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A Single Stiffener Parallel to Stress
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BUCKLING OF STIFFENED, FLAT, PLYWOOD PLATES IN COMPRESSION[1]

A Single Stiffener Parallel to Stress

Face Grain of Plywood at 45° to Its Edges

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Summary and Conclusions

The results of 38 tests of plywood plates subjected to edgewise compression are presented. The plates were stiffened by a single stiffener bisecting the plates and parallel to the direction of the stress. The face grain of each plate was inclined at an angle of 45° to its edges. An empirical analysis based on the analysis given in a previous report (Forest Products Laboratory Report No. 1553-B) was made and the following approximate formula for the critical stiffness of the stiffeners was obtained:

$$(EI)_{scr} = \frac{1.2\ hab^2\ (p_{cr2}-p_{cr1})}{8\pi^2}$$

The tests show that when a plywood plate is stiffened in the manner described, its critical buckling load may be increased about four times. The weight of the stiffener is found to be small compared to that of the plate.

Introduction

In closely designed plywood structures, such as aircraft, the great shear strength and rigidity of 45° plywood can economically be employed in places where the shear stress is high. Such a panel may, however, have to carry considerable compressive stress. A panel designed for the required shear stresses may not be adequate for the compressive stresses to which it will be subjected. This difficulty may be overcome by fixing a stiffener along the centerline of the panel and parallel to the direction of the

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compressive stress. Good design requires that the stiffener be just heavy enough to cause the panel to act essentially as two independent panels, so that it will thus greatly increase the critical buckling stress, while at the same time it will function as a column that will carry part of the compressive load. To determine the minimum size, and to present a method of computing a recommended adequate-size stiffener for this purpose, the study presented here was undertaken.

The test specimens were proportioned so that the ratios of the computed critical buckling stress of the half panel to the proportional-limit stresses of the material were 0.2, 0.4, and 0.6. A stiffener was glued to the vertical center line on one face of each panel. The specimens were tested in vertical edgewise compression.

Description of Specimens

The test plates were cut from 2- by 6-foot panels of three- and five-ply aircraft-grade yellow birch and yellow-poplar plywood. The grain direction of each ply was perpendicular to the adjacent plies.

In the tests, the load was applied to the ends of the plates. The direction of the grain of the plies was at an angle of 45° to the direction of the load.

The stiffeners were made of Sitka spruce 3/8 inch by 1-1/2 inches and fastened at one of their 3/8-inch edges to the panels by means of a cold-setting urea-formaldehyde glue, at a pressure of about 100 pounds per square inch.

The plates and stiffeners were stored in a conditioning room at 65 percent relative humidity and at a temperature of 70° F. both before and after assembly and until time of test.

The stiffened plywood plate is shown in figure 28 fitted in the test apparatus before clamping.

Test Procedure

The apparatus used and the test procedure were identical to those described in Forest Products Laboratory Report No. 1553-B, with the exception of the method for determining the critical load.

For the latter purpose two lateral-deflection dials were used as shown in figure 29. One dial was placed at the midpoint of the stiffener and plate and the other at one of the quarter points of the plate. These points were near the peaks of the buckles. The former was so placed in the event that the plate and stiffener buckled in a single half wave, and the latter in the event that the stiffener was stiff enough and each half plate buckled in a full wave. The critical loads were determined from the points of inflection of the load-lateral deflection curves, as shown in figure 30. This method is described in Forest Products Laboratory Report No. 1525-A.

Notation

The major symbols used in this report are as follows:

- a = the length of the loaded sides of a plywood plate.
- b = the length of the unloaded sides of a plywood plate.
- b' = the half-wave length of a buckled surface in the case of an infinitely long plate.
- d = the depth of the stiffener, measured perpendicular to the face of the plywood.
- h = the thickness of the plywood.
- t = the width of the stiffener, measured parallel to the face of the plywood.
- E_{fw} = effective modulus of elasticity of the plywood in bending, measured parallel to the grain direction of the face plies.
- E_{fx} = effective modulus of elasticity of the plywood in bending, measured perpendicular to the grain direction of the face plies.
- E_L = modulus of elasticity of wood in the direction parallel to the grain. $20/21\ (E_{fw} + E_{fx})$
- E_s = modulus of elasticity of the stiffener in the direction parallel to the grain, as determined from a compression test.
- $(EI)_s$ = a measure of the stiffness of the stiffener. Product of modulus of elasticity and moment of inertia about the neutral axis of the stiffener.
- $(EI)_{scr}$ = the minimum stiffness of stiffener necessary to stiffen effectively a plywood plate.
- K_c = coefficient in formula, $p_{cr} = K_c E_L \frac{h^2}{a^2}$
- $K_{c\infty}$ = value of the coefficient K_c for an infinitely long plate.
- P_{cr} = compressive load at which buckling occurs.
- p_{cr} = compressive stress at which buckling occurs.
- p_{cr2} = p_{cr} for a stiffened plywood plate with adequate stiffener.
- p_{cr1} = p_{cr} for a plywood plate with stiffener removed.

Explanation of Tables

Table 12 lists the construction, dimensions, and elastic properties of the plywood plates tested and the width and the compressive modulus of elasticity of the stiffeners. Columns 1 through 6 contain plate designation, species, constructions, and dimensions. Columns 7 and 8 contain the effective modulus of elasticity of the plywood in bending determined parallel and perpendicular, respectively, to the direction of the grain of the face plies. Column 9 contains the values of the modulus of elasticity of wood in the direction parallel to the grain as computed by the equation: $E_L = \frac{20}{21}(E_{fw} + E_{fx})$. The values in columns 10 and 11 are for the particular plywood and are obtained from the curves of figure 17 of Forest Products Laboratory Report No. 1553-A. Column 12 contains the width of the stiffeners. Column 13 shows the values of the modulus of elasticity of the stiffener material parallel to the grain as determined from a compression coupon cut from one end.

Table 13 lists the data obtained from tests of the various test specimens. Column 1 lists the successively reduced stiffener depth. Column 2 lists the loads carried by the stiffeners when the plates buckled. These values were computed by equation 68 of Forest Products Laboratory Report No. 1553-B. Column 3 lists the gross critical buckling load of the specimen. Column 4 contains the values for the net critical stress of the plywood plate only, obtained by subtracting the stiffener load from the gross critical load and dividing by the bearing area of the plate. Column 5 contains the values for the stiffness of the stiffener, which were obtained by $(EI)_S = \frac{E_s\ td^3}{12}$. Columns 6 and 7 contain values of the coordinates of figure 31, in which the abscissas x in column 6 were derived by the formula $\frac{8\pi^2\ (EI)_s}{hab^2\ (p_{cr2} - p_{cr1})}$, and the ordinates y in column 7 were given by the

formula $\frac{p_{cr} - p_{cr1}}{p_{cr2} - p_{cr1}}$

where p_{cr} = observed critical stress on plate for any size stiffener.

p_{cr1} = observed critical stress on plate with no stiffener.

p_{cr2} = observed average critical stress on plates for which the stiffener is stiff enough to remain straight when the plate buckles.

Analysis of Results

It seems impractical to make a mathematical analysis similar to that given in Report No. 1553-B for plywood plates having the face grain inclined at an angle of 45° to their edges. The data obtained are, therefore, analyzed by means of an empirical method. The method consists of plotting the results to such coordinates that all of the plotted points will fall on a single curve. The ordinate chosen is:

$$\frac{p_{cr} - p_{cr1}}{p_{cr2} - p_{cr1}} \tag{71}$$

in which

p_{cr} = the critical stress of the plate stiffened by a stiffener.

p_{cr1} = the critical stress of the plate with the stiffener removed.

p_{cr2} = the critical stress of the plate stiffened with a very stiff stiffener, i.e., the critical stress of half of the plate divided in the direction of the stiffener.

This parameter insures that all of the plotted points will lie between zero and one, because if the stiffener has no stiffness $p_{cr} = p_{cr1}$, and if the stiffener has infinite stiffness $p_{cr} = p_{cr2}$. If the stiffener is progressively increased in stiffness, its critical stiffness is defined as the least value which causes p_{cr} to equal p_{cr2}.

The abscissa chosen was suggested by the analysis in Report No. 1553-B, omitting a term in the denominator that takes into account the interaction of the plate with the stiffener, and is:

$$\frac{8\pi^2 (EI)_s}{hab^2 (p_{cr2} - p_{cr1})} \tag{72}$$

in which

$(EI)_s$ = the stiffness added to the plate by the stiffener.

h = thickness of plate.

a = width of plate, perpendicular to stiffener and direction of stress.

b = length of plate, parallel to stiffener and direction of stress.

It was also assumed that:

$$(EI)_s = \frac{E_s \; td^3}{12}$$

in which

E_s = modulus of elasticity of the stiffener in the direction of its length.
t = width of stiffener, parallel to surface of the plate.
d = depth of stiffener, perpendicular to surface of the plate.

This assumption is approximately correct because the modulus of elasticity of the plate at 45° to the direction of the face grain is low compared to that of the stiffener, and, therefore, the interaction between plate and stiffener can be neglected.

The values of the parameters (71) and (72) were computed for each plate from the data obtained by test. The load supported by the stiffener at the critical load of the plate or combination of plate and stiffener was computed by means of the mean strain in the stiffener at the critical load and the cross-sectional dimensions and the modulus of elasticity of the stiffener. This load was subtracted from the critical load and the remainder divided by the cross-sectional area of the plate to obtain the critical stress of the plate. In general, the value of p_{cr1} was obtained from a similar test of the plate after the stiffener was removed. The value of p_{cr2} was obtained from the plates in which the stiffener did not buckle. These plates are referred in column 4, table 13, to footnote 1 of the table. An average value from these plates was used.

The values of the parameters are plotted in figure 31. The points associated with the plates in which the stiffeners buckled are plotted by open circles. The points associated with the plates in which the stiffeners did not buckle are plotted by solid circles. The plotted points roughly follow the curve and the straight line drawn in the figure. It is evident that the critical stiffness of the stiffener is given by the equation:

$$\frac{8\pi^2 \; (EI)_{scr}}{hab^2 \; (p_{cr2} - p_{cr1})} = 1.2 \qquad (73)$$

This formula is empirical and rests upon the tests reported. The plates tested were clamped at their loaded edges and were of such dimensions that they buckled into a single half wave. However, it is assumed that equation (73) is approximately correct if it is applied to plates having other edge conditions and dimensions.

Table 12.--The construction, dimensions, elastic properties, and constants of test plates and stiffeners[1]

Panel No.	Species	No. and thickness of plies	h	b	a	E_{fw}	E_{fx}	E_L	$\frac{b'}{a}$	$(K_c)_\infty$	Stiffener Width: t	Stiffener E_s
1	2	3	4	5	6	7	8	9	10	11	12	13
			Inch	Inches	Inches	1,000 p.s.i.	1,000 p.s.i.	1,000 p.s.i.			Inch	1,000 p.s.i.
3xhl-2	Yellow birch	3 - 1/16"	0.173	22.12	16.895	2,173	187.4	2,248	0.848	0.872	0.390	1,810
3xhl-4	Yellow birch	3 - 1/16"	.176	17.63	13.275	2,173	187.4	2,248	.848	.872	.389	1,930
6xJ2	Yellow-poplar	3 - 1/16"	.181	25.59	22.000	1,790	120.0	1,819	.821	.786	.378	1,680
13xJ2	Yellow birch	5 - 1/48"	.108	31.62	22.000	1,690	525.0	2,109	.968	1.444	.375	1,680
13xh-1	Yellow birch	5 - 1/48"	.102	15.97	10.650	1,690	525.0	2,109	.968	1.444	.388	1,880
13xh-2	Yellow birch	5 - 1/48"	.101	22.12	16.900	1,690	525.0	2,109	.968	1.444	.388	1,880

[1]Depths of stiffeners are given in table 13.

Table 13.--The effect of the stiffener on the critical buckling load of the plywood plates (face grain 45° to the direction of the load)

Stiffener		Plate		Stiffener effect		
Depth	Load	Gross critical load	Net critical stress	Stiffness	Coordinates	
(d)	(P)	(L)	(p_{cr})	$(EI)_s$	(X)	(Y)
1	2	3	4	5	6	7
Inch	Pounds	Pounds	P.s.i.			
			3xh1-2			
1.007	997	4,700	[1]1,239	60,070	3.55	1.01
.750	726	4,400	[1]1,230	24,820	1.47	1.00
.500	355	2,900	852	7,350	.44	.59
.250	138	1,500	456	920	.05	.17
.000	0	900	301	....	.....	......
			3xh1-4			
1.004	1,050	5,700	[1]1,936	63,320	4.88	1.05
.750	578	4,900	[1]1,799	26,400	2.04	.95
.480	237	3,200	1,233	6,920	.53	.55
.240	125	1,700	656	860	.07	.14
.000	0	1,100	458	....	.....	......
			6xJ-2			
1.250	336	2,360	[1]500	103,360	8.74	1.03
1.000	385	2,350	[1]485	52,920	4.48	.99
.745	131	2,100	[1]486	21,880	1.85	.99
.620	147	1,800	408	12,610	1.07	.77
.500	74	1,500	352	6,615	.56	.62
.250	18	900	218	830	.07	.24
.128	18	650	156	110	.01	.07
.000	0	535	132	....	.....	......

Sheet 1 of 2

Table 13.--The effect of the stiffener on the critical buckling load of the plywood plates (face grain 45° to the direction of the load) (Continued)

Stiffener		Plate		Stiffener effect		
Depth (d)	Load (P)	Gross critical load (L)	Net critical stress (p_{cr})	Stiffness $(EI)_s$	Coordinates (X)	Coordinates (Y)
1	2	3	4	5	6	7
Inch	Pounds	Pounds	P.s.i.			
			13xJ-2			
1.493	399	1,500	[1]478	174,720	16.80	0.96
1.237	407	1,540	[1]492	99,370	9.56	1.00
1.120	449	1,590	[1]496	73,760	7.10	1.01
1.002	389	1,530	[1]496	52,815	5.08	1.01
.875	218	1,340	[1]487	35,170	3.38	.98
.750	117	1,300	[1]514	22,150	2.13	1.06
.615	74	1,200	[1]489	12,210	1.18	.99
.501	37	1,000	418	6,600	.64	.80
.247	14	350	146	790	.08	.04
.000	0	300	130	....	.....	.....
			13xh-1			
1.008	669	2,200	[1]1,362	62,260	17.90	1.02
.740	434	2,000	[1]1,393	24,630	7.09	1.05
.500	153	1,600	[1]1,287	7,600	2.19	.94
.250	81	750	595	950	.27	.24
.000	0	400	356	....	.....	.....
			13xh-2			
1.009	748	2,300	[1]890	62,440	8.52	.99
.750	491	2,000	[1]865	25,640	3.50	.96
.510	183	1,800	[1]927	8,070	1.10	1.05
.260	92	750	377	1,070	.15	.25
.000	0	350	201	....	.....	.....

[1]These values were used for average p_{cr2}.

Sheet 2 of 2

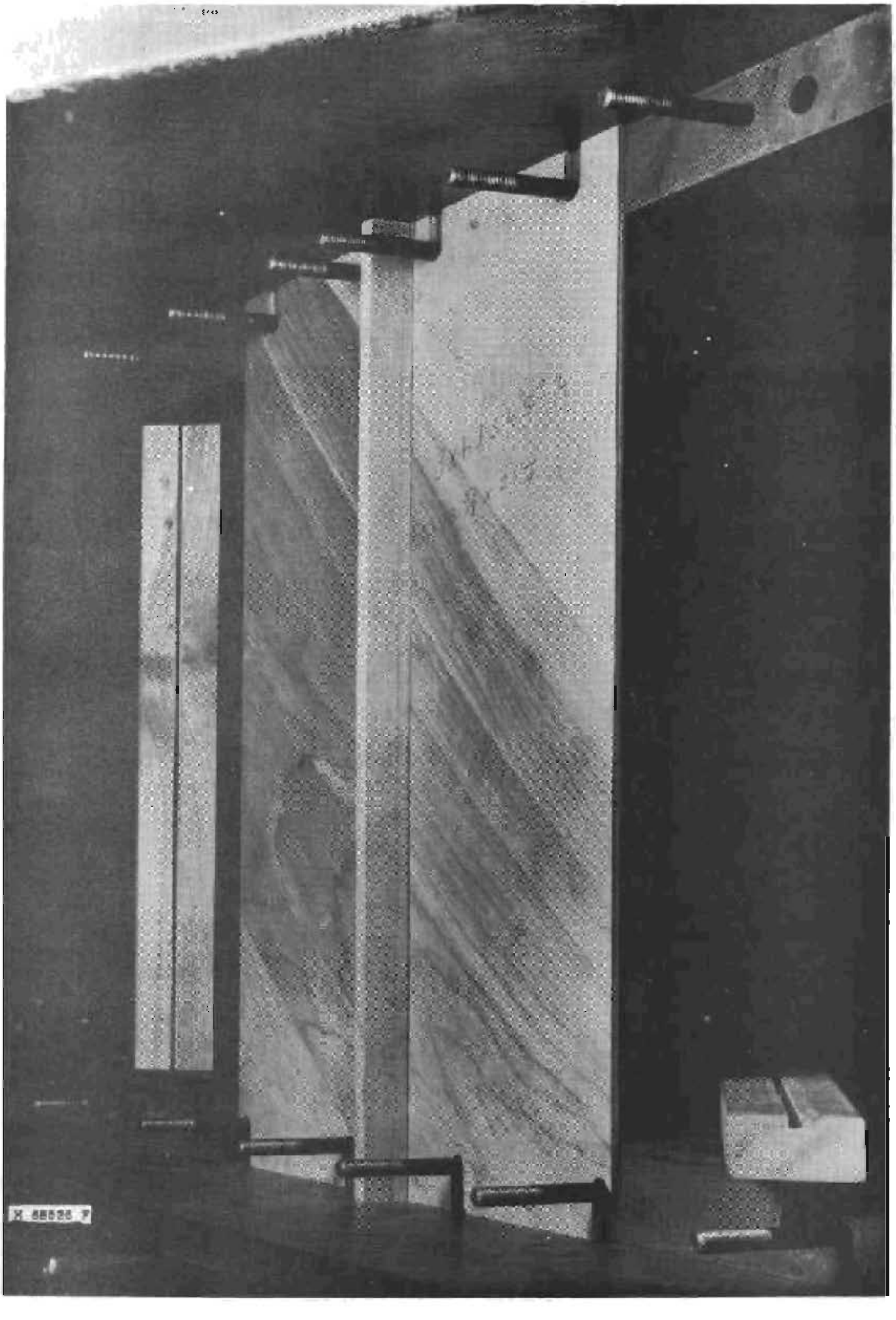

Figure 28.--Stiffened plywood plate shown fitted in test apparatus before clamping.
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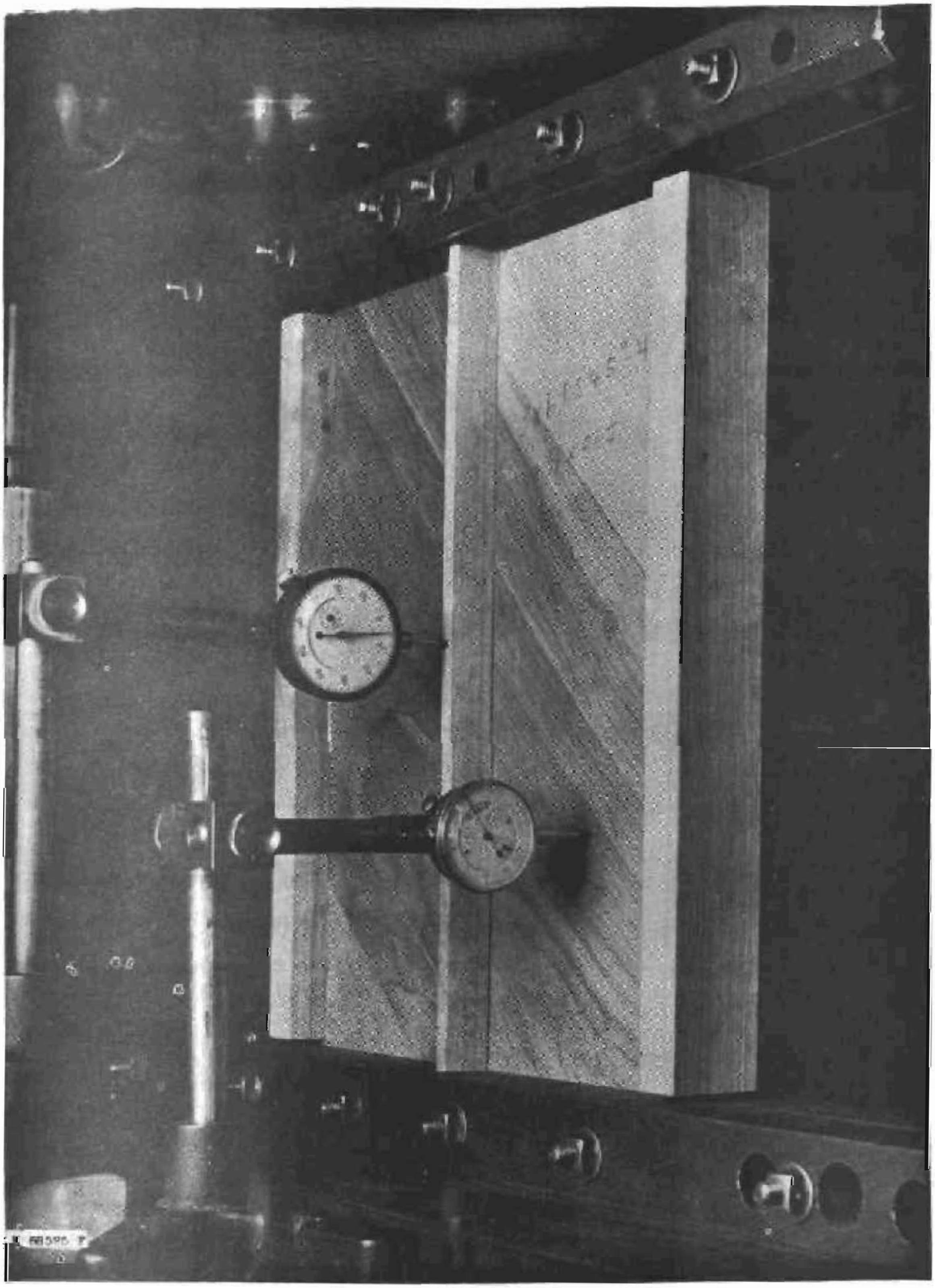

Figure 29.--Specimen ready for test. Loaded ends clamped, edges simply supported, and lateral-deflection indicating dials in position.

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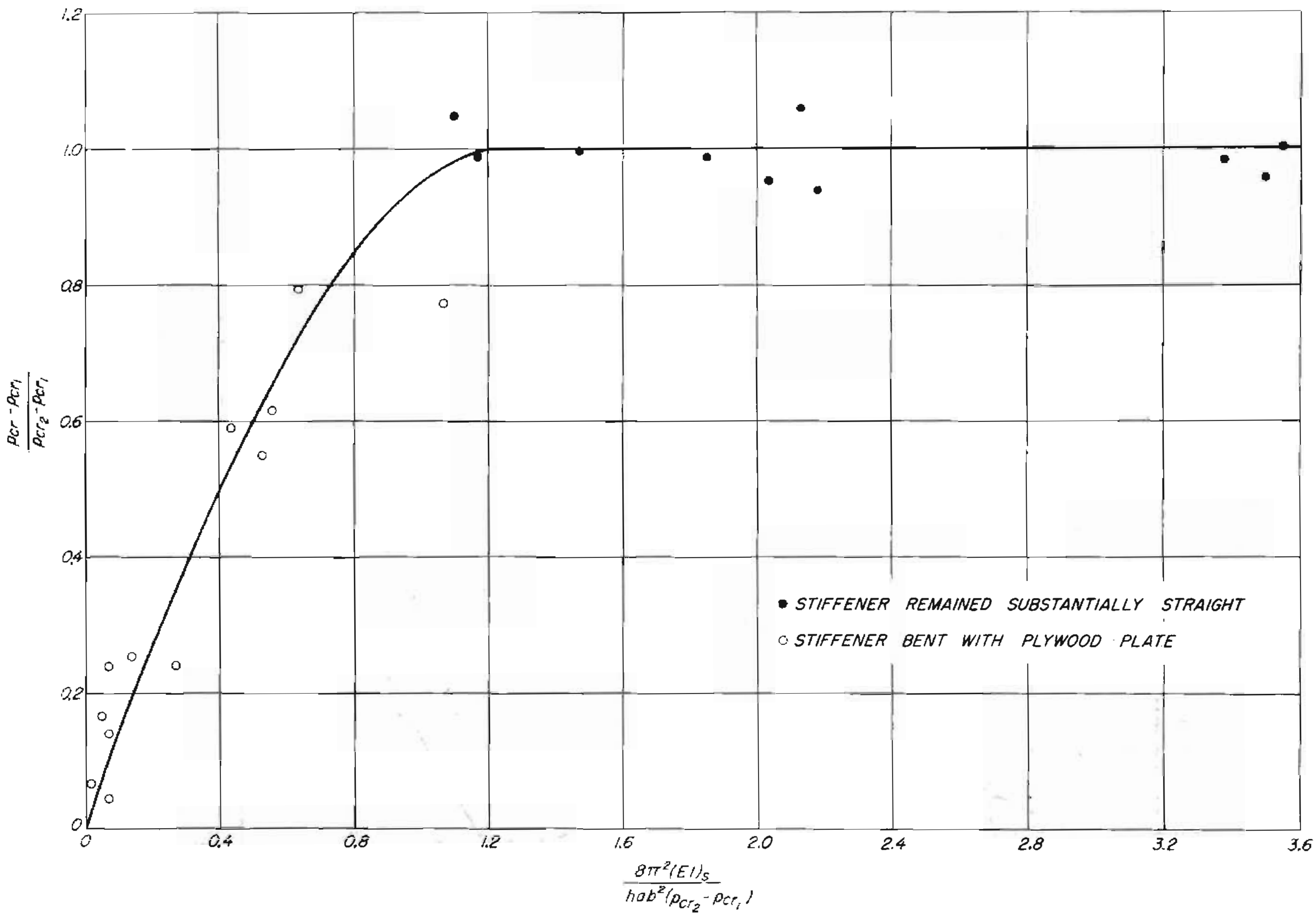

Figure 30.--Typical load-deflection curves plotted for the determination of the critical buckling load.

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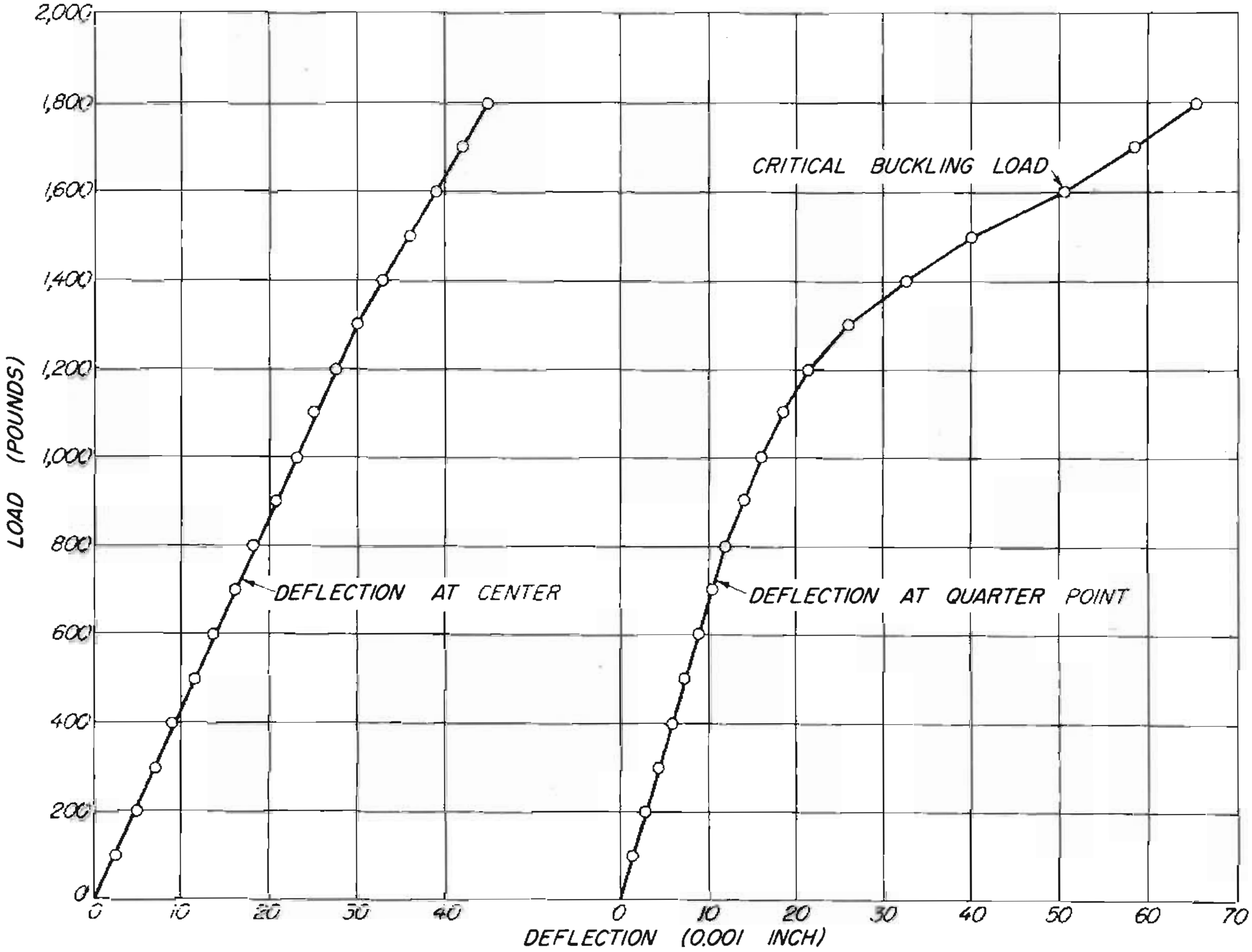

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Figure 31.--Plot of the experimental values from which the critical-size stiffener was determined.