15	97.0	101.7	96.3	1.048	1.055
		0.20	96.0	102.2	95.1	1.065	1.074
		0.25	95.0	102.8	94.0	1.081	1.093
		0.30	94.1	103.3	92.9	1.098	1.112
		0.35	93.2	103.8	91.8	1.114	1.131
		0.40	92.4	104.4	90.8	1.130	1.150

Table 5 continued: Fifth percentile strengths and ratios of fifth percentile strengths for a truncated lognormal distribution. Based on 100,000 trials.

ρ	Number of members	COV	$y_{\text{sngl}} =$ Single member	$y_{\text{assem}} =$ Load sharing	$y_{\text{mink}} =$ Weakest link	$R_{\text{ADS}} = y_{\text{assem}}/y_{\text{sngl}}$	$R_{\text{ADM}} = y_{\text{assem}}/y_{\text{mink}}$
0.500	3	0.05	95.2	93.0	92.9	0.977	1.001
		0.10	90.7	86.5	86.4	0.954	1.001
		0.15	86.4	80.5	80.4	0.933	1.002
		0.20	82.3	75.0	74.8	0.911	1.003
		0.25	78.5	70.0	69.7	0.892	1.004
		0.30	74.9	65.4	65.1	0.872	1.005
		0.35	71.6	61.2	60.8	0.854	1.006
		0.40	68.5	57.3	56.9	0.837	1.007
	7	0.05	95.2	91.6	91.5	0.963	1.001
		0.10	90.6	84.0	83.7	0.927	1.004
		0.15	86.3	77.0	76.7	0.892	1.004
		0.20	82.2	70.7	70.3	0.860	1.006
		0.25	78.4	65.2	64.5	0.831	1.010
		0.30	74.8	60.0	59.3	0.802	1.012
		0.35	71.5	55.3	54.6	0.774	1.013
		0.40	68.4	51.1	50.4	0.748	1.015
	11	0.05	95.1	90.8	90.7	0.955	1.000
		0.10	90.5	82.5	82.4	0.912	1.002
		0.15	86.1	75.0	74.8	0.870	1.002
		0.20	82.0	68.4	68.1	0.834	1.005
		0.25	78.1	62.3	62.0	0.798	1.005
		0.30	74.5	56.9	56.5	0.764	1.006
		0.35	71.1	52.1	51.7	0.733	1.010
		0.40	67.9	47.9	47.3	0.705	1.012
0.700	3	0.05	96.3	94.5	94.4	0.982	1.001
		0.10	92.7	89.4	89.1	0.965	1.003
		0.15	89.2	84.5	84.2	0.947	1.004
		0.20	85.9	80.0	79.6	0.930	1.005
		0.25	82.8	75.9	75.3	0.916	1.008
		0.30	79.9	72.1	71.3	0.902	1.012
		0.35	77.1	68.4	67.5	0.887	1.013
		0.40	74.5	65.1	64.1	0.873	1.015
	7	0.05	96.2	93.3	93.0	0.970	1.003
		0.10	92.5	87.1	86.6	0.941	1.006
		0.15	89.1	81.3	80.6	0.913	1.009
		0.20	85.7	76.0	75.1	0.887	1.012
		0.25	82.6	71.2	70.1	0.862	1.017
		0.30	79.6	66.8	65.4	0.839	1.021
		0.35	76.8	62.7	61.2	0.816	1.025
		0.40	74.1	58.9	57.3	0.794	1.028
	11	0.05	96.2	92.8	92.4	0.965	1.004
		0.10	92.6	86.2	85.5	0.931	1.008
		0.15	89.1	80.1	79.1	0.898	1.012
		0.20	85.8	74.5	73.2	0.868	1.017
		0.25	82.7	69.4	67.9	0.839	1.022
		0.30	79.7	64.6	63.0	0.811	1.026
		0.35	76.9	60.4	58.6	0.785	1.030
		0.40	74.3	56.4	54.5	0.759	1.034

Table 6: Tenth percentile strengths and ratios of tenth percentile strengths for a truncated lognormal distribution. Based on 100,000 trials.

ρ	Number of members	COV	$y_{\text{sngl}} =$ Single member	$y_{\text{assem}} =$ Load sharing	$y_{\text{mink}} =$ Weakest link	$R_{\text{ADS}} =$ $y_{\text{assem}}/y_{\text{sngl}}$	$R_{\text{ADM}} =$ $y_{\text{assem}}/y_{\text{mink}}$
		0.05	97.9	97.1	96.6	0.991	1.005
		0.10	95.9	94.3	93.4	0.983	1.010
		0.15	93.9	91.6	90.2	0.976	1.016
		0.20	92.0	89.0	87.2	0.967	1.020
	3	0.25	90.2	86.5	84.4	0.959	1.025
		0.30	88.4	84.1	81.7	0.952	1.030
		0.35	86.7	82.0	79.1	0.946	1.036
		0.40	85.1	80.0	76.7	0.940	1.043
		0.05	97.9	96.4	95.8	0.985	1.007
		0.10	95.9	93.1	91.8	0.971	1.014
		0.15	93.9	89.8	88.0	0.956	1.021
		0.20	92.0	86.8	84.3	0.943	1.029
0.900	7	0.25	90.2	83.9	80.9	0.930	1.036
		0.30	88.4	81.1	77.7	0.917	1.044
		0.35	86.7	78.4	74.7	0.904	1.050
		0.40	85.1	76.2	71.8	0.895	1.061
		0.05	97.9	96.1	95.4	0.982	1.008
		0.10	95.9	92.4	91.0	0.964	1.016
		0.15	93.9	88.9	86.8	0.948	1.025
		0.20	91.9	85.6	82.9	0.931	1.033
	11	0.25	90.1	82.4	79.2	0.915	1.041
		0.30	88.3	79.5	75.7	0.900	1.050
		0.35	86.6	76.7	72.4	0.886	1.059
		0.40	84.9	74.2	69.4	0.873	1.069
		0.05	99.3	100.2	98.9	1.009	1.013
		0.10	98.5	100.3	97.9	1.018	1.026
		0.15	97.8	100.5	96.8	1.028	1.038
		0.20	97.1	100.7	95.8	1.037	1.051
	3	0.25	96.4	100.9	94.8	1.046	1.064
		0.30	95.7	101.1	93.8	1.056	1.077
		0.35	95.1	101.3	92.9	1.066	1.091
		0.40	94.4	101.4	92.0	1.074	1.103
		0.05	99.2	100.6	98.8	1.013	1.018
		0.10	98.5	101.1	97.6	1.027	1.036
		0.15	97.7	101.7	96.4	1.040	1.054
		0.20	97.0	102.3	95.3	1.054	1.073
1.000	7	0.25	96.3	102.8	94.2	1.068	1.092
		0.30	95.6	103.4	93.1	1.081	1.110
		0.35	94.9	103.9	92.1	1.095	1.128
		0.40	94.3	104.5	91.1	1.108	1.147
		0.05	99.3	100.7	98.8	1.015	1.020
		0.10	98.5	101.5	97.6	1.030	1.040
		0.15	97.8	102.2	96.5	1.045	1.060
		0.20	97.1	103.0	95.3	1.060	1.080
	11	0.25	96.4	103.7	94.2	1.075	1.100
		0.30	95.8	104.4	93.2	1.090	1.121
		0.35	95.1	105.2	92.1	1.106	1.141
		0.40	94.5	105.9	91.1	1.121	1.162

Table 6 continued: Tenth percentile strengths and ratios of tenth percentile strengths for a truncated lognormal distribution. Based on 100,000 trials.

ρ	Number of members	COV	$y_{\text{sngl}} =$ Single member	$y_{\text{assem}} =$ Load sharing	$y_{\text{mink}} =$ Weakest link	$R_{\text{ADS}} = y_{\text{assem}}/y_{\text{sngl}}$	$R_{\text{ADM}} = y_{\text{assem}}/y_{\text{mink}}$
0.500	3	0.05	88.3	85.6	85.5	0.970	1.002
		0.10	77.4	72.8	72.4	0.941	1.005
		0.15	67.4	61.7	60.9	0.915	1.013
		0.20	58.4	51.4	50.9	0.880	1.011
		0.25	50.4	43.3	42.2	0.860	1.027
		0.30	43.2	35.7	34.8	0.826	1.026
		0.35	37.0	29.1	28.6	0.788	1.020
		0.40	31.5	24.2	23.4	0.767	1.034
	7	0.05	88.0	83.4	83.1	0.948	1.005
		0.10	76.9	68.9	68.3	0.896	1.009
		0.15	66.8	56.7	55.7	0.849	1.019
		0.20	57.7	46.1	45.0	0.800	1.026
		0.25	49.6	37.2	36.1	0.750	1.031
		0.30	42.4	29.8	28.7	0.704	1.038
		0.35	36.1	23.7	22.7	0.657	1.043
		0.40	30.6	18.9	17.9	0.617	1.055
	11	0.05	88.0	82.4	81.9	0.936	1.006
		0.10	76.9	67.1	66.3	0.873	1.011
		0.15	66.8	54.3	53.2	0.814	1.022
		0.20	57.6	43.4	42.3	0.753	1.026
		0.25	49.5	34.4	33.3	0.694	1.031
		0.30	42.3	27.1	26.1	0.640	1.039
		0.35	36.0	21.4	20.3	0.595	1.056
		0.40	30.6	16.7	15.7	0.545	1.062
0.700	3	0.05	90.7	89.0	88.4	0.981	1.007
		0.10	81.9	78.7	77.6	0.961	1.015
		0.15	73.6	69.0	67.7	0.938	1.019
		0.20	65.8	60.4	58.8	0.917	1.027
		0.25	58.6	52.2	50.7	0.891	1.029
		0.30	52.1	45.1	43.6	0.866	1.034
		0.35	46.1	38.9	37.3	0.843	1.040
		0.40	40.7	33.6	31.9	0.827	1.055
	7	0.05	90.8	87.2	86.6	0.960	1.006
		0.10	82.0	75.9	74.4	0.925	1.019
		0.15	73.7	64.9	63.5	0.880	1.021
		0.20	65.9	55.4	53.8	0.840	1.029
		0.25	58.8	47.2	45.4	0.803	1.040
		0.30	52.2	40.2	38.1	0.771	1.057
		0.35	46.2	33.6	31.8	0.726	1.057
		0.40	40.8	28.2	26.4	0.690	1.066
	11	0.05	91.0	86.5	85.9	0.951	1.007
		0.10	82.3	74.5	73.2	0.905	1.018
		0.15	74.2	63.7	61.9	0.859	1.029
		0.20	66.6	54.0	52.0	0.811	1.037
		0.25	59.5	45.4	43.4	0.764	1.046
		0.30	53.0	38.1	36.1	0.720	1.058
		0.35	47.0	31.9	29.8	0.678	1.071
		0.40	41.7	26.9	24.5	0.645	1.096

Table 7: First percentile strengths and ratios of first percentile strengths for a truncated Weibull distribution. Based on 100,000 trials.

ρ	Number of members	COV	$y_{\text{sngl}} =$ Single member	$y_{\text{assem}} =$ Load sharing	$y_{\text{mink}} =$ Weakest link	$R_{\text{ADS}} =$ $y_{\text{assem}}/y_{\text{sngl}}$	$R_{\text{ADM}} =$ $y_{\text{assem}}/y_{\text{mink}}$
0.900	3	0.05	95.0	94.5	93.9	0.995	1.007
		0.10	89.9	89.1	87.8	0.991	1.015
		0.15	84.9	83.7	81.9	0.985	1.022
		0.20	80.0	78.3	76.2	0.979	1.028
		0.25	75.2	73.1	70.6	0.972	1.035
		0.30	70.6	68.3	65.4	0.968	1.046
		0.35	66.2	63.5	60.4	0.960	1.053
		0.40	61.9	58.8	55.7	0.950	1.057
	7	0.05	95.0	94.0	92.9	0.990	1.012
		0.10	90.0	88.0	86.0	0.978	1.024
		0.15	85.0	82.2	79.3	0.967	1.038
		0.20	80.1	76.6	72.8	0.955	1.051
		0.25	75.4	70.9	66.7	0.940	1.062
		0.30	70.8	66.1	61.0	0.933	1.083
		0.35	66.4	61.1	55.6	0.920	1.098
		0.40	62.1	55.7	50.6	0.897	1.102
	11	0.05	94.9	93.4	92.2	0.985	1.013
		0.10	89.8	86.9	84.7	0.967	1.025
		0.15	84.8	80.6	77.5	0.951	1.040
		0.20	79.8	74.4	70.6	0.932	1.053
		0.25	75.0	68.7	64.2	0.915	1.070
		0.30	70.3	63.1	58.1	0.897	1.085
		0.35	65.8	57.6	52.5	0.875	1.098
		0.40	61.6	53.0	47.3	0.861	1.119
1.000	3	0.05	98.8	99.4	98.8	1.006	1.006
		0.10	97.6	98.7	97.5	1.012	1.013
		0.15	96.3	98.1	96.2	1.019	1.020
		0.20	95.0	97.5	94.8	1.026	1.028
		0.25	93.6	96.7	93.5	1.033	1.035
		0.30	92.3	95.9	92.1	1.040	1.042
		0.35	90.9	95.2	90.7	1.048	1.051
		0.40	89.5	94.5	89.2	1.056	1.059
	7	0.05	98.8	99.7	98.8	1.009	1.009
		0.10	97.6	99.6	97.6	1.020	1.020
		0.15	96.4	99.6	96.3	1.033	1.034
		0.20	95.1	99.7	94.9	1.049	1.050
		0.25	93.8	99.9	93.6	1.065	1.068
		0.30	92.5	99.8	92.2	1.080	1.083
		0.35	91.1	99.7	90.8	1.094	1.098
		0.40	89.8	99.6	89.4	1.109	1.113
	11	0.05	98.8	99.8	98.8	1.010	1.010
		0.10	97.6	99.9	97.6	1.023	1.024
		0.15	96.4	100.1	96.3	1.039	1.040
		0.20	95.1	100.6	94.9	1.057	1.059
		0.25	93.8	101.1	93.6	1.078	1.081
		0.30	92.5	101.4	92.2	1.096	1.100
		0.35	91.2	101.6	90.8	1.114	1.118
		0.40	89.8	101.6	89.4	1.131	1.136

Table 7 continued: First percentile strengths and ratios of first percentile strengths for a truncated Weibull distribution. Based on 100,000 trials.

ρ	Number of members	COV	$y_{\text{sngl}} =$ Single member	$y_{\text{assem}} =$ Load sharing	$y_{\text{mink}} =$ Weakest link	$R_{\text{ADS}} = y_{\text{assem}}/y_{\text{sngl}}$	$R_{\text{ADM}} = y_{\text{assem}}/y_{\text{mink}}$
0.500	3	0.05	92.6	89.5	89.4	0.967	1.001
		0.10	85.3	79.8	79.5	0.935	1.004
		0.15	78.3	70.5	70.2	0.899	1.004
		0.20	71.7	62.1	61.7	0.866	1.005
		0.25	65.4	54.8	54.0	0.837	1.013
		0.30	59.5	47.5	47.1	0.799	1.009
		0.35	54.0	41.4	40.9	0.766	1.011
		0.40	48.9	36.0	35.5	0.736	1.014
	7	0.05	92.6	87.4	87.2	0.944	1.002
		0.10	85.3	75.9	75.5	0.889	1.005
		0.15	78.3	65.3	64.9	0.834	1.007
		0.20	71.7	56.1	55.5	0.783	1.012
		0.25	65.4	47.8	47.1	0.732	1.015
		0.30	59.5	40.6	39.8	0.682	1.019
		0.35	54.0	34.2	33.5	0.634	1.020
		0.40	48.9	29.2	28.1	0.598	1.038
	11	0.05	92.6	86.1	86.0	0.930	1.001
		0.10	85.3	73.6	73.3	0.862	1.003
		0.15	78.4	62.3	62.1	0.795	1.003
		0.20	71.7	52.7	52.2	0.734	1.009
		0.25	65.4	44.3	43.6	0.677	1.016
		0.30	59.5	36.8	36.3	0.618	1.014
		0.35	54.0	30.4	30.0	0.563	1.014
		0.40	48.9	25.3	24.7	0.516	1.022
0.700	3	0.05	94.3	92.2	91.9	0.978	1.003
		0.10	88.7	84.6	84.1	0.954	1.006
		0.15	83.2	77.5	76.6	0.931	1.011
		0.20	77.8	70.5	69.6	0.906	1.013
		0.25	72.6	63.9	62.9	0.881	1.016
		0.30	67.6	57.9	56.8	0.858	1.021
		0.35	62.8	52.4	51.1	0.835	1.027
		0.40	58.3	47.0	45.8	0.807	1.027
	7	0.05	94.4	90.6	90.1	0.960	1.005
		0.10	88.8	81.8	80.8	0.921	1.012
		0.15	83.3	73.3	72.1	0.880	1.018
		0.20	78.0	65.6	64.0	0.841	1.026
		0.25	72.8	58.4	56.5	0.803	1.033
		0.30	67.8	51.8	49.8	0.763	1.040
		0.35	63.1	45.7	43.7	0.725	1.046
		0.40	58.6	40.2	38.3	0.687	1.052
	11	0.05	94.4	89.9	89.4	0.952	1.005
		0.10	88.8	80.3	79.5	0.905	1.010
		0.15	83.3	71.4	70.3	0.858	1.016
		0.20	77.9	63.1	61.8	0.810	1.020
		0.25	72.7	55.5	54.1	0.763	1.025
		0.30	67.7	48.8	47.2	0.721	1.034
		0.35	62.9	42.8	41.0	0.679	1.042
		0.40	58.4	37.2	35.6	0.636	1.045

Table 8: Fifth percentile strengths and ratios of fifth percentile strengths for a truncated Weibull distribution. Based on 100,000 trials.

ρ	Number of members	COV	$y_{\text{sngl}} =$ Single member	$y_{\text{assem}} =$ Load sharing	$y_{\text{mink}} =$ Weakest link	$R_{\text{ADS}} = y_{\text{assem}}/y_{\text{sngl}}$	$R_{\text{ADM}} = y_{\text{assem}}/y_{\text{mink}}$
		0.05	96.9	96.0	95.5	0.991	1.006
		0.10	93.7	92.0	91.0	0.982	1.012
		0.15	90.5	88.1	86.4	0.974	1.019
		0.20	87.2	84.0	82.0	0.963	1.025
	3	0.25	84.0	80.1	77.6	0.954	1.033
		0.30	80.8	76.2	73.3	0.943	1.039
		0.35	77.7	72.5	69.2	0.934	1.048
		0.40	74.6	68.9	65.2	0.924	1.056
		0.05	97.0	95.5	94.5	0.984	1.010
		0.10	93.9	90.9	89.1	0.968	1.021
		0.15	90.7	86.3	83.7	0.952	1.032
		0.20	87.6	81.8	78.4	0.934	1.043
0.900	7	0.25	84.4	77.4	73.3	0.917	1.055
		0.30	81.3	73.2	68.4	0.900	1.069
		0.35	78.2	68.9	63.7	0.881	1.080
		0.40	75.2	64.8	59.3	0.863	1.094
		0.05	97.0	95.1	94.2	0.981	1.010
		0.10	93.9	90.2	88.4	0.961	1.021
		0.15	90.7	85.3	82.7	0.940	1.031
		0.20	87.5	80.6	77.2	0.921	1.045
	11	0.25	84.4	75.8	71.8	0.899	1.056
		0.30	81.2	71.2	66.7	0.877	1.068
		0.35	78.2	66.9	61.8	0.857	1.082
		0.40	75.1	62.7	57.2	0.834	1.094
		0.05	99.1	99.8	98.9	1.007	1.009
		0.10	98.1	99.6	97.8	1.016	1.019
		0.15	97.1	99.5	96.6	1.025	1.031
		0.20	96.1	99.5	95.4	1.036	1.043
	3	0.25	95.0	99.3	94.1	1.045	1.055
		0.30	93.9	99.1	92.8	1.055	1.068
		0.35	92.8	98.9	91.6	1.066	1.081
		0.40	91.7	98.7	90.3	1.076	1.094
		0.05	99.1	100.0	98.8	1.009	1.011
		0.10	98.1	100.2	97.6	1.021	1.026
		0.15	97.1	100.5	96.4	1.036	1.043
		0.20	96.0	101.1	95.1	1.052	1.062
1.000	7	0.25	94.9	101.7	93.8	1.071	1.084
		0.30	93.8	102.1	92.5	1.088	1.104
		0.35	92.7	102.4	91.1	1.104	1.123
		0.40	91.6	102.5	89.8	1.119	1.142
		0.05	99.0	100.0	98.8	1.010	1.013
		0.10	98.0	100.3	97.6	1.024	1.028
		0.15	97.0	100.9	96.3	1.040	1.048
		0.20	95.9	101.6	95.0	1.060	1.070
	11	0.25	94.8	102.6	93.6	1.082	1.095
		0.30	93.7	103.2	92.3	1.101	1.118
		0.35	92.6	103.6	90.9	1.119	1.140
		0.40	91.4	104.1	89.5	1.138	1.163

Table 8 continued: Fifth percentile strengths and ratios of fifth percentile strengths for a truncated Weibull distribution. Based on 100,000 trials.

ρ	Number of members	COV	y_{sngl} = Single member	y_{assem} = Load sharing	y_{mink} = Weakest link	$R_{ADS} = y_{assem}/y_{sngl}$	$R_{ADM} = y_{assem}/y_{mink}$
0.500	3	0.05	94.6	91.6	91.5	0.968	1.001
		0.10	89.3	83.6	83.4	0.936	1.002
		0.15	84.0	75.9	75.7	0.904	1.003
		0.20	78.8	68.7	68.4	0.872	1.005
		0.25	73.8	61.9	61.5	0.838	1.005
		0.30	69.0	55.6	55.2	0.806	1.006
		0.35	64.3	50.1	49.4	0.778	1.013
		0.40	59.9	44.8	44.1	0.748	1.016
	7	0.05	94.6	89.3	89.1	0.944	1.002
		0.10	89.2	79.3	78.9	0.889	1.006
		0.15	83.9	70.1	69.4	0.835	1.009
		0.20	78.8	61.6	60.8	0.782	1.013
		0.25	73.7	53.9	53.0	0.731	1.016
		0.30	68.9	46.7	46.0	0.679	1.016
		0.35	64.2	40.6	39.8	0.632	1.020
		0.40	59.8	35.1	34.3	0.586	1.022
	11	0.05	94.6	88.3	88.1	0.933	1.002
		0.10	89.3	77.5	77.1	0.868	1.006
		0.15	84.1	67.5	67.0	0.803	1.007
		0.20	78.9	58.6	58.0	0.743	1.012
		0.25	73.9	50.5	49.9	0.683	1.012
		0.30	69.1	43.4	42.7	0.629	1.018
		0.35	64.5	37.2	36.4	0.577	1.022
		0.40	60.1	31.8	31.0	0.530	1.029
0.700	3	0.05	96.0	93.7	93.5	0.976	1.002
		0.10	91.9	87.4	87.1	0.950	1.004
		0.15	87.9	81.4	80.8	0.927	1.008
		0.20	83.9	75.5	74.8	0.900	1.009
		0.25	79.9	69.9	69.0	0.875	1.013
		0.30	76.0	64.6	63.5	0.850	1.017
		0.35	72.2	59.6	58.4	0.825	1.021
		0.40	68.5	54.8	53.5	0.801	1.025
	7	0.05	96.1	92.1	91.8	0.959	1.003
		0.10	92.1	84.5	84.0	0.917	1.007
		0.15	88.2	77.2	76.4	0.876	1.010
		0.20	84.2	70.2	69.3	0.834	1.013
		0.25	80.3	63.8	62.6	0.794	1.018
		0.30	76.5	57.7	56.4	0.754	1.022
		0.35	72.7	52.1	50.7	0.717	1.028
		0.40	69.1	46.8	45.5	0.678	1.031
	11	0.05	96.0	91.3	90.9	0.951	1.004
		0.10	92.0	82.9	82.2	0.901	1.008
		0.15	88.0	75.2	74.0	0.854	1.015
		0.20	84.0	67.6	66.4	0.805	1.018
		0.25	80.0	60.7	59.3	0.758	1.024
		0.30	76.1	54.4	52.7	0.714	1.030
		0.35	72.3	48.6	46.8	0.671	1.038
		0.40	68.7	43.3	41.4	0.630	1.045

Table 9: Tenth percentile strengths and ratios of tenth percentile strengths for a truncated Weibull distribution. Based on 100,000 trials.

ρ	Number of members	COV	$y_{\rm sngl}$ = Single member	$y_{\rm assem}$ = Load sharing	$y_{\rm mink}$ = Weakest link	$R_{\rm ADS} = y_{\rm assem}/y_{\rm sngl}$	$R_{\rm ADM} = y_{\rm assem}/y_{\rm mink}$
0.900	3	0.05	97.9	96.9	96.4	0.990	1.006
0.900	3	0.10	95.8	93.8	92.8	0.979	1.011
0.900	3	0.15	93.6	90.7	89.1	0.969	1.018
0.900	3	0.20	91.4	87.6	85.4	0.959	1.026
0.900	3	0.25	89.1	84.4	81.8	0.947	1.033
0.900	3	0.30	86.9	81.3	78.2	0.935	1.040
0.900	3	0.35	84.6	78.2	74.7	0.924	1.047
0.900	3	0.40	82.4	75.0	71.2	0.911	1.053
0.900	7	0.05	97.9	96.1	95.4	0.982	1.008
0.900	7	0.10	95.7	92.2	90.7	0.964	1.017
0.900	7	0.15	93.4	88.3	86.1	0.945	1.026
0.900	7	0.20	91.1	84.3	81.5	0.925	1.035
0.900	7	0.25	88.8	80.5	77.0	0.906	1.045
0.900	7	0.30	86.5	76.7	72.6	0.886	1.056
0.900	7	0.35	84.2	72.9	68.4	0.866	1.066
0.900	7	0.40	81.9	69.4	64.4	0.847	1.078
0.900	11	0.05	97.9	95.8	94.9	0.978	1.010
0.900	11	0.10	95.8	91.6	89.8	0.956	1.020
0.900	11	0.15	93.6	87.4	84.7	0.933	1.031
0.900	11	0.20	91.4	83.3	79.8	0.911	1.044
0.900	11	0.25	89.2	79.1	74.9	0.887	1.056
0.900	11	0.30	86.9	75.1	70.3	0.864	1.068
0.900	11	0.35	84.7	71.2	65.8	0.842	1.083
0.900	11	0.40	82.4	67.5	61.5	0.819	1.097
1.000	3	0.05	99.3	100.0	99.0	1.007	1.010
1.000	3	0.10	98.5	100.1	97.9	1.015	1.022
1.000	3	0.15	97.8	100.3	96.8	1.025	1.036
1.000	3	0.20	97.0	100.5	95.6	1.036	1.051
1.000	3	0.25	96.1	100.7	94.4	1.048	1.067
1.000	3	0.30	95.3	100.8	93.2	1.058	1.081
1.000	3	0.35	94.4	100.8	92.0	1.068	1.096
1.000	3	0.40	93.6	100.9	90.8	1.078	1.111
1.000	7	0.05	99.3	100.1	98.9	1.008	1.013
1.000	7	0.10	98.6	100.5	97.7	1.019	1.028
1.000	7	0.15	97.8	101.0	96.5	1.033	1.047
1.000	7	0.20	97.0	101.8	95.3	1.049	1.068
1.000	7	0.25	96.2	102.7	94.0	1.067	1.092
1.000	7	0.30	95.4	103.2	92.7	1.083	1.114
1.000	7	0.35	94.5	103.7	91.4	1.097	1.134
1.000	7	0.40	93.7	104.1	90.1	1.112	1.156
1.000	11	0.05	99.3	100.2	98.8	1.009	1.014
1.000	11	0.10	98.5	100.6	97.6	1.021	1.031
1.000	11	0.15	97.8	101.3	96.4	1.036	1.051
1.000	11	0.20	97.0	102.1	95.1	1.053	1.074
1.000	11	0.25	96.1	103.3	93.7	1.074	1.102
1.000	11	0.30	95.3	104.0	92.4	1.092	1.126
1.000	11	0.35	94.4	104.6	91.0	1.108	1.149
1.000	11	0.40	93.6	105.2	89.7	1.124	1.173

Table 9 continued: Tenth percentile strengths and ratios of Tenth percentile strengths for a truncated Weibull distribution. Based on 100,000 trials.

COV	Distribution	Nominal Correlation			
		0.5	0.7	0.9	1.0
.05	Normal	.500	.700	.900	1.000
	Lognormal	.499	.700	.899	.999
	Weibull	.489	.685	.881	.979
.10	Normal	.500	.700	.900	1.000
	Lognormal	.497	.698	.898	.998
	Weibull	.494	.690	.888	.986
.15	Normal	.499	.699	.900	1.000
	Lognormal	.498	.695	.895	.994
	Weibull	.498	.695	.893	.992
.20	Normal	.500	.699	.900	1.000
	Lognormal	.495	.693	.891	.990
	Weibull	.498	.697	.897	.996
.25	Normal	.500	.699	.900	1.000
	Lognormal	.492	.689	.886	.985
	Weibull	.497	.699	.899	.999
.30	Normal	.500	.700	.900	1.000
	Lognormal	.489	.685	.881	.979
	Weibull	.498	.699	.899	.999
.35	Normal	.499	.701	.900	1.000
	Lognormal	.485	.680	.874	.971
	Weibull	.499	.699	.899	.999
.40	Normal	.500	.700	.900	1.000
	Lognormal	.482	.675	.867	.963
	Weibull	.498	.698	.897	.997

Table 10: Sample correlations between MOE and MOR. Based on 1,000,000 trials.