BUCKLING LOADS OF FLAT SANDWICH PANELS IN COMPRESSION

VARIOUS TYPES OF EDGE CONDITIONS

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BUCKLING LOADS OF FLAT SANDWICH PANELS IN COMPRESSION

VARIOUS TYPES OF EDGE CONDITIONS[1]

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Introduction

This report presents approximate formulas and corresponding families of curves for determining the buckling loads in compression of flat sandwich panels of various constructions and subject to one of the following types of edge conditions:

- I. All edges simply supported.
- II. Loaded edges simply supported and the remaining edges clamped.
- III. Loaded edges clamped and the remaining edges simply supported.
- IV. All edges clamped.

The faces and core are assumed to be composed of either orthotropic or isotropic materials. The formulas are written for orthotropic materials having their axes of symmetry parallel to the edges of the panel. From them the corresponding formulas for isotropic materials are readily obtained. Curves are drawn for a series of values of a parameter whose values are determined by the elastic constants of the faces and core.

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[1] This is one of a series of progress reports prepared by the Forest Products Laboratory relating to the use of wood in aircraft issued in cooperation with the Army-Navy-Civil Committee on Aircraft Design Criteria. Original report published in 1945.

[2] Maintained at Madison, Wis., in cooperation with the University of Wisconsin.

Under assumptions that will be stated below, the buckling behavior of sandwich panels for each of the types of boundary conditions considered could be determined from papers dealing with plywood panels. Reference will be made to these papers in the discussion of the various cases. Because of the current interest in sandwich construction an explicit presentation of the results as applied to sandwich panels is needed.

In obtaining the approximate formulas of the present report, an energy method has been used. This method has the advantage of leading to formulas for the buckling load that are expressed in closed form for all cases. They thus exhibit the influence of the various characteristic constants of the panel in determining its buckling load. It has the disadvantage of leading to buckling loads which are somewhat too high in all cases except case I where the results are exact. Random checks with the results of exact formulas indicate that the greatest error is of the order of magnitude of 8 percent. Frequently, the error is much less.

Two assumptions are made throughout, viz., that the materials of the faces and core are stressed below the proportional limit and that the effect of shear deformation in the core may be neglected. It is hoped to investigate the modifications of the results that must be made when one or both of these assumptions are not warranted. For the effect of shear deformation when all edges are simply supported, reference may be made to U. S. Forest Products Laboratory reports Nos. 1504 and 1505 (restricted). In many instances, it appears that the effect is not important. When, however, two opposite edges are clamped there are points of contraflexure in the sections perpendicular to these edges and relatively near them, and a greater reduction in the effective stiffness in this direction is to be expected than when these edges are simply supported. It is also assumed that the faces of the panels do not wrinkle before buckling occurs.

Notation

The choice of axes and the dimensions of the panel are shown in figure 1.

c = thickness of core

f = thickness of each face

$h = 2f + c$ = thickness of panel

Subscripts c and f denote that the symbols to which they are attached refer to quantities that are measured in the core and faces, respectively.

E_x = Young's modulus in a direction parallel to the X-axis.

E_y = Young's modulus in a direction parallel to the Y-axis.

μ_{xy} = modulus of rigidity associated with a shearing strain corresponding to the axes of x and y.

σ_{yx} = the Poisson's ratio associated with a contraction parallel to the X-axis and a tensile stress parallel to the Y-axis.

$\lambda = 1 - \sigma_{xy}\sigma_{yx}$

z = coordinate measured perpendicular to the middle plane of the panel.

$$D_1 = \int_{-h/2}^{h/2} (E_x/\lambda)\, z^2\, dz$$

$$D_2 = \int_{-h/2}^{h/2} (E_y/\lambda)\, z^2\, dz$$

$$A = E_x\, \sigma_{yx} + 2\lambda\mu_{xy}$$

$$K = \int_{-h/2}^{h/2} (A/\lambda)\, z^2\, dz$$

$$\kappa = K/(D_1\, D_2)^{1/2}$$

$$r = (b/a)(D_1/D_2)^{1/4}$$

w = lateral deflection (namely, in the z direction) of points on the middle plane of the panel

P = load per inch of edge of panel

P_{cr} = buckling load per inch of edge of panel

k_{cr} = coefficient in buckling formulas

When faces and core are each composed of a single layer,

$$D_1 = \frac{(E_x/\lambda)_f\, h^3 - \left[(E_x/\lambda)_f - (E_x/\lambda)_c\right] c^3}{12}$$

$$D_2 = \frac{(E_y/\lambda)_f\, h^3 - \left[(E_y/\lambda)_f - (E_y/\lambda)_c\right] c^3}{12}$$

$$K = \frac{(A/\lambda)_f\, h^3 - \left[(A/\lambda)_f - (A/\lambda)_c\right] c^3}{12}$$

When faces and core are isotropic:

$$(E_x)_f = (E_y)_f = A_f = E_f$$

$$(E_x)_c = (E_y)_c = A_c = E_c$$

$$D_1 = D_2 = K = D = \frac{(E/\lambda)_f\, h^3 - \left[(E/\lambda)_f - (E/\lambda)_c\right] c^3}{12}$$

where

$\lambda_f = 1 - \sigma_f^2$, $\lambda_c = 1 - \sigma_c^2$, and σ_f and σ_c denote Poisson's ratio for the faces and core respectively. Further

$\kappa = 1$ and $r = b/a$.

When faces and core are isotropic and furthermore have the same Poisson's ratios

$$D_1 = D_2 = K = D = \frac{E_f\, h^3 - (E_f - E_c)\, c^3}{12\lambda}$$

where

$$\lambda = 1 - \sigma^2$$

Energy Method

In the energy method used to determine the buckling load an equation representing the form of the buckled surface of the panel is assumed for each of the types of edge conditions. The buckling load is determined[3] by equating

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[3] Timoshenko, S., Theory of Elastic Stability, New York, 1936, pp. 75-84 and 325.

the loss in potential energy of the load during buckling to the strain energy of bending of the panel. The work done by the load is given by the expression

$$V_L = \frac{P}{2} \int_0^a \int_0^b \left(\frac{\delta w}{\delta y}\right)^2 dy\, dx \qquad (1)$$

The strain energy of bending of the panel is given by the expression[4]

$$V_B = \frac{1}{2} \int_0^a \int_0^b \left\{ D_1 \left(\frac{\delta^2 w}{\delta x^2}\right)^2 + D_2 \left(\frac{\delta^2 w}{\delta y^2}\right)^2 + 2\left[\int_{-h/2}^{h/2} \frac{E_x \sigma_{yx}}{\lambda} z^2 dz\right] \frac{\delta^2 w}{\delta x^2} \frac{\delta^2 w}{\delta y^2} \right.$$

$$\left. + 4\left[\int_{-h/2}^{h/2} \mu_{xy} z^2 dz\right] \left(\frac{\delta^2 w}{\delta x\, \delta y}\right)^2 \right\} dy\, dx \qquad (2)$$

Case I. All Edges Simply Supported

$$\text{Let } w = w_o \sin\frac{\pi x}{a} \sin\frac{n\pi y}{b} \qquad (3)$$

where n is the number of half-waves in the direction of loading.

It is found by substituting (3) in (1) and (2) that

$$V_L = \frac{P\pi^2 w_o^2 n^2 a}{8b}$$

$$V_B = \frac{\pi^4 w_o^2 ab}{8} \left[\frac{D_1}{a^4} + \frac{D_2 n^4}{b^4} + \frac{2K n^2}{a^2 b^2}\right]$$

The following expression for P_{cr} is found by equating V_L and V_B and solving the resulting equation for P:

$$P_{cr} = \frac{\pi^2}{a^2}\left[D_1 \frac{b^2}{n^2 a^2} + D_2 \frac{n^2 a^2}{b^2} + 2K\right] \qquad (4)$$

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[4] See for example U. S. Forest Products Laboratory Rept. No. 1312. Equation following (3.20) and equations (2.3), (2.4), and (2.5).

Equation (4) is identical in form with the corresponding equation for the buckling of panels of homogeneous orthotropic material[5] and for plywood panels.[6] The energy method in this case leads to an exact result because the form (3) assumed for the buckled surface is the correct one.

In order to represent equation (4) by a family of curves from which the buckling load, P_{cr}, can be obtained for a panel of given construction, it will be written in the following forms:

$$P_{cr} = \frac{\pi^2}{a^2} \sqrt{D_1 D_2} \left[\frac{r^2}{n^2} + \frac{n^2}{r^2} + 2\kappa \right] \tag{5}$$

$$= k_{cr} \frac{12 \sqrt{D_1 D_2}}{a^2} \tag{6}$$

where

$$r = \frac{b}{a} \left(\frac{D_1}{D_2} \right)^{1/4} \tag{7}$$

$$\kappa = \frac{K}{\sqrt{D_1 D_2}} \tag{8}$$

$$k_{cr} = \frac{\pi^2}{12} \left[\frac{r^2}{n^2} + \frac{n^2}{r^2} + 2\kappa \right] \tag{9}$$

It is readily determined from either (5) or (9) that a panel will buckle in one half-wave if

$$r < \sqrt{2} \; ;$$

in two half-waves if

$$\sqrt{2} < r < \sqrt{6} \; ;$$

[5] Timoshenko, S., Theory of Elastic Stability (New York, 1936), p. 381.

[6] U. S. Forest Products Laboratory Rept. No. 1316 (April, 1942), p. 3.

and into n half-waves if

$$\sqrt{n\,(n-1)} < r < \sqrt{n\,(n+1)}$$

Case II. Loaded Edges Simply Supported. Remaining Edges Clamped.

$$\text{Let } w = w_o \sin^2 \frac{\pi x}{a} \sin \frac{n \pi y}{b} \tag{10}$$

where n is the number of half-waves into which the panel buckles.

After equating V_L and V_B as found by substituting (10) in (1) and (2), the following expression for P_{cr} is obtained:[7]

$$P_{cr} = \frac{16\,\pi^2}{3\,a^2}\left[D_1 \frac{b^2}{n^2\,a^2} + \frac{3}{16} D_2 \frac{n^2\,a^2}{b^2} + \frac{K}{2}\right] \tag{11}$$

For the purpose of constructing a family of curves to represent equation (11) it is written in the following form:

$$P_{cr} = k_{cr} \frac{12\sqrt{D_1\,D_2}}{a^2} \tag{12}$$

where

$$k_{cr} = \frac{4\pi^2}{9}\left[\frac{r^2}{n^2} + \frac{3}{16}\frac{n^2}{r^2} + \frac{\kappa}{2}\right] \tag{13}$$

r and κ being defined by (7) and (8). The plate will buckle in n half-waves when

$$\frac{1}{2}\sqrt{n\,(n-1)\sqrt{3}} < r < \frac{1}{2}\sqrt{n\,(n+1)\sqrt{3}}$$

[7]This equation is equivalent to equation (5) of U. S. Forest Products Laboratory Rept. 1316-B (Nov. 1942). In that equation, b represents the length of a single half-wave while here b/n is the length of a single half-wave. For an exact solution based on the differential equation of buckling, see Smith, R. C. T., "Buckling of Flat Plywood Plates in Compression II." Report S. M. 31(restricted) Div. of Aeronautics, Australian Council of Sci. and Ind. Res., Feb. 1944. For a different treatment of the homogeneous isotropic panel, see Timoshenko, S., Theory of Elastic Stability (New York, 1936), p. 345.

Case III. Two Loaded Edges Clamped. Remaining Edges Simply Supported

The following forms will be chosen for the buckled surface depending upon the number of half-waves assumed to be formed.

One half-wave

$$w = w_o \sin \frac{\pi x}{a} \sin^2 \frac{\pi y}{b}$$

Two half-waves

$$w = w_o \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} \sin \frac{2\pi y}{b}$$

Three half-waves

$$w = w_o \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} \sin \frac{3\pi y}{b}$$

Four half-waves

$$w = w_o \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} \sin \frac{4\pi y}{b}$$

Equating the values of V_L and V_B obtained by substituting these expressions for w in (1) and (2) leads to the following values of k_{cr} in the formula:[8]

$$P_{cr} = k_{cr} \frac{12\sqrt{D_1 D_2}}{a^2} \tag{14}$$

where for one half-wave

$$k_{cr} = \frac{\pi^2}{48}(3 r^2 + \frac{16}{r^2} + 8\kappa); \tag{15}$$

for two half-waves

$$k_{cr} = \frac{\pi^2}{60}(r^2 + \frac{41}{r^2} + 10\kappa); \tag{16}$$

[8] For an exact solution based on the differential equation of buckling, see Smith, R. C. T., "Buckling of Flat Plywood Plates in Compression I." Reports S & M 30 (Restricted) Div. of Aeronautics, Australian Council for Sci. and Ind. Res. February, 1944. For the homogeneous isotropic panel, see Timoshenko, S., Theory of Elastic Stability (New York, 1936), p. 364.

for three half-waves

$$k_{cr} = \frac{\pi^2}{120} \left(r^2 + \frac{136}{r^2} + 20\kappa\right); \tag{17}$$

for four half-waves

$$k_{cr} = \frac{\pi^2}{204} \left(r^2 + \frac{353}{r^2} + 34\kappa\right). \tag{18}$$

Case IV. All Edges Clamped

The following forms will be chosen for the buckled surface depending upon the number of half-waves assumed to be formed.

One half-wave

$$w = w_o \sin^2 \frac{\pi x}{a} \sin^2 \frac{\pi y}{b}$$

Two half-waves

$$w = w_o \sin^2 \frac{\pi x}{a} \sin \frac{\pi y}{b} \sin \frac{2\pi y}{b}$$

Three half-waves

$$w = w_o \sin^2 \frac{\pi x}{a} \sin \frac{\pi y}{b} \sin \frac{3\pi y}{b}$$

Four half-waves

$$w = w_o \sin^2 \frac{\pi x}{a} \sin \frac{\pi y}{b} \sin \frac{4\pi y}{b}$$

Equating the values of V_L and V_B obtained by substituting these expressions for w in (1) and (2) leads to the following values of k_{cr} in the formula,[9]

$$P_{cr} = k_{cr} \frac{12\sqrt{D_1 D_2}}{a^2} \tag{19}$$

[9] For an application of the Rayleigh-Ritz method to obtain close approximations to the solution of this problem, see Smith, R. C. T., "Buckling of Flat Plywood Plates in Compression III." Report S & M 32 (Restricted) Div. of Aeronautics, Australian Council for Sci. and Ind. Res., February 1944. For a reference to a treatment of the corresponding problem for homogeneous isotropic panels, see Timoshenko, S., Theory of Elastic Stability (New York, 1936), p. 365.

where for one half-wave

$$k_{cr} = \frac{\pi^2}{9}\left(3r^2 + \frac{3}{r^2} + 2\kappa\right); \qquad (20)$$

for two half-waves

$$k_{cr} = \frac{\pi^2}{180}\left(16\, r^2 + \frac{123}{r^2} + 40\kappa\right); \qquad (21)$$

for three half-waves

$$k_{cr} = \frac{\pi^2}{45}\left(2r^2 + \frac{51}{r^2} + 10\kappa\right); \qquad (22)$$

for four half-waves

$$k_{cr} = \frac{\pi^2}{612}\left[16\, r^2 + \frac{1059}{r^2} + 136\kappa\right] \qquad (23)$$

Isotropic Faces and Core

When the faces and the core are all isotropic

$$D_1 = D_2 = K = D = \frac{(E/\lambda)_f\, h^3 - \left[(E/\lambda)_f - (E/\lambda)_c\right] c^3}{12}$$

consequently,

$$\kappa = 1 \text{ and } r = \frac{b}{a}$$

If further the Poisson's ratios of the isotropic faces and core are the same, the expression for D simplifies further to

$$D = \frac{E_f\, h^3 - E_f - E_c\;\; c^3}{12\lambda}$$

where

$$\lambda = 1 - \sigma^2$$

and σ is the Poisson's ratio common to faces and core. If information regarding the Poisson's ratios of faces and core is lacking, it should be sufficient for preliminary calculations, at least, to choose them both equal to 0.25 or 0.3.

Orthotropic Faces and Cores

For orthotropic materials, the quantity $\lambda = 1 - \sigma_{xy}\,\sigma_{yx}$ is frequently nearly equal to unity. Considerable simplifications in the formulas for calculating D_1, D_2, and K will result if λ is replaced by unity for both faces and cores. But this should not be done without due realization of the approximations involved.

Conclusions

Formulas and curves are presented for determining approximately the buckling loads of sandwich panels under several types of edge conditions. An energy method was used and as a consequence the buckling loads given by the formulas will be somewhat too high except for a panel with all edges simply supported. The error involved is believed to be less than or equal to 8 percent. (Figures 2 - 5.)

The formulas were derived under the assumptions that stresses in all materials in the panels are below the proportional limit and that the effect of shear deformation in the core may be neglected. It is hoped to discover what modifications are necessary in case these assumptions do not apply. Further, it was assumed that the faces do not wrinkle. Reference to sources in which corresponding results were obtained by exact methods for plywood panels are given in connection with each case considered.

It is to be emphasized that the results presented were obtained by a theoretical analysis based upon certain assumptions and that they must be confirmed by test. Such a test program is now under way.

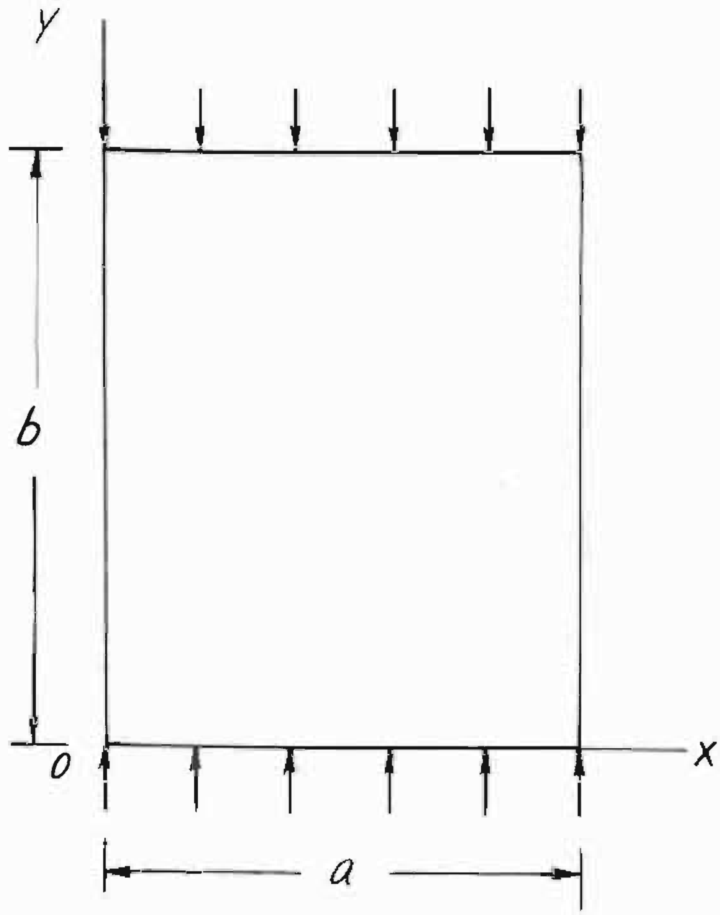

Figure 1.--Flat sandwich panel in compression.

ZM 59475 F

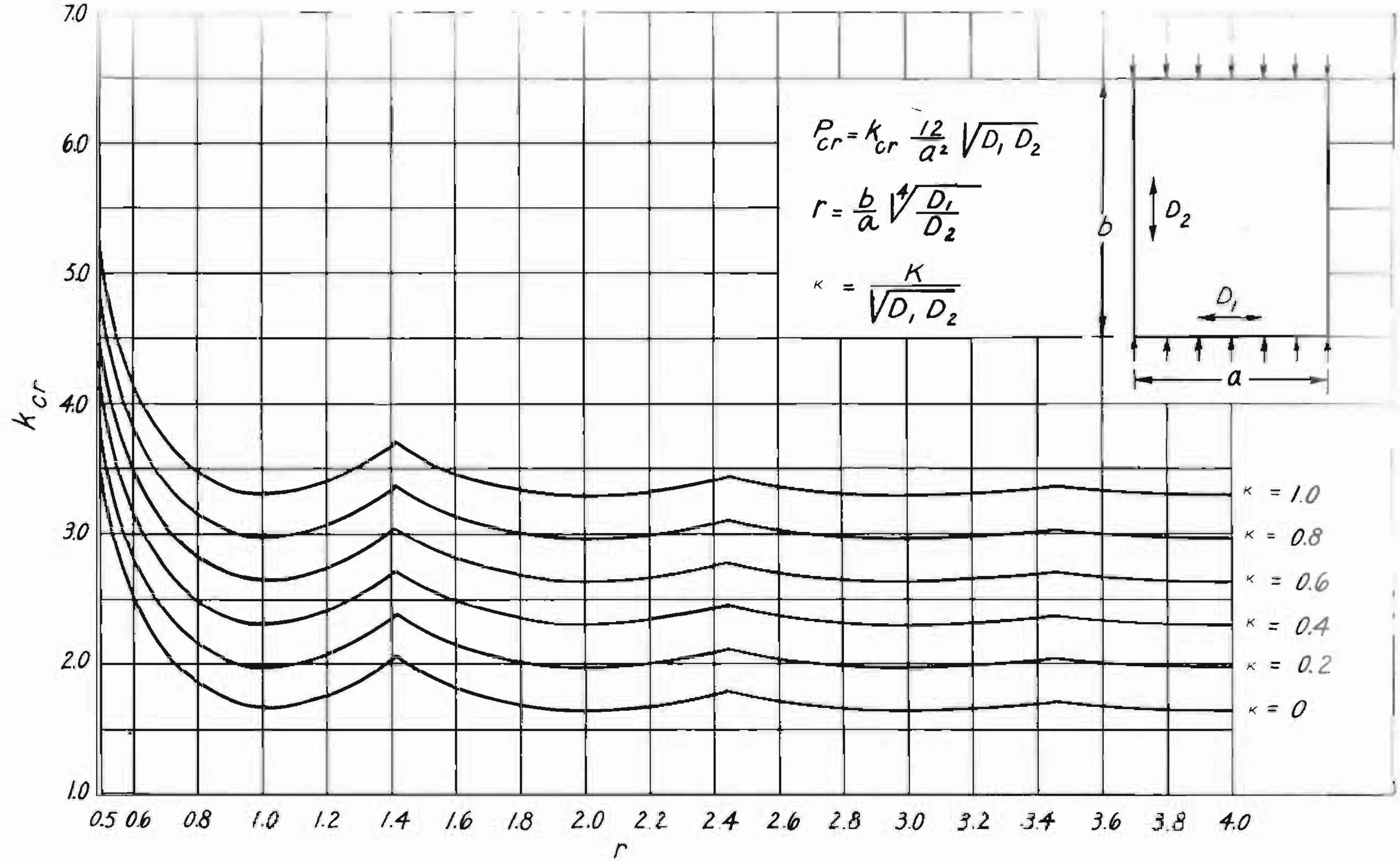

Figure 2.--Buckling of flat sandwich panels in uniform compression. All edges simply supported.

Z M 59476 F

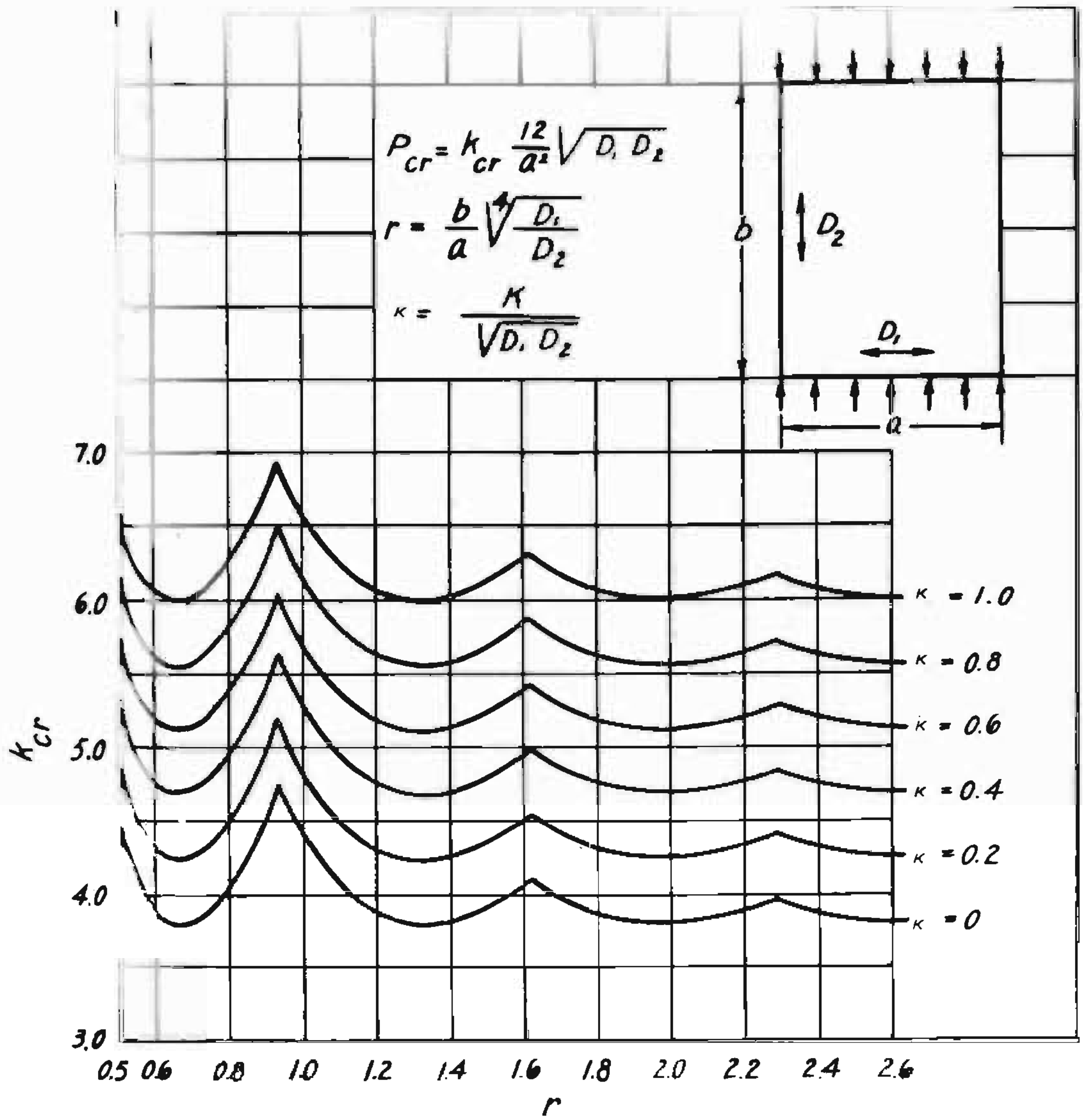

Figure 3.--Buckling of flat sandwich panels in uniform compression. Loaded edges simply supported. Remaining edges clamped.

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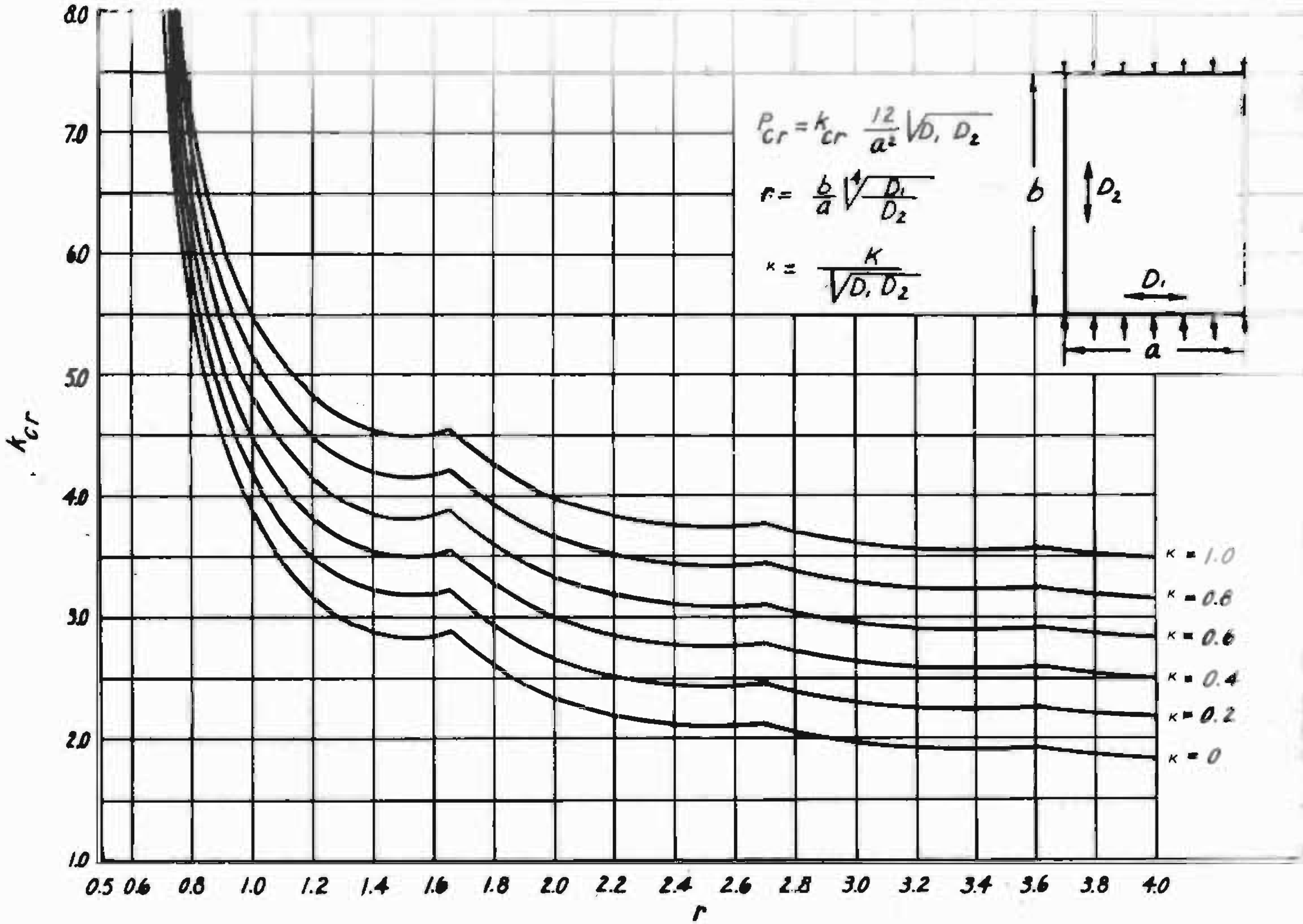

Figure 4.--Buckling of flat sandwich panels in uniform compression. Loaded edges clamped. Remaining edges simply supported.

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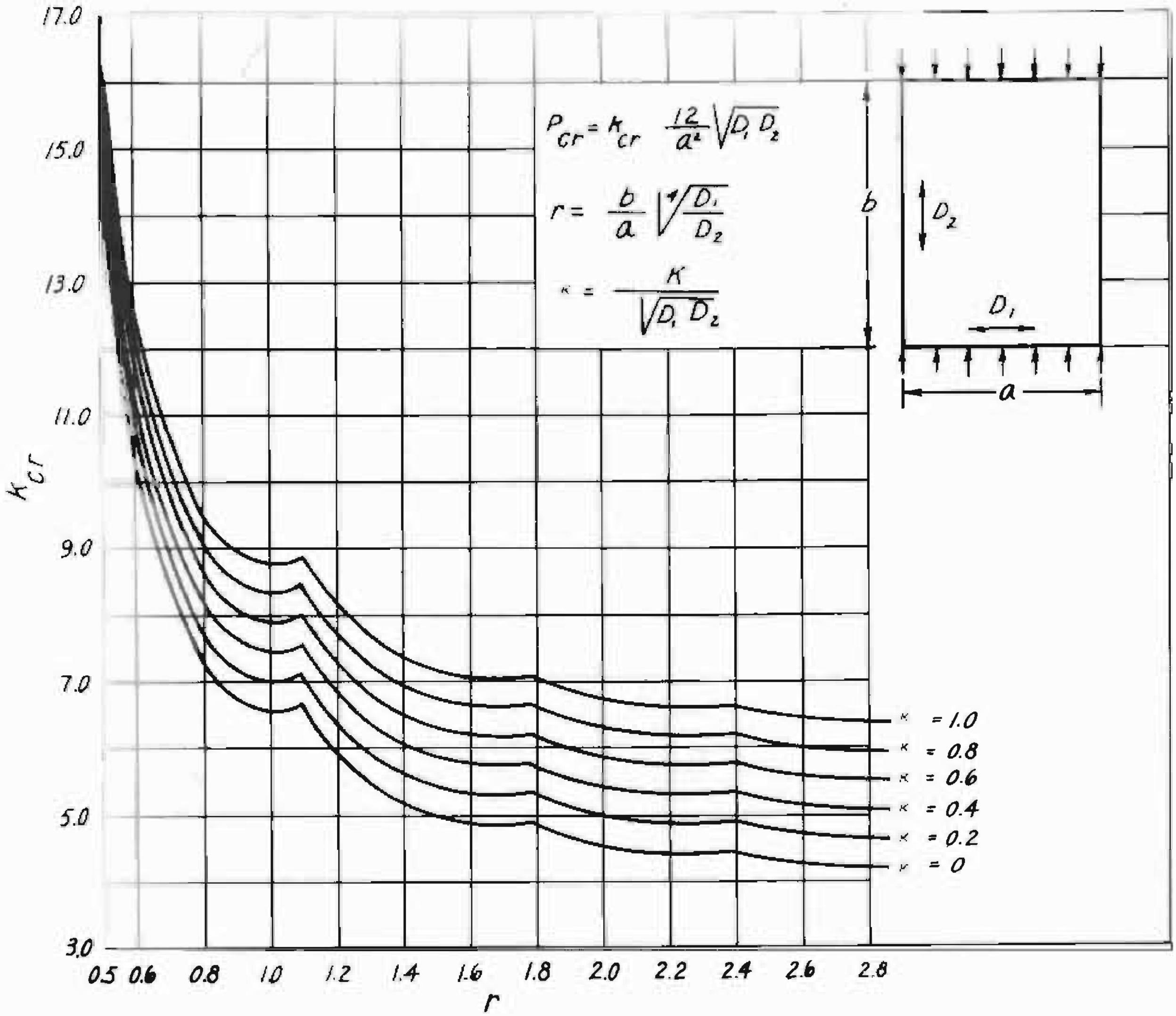

Figure 5.--Buckling of flat sandwich panels in uniform compression. All edges clamped.

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