TORSIONAL BUCKLING OF LONGITUDINALLY STIFFENED, THIN-WALLED, PLYWOOD CYLINDERS

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TORSIONAL BUCKLING OF LONGITUDINALLY STIFFENED, THIN-WALLED PLYWOOD CYLINDERS

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Summary

The results of torsion tests performed at the Forest Products Laboratory upon stiffened, thin-walled, plywood cylinders showed that the buckling stress of the portion of the shell between the stiffeners is about 85 percent of the buckling stress of an unstiffened cylinder of the same curvature and thickness. The cylinders tested were about 0.04 inch thick, were curved to a 9-inch radius, and had from 11 to 28 longitudinal stiffeners from 1/32 to 1/2 inch thick glued to the inner surface. The theoretical buckling torque of an unstiffened cylinder of the same weight and curvature as a stiffened cylinder was found to be greater than that for the stiffened cylinder.

Introduction

At the time this work was begun (July 1943), it was thought that more information on the behavior of longitudinally stiffened shells was needed for the design of aircraft structures. Previous work at the Forest Products Laboratory on unstiffened shells showed that plywood is particularly useful in the experimental determination of the buckling stress of shell structures since it is smooth and can be made thicker than metal shells of the same size and still exhibit buckling. Accordingly, plywood was used in the series of specimens included in the work covered by this report.

A theoretical analysis applicable to a stiffened structure should include the behavior of plates supported at the edges by stiffeners that are not rigid but that can bend and twist; but, as an analysis of this sort is exceedingly complicated,[2] it is not included in the work covered by this report,

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[1]This progress report is one of a series prepared and distributed by the Forest Products Laboratory under U. S. Navy, Bureau of Aeronautics No. NBA-PO-NAer00565 and Army Air Force No. AAF-PO-(33-038)46-1189. Results here reported are preliminary and may be revised as additional data become available.

[2]Timoshenko, S., "Theory of Elastic Stability," p. 337, 1936.
Chwalla, E., Ingenieur-Archiv. vol. 5, p. 54, 1934.

Description of Specimens

The specimens consisted of cylinders of yellow birch plywood having longitudinal yellow birch stiffeners glued to their inner surfaces. The shell and the stiffener sizes are given in table 1. The plywood was of four plies of 1/100-inch yellow birch veneer, in which the grain direction of face plies war circumferential and the grain direction of core plies was axial; it was thus equivalent to a three-ply, 1:2:1 construction. From eleven to twenty-eight longitudinal stiffeners of different widths and of thicknesses from 1/32 to 1/2 inch were glued to the inner surface of each cylinder. The method of manufacture employed was identical to that described in Forest Products Laboratory Report No. 1562.[3] This work included tests on 320 plywood cylinders. Coupons of the plywood and the stiffeners were also prepared and tested as described in that report.

Methods of Testing

The ends of the specimens were fitted with maple plugs 3 inches thick. The plugs were of such diameter that they fitted inside the ring of stiffeners, and the spaces between the stiffeners were filled with shims the same thickness as the stiffeners. The circumference of the plug was covered with a thin, rubberized, glass cloth to increase the friction between the plug and specimen. A slight taper on the plug insured a tight fit. After the plugs were placed, steel straps were clamped around the outside of the specimen. A couple was applied through cross arms attached to each end plug as shown in figure 1. The couple was slowly increased until failure occurred.

Computation of Results

The stresses in the plywood shell were computed by the formula

$$\tau = \frac{2R_3T}{\pi(R_3^4 - R_2^4) + \frac{nb}{R_1 + R_2}(R_2^4 - R_1^4)} \qquad (1)$$

where T = torque
R_3 = outside radius of curvature of shell
R_2 = inside radius of curvature of shell
R_1 = inside radius of curvature of stiffener
n = number of stiffeners
b = width of stiffeners

[3] Forest Products Laboratory Report No. 1662, "Longitudinally Stiffened Thin-walled Plywood Cylinders in Axial Compression." In press.

This formula is derived as follows:

Figure 2 shows the notation in which x, r, and θ are cylindrical coordinates.

From figure 2

$$\gamma = \phi r$$

If μ is the modulus of rigidity, then

$$\tau = \mu \frac{\gamma}{l} = \mu \frac{\phi r}{l}$$

where 7 is the shear stress.

The energy of a differential volume dV is

$$d\epsilon_i = 1/2\, \mu \frac{\phi^2 r^3}{l^2}\, dr\, d\theta\, dx .$$

Then the total internal energy is

$$\epsilon_i = 1/2\mu_c \frac{\phi^2}{l^2} \int_0^l \int_0^{2\pi} \int_{R_2}^{R_3} r^3 dr\, d\theta\, dx + 1/2\mu_s \frac{\phi^2}{l^2} \int_0^l \int_0^{\frac{2nb}{R_1+R_2}} \int_{R_1}^{R_2} r^3 dr\, d\theta\, dx$$

where μ_c and μ_s are the moduli of rigidity of the cylindrical shell and stiffener, respectively.

Integrating,

$$\epsilon_i = 1/4 \frac{\phi^2}{l} \left[\pi \mu_c (R_3^4 - R_2^4) + \frac{nb\mu_s}{R_1+R_2} (R_2^4 - R_1^4) \right] .$$

The external energy is

$$\epsilon_e = 1/2\, T\phi$$

where T is the torque,

Equating the external energy to the internal energy

$$\epsilon_e = \epsilon_i$$

or

$$1/2\ T\phi = 1/4\ \frac{\phi^2}{l}\left[\pi\mu_c(R_3^4 - R_2^4) + \frac{nb\mu_s}{R_1 + R_2}(R_2^4 - R_1^4)\right].$$

Using the relationship $\tau_c = \mu_c \frac{\phi R_3}{l}$

and solving for τ_c

$$\tau_c = \frac{2\mu_c TR_3}{\pi\mu_c(R_3^4 - R_2^4) + \frac{nb\,\mu_s}{R_1 + R_2}(R_2^4 - R_1^4)}$$

Now if it is assumed that $\mu_c = \mu_s$

then

$$\tau_c = \frac{2TR_3}{\pi(R_3^4 - R_2^4) + \frac{nb}{R_1 + R_2}(R_2^4 - R_1^4)}$$

which is equation (1). This equation is approximate because of the assumption that the shear strain in the cylinder and in the stiffeners is directly proportional to the coordinate r and independent of coordinate ϕ, which is not strictly true.

Previous work on the buckling of unstiffened thin-walled plywood cylinders in torsion is presented in Forest Products Laboratory Report No. 1529.[4] The buckling stress can be calculated by the formula

$$\tau_{cr} = k\ E_L \frac{h}{r}$$

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[4] Forest Products Laboratory Report No. 1529, "Buckling of Thin-walled Plywood Cylinders in Torsion." 1945.

where E_L is the modulus of elasticity in the direction of the grain of the veneer of the plywood, k is a coefficient depending on the size and construction of the cylinder, h is the thickness of the cylinder wall, and r is the mean radius of the cylindrical shell of plywood. Values of E_L were computed from data obtained from tests of coupons of the plywood.

For this report, values of k were taken from the curves of figure 3. These curves were computed by the method of Forest Products Laboratory Report No. 1529, for a three-ply plywood, having the face plies half as thick as the core and the grain direction of the face plies circumferential.

In order to compare a stiffened cylinder with an unstiffened one of the same weight and curvature but greater thickness, the buckling torque of such an equivalent cylinder was computed by the formula

$$T_{equiv.} = 2\pi k_e E_L r h_e^2$$

where $h_e = h + \frac{nbd}{2\pi r}$ and k_e was determined for a $J_e = \frac{l^2}{h_e r}$

Presentation of Results

The results of tests are presented in tabular and graphical form.

Table 1 gives the sizes of the specimens, the buckling torques, and the stresses in the plywood, It also contains the moduli of elasticity, as determined from the test data of coupons, and the computed buckling stresses. The ratios of $T_{equiv.}/T$ are tabulated beside values of $T_{equiv.}$

The graph of figure 4 shows the experimental buckling stresses in the plywood plotted against theoretical values for an unstiffened cylinder of the same dinensions and plywood construction.

Discussion of Results

All the specimens failed by buckling in the space between the stiffeners. The thinner stiffeners allowed the buckles to grow across the stiffener. Most of the specimens failed immediately at the time of buckling. No additional load greater than that required to cause buckling was necessary to promote failure. The failures were usually explosive in nature, causing the plywood to be broken into small pieces and separating the stiffeners from the cylinder if they were too rigid to buckle.

A comparison of the experimental with the theoretical buckling stresses, as shown in figure 4, indicates that the experimental buckling stresses are about 15 percent lower than the computed values. No satisfactory explanation has been found for this discrepancy. The scatter of the points is typical of data pertaining to the buckling of cylinders.

The buckling torques of stiffened cylinders and those of unstiffened cylinders of the same weight may be compared by referring to the column headed $T_{equiv.}/T$ in table 1. The equivalent cylinder is stronger for all the specimens, except one, and the ratios $\frac{T_{equiv.}}{T}$ range from 0.984 to 2.539.

Conclusions

The torsional buckling stress of the curved shell between the stiffeners of a stiffened plywood cylinder is about 85 percent of that Of an unstiffened cylinder of the same dimensions and plywood construction.

The theoretical buckling torque of an unstiffened cylinder of the same weight and curvature as a stiffened cylinder is greater than or equal to that for the stiffened cylinder.

Table 1.--Tests of stiffened plywood cylinders in torsion[1]

Specimen number	Number of stiffeners (n)	Thickness of stiffener (d)	Width of stiffener (b)	Length of cylinder (l)	Thickness of plywood (h)	Radius of curvature (r)	Buckling torque (T)	Shear stress in plywood at buckling (τ_c)	$J = \frac{l^2}{hr}$	Elastic Moduli: Plywood: Circumferential bending modulus of elasticity	Elastic Moduli: Plywood: Axial bending modulus of elasticity	Elastic Moduli: Plywood: Axial compression modulus of elasticity	Elastic Moduli: Plywood: Modulus of elasticity of veneer (E_L)	Elastic Moduli: Stiffener: Modulus of elasticity (E_L)	Buckling constant (k)	Computed buckling stress (τ_{cr})	Buckling torque of an equivalent unstiffened cylinder ($T_{equiv.}$)	$\frac{T_{equiv.}}{T}$
		Inch	Inch	Inches	Inch	Inches	Inch-pound	P.s.i.		1,000 p.s.i.	1,000 p.s.i.	1,000 p.s.i.	1,000 p.s.i.	1,000 p.s.i.		P.s.i.	Inch-pounds	
SCT 1	0	....	....	29	0.037	9.15	10,630	510	2,480	425	1,956	1,189	2,278	....	0.0588	540	....	....
SCT 2	11	0.029	0.246	29	.037	9.15	10,320	470	2,480	428	1,938	1,171	2,264	2,026	.0588	540	11,320	1.096
SCT 3	11	.030	1.000	29	.037	9.15	10,570	440	2,480	404	1,762	1,141	2,073	2,053	.0588	490	13,040	1.234
SCT 4	11	.029	2.007	28	.037	9.15	14,350	530	2,320	433	2,244	1,318	2,562	2,128	.0592	610	21,060	1.466
SCT 5	11	.030	3.010	29	.037	9.15	15,150	510	2,480	446	2,012	1,358	2,352	2,521	.0588	560	24,760	1.634
SCT 6	19	.029	.254	29	.036	9.15	11,810	530	2,550	409	2,135	1,278	2,434	2,026	.0585	560	12,170	1.031
SCT 7	19	.030	1.002	28	.037	9.15	14,840	570	2,320	498	1,900	1,244	2,295	2,053	.0592	550	17,820	1.200
SCT 8	19	.029	2.001	29	.037	9.15	16,330	520	2,480	335	2,258	1,192	2,481	2,128	.0588	590	28,020	1.716
SCT 10	28	.030	.254	28	.037	9.15	11,510	500	2,320	475	2,091	1,163	2,456	2,026	.0592	590	14,060	1.221
SCT 11	28	.030	1.001	28	.038	9.15	15,590	540	2,250	396	2,044	1,222	2,335	2,053	.0597	580	23,180	1.486
SCT 13	0	....	....	17	.037	9.15	12,270	580	850	445	2,217	1,374	2,547	....	.0672	690	....	....
SCT 14	11	.029	.249	18	.038	9.15	13,990	640	930	481	2,282	1,284	2,644	2,026	.0666	730	15,790	1.129
SCT 15	11	.030	1.005	18	.038	9.15	13,650	570	930	417	2,388	1,267	2,684	2,047	.0666	740	20,010	1.466
SCT 16	11	.029	2.006	18	.037	9.15	17,180	640	960	518	2,188	1,382	2,589	2,154	.0664	700	23,490	1.368
SCT 17	11	.030	2.965	18	.038	9.15	19,290	650	930	546	2,547	1,462	2,960	1,957	.0666	820	35,880	1.860
SCT 19	19	.029	1.001	17	.036	9.13	14,670	560	880	517	2,263	1,322	2,660	2,047	.0670	700	21,860	1.489
SCT 20	19	.029	2.004	17	.036	9.13	20,140	640	880	511	2,450	1,430	2,833	2,134	.0670	750	34,860	1.730
SCT 22	28	.029	.251	17	.036	9.13	12,970	570	880	539	2,157	1,382	2,580	2,026	.0670	680	15,710	1.211
SCT 23	28	.029	1.000	17	.037	9.14	17,660	620	850	504	2,063	1,349	2,456	2,047	.0672	670	25,740	1.458
SCT 25	0	....	....	12	.036	9.13	13,890	660	440	509	2,333	1,436	2,725	....	.0726	780	....	....
SCT 26	11	.029	.249	12	.036	9.13	15,130	700	440	611	2,433	1,512	2,913	2,026	.0726	830	17,060	1.128
SCT 27	11	.030	1.010	12	.036	9.13	17,740	740	440	564	2,378	1,486	2,815	1,925	.0726	810	20,880	1.176
SCT 28	11	.029	2.010	12	.037	9.14	19,580	730	430	542	2,341	1,548	2,759	2,042	.0728	810	27,640	1.411
SCT 29	11	.029	2.987	12	.036	9.13	19,650	660	440	510	1,941	1,316	2,345	2,121	.0726	670	28,290	1.440
SCT 30	19	.029	.250	12	.037	9.13	16,400	740	430	545	2,368	1,496	2,788	2,026	.0728	820	18,250	1.113
SCT 31	19	.029	1.001	12	.036	9.13	19,080	730	440	948	2,010	1,548	2,831	1,925	.0726	810	25,190	1.320
SCT 32	19	.029	2.006	12	.037	9.13	23,990	770	430	910	2,204	1,439	2,980	2,042	.0728	880	41,400	1.725
SCT 34	28	.029	.249	12	.037	9.13	16,060	700	430	571	2,031	1,567	2,490	2,026	.0728	730	17,290	1.076

Table 1.--Tests of stiffened plywood cylinders in torsion[1] (continued)

Specimen number	Number of stiffeners (n)	Thickness of stiffener (d)	Width of stiffener (b)	Length of cylinder (l)	Thickness of plywood (h)	Radius of curvature (r)	Buckling torque (T)	Shear stress in plywood at buckling (τ_c)	$J = \frac{l^2}{hr}$	Elastic Moduli: Plywood: Circumferential bending modulus of elasticity	Elastic Moduli: Plywood: Axial bending modulus of elasticity	Elastic Moduli: Plywood: Axial compression modulus of elasticity	Elastic Moduli: Plywood: Modulus of elasticity of veneer (E_L)	Elastic Moduli: Stiffener: Modulus of elasticity (E_L)	Buckling constant (k)	Computed buckling stress (τ_{cr})	Buckling torque of an equivalent unstiffened cylinder ($T_{equiv.}$)	$\frac{T_{equiv.}}{T}$
		Inch	Inch	Inches	Inch	Inches	Inch-pounds	P.s.i.		1,000 p.s.i.	1,000 p.s.i.	1,000 p.s.i.	1,000 p.s.i.	1,000 p.s.i.		P.s.i.	Inch-pounds	
SCT 35	28	0.029	1.001	12	0.037	9.13	22,320	780	430	592	2,284	1,597	2,752	1,925	0.0728	810	31,290	1.402
SCT 37	0	....	....	28	.038	9.13	10,930	520	2,260	500	1,794	1,328	2,195	....	.0596	540	....	....
SCT 38	11	.057	.249	28	.037	9.14	11,540	510	2,320	564	1,740	1,407	2,205	2,228	.0592	530	11,950	1.035
SCT 39	11	.056	.998	28	.037	9.13	14,090	520	2,320	487	1,852	1,301	2,238	1,690	.0592	540	17,990	1.276
SCT 40	11	.057	1.994	28	.036	9.13	16,620	510	2,390	525	1,884	1,330	2,305	1,624	.0590	540	27,920	1.680
SCT 41	11	.058	2.990	28	.036	9.14	19,550	500	2,380	474	1,875	1,233	2,248	1,578	.0591	520	40,000	2.046
SCT 42	19	.056	.250	28	.036	9.13	12,890	550	2,390	494	1,840	1,286	2,253	2,228	.0590	520	12,690	.984
SCT 43	19	.057	.992	28	.036	9.13	15,880	510	2,390	500	2,001	1,286	2,393	1,690	.0590	560	25,790	1.625
SCT 44	19	.056	1.999	28	.036	9.13	21,020	510	2,390	531	1,860	1,268	2,288	1,624	.0590	530	45,590	2.169
SCT 46	28	.059	.250	28	.039	9.16	12,420	500	2,190	534	2,466	1,361	2,871	2,228	.0598	730	21,620	1.741
SCT 47	28	.059	1.001	28	.040	9.16	19,440	540	2,140	510	2,430	1,366	2,813	1,690	.0600	740	49,360	2.539
SCT 49	0	....	....	12	.038	9.15	14,560	690	410	511	2,397	1,294	2,783	....	.0732	850	....	....
SCT 50	11	.059	.249	12	.037	9.14	14,780	660	430	446	2,644	1,242	2,957	2,228	.0728	870	19,770	1.338
SCT 51	11	.059	1.003	12	.039	9.15	18,740	690	400	490	2,371	1,393	2,738	1,952	.0735	860	30,110	1.606
SCT 52	11	.059	1.999	12	.038	9.15	20,930	640	410	466	2,693	1,346	3,023	1,564	.0732	920	49,140	2.348
SCT 53	11	.058	3.001	12	.038	9.15	25,550	660	410	495	2,422	1,205	2,791	2,009	.0732	850	63,870	2.500
SCT 54	19	.058	.249	12	.038	9.15	16,180	690	410	444	2,322	1,178	2,647	2,228	.0732	800	20,660	1.276
SCT 55	19	.058	1.001	12	.039	9.16	22,430	720	400	427	2,440	1,206	2,744	1,952	.0735	860	41,080	1.831
SCT 56	19	.059	1.998	12	.039	9.15	33,710	810	400	546	2,440	1,298	2,857	1,564	.0735	900	79,150	2.348
SCT 58	28	.058	.247	12	.039	9.14	16,150	650	400	485	2,364	1,261	2,726	2,228	.0735	850	24,810	1.536
SCT 59	28	.059	1.003	12	.040	9.15	27,770	770	390	404	2,320	1,321	2,607	1,952	.0738	840	55,340	1.993
SCT 61	0	....	....	27	.041	9.15	11,510	550	1,940	406	1,847	1,433	2,156	....	.0610	590	12,710	1.104
SCT 62	19	.376	.246	27	.038	9.14	22,000	600	2,100	392	2,478	1,440	2,746	2,234	.0602	690	48,390	2.200
SCT 63	19	.497	.506	27	.039	9.16	34,400	550	2,040	422	1,887	1,326	2,210	2,344	.0604	570	131,390	3.823
SCT 64	28	.374	.248	27	.041	9.14	28,130	650	1,950	453	2,026	1,214	2,372	2,366	.0608	650	67,410	2.398
SCT 65	28	.494	.498	27	.041	9.15	42,250	530	1,940	338	2,165	1,261	2,395	2,119	.0610	650	255,270	6.044
SCT 66	19	.369	.249	12	.041	9.15	33,150	910	380	423	1,900	1,231	2,223	2,266	.0740	740	51,020	1.540

[1]Each value represents the average of five specimens.

Z M 76136 F

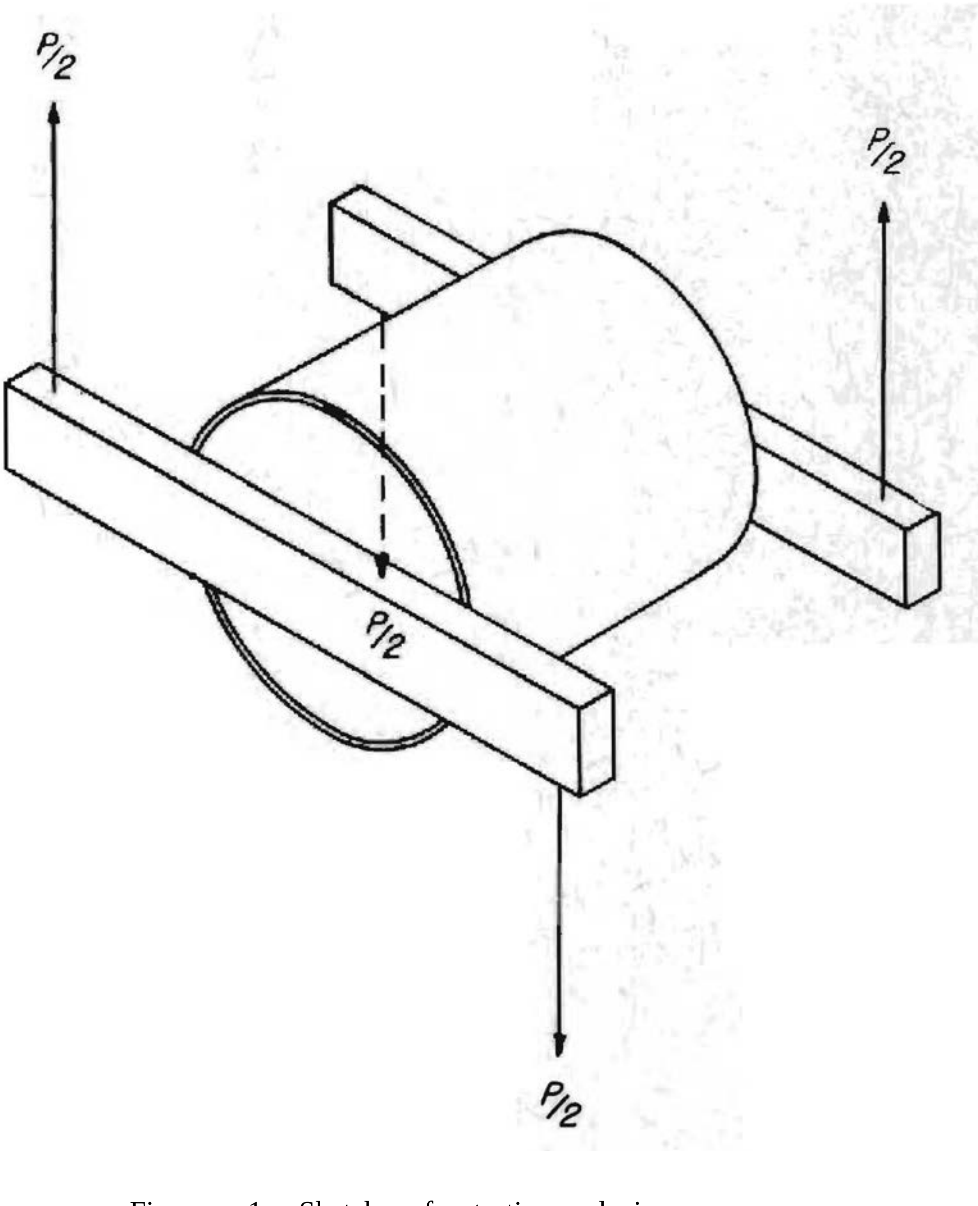

Figure 1.--Sketch of testing device.

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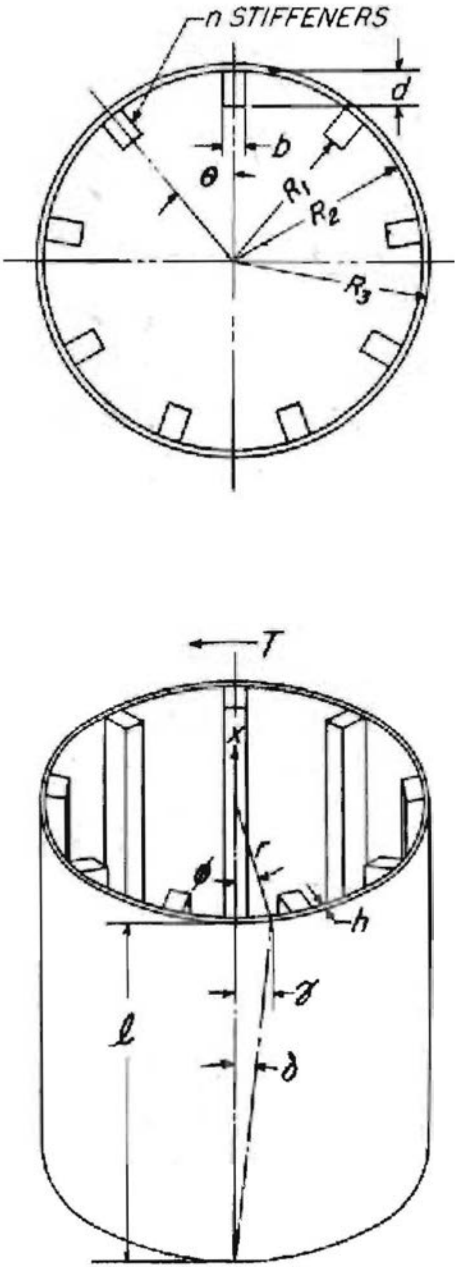

Figure 2.--Notation for stiffened cylinders in torsion.

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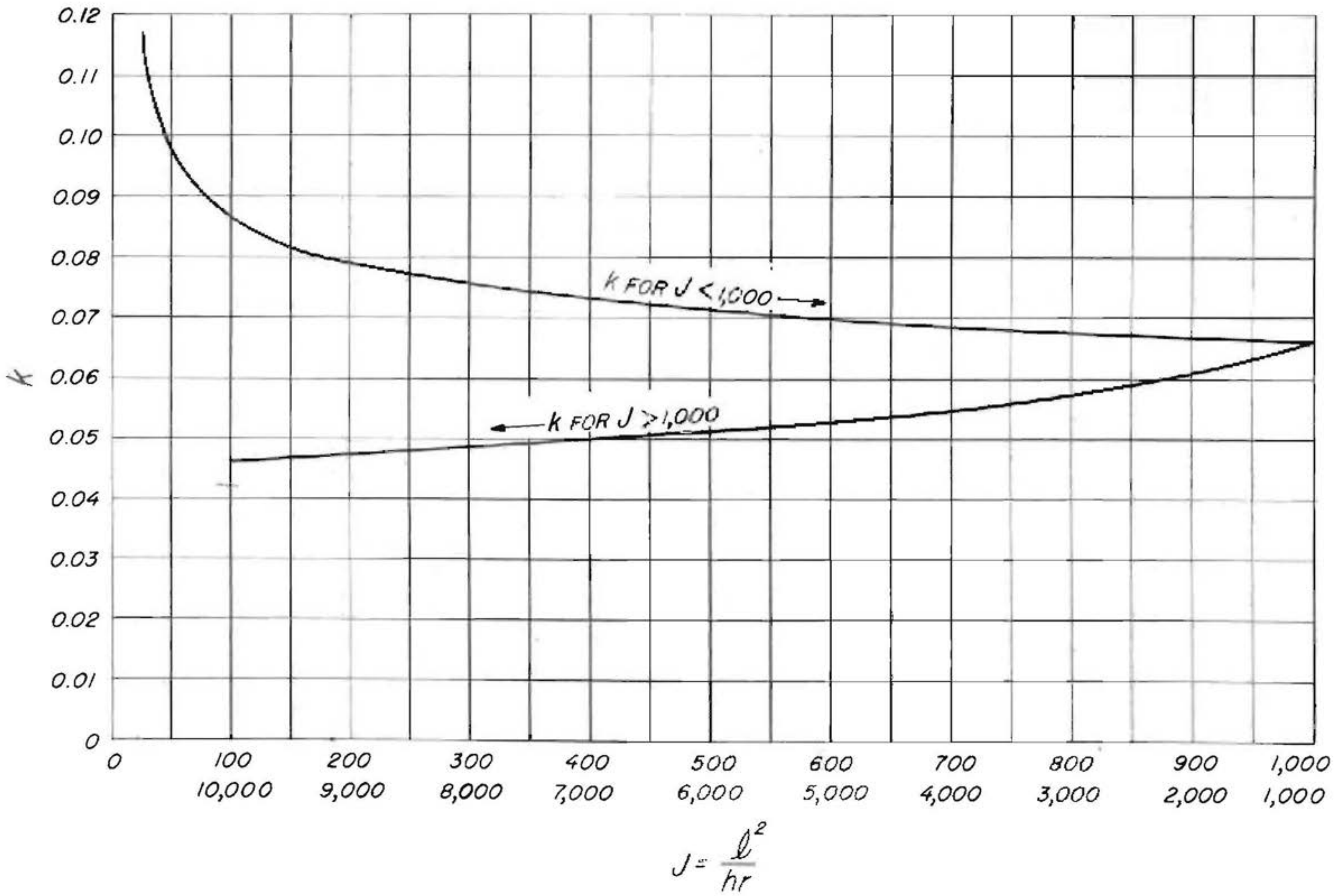

Figure 3.--Values of k for three-ply plywood having the face plies one-half as thick as the core ply and the face-grain direction circumferential.

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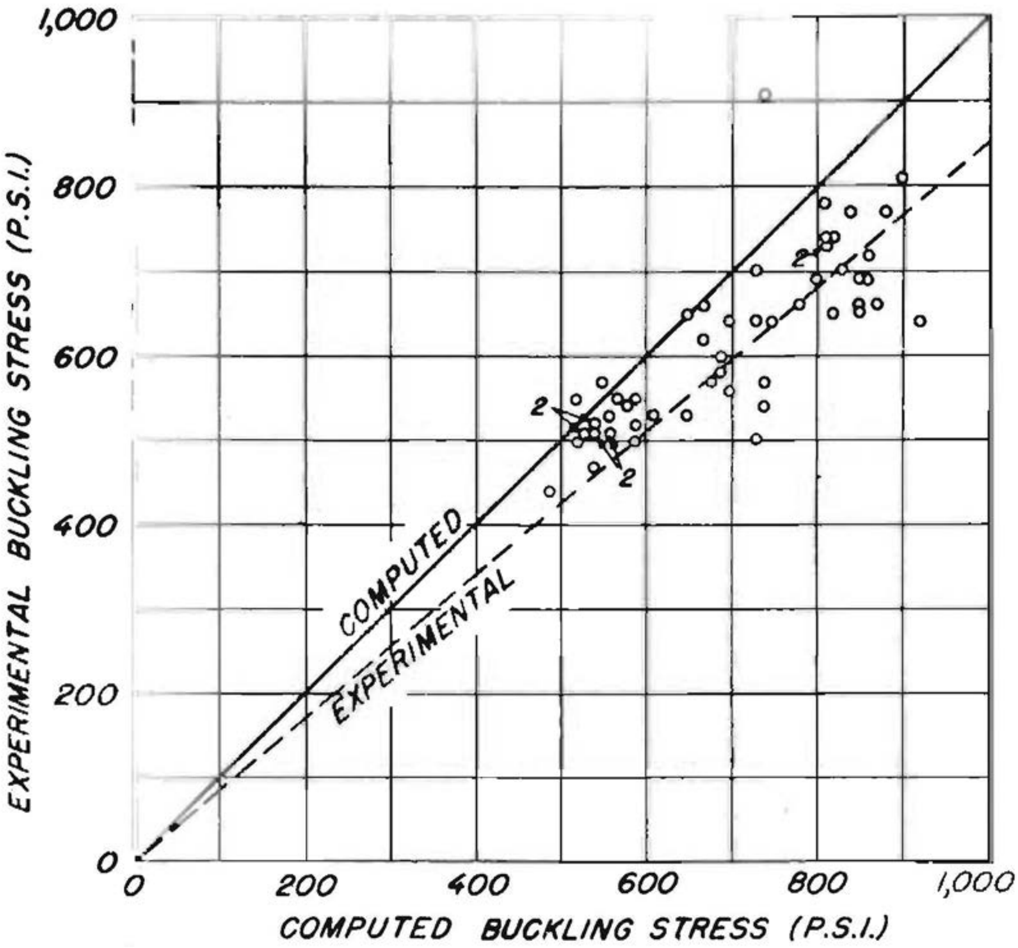

Figure 4.--Experimental vs. computed buckling stresses of stiffened plywood cylinders in torsion.

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