Method for Adjusting Warp Measurements to a Different Board Dimension

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Abstract

Warp in lumber is a common problem that occurs while lumber is being dried. In research or other testing programs, it is sometimes necessary to compare warp of different species or warp caused by different process variables. If lumber dimensions are not the same, then direct comparisons are not possible, and adjusting warp to a common dimension would be desirable so that these comparisons are possible. In this report, two methods of adjusting warp to different dimensions are developed—one based on the geometry of bow and crook and another based on the geometry of twist. These methods provide a way to adjust crook and bow measurements to a different length and to adjust twist to a different length or width so that comparisons are possible.

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Method for Adjusting Warp Measurements to a Different Board Dimension

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Introduction

For many years, numerous researchers have conducted studies investigating warp (crook, bow, and twist) that occurs in lumber during drying. Their studies have generally focused on characterizing the amount of warp that occurs in a particular species, as well as identifying processing methods that might reduce the amount of warp that occurs. These studies have usually been done in a laboratory setting, and the boards used in any one study have typically all been the same dimensions. However, there are potential occasions where it is not realistic for all the study boards to be the same length and width. For example, in studies conducted at a lumber mill under typical production conditions, it is likely that boards of different lengths and widths will be produced. The purpose of such a study might be to compare warp in different species or to determine if certain processing variables affect the amount of warp, but the amount of warp in boards of different lengths and widths cannot be compared directly because the amount of warp is dependent on board length or width. The purpose of this report is to present a way to adjust measurements of crook and bow to a different board length and also to adjust twist to a different board length or width so that they can be compared.

Crook and Bow

Crook and bow can be adjusted for length in the same way. No adjustment for width is necessary. The basic assumption is that crook and bow follow the arc of a circle. This assumption is necessary to develop the equations used in the adjustment. If localized grain deviations, like large knots, are common on board edges and faces, this assumption may not be correct.

The equations for length adjustment for crook and bow are developed from Figure 1. Two circles are shown, and the arcs $a_1$ and $a_2$ represent boards of different lengths. For the purpose of developing the equations, we will take $a_1$ to be the length of the board whose crook or bow we know, and $a_2$ to be the length of the board to which we want to adjust the crook or bow. Likewise, crook or bow of these two boards is represented by $w_1$ and $w_2$, respectively. The radius $r_1 = r_2 = r$.

From triangle ABC, we can write the following equations:

\[ w_1 = r(1 - \cos \frac{\theta_1}{2}) \quad (1) \]

and

\[ w_2 = r(1 - \cos \frac{\theta_2}{2}) \quad (1a) \]

Since

\[ r = \frac{a_1}{\theta_1} = \frac{a_2}{\theta_2}, \]

\[ w_1 = \frac{a_1}{\theta_1} (1 - \cos \frac{\theta_1}{2}) \quad (2) \]

and

\[ w_2 = \frac{a_2}{\theta_2} (1 - \cos \frac{\theta_2}{2}) \quad (2a) \]

The value of $\theta_1$ is unknown but can be determined from Equation (2). Equation (2) cannot be solved explicitly for $\theta_1$ but can quickly be solved by trial and error in a spreadsheet, for example, by inserting different $\theta_1$ values until the calculated value of $w_1$ equals the measured value of crook or bow.

We want to determine the value of $w_2$, which is the length-adjusted crook or bow. Equations (1) and (1a) can be solved for $r$, equated, and then solved for the length-adjusted crook or bow, $w_2$. 
The value of $\theta_1$ is not known, but the second equation that can be written from the equality of $r_1 = r_2 = r$ is derived from the equality

$$r = \frac{a_1}{\theta_1} = \frac{a_2}{\theta_2}$$

and therefore,

$$\theta_2 = \theta_1 \frac{a_2}{a_1} \tag{4}$$

Substituting Equation (4) into Equation (3) results in the final equation for calculating the length-adjusted crook or bow:

$$w_2 = w_1 \left(1 - \cos \left(\frac{\theta_1 a_2}{2a_1} \right) \right) \left(1 - \cos \frac{\theta_1}{2} \right) \tag{5}$$

Equation (5) can be simplified using the Taylor expansion for $\cos(\theta)$:

$$\cos(\theta) = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \ldots \tag{6}$$

for small values of $\theta$,

$$\cos(\theta) = 1 - \frac{\theta^2}{2} \tag{7}$$

Substituting Equation (7) into Equation (5),

$$w_2 = w_1 \left(\frac{a_2}{a_1} \right)^2 \tag{8}$$

The error caused by the Equation (7) approximation is illustrated in Figure 2, where the measured bow or crook $w_1$ is plotted against the error in the adjusted value $w_2$, that is, the difference between the $w_2$ values calculated by Equations (5) and (8). Figure 2 includes the errors in adjusting from $a_1 = 8$ ft (2.4 m) to $a_2 = 12$, 16, and 24 ft (3.7, 4.9, and 7.3 m). The errors are insignificant in adjusting from 8 to 12 ft (2.4 to 3.7 m) but may become significant at $w_1$ values of 2 to 3 in. (51 to 76 mm) in adjusting from 8 to 16 ft (2.4 to 4.9 m) and at $w_1$ values of 1 to 2 in. (25.4 to 51 mm) in adjusting from 8 to 24 ft (2.4 to 7.3 m). Adjustments in bow or crook to shorter lengths, that is when $a_2 < a_1$, do not become significant within their practical range. Figure 3 shows adjustment of bow and crook measurements on 8-ft- (2.4-m-) long boards to 10- to 16-ft- (3.1- to 4.9-m-) long boards (in 2-ft (0.6-m) increments). The exact calculation was made with Equation (5).

Equation 8 can also be used to adjust cup for different board widths. However, caution should be used in interpreting the results because of the possible effect of board width itself on warp; that is, the unit cup of a board may be width dependent.

**Twist**

Twist can be adjusted for both length and width with the assumption that the twisted board axis can be approximated by a straight line (Fig. 4a and 4b). From Figure 4a and 4b, we can write the following equations:
Figure 3—Adjustment of bow or crook measured on 8-ft-long boards to 10-, 12-, 14-, and 16-ft-long boards (1 in. = 25.4 mm; 1 ft = 0.3048 m).

For length adjustment,

\[
\sin \alpha = \frac{t_1}{a_1} = \frac{t_2}{a_2} \tag{9a}
\]

and for width adjustment,

\[
\sin \alpha = \frac{t_1}{w_1} = \frac{t_2}{w_2} \tag{9b}
\]

where \( \alpha \) is the angle of twist, \( t_1 \) and \( t_2 \) the amount of twist, \( a_1 \) and \( a_2 \) board lengths, and \( w_1 \) and \( w_2 \) board widths.

From Equation (9),

\[
t_2 = t_1 \frac{a_2}{a_1} \tag{10a}
\]

and

\[
t_2 = t_1 \frac{w_2}{w_1} \tag{10b}
\]

which adjust twist for board length and width.

Figure 4—Relationship between twist and (a) board length and (b) board width.