DESIGN CRITERIA FOR LONG CURVED PANELS OF SANDWICH CONSTRUCTION IN AXIAL COMPRESSION

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DESIGN CRITERIA FOR LONG CURVED PANELS OF SANDWICH CONSTRUCTION IN AXIAL COMPRESSION[1]

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Summary

This investigation was conducted at the Forest Products Laboratory to establish design criteria for curved plates of sandwich construction under axially compressive loads.

The axial buckling strength of a well-made, long, curved plate of sandwich material may be computed by adding the critical stress of a complete cylinder, of which the plate may be considered a part, to the critical stress of a flat plate having the same dimensions as the curved plate. The stress at which crimping of the entire sandwich will occur is equal to or greater than the computed critical stress, provided there are no structural defects. The analysis presented includes methods of calculating the critical stresses when the facings are stressed beyond the proportional limit.

Introduction

Attempts have been made to analyze mathematically the behavior of a curved plate under axially compressive loads. No adequate mathematical analysis has yet been developed that enables the designer to calculate the critical loads of curved sections. Available formulas are based upon the assumption that the critical stress of the curved plate is determined by some combination of the complete-cylinder and flat-plate theories. Lundquist[2] stated that the critical stress is equal to either the critical stress of an unstiffened cylinder of the same radius-thickness ratio as that

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[1]This report is one of a series of progress reports prepared by the Forest Products Laboratory. Results here reported are preliminary and may be revised as additional data become available.

[2]Lundquist, Eugene E., "Preliminary Data on Buckling Strength of Curved Sheet Panels in Compression," NACA, November, 1941.

of the curved plate, or the critical stress for the same plate when flat, whichever is the larger. Redshaw[3] arrived at an expression that can be written

$$p_{cr} = \sqrt{p_1^2 + \frac{1}{4} p^2} + \frac{1}{2} p$$

where,

p_{cr} = critical stress of a curved plate

p_1 = critical stress of a complete cylinder of same radius as the curved plate

p = critical stress of a flat plate of the same size as the curved plate

Wenzek[4] presented the empirical relation that the critical stress of the curved plate is equal to the sum of the critical stress of a complete cylinder and that of a flat plate of the same size as the curved plate.

Lundquist, Redshaw, and Wenzek were concerned with plates of solid, isotropic materials. The behavior of a plate of sandwich construction involves the possibilities of crimping (fig. 1) and wrinkling of the facing in addition to the formation of large buckles, the buckling at facing stresses above the proportional limit, and of the reduction of critical stresses due to the low shear modulus of the core.

The object of the work reported herein was to establish design criteria for curved plates of sandwich construction. Formulas are developed for calculating the compressive strength when buckling or crimping failures occur either below or above the proportional limit stress of the facing material.

Development of Formulas

Available theories assume the critical stress of a curved plate to be some combination of the critical stresses of a complete cylinder and a flat plate. The theory of Lundquist[2] gave values that are low compared to those of experimental data in which the computed critical stresses of the equivalent flat plate and cylinder are nearly equal. Values by Redshaw's theory are also too low, compared to such data, although they are higher than those given by Lundquist. The formula that agrees best with the experimental data of this report is that presented by Wenzek.

[3]Redshaw, S. C., "The Elastic Stability of a Thin Curved Panel Subjected to an Axial Thrust, Its Axial and Circumferential Edges Being Simply Supported," R. & M. 1565.

[4]Wenzek, W. A., "The Effective Width of Curved Sheet after Buckling," NACA. Tech. Memo. No. 880, November, 1938.

Notation

The following notation is used:

a = width of plate, measured in the circumferential direction.

b = length of plate, measured in the axial direction.

c = core thickness. As a subscript, "c" refers to the core.

E = Young's modulus of elasticity of the facing material.

E_t = tangent modulus of elasticity of the facing material.

E_a = apparent compressive modulus of elasticity of the sandwich, measured in the axial direction.

E_1 = apparent bending modulus of elasticity of the sandwich, measured in the axial direction.

E_2 = apparent bending modulus of elasticity of the sandwich measured in the circumferential direction.

f = facing thickness. As a subscript "f" refers to the facing.

h = total thickness of the sandwich.

p = mean theoretical buckling stress of a flat plate of width "a" and length "b".

p_1 = mean theoretical buckling stress of a thin-walled cylinder of radius of curvature "r".

p_{cr} = mean theoretical buckling stress of a curved plate.

r = mean radius of curvature.

S = mean compressive strength, over thickness "h", at the compressive strength of the facing material.

S_{pl} = mean compressive stress, over thickness "h", at the proportional limit stress of the facing material.

λ = $(1 - \sigma^2)$, where σ is Poisson's ratio.

According to Wenzek the buckling stress of a curved plate is given by the formula

$$p_{cr} = p_1 + p \qquad (1)$$

where p_1 is the critical stress of a complete cylinder of which the curved plate can be considered a part, and p is that of a flat plate of the same dimensions and materials as the curved plate.

Cylinder Theory

The theoretical as well as the experimental treatment of the buckling of plywood cylinders under axial compression has been published in Forest Products Laboratory reports Nos. 1322, 1322-A, and 1322-B. The resulting form of the equation giving the critical stress is

$$p_1 = kE\frac{h}{r}$$

If the facings and core of the sandwich are isotropic, the theory of Report No. 1322-A leads to the formula (see derivation in Appendix)

$$p_1 = 0.2426 \sqrt{E_a E_1}\frac{h}{r} \qquad (2)$$

If it is considered that the core positions the facings but does not contribute to the stiffness of the sandwich,

$$E_a = E(1 - \frac{c}{h})$$

$$E_1 = E(1 - \frac{c^3}{h^3})$$

The accuracy of these formulas can be illustrated by comparing the computed values of E_a and E_1 with the values obtained from tests of coupons. As an example the values of E_a and E_1 will be computed for a sandwich having 0.012 inch aluminum facings on a 1/8-inch core. The total thickness will be about 0.153 inch allowing 0.002 inch for each glue line. Then

$$E_a = 10^7 \left[1 - (\frac{0.129}{0.153})\right] = 1{,}570{,}000 \text{ pounds per square inch.}$$

$$E_1 = 10^7 \left[1 - (\frac{0.129}{0.153})^3\right] = 4{,}010{,}000 \text{ pounds per square inch.}$$

The average values obtained from test coupons are

$$E_a = 1{,}490{,}000 \text{ pounds per square inch}$$

$$E_1 = 4{,}110{,}000 \text{ pounds per square inch}$$

The theory (Report No. 1322-A) presents the following formula for plywood cylinders, for which the value of the ratio $\frac{E_1}{E_1 + E_2}$ lies between 0.3 and 0.6 (Fig. 1 of Forest Products Laboratory Report No. 1322),

$$p_1 = 0.12\, E_L \frac{h}{r}$$

For plywood it is known that $E_1 + E_2 = E_L + E_T$, where E_L and E_T are the elastic moduli of the wood in the longitudinal and tangential directions, respectively. Since E_T is only about 5 percent of E_L, the value of E_T may be neglected and the formula becomes

$$P_1 = 0.12\, (E_1 + E_2) \frac{h}{r} \qquad (3)$$

This equation can be obtained from the theory in Report No. 1322-A by replacing E_L by $(E_1 + E_2)$ in formulas 48, 49, and 50 of that report. Formula (3) can be applied to sandwich combinations having facings of plywood if the values of E_1 and E_2 are those of the entire sandwich.

Flat-plate Theory

The theoretical treatment of the buckling of flat plates has been published in Forest Products Laboratory Report No. 1525. The theory includes isotropic and orthotropic materials.

The theoretical critical stress of a flat plate is given by the equation

$$p = \frac{k}{\lambda} \sqrt{E_1 E_2}\, \frac{h^2}{a^2} \qquad (4)$$

This equation is obtained from formula 6 of Forest Products Laboratory Report No. 1525 by replacing D_1 and D_2 (notation from Report No. 1525) by the equivalent expressions $\frac{E_1 h^3}{12\,\lambda}$ and $\frac{E_2 h^3}{12\,\lambda}$, respectively, and dividing the load (p_{cr}) per inch of edge by the thickness (h) to obtain p in mean stress units. The constant k was determined for panels simply supported on four edges by use of the curves of figure 2 of Report No. 1525. For this purpose values of κ were computed from formula 8 of Report No. 1525. ($\kappa = \frac{K}{\sqrt{D_1 D_2}}$ from formulas on page 3 of Report No. 1525.) Values of D_1 and D_2 were computed from the average values of E_1 and E_2 obtained from tests of coupons of sandwich. Values of K were computed from well-established values of E_x, μ_{yx}, and σ_{xy} for the materials in question as follows.

For birch plywood facings on a quipo core, the value of κ was computed to be 0.37 by assuming for the computation of K that

E_{xf} = 2,300,000 pounds per square inch, σ_{yxf} = 0.02,

λ_f = 0.99, μ_{xyf} = 180,000 pounds per square inch

E_{xc} = 40,000 pounds per square inch, σ_{yxc} = 0.20, λ_c = 0.99,

μ_{xyc} = 26,000 pounds per square inch.

For birch plywood facings on a 1/10- and 2/10-inch pulpboard core, the values of κ were computed to be 0.29 and 0.24 respectively, by assuming for the computation of K that E_{xf} = 2,300,000 pounds per square inch, σ_{yxf} = 0.02, λ_f = 0.99, μ_{xyf} = 180,000 pounds per square inch, and that the contribution of the core to K could be neglected.

For specimens with fiberglas facings the value of κ was computed to be 0.55 by assuming for the computation of K that $E_{xf} = E_{yf}$ = 2,200,000 pounds per square inch, σ_{yxf} = 0.20, λ_f = 0.96, μ_{xyf} = 400,000 pounds per square inch, and that the contribution of the core to K could be neglected.

For sandwich constructions having aluminum facings, the value of κ is 1.00 as a consequence of the assumption that the facings and core are isotropic.

Curved-plate Theory

The critical stress of a curved plate was calculated as the sum of the critical stress of a complete cylinder of which the curved plate could be a part, and the critical stress of a flat plate the same size as the curved plate; that is,

$$p_{cr} = p_1 + p \qquad (1)$$

where p_1 and p were obtained as outlined in the preceding paragraphs.

The discussion up to this point has assumed that the stresses in the facings are below the proportional limit. When the stresses exceed the proportional limit, it is necessary to replace the moduli of elasticity in the formulas by reduced moduli, one for the modulus in bending and another for the modulus in compression. For the bending of sandwich construction having isotropic facings, the modulus applicable when the stresses in the facings are below the proportional limit, is to be replaced by the reduced modulus $E_r = \frac{2EE_t}{E + E_t}$ when these stresses are above the proportional limit. This expression was derived at the Forest Products Laboratory for sandwich constructions in the same way as was the similar formula given by Timoshenko for solid plates.[5] For the behavior under compressive stresses, the modulus E_a is to be replaced by the reduced modulus $\frac{E + E_t}{2}$, which is the average of the moduli in the two faces at the instant buckling begins. On the concave side the modulus of the face is E_t, while on the convex side it is E.

By replacing E_1 and E_a in formula 2 and E_1 in formula 4 by the corresponding reduced moduli, the following formulas were obtained for calculating the critical stresses when the facings are stressed beyond their proportional limit. It was assumed that the facings were isotropic, that the contributions of the core to the load-carrying ability of the sandwich and to its stiffness could be neglected, and that the reduction in shear modulus corresponds to the reduction in bending modulus.

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[5] Timoshenko, S. "Theory of Elastic Stability" p. 156, Art. 29 1936.

$$p'_1 = 0.2426 \frac{h}{r} \sqrt{\frac{E + E_t}{2} (1 - \frac{c}{h}) \cdot \frac{2EE_t}{E + E_t} (1 - \frac{c^3}{h^3})}$$ for cylinders and

$$p' = \frac{k}{\lambda} \cdot \frac{2EE_t}{E + E_t} (1 - \frac{c^3}{h^3}) \frac{h^2}{a^2}$$ for flat plates. These expressions were further simplified to

$$p'_1 = 0.2426 \sqrt{E_a E_1} \frac{h}{r} \sqrt{\frac{E_t}{E}}$$

$$p' = \frac{k}{\lambda} E_1 \frac{h^2}{a^2} \frac{2 \frac{E_t}{E}}{1 + \frac{E_t}{E}} \quad ;$$

or

$$p'_1 = p_1 \sqrt{\frac{E_t}{E}}$$

$$p' = p \frac{2 \frac{E_t}{E}}{1 + \frac{E_t}{E}}$$

where p_1 and p are the stresses computed by formulas 2 and 4, respectively, and p'_1 and p' represent the critical stresses of the cylinder and flat plate above the proportional limit stress of the facing material.

The expression for the critical stress of a curved panel is again given by the sum of the two previous equations as indicated by equation (1)

$$p_{cr} = p_1 \sqrt{\frac{E_t}{E}} + p \frac{2 \frac{E_t}{E}}{1 + \frac{E_t}{E}} \qquad (5)$$

Since the magnitude of the stress in the facings is the factor under consideration, however, this expression can be changed to

$$p_{cr.f} = \frac{h}{2f} \left[p_1 \sqrt{\frac{E_t}{E}} + p \frac{2 \frac{E_t}{E}}{1 + \frac{E_t}{E}} \right] \qquad (6)$$

where $p_{cr.f}$ is the stress in the facing at which buckling of the sandwich will occur.

There are two unknowns in these equations, the stress that is desired, and the tangent modulus of the facing material at that stress. Their solution depends, therefore, upon knowledge of the relation between stress and the tangent modulus. Curves showing the relation between $\frac{E_t}{E}$ and the facing stress ($p_{cr.f}$) for aluminum facing materials are shown in figure 2. The points plotted in that figure represent values obtained from the stress-strain data of compression tests. The curves are drawn to represent the plotted points. Thus it is seen that the solution of equation 6 must be such that the relation between $p_{cr.f}$ and $\frac{E_t}{E}$ given by this equation is also satisfied by the curve between $p_{cr.f}$ and $\frac{E_t}{E}$. A curve representing the relation of $p_{cr.f}$ to values of $\frac{E_t}{E}$ can be determined by means of equation (6) and plotted in figure 2. The intersection of this curve with the appropriate curve of figure 2 will give the stress in the facing at which buckling will occur.

The preceding discussion supposes that a stress-strain curve of the facing material is available. If such a curve is not available, an approximate solution can be obtained by assuming the relation of both $\sqrt{\frac{E_t}{E}}$ and $\frac{2\frac{E_t}{E}}{1+\frac{E_t}{E}}$ to $p_{cr.f}$ to be represented by a straight line between the proportional limit stress and the maximum stress. The accuracy of this assumption can be seen by referring to figure 3. The curves show that both the $\sqrt{\frac{E_t}{E}}$ and $\frac{2\frac{E_t}{E}}{1+\frac{E_t}{E}}$ ratios are fairly well represented by the straight line MN. The straight line MN is defined by the equations

$$p_{cr.f} = -(S_f - S_{p\,lf})\sqrt{\frac{E_t}{E}} + S_f$$

or

$$p_{cr.f} = -(S_f - S_{p\,lf})\frac{2\frac{E_t}{E}}{1+\frac{E_t}{E}} + S_f$$

where $S_{p\,lf}$ and S_f are the proportional limit stress and the maximum stress of the facing material. Solving these equations for $\sqrt{\frac{E_t}{E}}$ and $\frac{2\frac{E_t}{E}}{1+\frac{E_t}{E}}$, substituting the expressions obtained in equation 6, and then solving for $p_{cr.f}$ results in:

$$p_{cr.f} = \frac{(p_1 + p) \frac{h}{2_f} S_f}{(p_1 + p) \frac{h}{2_f} + S_f - S_{plf}} \qquad (7)$$

Theoretical buckling stresses for panels having aluminum facings were computed by using each of equations 6 and 7. A comparison of the results obtained by the two equations can be seen by referring to figure 4, which shows that stresses computed by equation 7, in which S_f = 57,000 pounds per square inch, and S_{plf} = 27,000 pounds per square inch, do not differ greatly from the values given by equation 6. The line MN was used for both the 24ST and the 24SH aluminum alloys. Good agreement could be expected at stresses below about 50,000 pounds per square inch because the straight line is a good approximation of the curve in that range.

No attempt was made to develop a means of including the effect on the critical stresses of shear deformations in the core. Because the plate is curved, the wave length of the buckle pattern is unknown; and, therefore, an analysis of the deflections due to shear is exceedingly difficult.

Preparation of Materials

This study was undertaken primarily to investigate the buckling of curved panels of sandwich construction. Therefore the specimens were designed so that they would buckle before the compressive strength of the material was reached. For this reason it was necessary to use rather thin cores so that specimens of small enough size in width and length to fit the testing machine could be employed. Even though cores of 1/8 inch thickness were used it was necessary to make several specimens as large as 6 feet square to determine buckling characteristics of some of the panels having large radii of curvature.

Facing Materials

The materials that were used for the facings of the sandwich are listed as follows:

Plywood.--The plywood was made of two plies of yellow birch veneer 1/100 inch thick bonded together with a film glue. The grain of adjacent plies was placed at right angles.

Aluminum.--Sheets of alclad aluminum alloy 24ST were used in thicknesses of 0.012, 0.020, and 0.032 inch. Sheets of aluminum alloy 24SH were used in a thickness of 0.005 inch.

Fiberglas.--The glass cloth used to make the facings was a continuous filament cloth 0.003 inch thick and of a plain-type weave with 40 ends to the inch in the warp and 39 ends to the inch in the fill direction. The cloth had been treated to remove lubricants. The facings were made of either 3, 6, 10, or 16 layers of cloth impregnated with a contact pressure type of resin. The resin also acted as bonds between the facings and the core. The layers of cloth were placed so that the warp of one piece was always at right angles to the warp of the adjacent piece.

Core Materials

Quipo.--Quipo was used in quarter-sawn sheets 1/10 inch thick. The density was from 6 to 11 pounds per cubic foot.

Impregnated fiberboard.--The impregnated fiberboard was a special lightweight insulating type of fiberboard containing 50 to 65 percent thermosetting, spirit-soluble, phenolic resin. The board was used in thicknesses of 1/10 or 2/10 inch. The density was from 10-1/2 to 12-1/2 pounds per cubic foot.

Balsa.--The balsa that was used was fabricated to sheets 1/8, 1/4, or 1/2 inch thick and was placed so that the grain direction was normal to the surface of the sheet. The sheet was made up of blocks about 2 by 4 inches in size that were edge-glued to each other with a thermosetting synthetic resin glue. The density of the cores was from 5 to 9 pounds per cubic foot.

Cellular cellulose acetate.--The cellular cellulose acetate was an extruded and expanded cellular cellulose acetate containing about 3 percent chopped-glass fibers. The cores were made up of strips 1/8 inch thick and about 2 inches wide, edge-glued together with a thermosetting synthetic resin glue. The density of the cores was 6 to 7 pounds per cubic foot.

Hard sponge rubber.--The hard sponge rubber was an expanded, hard, synthetic rubber sponge. The cores were made up of strips 1/8 inch thick and about 2 inches wide, edge-glued together with a thermosetting synthetic resin glue. The density of the cores was 6.2 to 7.2 pounds per cubic foot.

Manufacture of Specimens

All specimens, and matched coupons, were made by the bag-molding process. The specimens were bag-molded to the desired curvature on steel molds. The coupons were bag-molded on a flat steel sheet. A more detailed description of manufacturing technique and types of bonding materials is discussed in the Forest Products Laboratory Report, "The Manufacture of Lightweight Sandwich Test Panels."

Test specimens.--The sandwich specimens made of combinations of facings and cores as described previously are listed as follows. The panel sizes and radii of curvature are shown in tables 1, 2, and 3.

(1) Plywood facings; quipo core.--The plywood was placed so that the grain of the face plies was parallel to the axis of the curved plate. The grain of the core was placed in the axial direction.

(2) Plywood facings; impregnated fiberboard core.--The plywood was placed with the face grain either parallel or perpendicular to the axis of the curved plate.

(3) Aluminum facings; balsa core.

(4) Aluminum facings; cellular cellulose acetate core.

(5) Aluminum facings; hard sponge rubber core.

(6) Fiberglas facings; balsa core.

(7) Fiberglas facings; cellular cellulose acetate core.

(8) Fiberglas facings; hard sponge rubber core.

Coupons.--The coupons were made of the same combination of materials and by the same manufacturing technique as the specimens. They were made in a single sheet and finally cut to sizes of 1 by 4-1/2 inches for compression specimens and 1 by 18 inches for bending specimens.

Preparation for Testing

The edges and ends of the specimens with plywood facings were sawed square and the edges were fitted with maple guides.

The specimens with aluminum facings were each fitted with four strips of thin aluminum 1 inch wide and 0.02 inch thick, bonded to the facings at the loaded edges. These strips were then covered with 1/8- by 1-inch steel bars, which were fastened to the sandwich by means of 1/4 inch bolts spaced about 4 inches on centers. The ends of the specimens were then machined. The addition of the strips of aluminum and steel prevented the formation of sharp wrinkles or folding under of the facings at the ends of the specimens. Maple guides were fitted to the unloaded edges of the specimens.

The specimens having fiberglas facings were fitted with strips of thin plywood 1 inch wide bonded to the facings at the loaded edges. The ends of the specimen were then sawed square and true. The plywood strip was added to prevent the facings from folding under at the ends of the specimen. Maple guides were fitted to the unloaded edges of the specimens.

The edge guides were pieces of maple about 2 by 2 inches in cross section with a length about 1/4 inch shorter than the length of the test specimen. The guides were grooved in the lengthwise direction with grooves 1/4 inch deep and wide enough to allow them to be slipped onto the edges of the test specimen.

Testing Methods

Specimens that were not wider than 30 inches were placed on a heavy flat plate, which was supported by a spherical bearing placed on the lower head of a hydraulic testing machine (figs. 5 and 6). The heads of the testing machine were then brought together until the specimen just touched the upper platen with no load indicated. Adjustments were made on the spherical base until no light could be seen between the ends of the specimen and the loading heads. Screw jacks were then placed under the lower loading plate to prevent tilting of the plate while the load was being applied to the specimen. The load was then applied slowly until failure occurred.

Specimens wider than 30 inches were tested between the heads of a four-screw, mechanically operated, testing machine. No spherical bearing was used. The specimens were cut as true as possible. If light could be seen between the ends of the specimen and the heads of the testing machine, shims of paper or brass were inserted until the gap was closed. The wide specimens were also very long; therefore, small irregularities at the ends were taken up as the load was applied without causing large variations from uniformity in the stresses in the facings.

The coupons were tested in bending and compression to determine the moduli of elasticity. The bending specimens were tested over a long span so that deflections due to shear were negligible.

Description of Test Failures

A curved panel of sandwich material loaded in axial compression may fail in one of five different ways: (1) buckling, (2) crimping (fig. 1), (3) compression failure in the facings, (4) wrinkling of the facings, or (5) by separation of the facing from the major part of the core. The buckling type of failure is of the general instability type involving the facings and the core and may be relieved by reducing the applied load. The crimping failure is more of a localized bend resulting in shear failure of the core (figs. 1, 5, and 6). Wrinkling of the facings can occur on specimens with relatively thick and weak cores. The wrinkle in the facings moves into or away from the core. Separation of the facings from the core appears as a buckle of the face and occurs when bonding between the facings and core is poor. The specific types of failures of the panels are tabulated in tables 1, 2, and 3.

The panels with plywood facings failed by buckling. The buckles were small compared to those observed in the specimens with aluminum or fiberglas facings. The failure was sudden, and, since the travel of the movable head of the testing machine could not be stopped instantaneously, the buckles observed were very sharp and crinkles in the plywood appeared at the edges of the buckles.

The most typical failures of the specimens with aluminum facings were buckling or crimping. Figures 5 and 6 show the crimping type of failure, which occurred in many panels of sharp curvature or small size. The size or sharpness of the crimp seemed to depend somewhat on the thickness of the facings of the specimen. This can be seen by comparing the failure of the panel having facings 0.012 inch thick (fig. 5) with the panel having facings 0.005 inch thick (fig. 6). Large, slightly curved panels failed by buckling. The aspect ratio of the buckles was about 1.0. Either type of failure caused an immediate drop in the load. Many of the specimens were so damaged by the failure that the load dropped to zero after failure.

The panels having fiberglas facings failed by buckling, by crimping, or by compression failure of the facings. The compression failures sometimes occurred after buckling. Most of the panels having facings of fiberglas failed by buckling.

The failures just described were for panels that were sound. The results for all specimens, sound or defective, are presented in the tables; but the curves of figures 7, 8, 9, and 10 show only the results of the tests on sound panels. Some of the defective panels were known to contain unbonded areas before they were tested. Other specimens that exhibited no defects prior to testing, failed by facing separation or crimping during test, and their defects were found by examination of the panel after testing. The only defect responsible for the failures at low stresses was that of poor bonding of the facings to the core. In some instances where unbonded areas were known to exist, attempts were made to reglue these areas, but the attempts were not always successful. The defective specimens were those that were manufactured early in the investigations of sandwich constructions, during the period when manufacturing techniques were being developed. Panels made at a later date were not defective if the proper materials were used and proper manufacturing techniques carefully followed.

Presentation and Discussion of Data

The experimental results and the results of theoretical computations are presented in tables 1, 2, and 3. The formulas and constants used to obtain the theoretical values in the tables were given in the section on development of formulas. Table 1 contains the data for sandwiches with plywood facings; table 2, aluminum facings; and table 3, fiberglas facings. The resulting theoretical stresses are plotted against the experimental values in figures 7, 8, 9, and 10.

A preliminary analysis of the data was made on the assumption that the curved plate would behave the same as a complete cylinder. The buckling stresses were computed by the formula

$$p'_{1f} = 0.2426 \sqrt{E_a E_1}\, \frac{h}{r} \cdot \frac{h}{2f} \sqrt{\frac{E_t}{E}}$$

A comparison between this computed stress and the experimental value may be seen by referring to figure 7. The scatter of points above the line representing (experimental stress) = (theoretical stress) indicates that the theoretical values are too low.

The results of the final analysis, which assumes that the critical stress of a curved plate is equal to the critical stress of a complete cylinder plus the critical stress of a flat plate (Wenzek's theory), are presented in figures 8, 9, and 10. A comparison of the position of the points shown on figure 7 with those shown on figure 8 indicates that the addition method is not too far in error.

The test data exhibit considerable variability. These variations are usually due to the presence of small irregularities in the surface or in the cross section of the specimen. A considerable discussion of the effect of these irregularities upon the critical stresses was given in Forest Products Laboratory Report No. 1322-A, "Buckling of Long, Thin Plywood Cylinders in Axial Compression." Graphs of the results of tests on plywood cylinders and plywood curved plates are reproduced in figures 11 and 12 herein.

A review of the results of the tests on sandwich materials as presented on the graphs of figures 8, 9, 10 shows that the magnitude of the scatter is somewhat different for different sandwich constructions. For sandwiches of aluminum or fiberglas facings on balsa cores (fig. 8) the trend of the data may be fairly represented by the theoretical line. The data representing specimens with the same kinds of facings on cellular cellulose acetate or hard sponge rubber cores (fig. 9) show experimental values to be a considerable amount lower than the theoretical ones. The results of tests of sandwich materials having plywood facings (fig. 10) show experimental values about equal to theoretical values for low stresses but much higher than theoretical values for higher stresses.

The data for the specimens having balsa cores agree well with theory except for one point on the extreme right of figure 8. This point represents a specimen having a 1/2-inch core. The ratio of the experimental critical stress to the theoretical critical stress for this specimen was less than similar ratios for thinner specimens with the same length, width, and curvature. Therefore, the correction due to shearing deformations is likely to be greater for this specimen than that for the specimens having the thinner cores. No method has yet been devised to correct the theoretical critical stress of a curved plate for the effect of shear deformation. This specimen with a 1/2-inch core might need a correction of about 20 percent.

The specimens having cellular cellulose acetate or hard sponge rubber cores showed lower critical stresses than the theory predicts (fig. 9). The shear moduli of the cores of these specimens are lower than that of balsa wood; therefore some correction for shear deformation may be needed. The modulus of elasticity in the direction normal to the plane of the sheet of these cores is also lower than that of balsa; therefore, incipient dents or buckles, which may have caused the early failures, were probably larger in these specimens than in those with balsa cores.

The experimental values of the critical loads of the specimens with plywood facings were somewhat higher than theoretical values. These plywood facings were relatively thicker than the other types of facings and, therefore, had smaller incipient dents or buckles. A discussion on page 25 of Forest Products Laboratory Report No. 1322-A shows that it is possible for the critical stress of a specimen to exceed the minimum as determined by formula (3), provided the initial imperfections are very small. The theoretical values of the stresses also may be low because the behavior of the plywood at stresses greater than the proportional limit may not have been adequately taken into account by the use of equation (7).

The type of failure of the sandwich depends upon the elastic properties of the facings and core, the relative thickness of the facings and core, and the magnitude of the small irregularities of the facings. The crimping type of failure, which was observed in many of the sandwiches with aluminum facings and in some panels of other constructions, occurred at or above the critical load of the panel. A crimp can appear if a large imperfection develops and causes severe bending stresses in the sandwich and, therefore, high shear stresses in the core. The crimp may cause a shear failure in the core. Most of the sandwich specimens were well-constructed, and the initial irregularities were small; therefore the facings did not crimp until buckling occurred because no appreciable amount of bending was developed before buckling.

Some of the panels with cellular cellulose acetate or hard sponge rubber cores failed by crimping at very low loads. These panels had blisters in them immediately after manufacture, but after the panels were cooled the blister contracted and could not be detected. It was not ascertained whether the blister was located between the facings and cores or whether the cores failed in tension normal to their plane. These specimens were manufactured during the early period of the study of sandwich constructions. The best gluing techniques were not established at that time.

No theoretical estimate of the loads at which separation of the facings occurred could be made because of the great number of responsible causes, such as insufficient strength of the bond between the facings and the core, weak core, large initial imperfections, and localized weakness in the facings.

Conclusions

Results obtained in the study, as given in the foregoing discussion, lead to the following conclusions.

The axial compressive strength of a long curved plate of sandwich material may be computed by adding the critical stress of a complete cylinder, of which the panel can be considered a part, to that of a flat plate identical in size and construction to the curved plate.

Buckling at stresses greater than the proportional limit may be computed by either of the two methods presented.

Panels of sandwich material with weak cores may buckle at stresses lower than stresses computed by these methods.

Poor bonding of the facings of a sandwich to the core will cause failure at very low loads.

Crimping types of failure occur at loads equal to or greater than the computed critical loads, provided the panels have no structural defects.

Appendix

The derivation of formula 2 from the mathematics of Forest Products Laboratory Report No. 1322-A is presented in the following.

If it is assumed that the facings are isotropic and that the contribution of the core to the stiffness and load-carrying ability of the sandwich can be neglected, then

$$E_c = 0 \qquad \sigma_c = 0 \qquad \mu_c = 0$$

$$E_x = E_y = E_f$$

$$E_a = E_b = E_f \quad (1 - \frac{c}{h}) = E_f \frac{2f}{h}$$

$$E_1 = E_2 = E_f \ (1 - \frac{c^3}{h^3}),$$

and equations 2 of Forest Products Laboratory Report No. 1322-A become

$$(X'_x)_f = \frac{E_f}{\lambda_f} \quad (e'_{xx} + \sigma_f \ e'_{yy})$$

$$(Y'_y)_f = \frac{E_f}{\lambda_f} \quad (e'_{yy} + \sigma_f \ e'_{xx}) \qquad (2)$$

$$(X'_y)_f = \mu_f \ e'_{xy},$$

and equations 4 become

$$\bar{X}'_x = \frac{E_a}{\lambda_f} \left(e'_{xx} + \sigma_f \, e'_{yy}\right)$$

$$\bar{Y}'_y = \frac{E_a}{\lambda_f} \left(e'_{yy} + \sigma_f \, e'_{xx}\right) \qquad (4)$$

$$\bar{X}'_y = \mu_f \frac{E_a}{E_f} \, e'_{xy}$$

Then solving for e',

$$e'_{xx} = \bar{X}'_x \frac{1}{E_a} - \bar{Y}'_y \frac{\sigma_f}{E_a}$$

$$e'_{yy} = \bar{Y}'_y \frac{1}{E_a} - \bar{X}'_x \frac{\sigma_f}{E_a} \qquad (7)$$

$$e'_{xy} = \bar{X}'_y \cdot \frac{E_f}{E_a} \frac{1}{\mu_f}$$

From equations 10 of Forest Products Laboratory Report No. 1322-A,

$$\bar{X}'_x = \frac{\delta^2 F}{\delta y^2}, \qquad \bar{Y}'_y = \frac{\delta^2 F}{\delta x^2}, \qquad \bar{X}'_y = -\frac{\delta^2 F}{\delta x \, \delta y} \qquad (10)$$

Then by introducing (10) in (7) and substituting the results in (11) of Report No. 1322-A, the left-hand member of the equation for the stress function (equation 12, Report No. 1322-A) F is obtained as follows:

$$A \frac{\delta^4 F}{\delta x^4} + B \frac{\delta^4 F}{\delta y^4} + C \frac{\delta^4 F}{\delta x^2 \delta y^2},$$

where

$$A = B = \frac{1}{E_a}$$

and

$$C = \frac{E_f}{E_a \mu_f} - \frac{2\sigma_f}{E_a} = \frac{2}{E_a}, \quad \text{since} \quad \mu_f = \frac{E_f}{2(1 + \sigma_f)}$$

for the sandwich construction.

The next step is to evaluate the constants in terms of properties of the sandwich construction so as to determine the values of K_1, K_2, etc., of equation 39 of Report No. 1322-A. A, B, and C have been found. The constant N is found as follows: For plywood, $N = E_L \sigma_{tL} + 2\lambda\mu_{Lt}$ from equation 33 of Report No. 1322-A. Since N is associated with flexural energy of deformation, it becomes for sandwich construction, if the contribution of the core is neglected,

$$N = \left[E_f \sigma_f + 2\lambda\mu_f\right](1 - \frac{c^3}{h^3})$$

But since $\lambda = (1 - \sigma^2{}_f)$ and $\mu_f = \dfrac{E_f}{2(1 + \sigma_f)}$

$$N = E_f(1 - \frac{c^3}{h^3}) = E_1 = E_2$$

Since the material is assumed to be isotropic, the aspect ratio (Z) of the buckle is unity, and equations 39 of Report No. 1322-A become

$$K_1 = \frac{100}{E_a}$$

$$K_2 = \frac{100}{E_a}$$

$$K_3 = \frac{4}{E_a}$$

$$K_4 = 8E_1$$

If γ_i (i = 1, 2, 3, 4) is substituted for $\dfrac{c_i}{E_f}$ in equation 44 of Report No. 1322-A, there results

$$\frac{pr}{E_f h} = \left[2\gamma_1 \eta (\xi + \xi_o) - \gamma_2 \frac{(3\xi + \xi_o)}{2\xi} + \frac{\gamma_3}{\eta\xi} + \frac{\gamma_4 \eta}{\xi}\right] \frac{(\xi - \xi_o)}{c_5} \qquad (44)$$

Let $\dfrac{pr}{E_f h} = k$,

and equation (44) can be reduced to equation (52) on page 26 of Report No. 1322-A by the methods described therein and is:

$$k = \left[2\gamma_1\gamma_4 \left(32\gamma_1\gamma_3 - 9\gamma_2^2\right)\right]^{1/2} \frac{1}{4\gamma_1 c_5} \tag{52}$$

By determining the values of c_1, c_2, c_3, c_4, and c_5 by means of equations (41) of Report No. 1322-A, and then determining values of γ_1, γ_2, γ_3, and γ_4 from the relation $\gamma_i = \frac{c_i}{E_f}$ and substituting in equation (52),

$$k = 0.2426 \frac{\sqrt{E_a E_1}}{E_f}$$

Then since $\frac{pr}{E_f h} = k$,

$$p_1 = 0.2426 \sqrt{E_a E_1} \frac{h}{r}$$

If the center material has the same properties as the material in the facings, the value of $E_a = E_1 = E$, and the formula reduces to $p = 0.2426E \frac{h}{r}$, which is that derived for isotropic cylinders (page 28 of Report No. 1322-A).

Table 1.--Axial-compression tests of curved panels of sandwich construction having plywood facings

Specimen number	Total thickness h	Width a	Length b	Mean radius of curvature r	Mean failing stress	Type of failure	Data from tests of coupons E_a	E_1	E_2	s_{pl}	s	Theoretical stresses p_1	p	p_{cr}
	Inches	Inches	Inches	Inches	Lb. per sq. in.		1,000 lb. per sq. in.	1,000 lb. per sq. in.	1,000 lb. per sq. in.	Lb. per sq. in.	Lb. per sq. in.	Lb. per sq. in.	Lb. per sq. in.	Lb. per sq. in.
Facing: Two plies of 1/100-inch yellow birch. Grain of face plies, axial. Grain of crossbanded plies, circumferential. Core: 1/10-inch quipo. Grain, axial														
2	0.135	17.6	30.0	9.8	1,260	Crimping	540	1,000	660	1,570	2,450	2,744	107	1,872
3	.132	19.9	30.0	15.2	1,200	....do...	540	1,000	660	1,570	2,450	1,730	87	1,651
4	.140	19.8	30.0	24.5	1,180	Buckling	540	1,000	660	1,570	2,450	1,138	98	1,236
5	.138	19.9	30.0	29.7	910	....do...	540	1,000	660	1,570	2,450	926	95	1,021
6	.137	17.8	30.0	10.2	2,410	....do...	540	1,000	660	1,570	2,450	2,676	109	1,860
7	.139	20.8	30.0	15.0	1,770	....do...	540	1,000	660	1,570	2,450	1,846	90	1,685
8	.140	20.7	30.0	24.6	1,270	....do...	562	1,000	660	1,540	2,450	1,134	90	1,224
9	.142	20.7	30.0	29.5	980	....do...	522	1,000	660	1,600	2,450	959	93	1,052
Facing: Two plies of 1/100-inch yellow birch. Grain of face plies, axial. Grain of crossbanded plies, circumferential. Core: 1/10-inch impregnated fiber board														
10-1	.124	18.0	30.0	10.5	1,660	Buckling	394	939	811	990	2,110	2,480	94	1,470
10-3	.128	18.0	30.0	10.1	2,020	....do...	394	939	811	990	2,110	2,661	100	1,500
11-1	.136	18.0	30.0	15.1	1,730	....do...	514	1,070	727	1,190	2,280	1,942	114	1,490
11-2	.129	18.0	30.0	15.4	1,610	....do...	514	1,070	727	1,190	2,280	1,806	103	1,450
11-3	.132	18.0	30.0	15.7	1,690	....do...	514	1,070	727	1,190	2,280	1,813	108	1,455
12-1	.134	18.0	30.0	24.4	1,290	....do...	428	1,024	792	940	1,580	1,197	113	1,060
12-2	.132	18.0	30.0	24.1	1,290	....do...	428	1,024	792	940	1,580	1,194	110	1,060
13-1	.134	18.0	30.0	31.4	960	....do...	437	936	790	1,030	1,730	884	108	992
13-2	.132	18.0	30.0	30.3	910	....do...	437	936	790	1,030	1,730	902	105	1,007
13-3	.130	18.0	30.0	33.6	1,030	....do...	437	936	790	1,030	1,730	801	102	903
Facing: Two plies of 1/100-inch yellow birch. Grain of face plies, axial. Grain of crossbanded plies, circumferential. Core: 2/10-inch impregnated fiber board														
16-1	.226	18.0	30.0	27.2	950	Buckling	302	789	597	840	1,460	1,382	235	1,055
16-2	.228	18.0	30.0	26.1	950	....do...	302	789	597	840	1,460	1,453	239	1,070
16-3	.226	18.0	30.0	27.6	990	....do...	302	789	597	840	1,460	1,362	235	1,050
17-1	.233	18.0	30.0	32.5	920	....do...	288	723	620	660	1,230	1,155	244	870
17-2	.229	18.0	30.0	33.0	980	....do...	288	723	620	660	1,230	1,118	235	865
17-3	.226	18.0	30.0	30.8	1,010	....do...	288	723	620	660	1,230	1,183	229	880
Facing: Two plies of 1/100-inch yellow birch. Grain of face plies, circumferential. Grain of crossbanded plies, axial. Core: 1/10-inch impregnated fiber board														
14-1	.127	18.0	30.0	10.2	1,820	Buckling	452	730	1,004	1,200	1,650	2,591	97	1,410
14-2	.132	18.0	30.0	10.0	1,690	....do...	452	730	1,004	1,200	1,650	2,747	105	1,425
14-3	.129	18.0	30.0	10.2	1,940	....do...	452	730	1,004	1,200	1,650	2,632	100	1,420
15-1	.135	18.0	30.0	26.8	1,010	....do...	475	848	1,028	1,240	1,970	1,134	119	1,198
15-2	.133	18.0	30.0	25.7	1,010	....do...	475	848	1,028	1,240	1,970	1,165	116	1,211
15-3	.132	18.0	30.0	24.1	1,050	....do...	475	848	1,028	1,240	1,970	1,233	114	1,237

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Table 2.--Axial-compression tests of curved panels of sandwich construction having aluminum facings

Specimen number	Total thickness h	Width a	Length b	Mean radius of curvature r	Facing stress at failure	Type of failure	Data from tests of coupons: E_a	E_l	S_{pl}	S	Theoretical stresses: p_l	p	p_{cr}	$p_{cr.f}$
	Inches	Inches	Inches	Inches	Lb. per sq. in.		1,000 lb. per sq. in.	1,000 lb. per sq. in.	Lb. per sq. in.	Lb. per sq. in.	Lb. per sq. in.	Lb. per sq. in.	Lb. per sq. in.	Lb. per sq. in.
						Facing: 0.005-inch 24SH aluminum alloy Core: 1/8-inch end-grain balsa								
18-1	0.147	11.7	30.0	9.4	[1]27,000	Crimping - many unbonded areas present before testing	742	2,109	1,830	3,880	4,746	1,244	3,180	46,700
18-2	.133	11.6	26.5	10.4	48,400	Crimping - good bond	752	2,030	2,030	4,280	3,833	982	3,286	43,700
18-3	.135	11.6	30.0	10.1	45,100	............do............	675	1,928	2,000	4,220	3,699	970	3,222	43,500
19-1	.146	16.7	30.0	16.4	46,900	............do............	743	1,989	1,850	3,900	2,626	558	2,712	39,600
19-2	.137	16.6	30.0	15.5	[1]7,800	Separation of facing - poor bond	717	2,085	1,970	4,160	2,622	520	2,810	38,500
19-3	.140	16.6	26.6	16.1	[1]26,000	Crimping - attempt was made to reglue unbonded areas before testing	728	2,349	1,930	4,070	2,759	633	2,854	40,000
20-1	.135	19.6	30.0	28.2	[1]24,800	Separation of facing - attempt was made to reglue unbonded areas before testing	850	1,995	2,000	4,220	1,512	367	1,879	25,400
20-2	.138	19.6	30.0	28.5	34,800	Buckling and crimping - bond under reglued areas was fair	854	2,276	1,960	4,130	1,638	438	2,069	28,600
20-3	.141	19.6	30.0	28.5	[1]14,900	Separation of facing - many unbonded areas were present before testing	881	2,207	1,910	4,040	1,674	443	2,110	29,800
55-1	.139	19.2	29.0	63.2	16,500	Sudden buckling followed by end crimping	728	2,214	1,940	4,100	677	449	1,126	15,700
55-2	.141	19.0	29.0	63.1	18,300	Sudden buckling	681	2,140	1,910	4,040	654	453	1,107	15,600
56-1	.139	14.6	28.9	59.9	22,000	Buckling followed by crimping	764	2,181	1,940	4,100	727	717	1,444	20,100
56-2	.141	14.6	28.9	63.2	20,000	Sudden buckling	717	2,058	1,910	4,040	657	696	1,353	19,100
						Facing: 0.012-inch 24ST aluminum alloy Core: 1/8-inch end-grain balsa								
26-1	.155	15.7	29.0	15.0	37,100	Crimping at end - good bond	1,598	4,163	4,180	8,820	6,466	1,471	5,613	36,300
26-2	.158	16.6	29.3	15.0	[1]25,700	Crimping at end - poor bond	1,618	4,337	4,100	8,660	6,769	1,451	5,597	36,800
26-3	.153	16.6	28.0	14.5	[1]25,100	Crimping - poor bond	1,472	4,233	4,240	8,940	6,390	1,336	5,600	35,700
27-1	.151	31.2	30.0	38.4	15,000	Buckling and crimping	1,394	4,647	4,290	9,060	2,428	395	2,823	17,800
27-2	.152	31.0	30.0	38.2	18,700	............do............	1,655	3,920	4,260	9,000	2,459	342	2,801	17,700
27-3	.154	30.8	30.0	37.6	20,000	Crimping at end	1,536	4,379	4,210	8,880	2,577	397	2,974	19,100
58-1	.155	41.2	41.1	51.0	19,700	Buckling	1,294	4,023	4,180	8,830	1,682	206	1,888	12,200
58-2	.156	41.2	41.0	48.3	17,900	............do............	1,354	3,847	4,150	8,770	1,788	200	1,988	12,900
58-3	.160	41.3	41.0	51.7	20,300	............do............	1,382	3,841	4,050	8,550	1,730	209	1,939	12,900
61-1	.157	69.2	70.0	92.9	7,100	............do............	1,531	4,343	4,130	8,710	1,057	82	1,139	7,500
61-2	.160	59.2	59.9	104.0	8,300	............do............	1,439	3,711	4,050	8,550	862	99	961	6,400
61-3	.153	69.2	69.9	73.2	8,300	............do............	1,483	4,004	4,240	8,940	1,236	71	1,307	8,300
72	.150	11.5	30.2	20.6	33,100	Crimping - good bond	1,600	4,073	4,320	9,120	4,510	2,513	5,488	34,300
						Facing: 0.012-inch 24ST aluminum alloy Core: 1/4-inch balsa								
71	.275	11.6	29.1	23.4	39,100	Buckling and crimping	873	2,396	2,360	4,970	4,123	4,957	3,753	43,000
						Facing: 0.012-inch 24ST aluminum alloy Core: 1/2-inch balsa								
70	.530	11.5	29.2	20.8	44,200	Crimping	453	1,298	1,220	2,580	4,740	10,149	2,468	54,500
						Facing: 0.020-inch 24ST aluminum alloy Core: 1/8-inch end-grain balsa								
28-1	.165	16.7	29.9	16.0	34,600	Crimping - good bond	2,396	5,053	6,550	13,810	8,705	1,810	8,267	34,100
28-2	.164	16.7	30.0	15.9	35,600	............do............	2,428	5,593	6,580	13,900	9,221	1,980	8,512	34,900
28-3	.168	16.8	29.9	15.9	30,800	............do............	2,396	5,367	6,430	13,570	9,192	1,970	8,381	35,200
29-1	.166	31.2	30.0	38.1	14,800	Crimping - fair bond	2,376	5,473	6,500	13,740	3,812	562	4,374	18,200
29-2	.165	31.2	30.0	39.6	22,200	Buckling and crimping - fair bond	2,374	5,757	6,540	13,810	3,737	584	4,321	17,800
29-3	.167	31.7	29.8	39.8	23,500	Crimping - good bond	2,689	6,013	6,470	13,660	4,093	605	4,698	19,600
59-1	.175	34.9	50.0	64.5	15,100	Buckling and crimping	1,879	4,687	6,170	13,030	1,953	473	2,426	10,600
59-2	.178	35.0	50.0	65.9	14,700	Crimping	2,071	5,363	6,070	12,810	2,184	556	2,740	12,200
59-3	.168	34.8	49.9	50.2	13,900	............do............	2,552	5,493	6,430	13,570	3,040	513	3,553	14,900
						Facing: 0.032-inch 24ST aluminum alloy Core: 1/8-inch end-grain balsa								
60-1	.200	57.9	60.1	79.2	9,100	Crimping at corner	3,247	6,810	8,640	18,240	2,881	296	3,177	9,900
60-2	.201	57.9	59.9	77.4	8,100	Buckling	2,969	7,167	8,600	18,150	2,906	315	3,221	10,100
60-3	.207	59.7	60.0	75.4	10,500	Crimping at corner	2,896	6,513	8,350	17,620	2,893	286	3,179	10,300
						Facing: 0.005-inch 24SH aluminum alloy Core: 1/8-inch cellular cellulose acetate								
38-1	.132	16.7	29.9	15.7	28,600	Crimping - good bond	770	2,131	2,050	4,320	2,613	487	2,848	37,600
38-3	.130	16.8	30.0	15.0	33,900	............do............	840	2,986	2,080	4,380	3,330	654	3,162	41,100
39-2	.138	19.5	30.0	28.0	[1]11,800	Separation of facing - good bond	780	2,201	1,960	4,130	1,567	424	1,986	27,400
						Facing: 0.005-inch 24SH aluminum alloy Core: 1/8-inch hard sponge rubber								
34-1	.142	16.6	30.0	16.0	14,500	Crimping at end - good bond	805	2,029	1,900	4,010	2,752	543	2,799	39,700
34-2	.140	16.7	30.0	15.6	22,400	............do............	844	2,082	1,930	4,070	2,886	537	2,857	40,000
34-3	.141	16.7	30.1	16.1	[1]17,900	Dent near end - grew to crimping	889	2,105	1,920	4,040	2,906	551	2,855	40,300
35-1	.142	19.6	30.0	28.0	23,000	Crimping - fair bond	811	1,983	1,900	4,010	1,560	404	1,951	27,700
35-2	.144	19.7	30.0	28.0	26,600	Crimping at end - good bond	762	2,104	1,870	3,960	1,580	436	2,007	28,900
35-3	.141	19.6	30.0	27.7	21,400	............do............	739	2,078	1,920	4,040	1,530	417	1,936	27,300

[1]Points representing these specimens are not shown on figures 8, 9, or 10 because these specimens were defective.

Table 3.--Axial-compression tests of curved panels of sandwich construction having fiberglas facings

Specimen number	Total thickness h	Width a	Length b	Mean radius of curvature r	Facing stress at failure	Type of failure	Data from tests of coupons E_a	E_l	S	Theoretical stresses p_l	p	p_{cr}	$p_{cr.f}$
	Inches	Inches	Inches	Inches	Lb. per sq. in.		1,000 lb. per sq. in.	1,000 lb. per sq. in.	Lb. per sq. in.	Lb. per sq. in.	Lb. per sq. in.	Lb. per sq. in.	Lb. per sq. in.
						Facing: 0.009-inch fiberglas Core: 1/8-inch end-grain balsa							
22-1	0.140	11.5	30.0	10.0	16,650	Facing compression	319	862	1,930	1,781	339	1,930	15,000
22-2	.140	11.5	30.0	10.4	14,000	........do........	313	819	1,930	1,653	322	1,930	15,000
23-1	.142	16.4	30.0	15.4	9,950	Buckling and breaking	322	836	1,900	1,161	170	1,331	10,500
23-2	.144	16.3	29.5	15.6	11,300	End compression	339	870	1,875	1,216	184	1,400	11,200
24-1	.138	25.2	29.5	25.9	6,600	Buckling	338	872	1,955	702	71	773	5,900
24-2	.138	25.1	29.5	28.2	6,450	........do........	344	855	1,955	644	70	714	5,500
24-3	.140	25.5	29.6	29.0	5,900	........do........	351	908	1,930	661	74	735	5,700
25-1	.142	30.7	29.6	40.2	3,800	........do........	352	890	1,900	480	50	530	4,200
25-2	.138	30.9	29.5	39.6	4,150	........do........	349	908	1,955	476	47	523	4,000
25-3	.141	30.8	29.5	41.0	3,450	........do........	338	836	1,915	444	46	490	3,800
47-1	.140	31.8	29.7	37.6	4,900	........do........	325	866	1,930	479	45	524	4,100
47-2	.148	31.8	29.8	38.3	4,700	........do........	304	877	1,825	484	50	534	4,400
47-3	.145	31.6	29.8	36.6	4,850	........do........	326	919	1,860	526	51	577	4,650
48-1	.144	25.6	31.0	35.3	4,800	........do........	250	646	1,875	398	57	455	3,650
48-2	.147	25.1	31.0	37.0	4,750	........do........	293	803	1,835	468	77	545	4,450
48-3	.148	25.3	31.0	39.7	4,300	........do........	293	829	1,825	446	80	526	4,300
49-1	.146	20.4	31.0	38.2	5,050	........do........	305	819	1,850	463	121	584	4,750
49-2	.148	20.4	31.1	40.0	4,850	........do........	307	794	1,825	443	121	564	4,650
49-3	.147	20.5	31.0	36.1	5,400	........do........	303	809	1,835	489	120	609	4,950
50-1	.149	15.4	31.1	39.5	5,450	........do........	250	763	1,810	400	190	590	4,900
50-2	.149	15.3	31.0	37.4	6,450	........do........	276	752	1,810	440	189	629	5,200
50-3	.148	15.1	31.0	39.8	6,000	........do........	271	750	1,825	407	191	598	4,900
51-1	.148	10.3	31.0	32.8	5,450	........do........	250	763	1,825	478	418	896	7,350
51-2	.150	10.4	31.0	37.4	5,100	........do........	276	752	1,800	443	416	859	7,150
51-3	.146	10.2	31.0	32.2	5,750	........do........	271	750	1,850	496	408	904	7,350
52-1	.147	5.3	31.0	40.0	10,300	........do........	305	819	1,835	446	1,674	1,835	15,000
52-2	.147	5.3	31.0	40.0	11,500	........do........	307	794	1,835	440	1,622	1,835	15,000
53-1	.142	3.0	30.5	55.0	11,850	Facing compression	250	646	1,900	252	3,844	1,900	15,000
53-2	.146	2.9	31.0	39.0	16,000	........do........	293	803	1,850	441	5,406	1,850	15,000
53-3	.150	3.0	30.5	50.0	11,100	........do........	293	829	1,800	359	5,505	1,800	15,000
54-1	.147	20.2	31.1	46.9	4,900	Buckling	281	784	1,835	357	119	476	3,900
54-2	.149	20.3	31.1	48.7	4,700	........do........	256	658	1,810	305	102	407	3,350
54-3	.149	20.3	31.1	46.3	4,800	........do........	264	701	1,810	336	108	444	3,700
57-1	.146	7.5	31.4	29.4	7,400	........do........	317	883	1,850	637	889	1,526	12,400
57-2	.146	7.5	31.4	33.5	6,500	........do........	317	883	1,850	559	889	1,448	11,750
57-3	.146	7.5	31.4	43.8	7,850	........do........	317	883	1,850	428	889	1,317	10,700
64-1	.136	70.4	70.5	81.9	1,900	........do........	321	830	1,985	208	8	216	1,650
64-2	.138	70.5	70.5	74.5	1,600	........do........	311	856	1,955	232	9	241	1,850
64-3	.137	70.5	70.5	72.3	1,650	........do........	316	836	1,970	236	8	244	1,850
						Facing: 0.018-inch fiberglas Core: 1/8-inch end-grain balsa							
30-1	.159	15.8	29.5	15.1	13,400	Buckling and breaking	598	1,419	3,395	2,353	389	2,742	12,100
30A-1	.152	16.6	30.8	14.9	11,250	Breaking	531	1,330	3,555	2,080	302	2,382	10,050
30-2	.160	15.7	29.5	15.8	19,900	Separation of facing - poor bond	593	1,329	3,375	2,181	367	2,548	11,300
30A-2	.152	16.4	30.8	14.5	13,200	Breaking	551	1,321	3,555	2,170	301	2,471	10,450
30-3	.159	16.1	30.0	15.7	14,500	Separation of facing - poor bond	590	1,347	3,395	2,190	356	2,546	11,250
30A-3	.152	16.5	30.8	14.8	11,450	Breaking	585	1,495	3,555	2,330	344	2,674	11,300

Sheet 1 of 2

Table 3.--Axial-compression tests of curved panels of sandwich construction having fiberglas facings (Continued)

Specimen number	Total thickness h	Width a	Length b	Mean radius of curvature r	Facing stress at failure	Type of failure	Data from tests of coupons E_a	E_1	S	Theoretical stresses p_1	p	p_{cr}	$p_{cr.f}$
	Inches	Inches	Inches	Inches	Lb. per sq. in.		1,000 lb. per sq. in.	1,000 lb. per sq. in.	Lb. per sq. in.	Lb. per sq. in.	Lb. per sq. in.	Lb. per sq. in.	Lb. per sq. in.
						Facing: 0.018-inch fiberglas **Core: 1/8-inch end-grain balsa**							
31A-1	0.153	37.2	29.8	40.4	3,250	Buckling	573	1,452	3,530	838	70	908	3,850
31-2	.157	30.8	29.6	39.3	4,250	..........do.........	588	1,474	3,440	902	102	1,004	4,400
31-3	.156	31.2	29.6	38.6	[1]3,600	Separation of facing - poor bond	598	1,235	3,460	843	82	925	4,000
31A-3	.147	31.6	29.8	40.0	4,000	Buckling	573	1,395	3,675	797	80	877	3,600
62-1	.150	41.4	39.8	49.0	4,150	..........do.........	609	1,601	3,600	733	56	789	3,300
62-2	.148	41.4	39.8	45.5	3,600	..........do.........	606	1,556	3,650	766	53	819	3,350
62-3	.149	41.3	40.0	45.6	4,650	..........do.........	611	1,510	3,625	761	52	813	3,350
						Facing: 0.033-inch fiberglas **Core: 1/8-inch end-grain balsa**							
32-1	.182	16.6	30.0	15.2	14,050	Breaking	837	1,862	5,440	3,626	606	4,232	11,650
32-2	.182	16.6	30.0	15.1	13,950	..........do.........	856	1,969	5,440	3,796	641	4,437	12,250
32-3	.175	16.5	30.0	15.4	12,500	..........do.........	896	1,932	5,655	3,627	589	4,216	11,200
33-1	.181	31.4	29.5	37.7	[1]2,150	Separation of facing - poor bond	805	1,672	5,470	1,351	148	1,499	4,100
33A-1	.172	31.5	29.8	40.0	4,250	Buckling	950	1,749	5,755	1,345	139	1,484	3,850
33-2	.183	31.2	29.6	38.4	[1]3,050	Separation of facing - poor bond	857	1,740	5,410	1,412	159	1,571	4,350
33A-2	.173	31.8	29.9	40.4	4,750	Buckling	944	2,271	5,725	1,521	179	1,700	4,450
33-3	.183	31.7	29.6	37.4	[1]1,150	Separation of facing - poor bond	829	1,803	5,410	1,451	160	1,611	4,450
33A-3	.175	31.9	29.9	38.4	4,500	Buckling and breaking	857	2,075	5,655	1,474	166	1,640	4,350
						Facing: 0.048-inch fiberglas **Core: 1/8-inch end-grain balsa**							
63-1	.196	60.9	60.6	62.5	2,900	Buckling	1,143	2,333	7,345	1,242	64	1,306	2,650
63-2	.195	60.9	60.6	62.3	2,650	..........do.........	1,154	2,387	7,385	1,260	65	1,325	2,700
63-3	.196	60.8	60.6	60.9	2,850	..........do.........	1,273	2,513	7,345	1,396	69	1,465	3,000
						Facing: 0.009-inch fiberglas **Core: 1/8-inch cellular cellulose acetate**							
40-1	.143	16.6	29.8	15.2	[1]8,350	Crimping - poor bond	349	840	1,890	1,236	169	1,405	11,150
40-2	.149	16.0	23.1	15.6	[1]8,200	Separation of facing - poor bond	322	790	1,810	1,169	201	1,370	11,350
40-3	.145	16.3	29.8	14.9	12,650	Crimping	331	828	1,860	1,236	178	1,414	11,400
41-1	.143	31.9	29.8	37.6	5,500	Buckling	215	733	1,890	366	39	405	3,200
41-2	.143	31.4	29.6	38.8	4,750	..........do.........	278	842	1,890	433	46	479	3,800
41-3	.144	31.6	28.9	38.5	5,200	..........do.........	290	883	1,875	459	50	509	4,050
						Facing: 0.009-inch fiberglas **Core: 1/8-inch hard sponge rubber**							
36-1	.139	16.5	29.6	15.6	[1]3,650	Separation of facing - poor bond	310	744	1,940	1,038	146	1,184	9,150
36-2	.137	16.5	29.5	15.7	6,300	Crimping	276	790	1,970	988	150	1,138	8,650
36-3	.130	16.4	29.5	15.7	[1]8,750	Separation of facing - fair bond	302	795	2,075	984	138	1,122	8,100
37-1	.131	31.3	29.6	37.2	3,650	Buckling	297	744	2,060	402	35	437	3,200
37-2	.136	31.6	29.6	36.9	4,100	..........do.........	333	885	1,985	485	44	529	4,000
37-3	.134	31.6	29.5	35.6	4,400	..........do.........	297	689	2,015	413	33	446	3,300

[1]Points representing these specimens are not shown on figures 8, 9, or 10 because these specimens were defective.

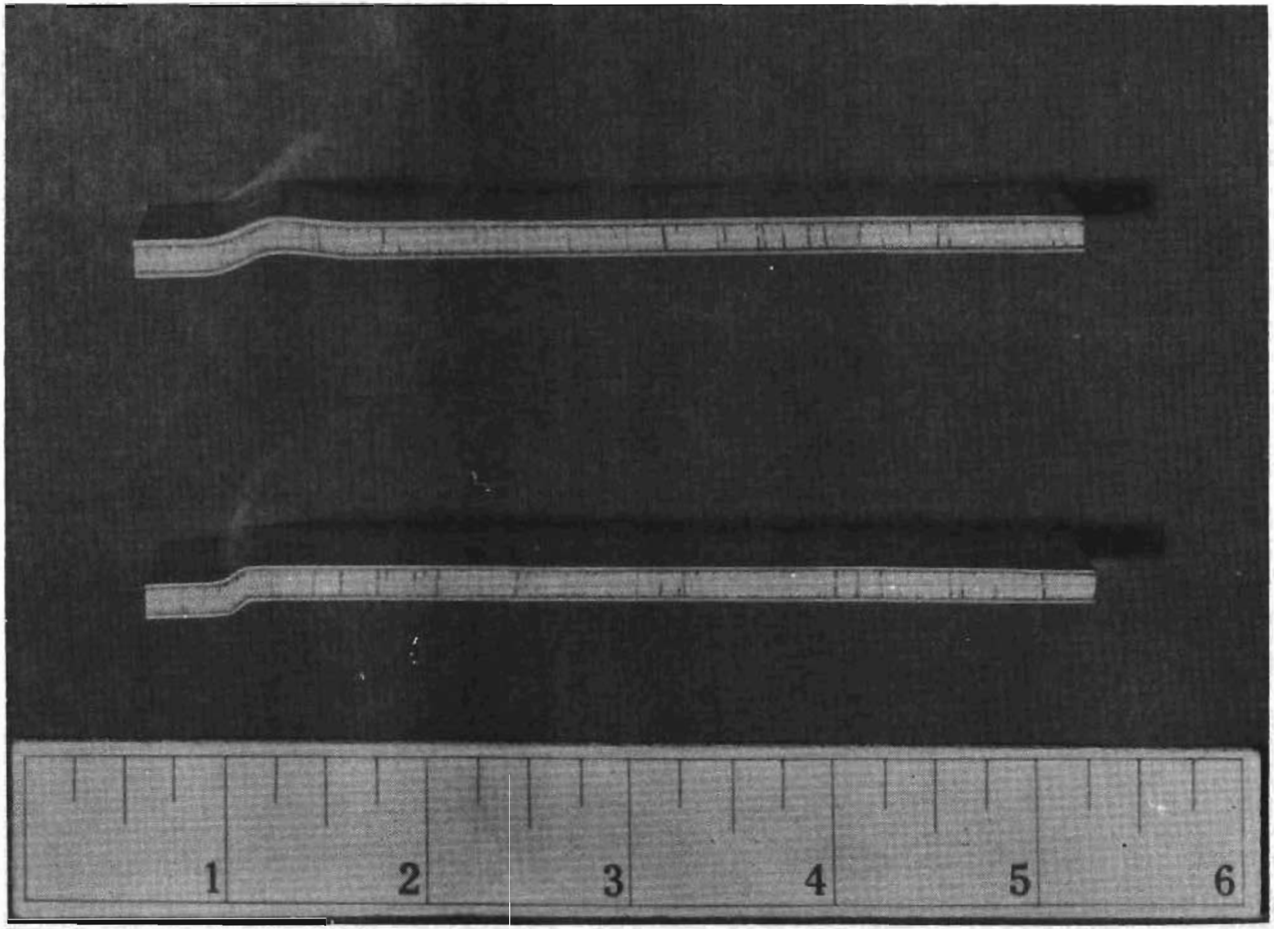

Figure 1.--Edge view of crimping failure of minor specimens. The appearance of this failure in the curved panels is similar.

Z M 71924 F

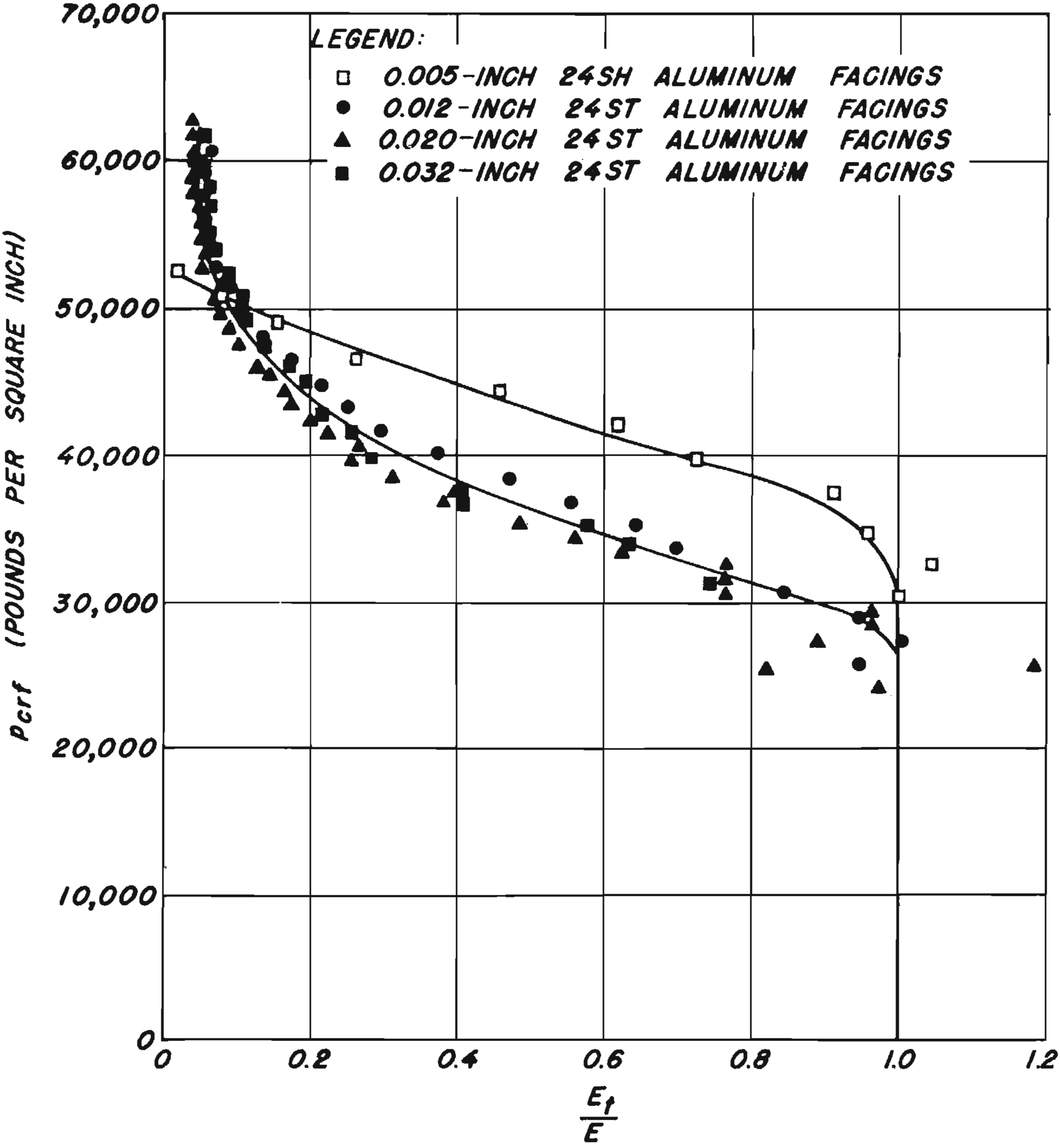

Figure 2.--Variation of $\frac{E_t}{E}$ with facing stress σ_{crf} for aluminum facing materials.

Z M 70846 F

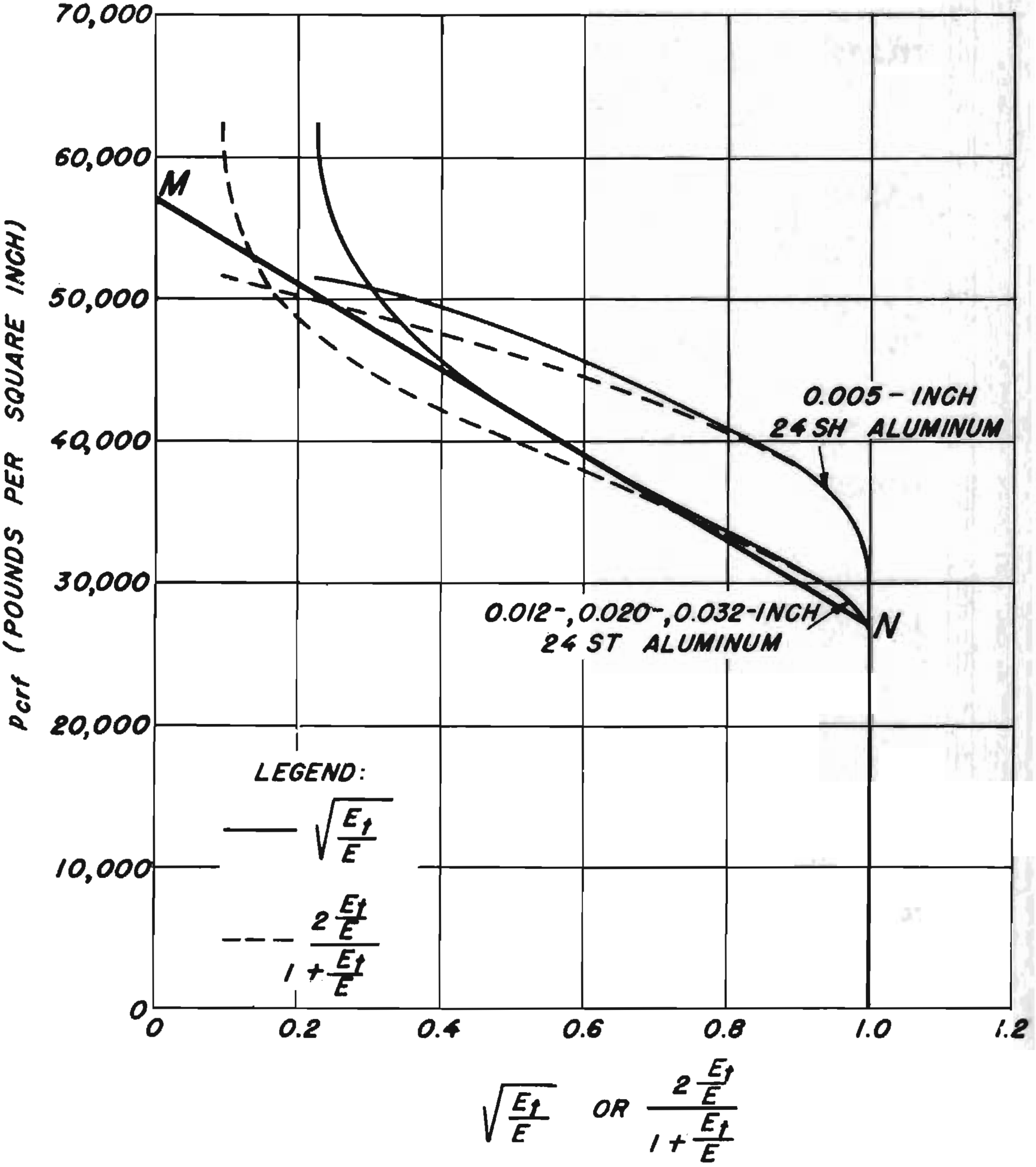

Figure 3.--Variation of $\sqrt{\frac{E_t}{E}}$ or $\frac{2\frac{E_t}{E}}{1+\frac{E_t}{E}}$ with facing stress, σ_{crf}, for aluminum facing materials.

Z M 70847 F

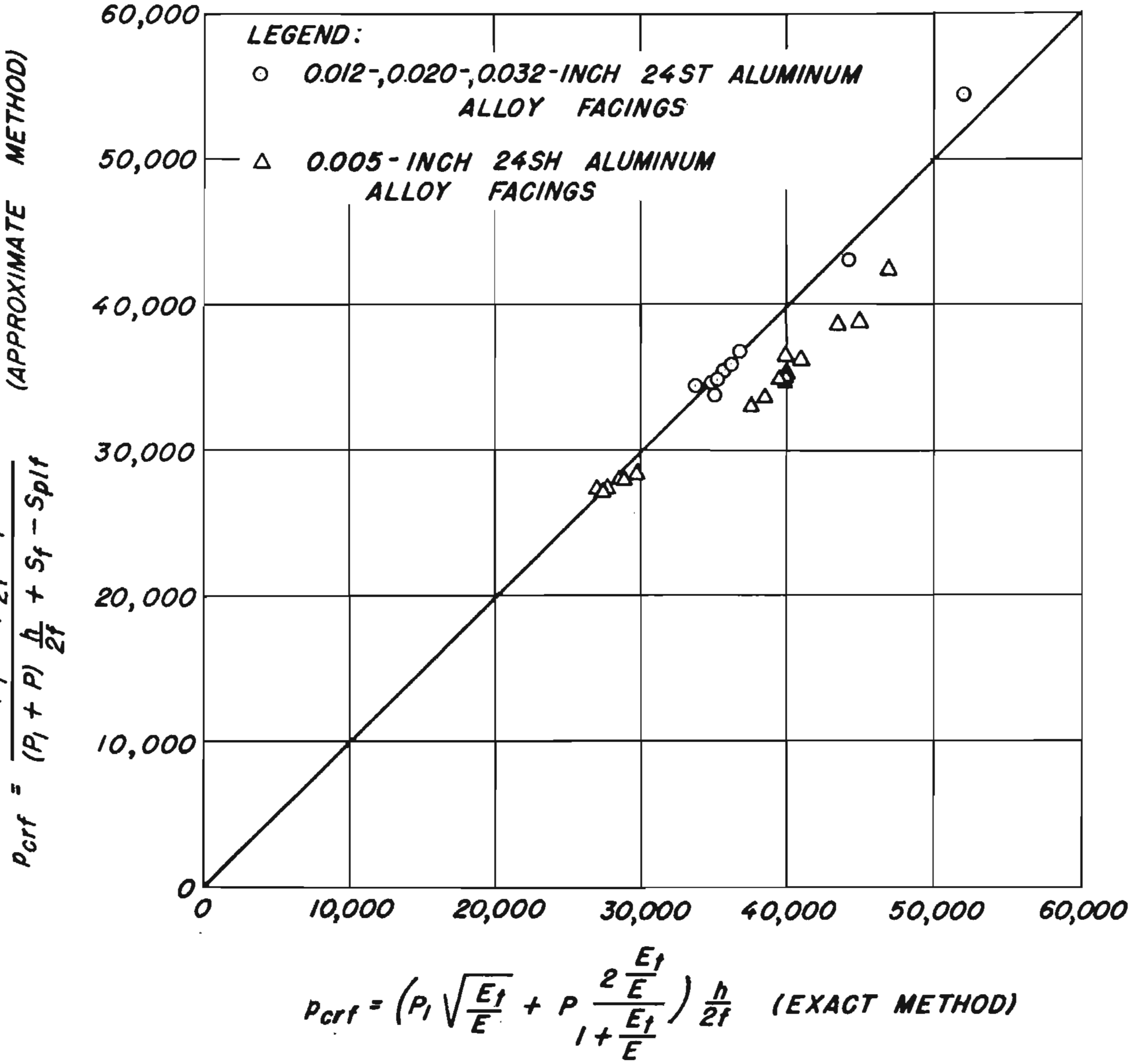

Figure 4.--Comparison of the approximate with the exact method for computing the buckling stress beyond the proportional limit.

Z M 70848 F

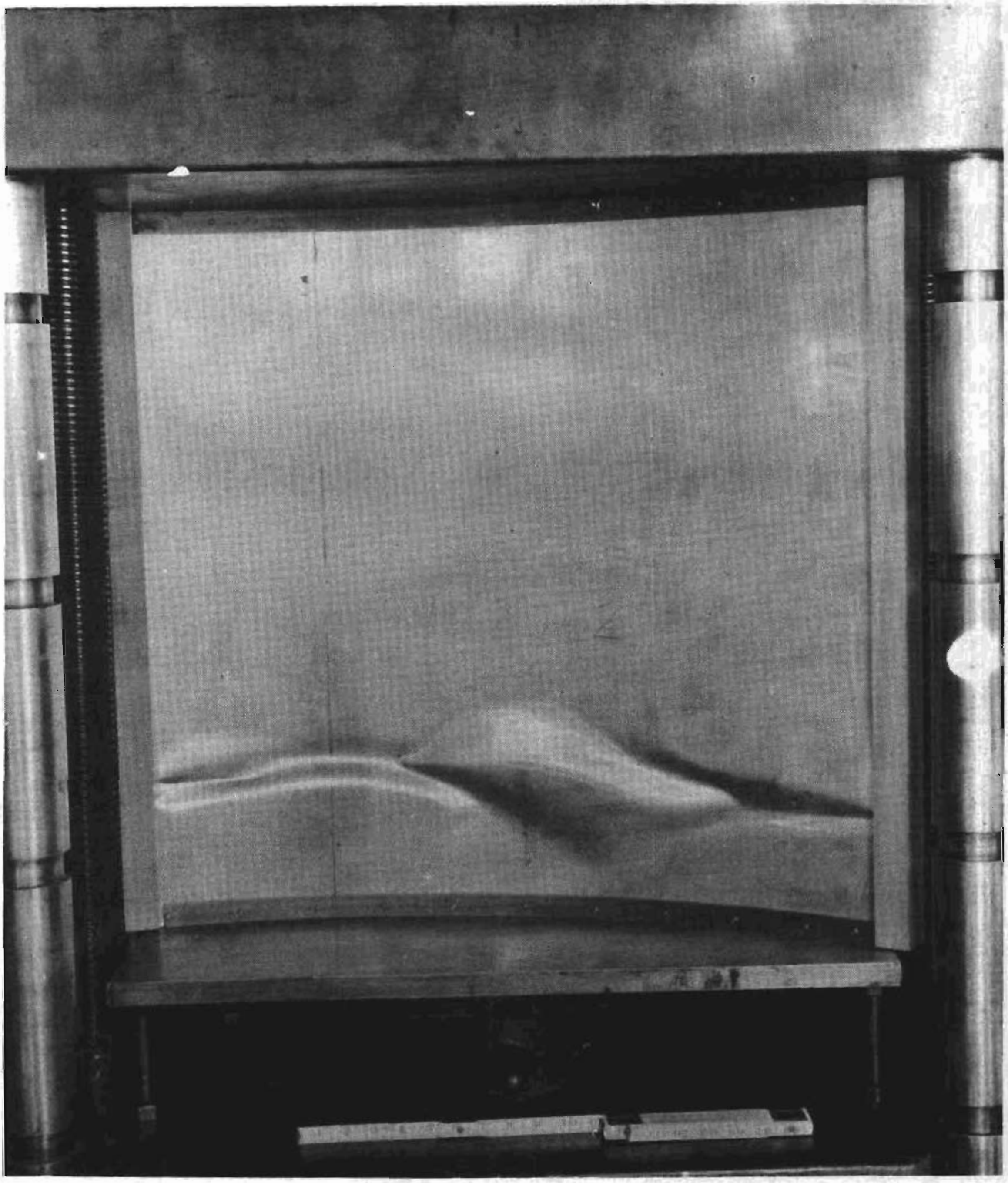

Figure 5.--Crimping type of failure of a panel having 0.012-inch aluminum facings on a 1/8-inch balsa core.

Z M 71925

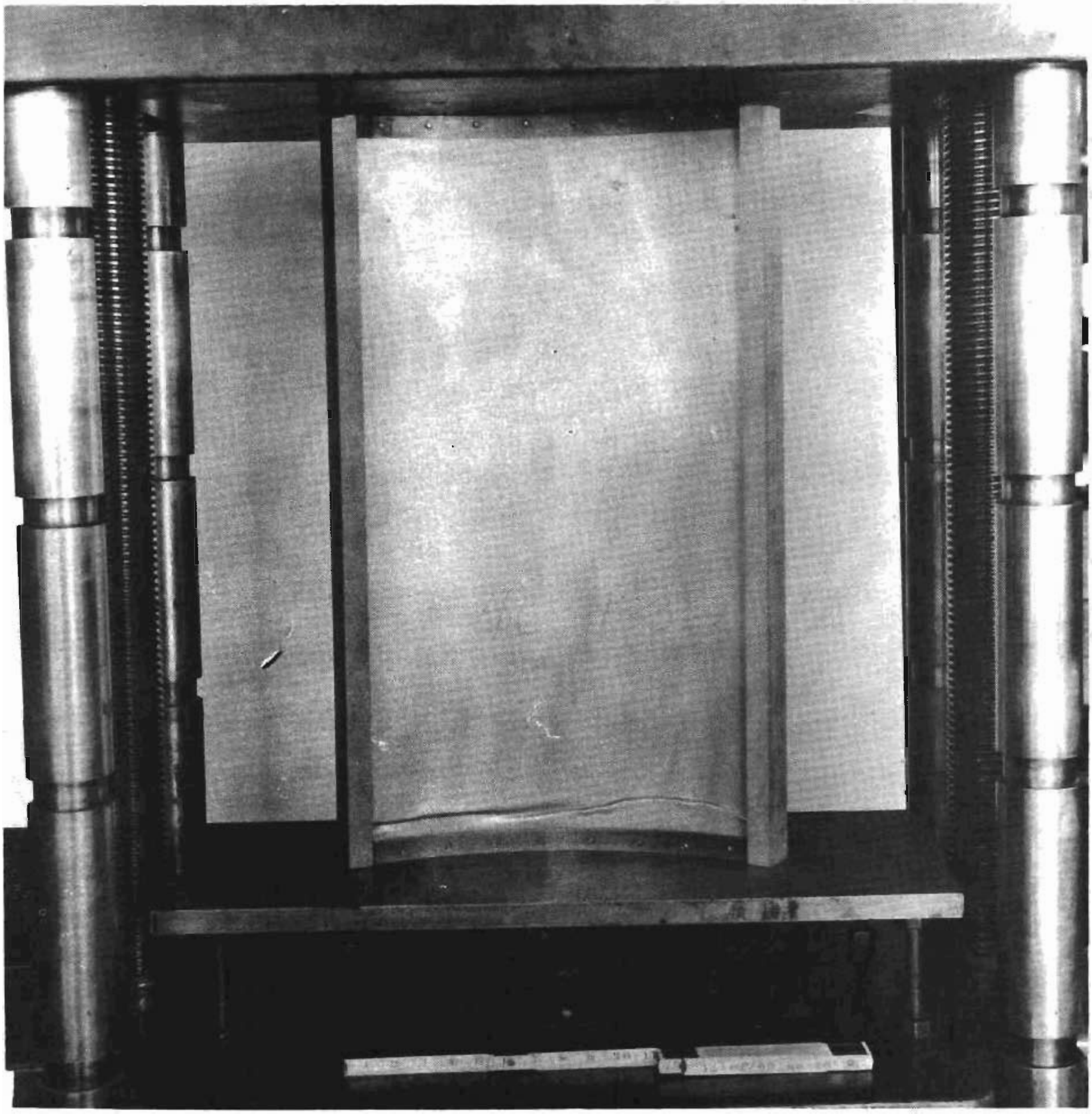

Figure 6.--Crimping type of failure of a panel having 0.005-inch aluminum facings on a 1/8-inch balsa core.

Z M 71926 F

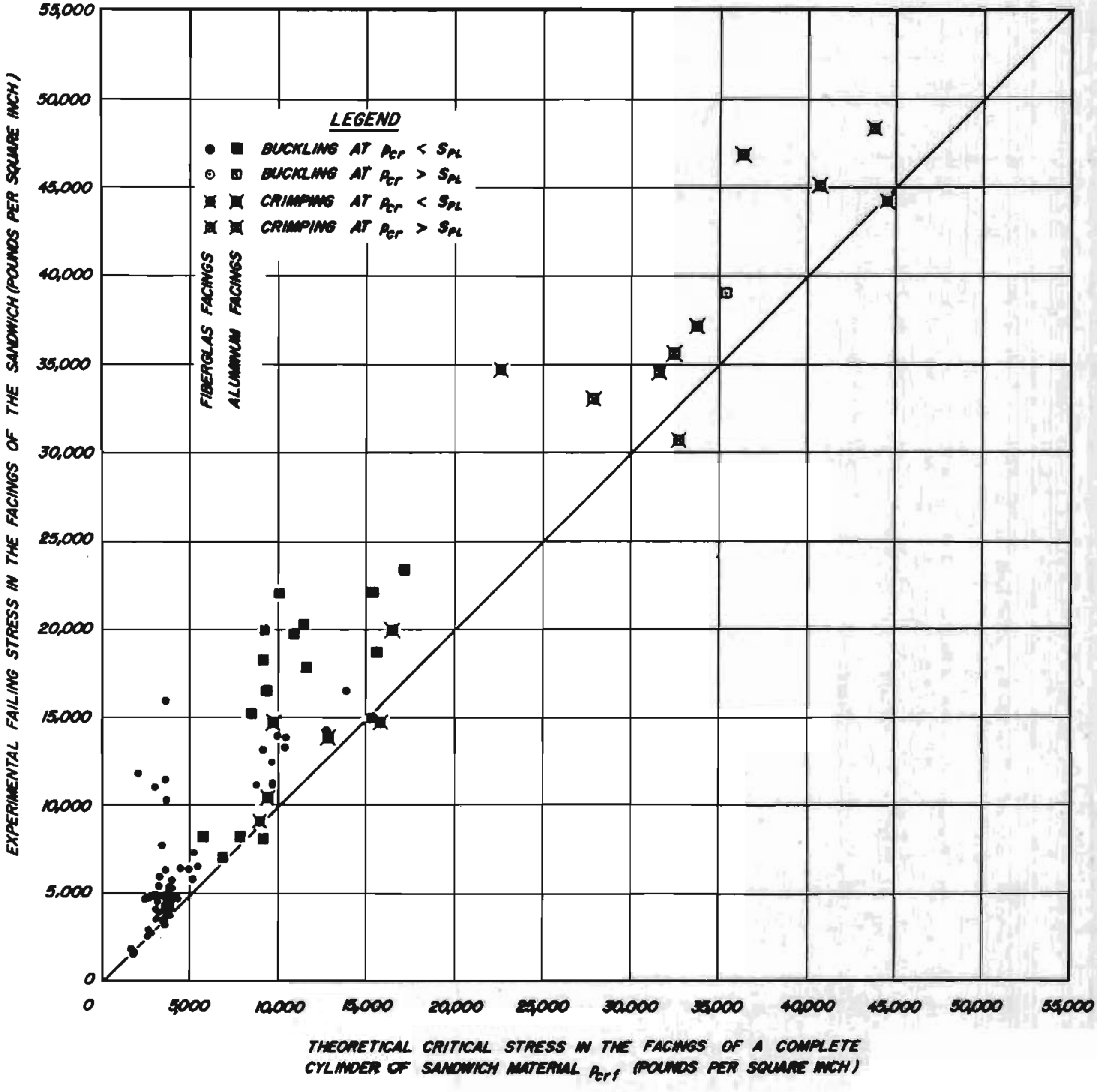

Figure 7.--A comparison of experimental failing stress of a curved plate with the theoretical critical stress of a complete cylinder of sandwich construction under axial compression loads. The sandwiches were constructed of aluminum or fiberglas facings on end-grain balsa cores.

Z M 70849 F

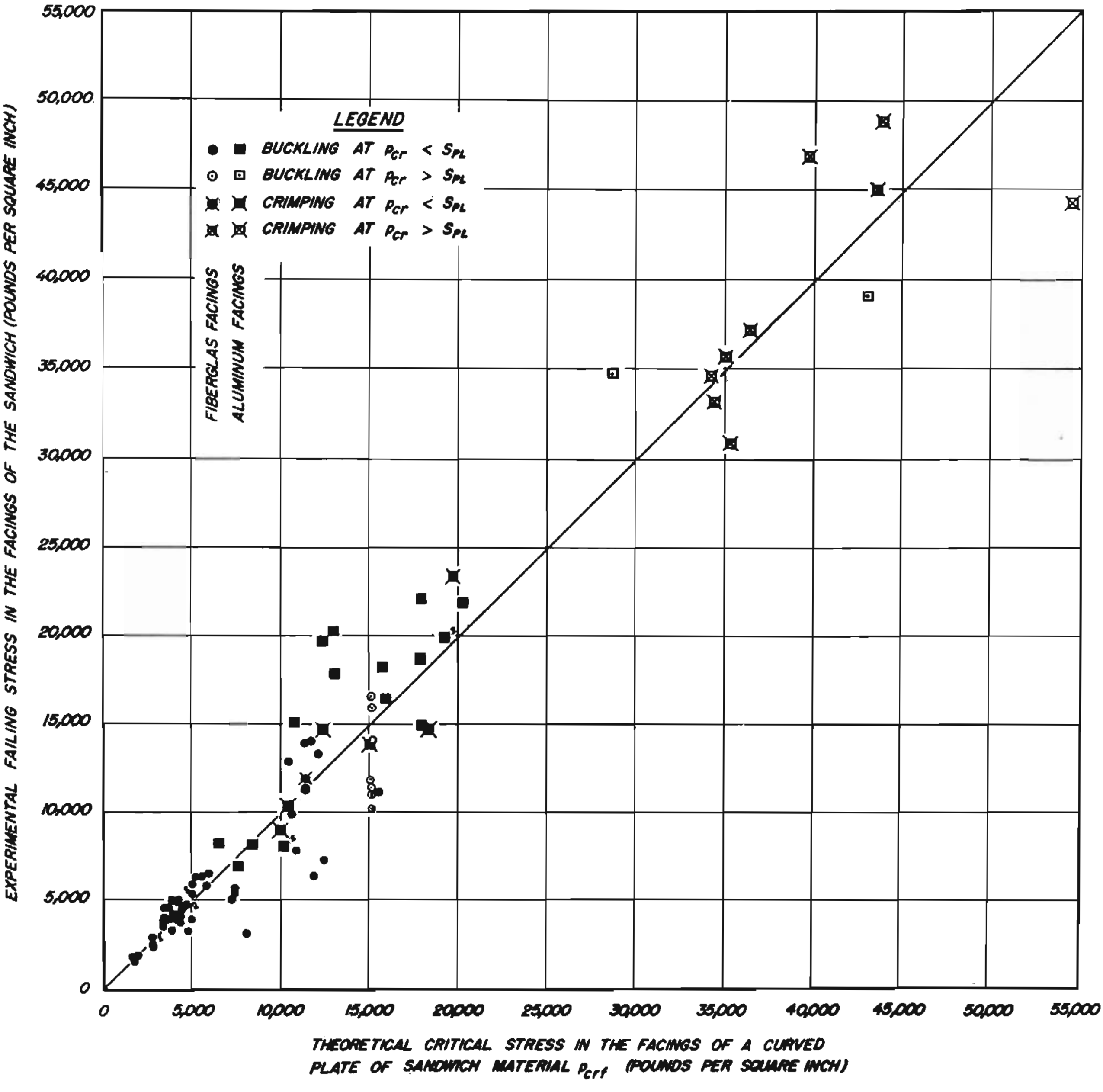

Figure 8.--A comparison of experimental with theoretical critical stresses of curved panels of sandwich construction under axial-compression loads. The sandwiches were constructed of aluminum or fiberglas facings on end-grain balsa cores.

Z M 70850 F

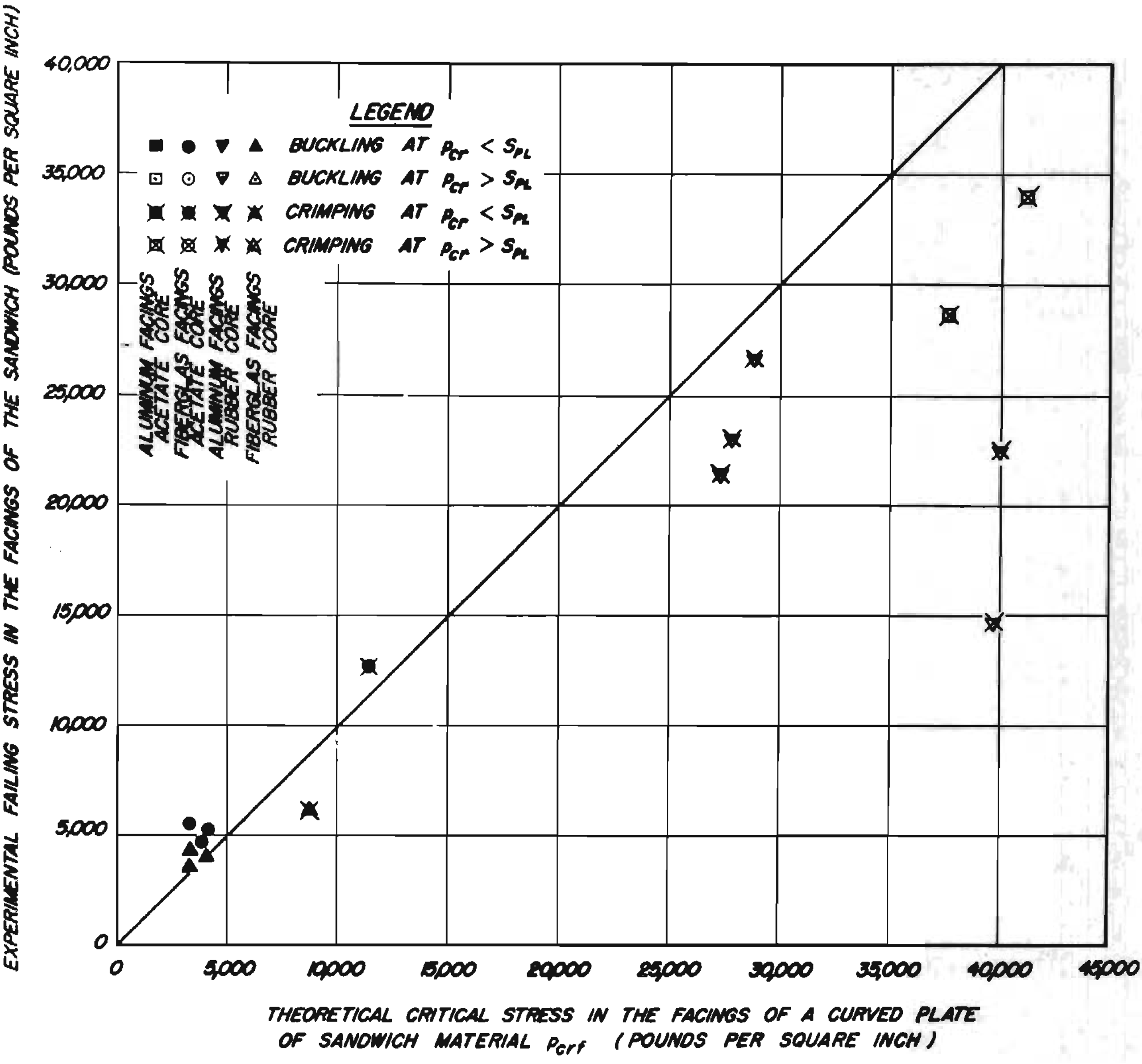

Figure 9.--A comparison of experimental with theoretical critical stresses of curved panels of sandwich construction under axial-compression loads. The sandwiches were constructed of aluminum or fiberglas facings on cellular cellulose acetate or hard sponge rubber cores.

Z M 70851 F

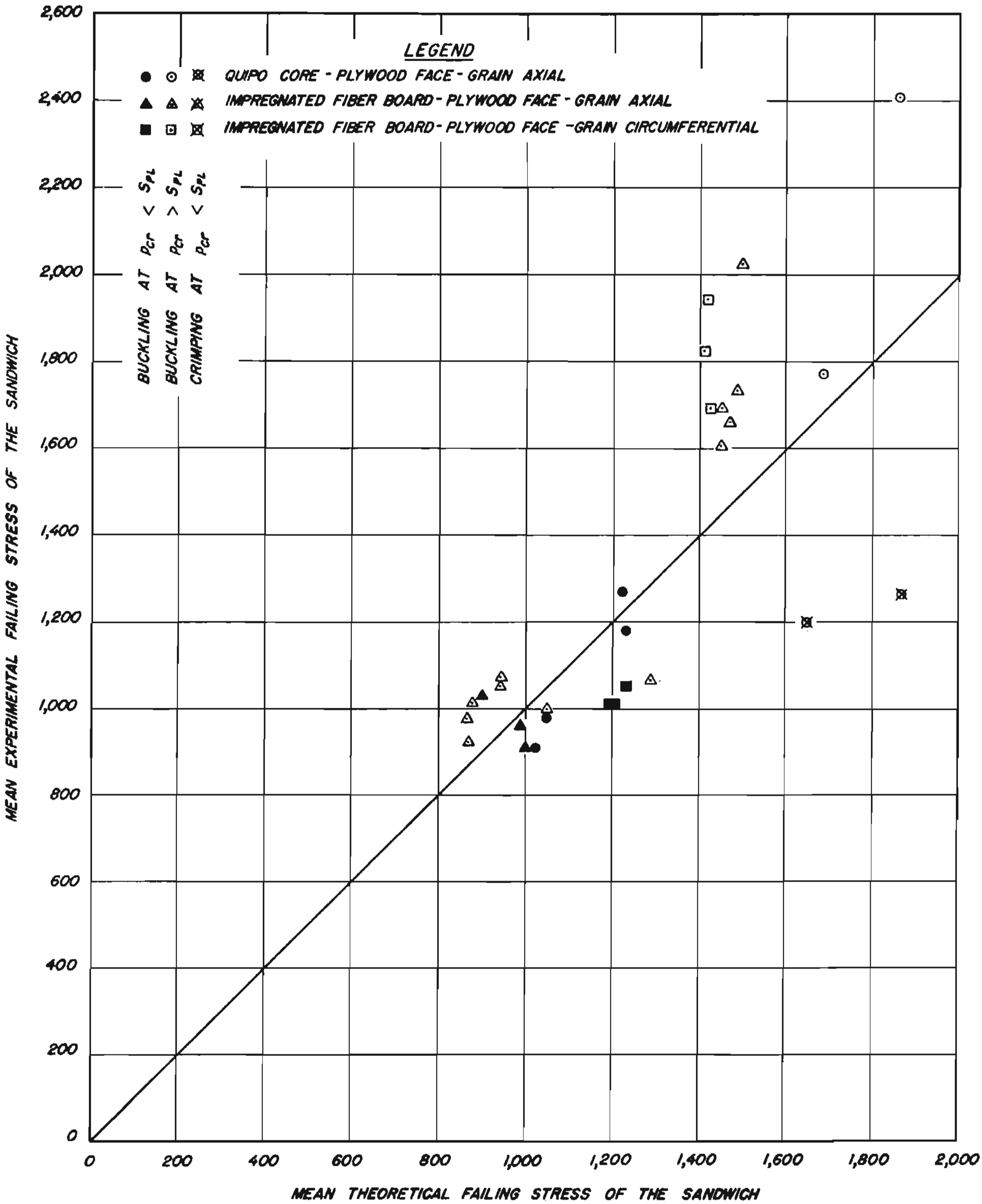

Figure 10.--A comparison of experimental with theoretical critical stresses of plywood-faced curved sandwich panels under axial-compression loads.

Z M 70852 F

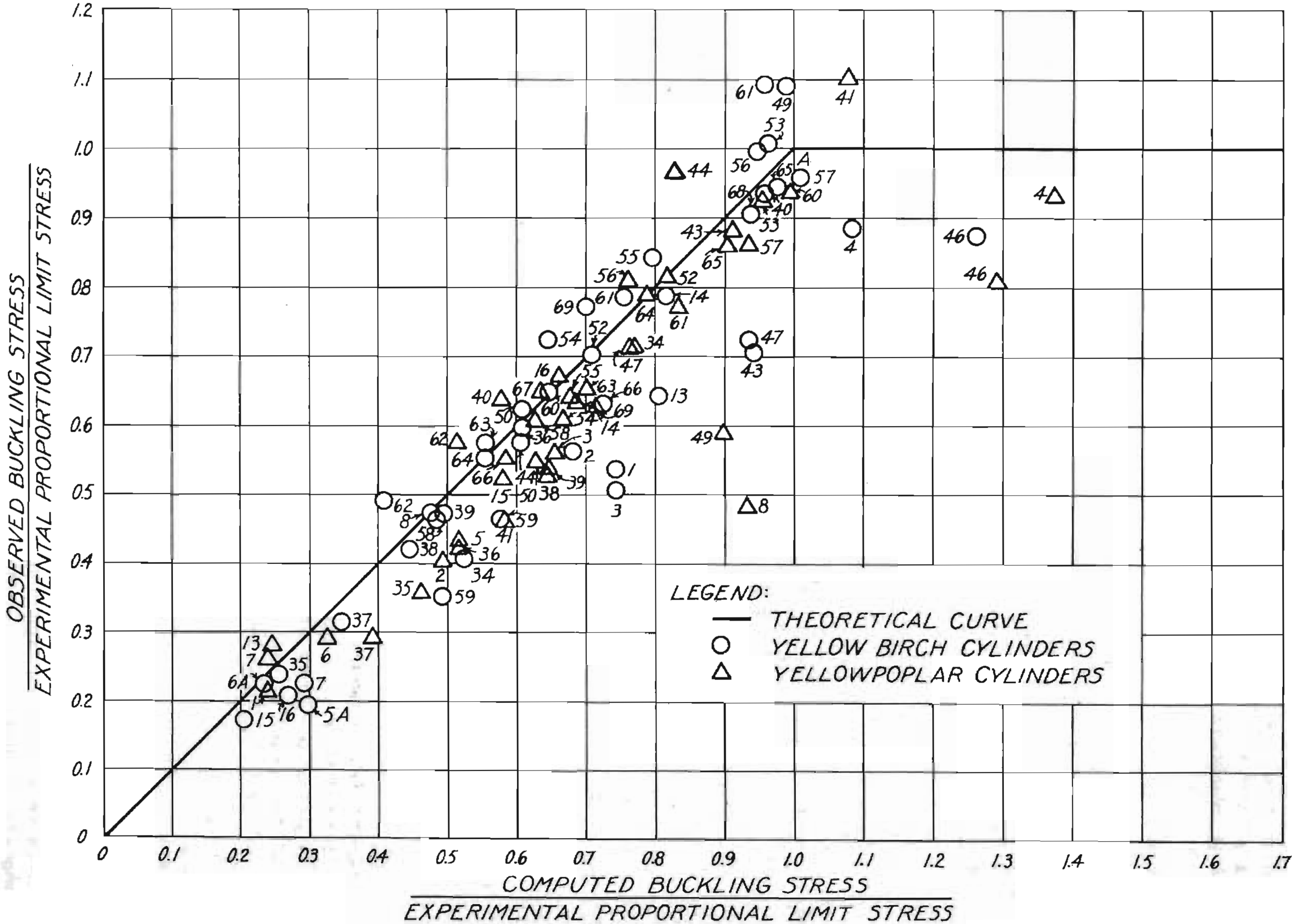

Figure 11.--Comparison of test and computed buckling stress of plywood cylinders under axial-compression loads. Direction of grain of face plies either axial or circumferential. (From fig. 3 of Report 1322)

M 47409 F

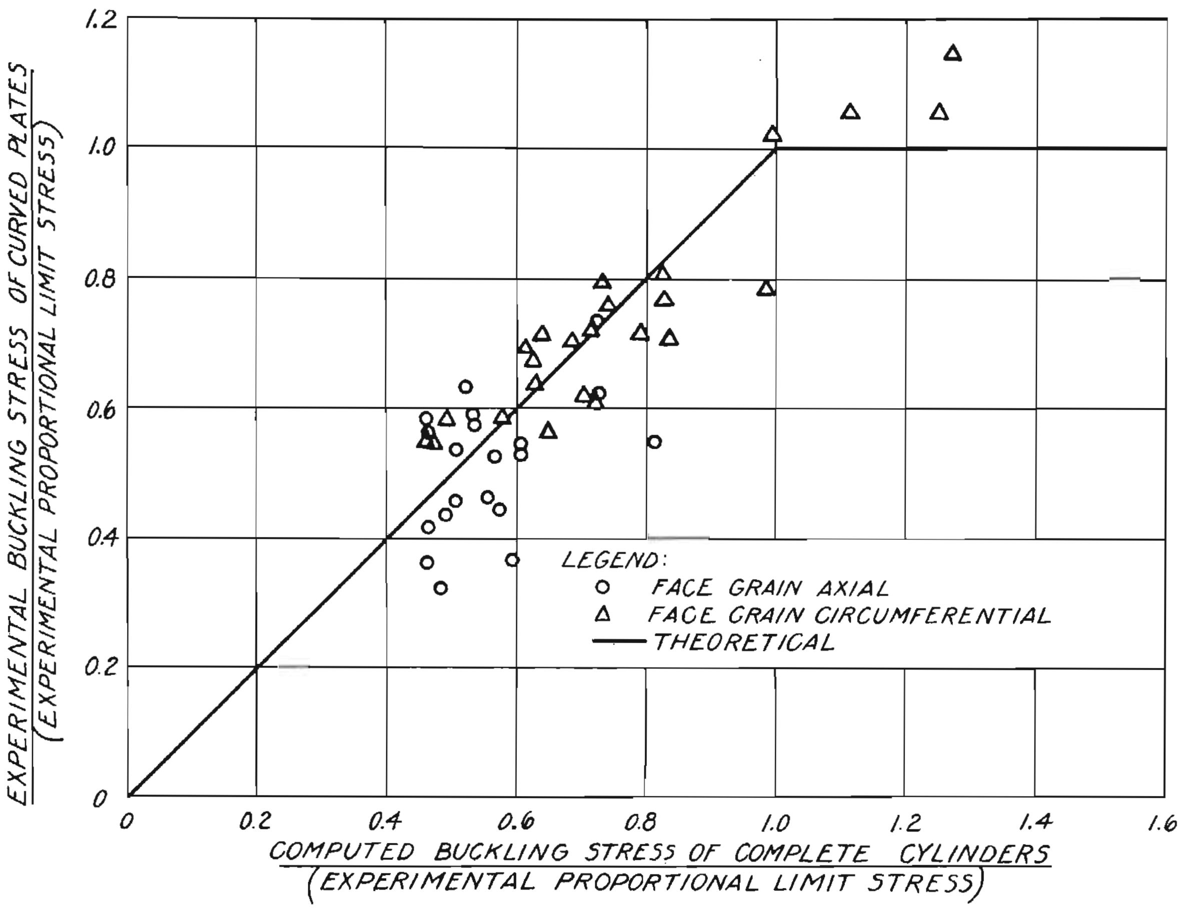

Figure 12.--Comparison of test and computed buckling stress of curved plywood plates in axial compression. All plates were 3-ply with faces of 0.010-inch and cores of 0.025-inch yellow birch veneers, and were formed to a 5-1/4-inch radius. (From fig. 9 of Report 1508).

Z M 54167 F