Bilinear Modelling of Cellulosic Orthotropic Nonlinear Materials

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The proposed method of modelling orthotropic solids that have a nonlinear constitutive material relationship affords several advantages. The first advantage is the application of a simple bilinear stress–strain curve to represent the material response on two orthogonal axes as well as in shear, even for markedly nonlinear materials. The second advantage is that this method correlates yield strengths in the orthogonal directions, which is necessitated by some finite-element programs. Herein, we provide an algorithm for fitting bilinear stress versus strain curves to data, such that the derived stress–strain law on each orthogonal axis is coupled by means of a common strain energy density function. Nonlinear stress–strain data for paper and paperboard illustrate the procedure. This method can be readily implemented in finite element applications.

La méthode proposée pour la modélisation des solides orthotropes ayant une relation matérielle constitutive non linéaire offre plusieurs avantages. Le premier est l'application d'une simple courbe effort-allongement bilinéaire pour représenter la réponse matérielle de deux axes orthogonaux comme duns le cisaillement, même pour les matières notablement non linéaires. Le second est que cette méthode met en corrélation la limite conventionnelle d'élasticité dans les sens orthogonaux, ce qui est requis par certains programmes d'éléments finis. Nous fournissons ici un algorithme pour adapter aux données l'effort bilinéaire par rapport aux courbes d'allongement, de façon à ce que la loi effort-allongement dérivée de chaque axe orthogonal soit associée au moyen d'une fonction de densité de l'énergie d'allongement commune. Les données sur l'effort-allongement non linéaire pour le papier et le carton illustrent la méthode. Cette derniére peut être facilement employée dans les applications d'éléments finis.

INTRODUCTION

There are many cellulosic materials that can be modelled as orthotropic solids with nonlinear constitutive relationships. Paper, paperboard, wood-based panels are several such materials, which engineers and analysts have modelled by means of continuum-mechanics methods or the finite-element method. Finite-element software readily allows engineers to perform nonlinear analyses. For the case of isotropic materials, nonlinear material property laws are easily input. The situation is more complicated for orthotropic plates that exhibit nonlinear stress versus strain responses. Some researchers choose to create constitutive models that do not obey invariance laws of tensor transformation [1]. Others have created nonlinear orthotropic models which couple the yield stresses on orthogonal axes, but do not couple the material moduli [2]. Still others have proposed nonlinear orthotropic constitutive laws that either do not include shear [3] or do include shear but do not couple orthogonal responses [4]. Johnson and Urbanik [5] have shown that the material moduli on orthogonal axes must be coupled to each other. We present a modification of their theory by creating bilinear constitutive models of orthotropic materials subjected to plane stress. The proposed method can be readily incorporated into finite-element modelling of cellulosic materials such as paper and paperboard. Others have also incorporated nonlinear orthotropic constitutive models into finite element codes [6], but their model does not couple material moduli or yield stresses as does ours.

We will demonstrate our model's effectiveness by examining six different experimental data sets. The first set examined is the most complete set available to us. It is a study of nonlinear properties of high-strength paperboards, conducted by Qiu et al. [7]. Laminates made up of paperboard were fabricated to create orthotropic specimens with a pronounced strong axis (1 axis) and a weak axis (2 axis). Compression tests along each axis, as well as shear tests in the 12 plane, were conducted. The data sets from Suhling [8] and Gerhardt [9] are also on paperboard, with Suhling using individual sheets and Gerhardt using laminates, as did Qiu. The remaining two data sets are composite materials that exhibit nonlinear orthotropicity.

BACKGROUND THEORY

Some researchers have used linear orthotropic elasticity when analyzing paperboard structural systems, although the inherent limitation of this approach excludes it from our consideration. Thorpe and Yang [10] presented tangential nonlinear elastic finite-element analyses of paper sheets. Their work updated the modulus of elasticity as a function of stress, but their work was limited to isotropic modelling. Suhling [8] has modelled nonlinear elastic behaviour by means of a hyperelastic formulation and applied the model to paperboard. Such an approach requires a strain energy density function to characterize the plate's constitutive response. Suhling's work was based on a special assumed form for the strain energy density

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function in terms of an effective strain variable, as suggested by Johnson and Urbanik [5].

This research also begins with the approach of a single effective strain. The required expression for strain energy can be found by the uniaxial response in either orthogonal direction or by shear response. The ability to extract the strain energy by either of these tests links the responses in the orthogonal directions as well as to the shear response.

We start by expressing the in-plane stresses s $_{ij}$ in terms of the strain energy density H:

$$\sigma_{11} = \frac{N_{11}}{h} = \frac{\partial H}{\partial \varepsilon_{11}}$$
$$\sigma_{22} = \frac{N_{22}}{h} = \frac{\partial H}{\partial \varepsilon_{22}}$$
$$\sigma_{12} = \frac{N_{12}}{h} = \frac{1}{4}\frac{\partial H}{\partial \gamma_{12}} \tag{1}$$

where h is the plate thickness and N_{ij} is the force per unit width and where

$$H = \frac{v_2 E_1}{2(1 - v_1 v_2)} e \tag{2}$$

and

$$e = \frac{1}{v_2}\varepsilon_{11}{}^2 + \frac{1}{v_1}\varepsilon_{22}{}^2 + 2\varepsilon_{11}\varepsilon_{22} + c\gamma_{12}{}^2 \tag{3}$$

where

$$c = \frac{4(1 - v_1 v_2)}{v_1 E_2} G \tag{4}$$

In [5], Johnson and Urbanik recognized that the form of $H(e)$ for nonlinear orthotropic elasticity would necessitate a complete set of all possible loading conditions in terms of middle surface strains. Clearly, then as now, such data does not exist. Our assumption here is that one could always develop some $H(e)$ based on experimental results and in this paper we work with a limited set of data. As was done in [5], we assume that $H(e)$ is valid for linear as well as for nonlinear experimental data. Either term in Eq. (1) could be used to quantify the term $H'(e)$. Of course, other loading conditions could also be analyzed, such as biaxial loading or twisting loading. The term $H'(e)$ could be extracted from these loadings as well, if they were available. The most convincing description of $H'(e)$ would arise from a truly combined loading case where biaxial in-plane loads are combined with twisting loads. For now, we limit ourselves to an investigation of uniaxial tests and we seek a proper mathematical fit to describe $H'(e)$.

In constructing this bilinear model, we have incorporated the modifications to Hill's yield criterion, which accounts for differences in yield strengths in orthogonal directions that have been proposed by Shih and Lee [2]. This takes into account possible differences in yield strength in tension and compression.

One result of this criterion is that the stress versus strain response on each of the three orthogonal axes can be bilinear, if we provide an initial modulus of elasticity and a secondary modulus of elasticity for stresses higher than the yield stress. This secondary modulus can be zero, resulting in an elastic–plastic model or it can be very nearly equal to the initial modulus, resulting in a nearly linear constitutive model. The breakpoint in this bilinear model is located by the yield point. We incorporate this same bilinear stress versus strain law in the shear response.

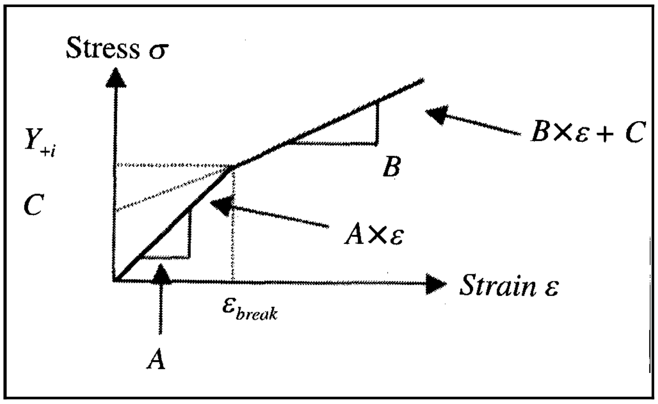

Fig. 1. Bilinear constitutive law.

APPLICATION TO CELLULOSIC ORTHOTROPIC PLANE STRESS STRUCTURE

Let the initial portion of the bilinear curve be defined as s $= A$e where A is the initial slope of the curve (Fig. 1). The second portion of the curve is defined as s = Be + C, where B is the slope of the second straight line, and C is the y intercept. The strain corresponding to the breakpoint is found by equating the two straight lines.

When

$$A\varepsilon_{break} = B\varepsilon_{break} + C$$

then

$$\varepsilon_{break} = \frac{C}{A - B} \tag{5}$$

Referring back to Eq. (1), we can extract the term $H(e)$ by integrating the expression for s (e). In the bilinear case shown in Fig. 1, the integration will have to be broken up into two cases, one before the breakpoint and one after the breakpoint. Having $H(e)$, we can then obtain $H'(e)$.

UNIAXIAL LOADING IN THE 2 DIRECTION

First, we consider the case of uniaxial loading in the 2 direction. The 1 direction could also be chosen as the starting point of an analysis; we choose the 2 direction to make our research consistent with the original Johnson and Urbanik study [5]. Also, the large nonlinearity of the 2 direction makes it a convenient first choice for modelling.

$$\sigma_2(\varepsilon_2) = \frac{N_{22}}{h} = A_2\varepsilon_2 \qquad \text{for } \varepsilon_2 \le \frac{C_2}{A_2 - B_2}$$
$$\sigma_2(\varepsilon_2) = \frac{N_{22}}{h} = B_2\varepsilon_2 + C_2 \text{ for } \varepsilon_2 \ge \frac{C_2}{A_2 - B_2} \tag{6}$$

where A_2, B_2 and C_2 describe the two straight lines of the stress versus strain curve in the 2 direction. By the second line of Eq. (1), this is rewritten as:

$$\frac{\mathrm{d}H}{\mathrm{d}\varepsilon_2} = A_2\varepsilon_2 \qquad \text{for } \varepsilon_2 \le \frac{C_2}{A_2 - B_2}$$
$$\frac{\mathrm{d}H}{\mathrm{d}\varepsilon_2} = B_2\varepsilon_2 + C_2 \qquad \text{for } \varepsilon_2 \ge \frac{C_2}{A_2 - B_2} \tag{7}$$

Integrating dH / de$_2$ produces the two-component strain energy density function:

$$H = \frac{1}{2}A_2\varepsilon_2{}^2 \qquad \text{for } \varepsilon_2 \le \frac{C_2}{A_2 - B_2}$$
$$H = \frac{1}{2}\frac{C_2{}^2}{B_2 - A_2} + \frac{1}{2}B_2\varepsilon_2{}^2 + C_2\varepsilon_2 \qquad \text{for } \varepsilon_2 \ge \frac{C_2}{A_2 - B_2} \tag{8}$$

We seek to derive $H'(e)$, so first we make H a function of e rather than of e.

Setting $N_{11} = 0$, e$_1$ = $-v_2$e$_2$ and g$_{12}$ = 0, and solving Eq. (3) for e$_2$ as a function of e gives the strain energy density in terms of e.

$$H(e) = \frac{1}{2}A_2\frac{v_1 e}{1 - v_1 v_2} \qquad \text{for } e \le \left(\frac{C_2}{A_2 - B_2}\right)^2 \frac{1 - v_1 v_2}{v_1}$$
$$H(e) = \frac{1}{2}\frac{C_2{}^2}{B_2 - A_2} + \frac{1}{2}B_2\frac{v_1 e}{1 - v_1 v_2} + C_2\sqrt{\frac{v_1\varepsilon}{1 - v_I v_2}} \qquad \text{for } e \ge \left(\frac{C_2}{A_2 - B_2}\right)^2 \frac{1 - v_1 v_2}{v_1} \tag{9}$$

Now, differentiate the above expression with respect to e, to obtain $H'(e)$.

$$H'(e) = \frac{1}{2}A_2\frac{v_1}{1 - v_1 v_2} \qquad \text{for } e \le \left(\frac{C_2}{A_2 - B_2}\right)^2 \frac{1 - v_1 v_2}{v_1}$$
$$H'(e) = \frac{1}{2}B_2\frac{v_1}{1 - v_1 v_2} + \frac{1}{2}C_2\sqrt{\frac{v_1 / e}{1 - v_1 v_2}} \qquad \text{for } e \ge \left(\frac{C_2}{A_2 - B_2}\right)^2 \frac{1 - v_1 v_2}{v_1} \tag{10}$$

UNIAXIAL LOADING IN THE 1 DIRECTION

Next, we consider the case of uniaxial loading in the 1 direction. We will see how the response on the 1 axis is related to A_2, B_2 and C_2, which are the bilinear curve parameters on the 2 axis. We follow the same steps as were performed in Eqs. (6) through (10).

$$\sigma_1(\varepsilon_1) = \frac{N_{11}}{h} = A_2\frac{\nu_1}{\nu_2}\varepsilon_1 = A_1\cdot\varepsilon_1$$

$$\text{for} \quad \varepsilon_1 \le \frac{C_2}{A_2 - B_2}\sqrt{\frac{\nu_2}{\nu_1}}$$

$$\sigma_1(\varepsilon_1) = \frac{N_{11}}{h} = B_2\frac{\nu_1}{\nu_2}\varepsilon_1 + C_2\sqrt{\frac{\nu_1}{\nu_2}}$$

$$= B_1\varepsilon_1 + C_1$$

$$\text{for} \quad \varepsilon_1 \ge \frac{C_2}{A_2 - B_2}\sqrt{\frac{\nu_2}{\nu_1}} \tag{11}$$

By inspection of Eq. (11), it becomes immediately obvious that the parameters describing the bilinear curve on the 1 axis are coupled to the parameters of the 2 axis curve by

$$A_1 = A_2\frac{\nu_1}{\nu_2} = A_2 O_R$$

$$B_1 = B_2\frac{\nu_1}{\nu_2} = B_2 O_R$$

$$C_1 = C_2\sqrt{\frac{\nu_1}{\nu_2}} = C_2\sqrt{O_R} \tag{12}$$

Further inspection of Eq. (11) compared with Eq. (6) shows also how the breakpoints are coupled to each other. In Eq. (12), it was useful to define the ratio ν_1/ν_2 as the orthotropicity ratio O_R.

$$O_R = \frac{\nu_1}{\nu_2} = \frac{A_1}{A_2} \tag{13}$$

SHEAR LOADING IN THE 12 PLANE

Finally, we consider the case of shear loading in the 12 plane. As we did above, we determine the coupled form of $\sigma_{12}(\gamma_{12})$ given $\sigma_2(\varepsilon_2)$. If the initial shear modulus G_{12} is not known, one can estimate it by means of the empirical relationship proposed by Panc [11]:

$$G_{12} = \frac{\sqrt{A_1 A_2}}{2\left(1+\sqrt{\nu_1\nu_2}\right)} = \frac{\sqrt{(\nu_1/\nu_2)A_2\cdot A_2}}{2\left(1+\sqrt{\nu_1\nu_2}\right)}$$

$$= \frac{A_2\sqrt{(\nu_1/\nu_2)}}{2\left(1+\sqrt{\nu_1\nu_2}\right)} \tag{14}$$

Note the substitution for A_1 in the second step of the above equation. This arises from Eq. (12). Proceeding, we obtain

$$\sigma_{12}(\gamma_{12}) = \frac{N_{12}}{h} = G_{12}\gamma_{12} = A_{12}\cdot\gamma_{12}$$

$$\text{for } \gamma_{12} \le \frac{C_2}{2(A_2 - B_2)}\sqrt{\frac{A_2}{G_{12}}}$$

$$\sigma_{12}(\gamma_{12}) = \frac{N_{12}}{h} = G_{12}\frac{B_2}{A_2}\gamma_{12}$$

$$+\frac{C_2}{2}\sqrt{\frac{G_{12}}{A_2}} = B_{12}\gamma_{12} + C_{12}$$

$$\text{for } \gamma_{12} \ge \frac{C_2}{2(A_2 - B_2)}\sqrt{\frac{A_2}{G_{12}}} \tag{15}$$

Inspection of Eq. (15) leads to the conclusion that the shear response is correlated to the axial response along one of the orthogonal axes, in this case axis 2. Thus:

$$A_{12} = G_{12}$$

$$B_{12} = G_{12}\frac{B_2}{A_2}$$

$$C_{12} = \frac{C_2}{2}\sqrt{\frac{G_{12}}{A_2}} \tag{16}$$

and the break in the shear bilinear curve is also linked to the normal bilinear curve

$$\gamma_{12\,break} = \frac{C_2}{2(A_2 - B_2)}\sqrt{\frac{A_2}{G_{12}}} \tag{17}$$

An advantage of this model is that it is simple to use and that the required data are readily available either from uniaxial tests or from biaxial tests. The model, as such, has been developed to solve planar problems. However, Chen and Saleeb [12] have demonstrated that such models have limited validity in three-dimensional situations. This is so because experimental evidence points to abrupt volume changes near the peak stresses in multiaxial compression tests.

APPLICATION OF THE COUPLED BILINEAR CONSTITUTIVE MODEL

Algorithm for Creating Coupled Bilinear Constitutive Model

The technique we used to construct the coupled bilinear constitutive model is governed by Eqs. (6), (11) and (15). These describe the stress versus strain response on the 2, 1 and 12 axes, respectively and they are coupled to each other by common constants describing the slopes of the bilinear curves. Four parameters are needed to construct the two normal stress versus strain curves on orthogonal axes: A_i, B_i, C_i and O_R. Five parameters are needed if one wants to model both normal and shear responses: A_i, B_i, C_i, O_R and G_{12}. However, if G_{12} is unavailable, it can be approximated by Eq. (14), requiring only four independent parameters provided that the two Poisson ratios are available.

Demonstrating the Algorithm's Effectiveness by Comparison to Experimental Data

Data from Qiu et al. [7]

Since this data set was the most complete available to us, including two axial responses as well as a shear response, our technique for arriving at the constants called for finding five constants. We minimized the error of the two axial response predictions (Eqs. 6,11) and the error of the shear response prediction (Eq. 15) simultaneously, by means of a spreadsheet calculation. The spreadsheet approach to iterating these five constants was advantageous because those five constants affect the constitutive response on the 1 axis, the 2 axis, as well as the 12 axis. By iteration, the best values for the five parameters A_2, B_2, C_2, O_R and $G_{12} = A_{12}$ were found, which provided optimum fits for axes 1, 2 and 12. These constants are shown in Table I. These five constants lead to values for ε_{2break} in the 2 direction; A_1, B_1, C_1 and ε_{1break} in the 1 direction; and B_{12}, C_{12} and $\gamma_{12break}$ in the 12 direction (Table I). Interestingly, our obtained orthotropicity ratio of O_R = 3.1 is close to the experimentally obtained value O_R = 3.3 reported in Qiu et al. [7]. Figure 2 depicts the applicability of the coupled bilinear model.

Qiu et al. also experimentally obtained the shear stress versus strain response and, by determining the initial slope of their published data, we can arrive at an initial shear modulus of $A_{12} = G_{12}$ = 1.81 GPa. The disparity between this experimental G_{12} and our evaluation of 3.30 GPa (Table I) can be dealt with by further generalizing our curve-fitting procedure. We can thus tit our model to all available data and minimize the expression

$$w_1\sum {r_1}^2 + w_2\sum {r_2}^2 + w_{12}\sum {r_{12}}^2 = \text{Minimum} \tag{18}$$

where r_i is a prediction error associated with data in the i direction and w_i is an assigned weighting factor between 0 and 1. Our first evaluation (Table I) is with the weighting $w_1 = w_2 = w_{12} = 1$.

However, if the data along a particular

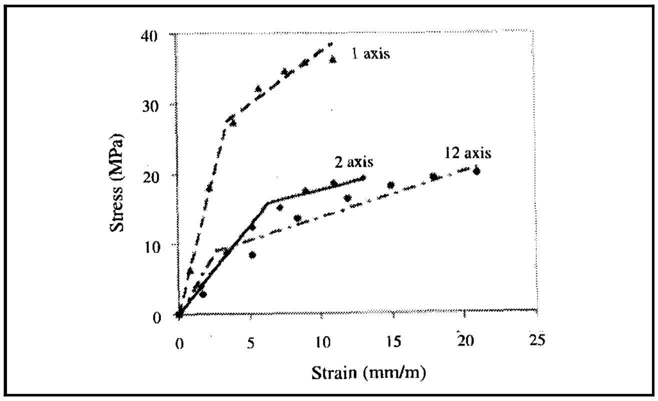

Fig. 2. Data from Qiu et al. [7] with predictions.

axis is suspect, other possible weighting combinations could be used. If the material was truly bilinear, all weighting procedures should be equal. If the model could be expanded to include other loading cases, then Eq. (18) could be expanded as well.

Data from Suhling [8]

Suhling examined paperboard subjected to axial loads. Nonlinear shear data was not available, although they did experimentally obtain the initial shear modulus by means of a torsion test. We arrived at evaluations for the four parameters A_2, B_2, C_2 and O_R for this data set. Recall that a case without shear data necessitates four parameters. The fit led to e_{2break} in the 2 direction and A_1, B_1, C_1 and e_{1break} in the 1 direction (Table I).

We can compare the initial slope of our generated shear stress versus strain response with the value obtained by Suhling's. Our model predicts an initial slope $A_{12} = G_{12} = 1.75$ GPa, whereas Suhling experimentally obtained $G_{12} = 1.69$ GPa. Figure 3 depicts the applicability of the coupled bilinear model.

Data from Gerhardt [9]

Gerhardt analyzed paperboard laminates as did Qiu. No shear response was determined. However, axial data and Poisson ratio data was found. As was done previously, we fit the 2 direction parameters from which we determined e_{2break} and the 1 direction parameters (Table I). Our calculated O_R = 3.6 compares well with Gerhardt's average experimental O_R = 3.1. Figure 4 depicts the applicability of the coupled bilinear model. In Fig. 4, we show the predicted shear response, even though there is no experimental shear data.

Data from Erickson and Boller [13]

These data tested plastic impregnated paper, which approaches more traditional composites. Again, there are only axial data on two orthogonal axes to which we fit the 2 direction parameters and determined e_{2break} and the 1 direction parameters (Table I). Figure 5 depicts the applicability of the coupled bilinear model.

Data from Kuenzi and Jenkinson [14]

These data report on a panel constructed of honeycombed sections of reinforced polyamide. The experimental data presents 1

TABLE I
EVALUATIONS OF PARAMETERS IN BILINEAR CONSTITUTIVE MODEL FIT TO DATA

	1 direction				2 direction					12 direction			
Data Source	A_1 (GPa)	B_1 (GPa)	C_1 (MPa)	e_{1break} (mm/m)	A_2 (GPa)	B_2 (GPa)	C_2 (MPa)	e_{2break} (mm/m)	O_R	A_{12} (GPa)	B_{12} (GPa)	C_{12} (MPa)	$g_{12break}$ (mm/m)
[7]	7.68	1.49	22.2	3.59	**2.50**	**0.485**	**12.7**	6.29	**3.1**	**3.30**	0.641	7.28	2.73
[8]	6.93	4.94	4.91	2.47	**2.80**	**2.00**	**3.12**	3.88	**2.5**	1.75	1.25	1.23	2.45
[9]	6.88	2.86	8.76	2.18	**1.93**	**0.801**	**4.64**	4.11	**3.6**	1.50	0.622	2.04	2.33
[13]	15.1	1.98	91.4	6.95	**9.77**	**1.28**	**73.4**	8.65	**1.5**	—	—	—	—
[14]	**0.139**	**0.0397**	**1.44**	14.5	—	—	—	—	—	**0.0546**	**0.0156**	**0.453**	11.6

Note: Figures in bold are solved for via regression; others are calculated dependencies.

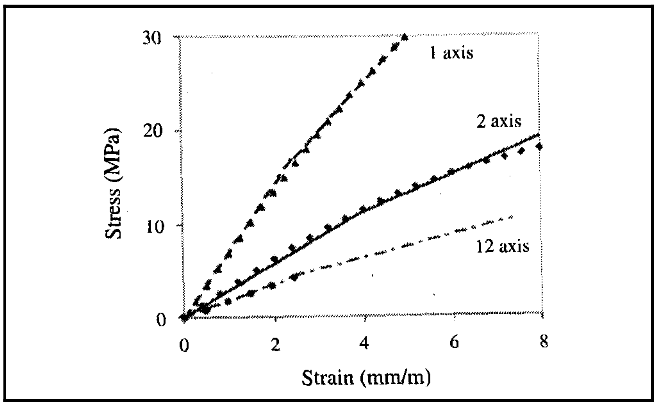

Fig. 3. Data from Suhling [8] with predictions.

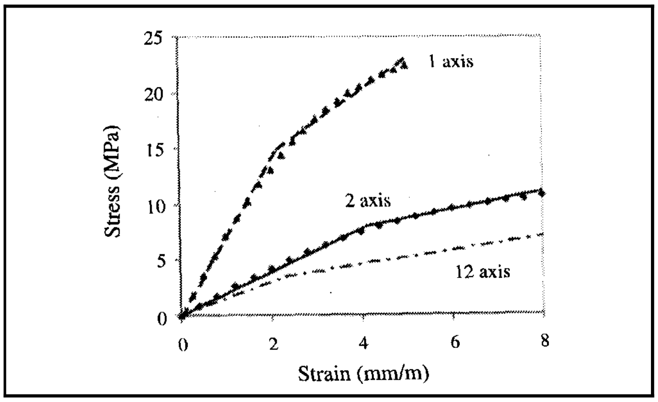

Fig. 4. Data from Gerhardt [9] with predictions.

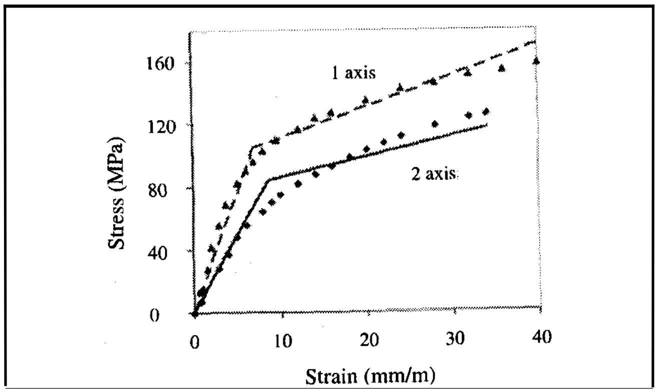

Fig. 5. Data from Erickson and Boller [13] with predictions.

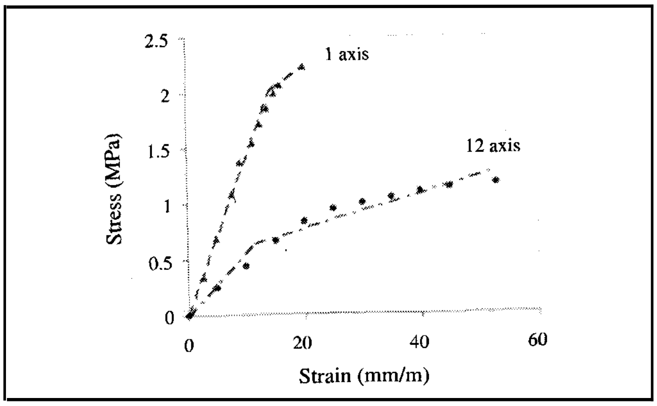

Fig. 6. Data from Kuenzi and Jenkinson [14] with predictions.

axis axial stress versus strain, as well as shear stress versus shear strain (12 axis data). Consequently, we started by analyzing the 1 axis and then we predicted the shear response. We arrived at values for A_1, B_1, C_1, G_{12} and O_R and then determined ε_{1break}, B12, C_{12} and "?break (Table I). Figure 6 depicts the applicability of the coupled bilinear model.

CONCLUSIONS

Examination of Figs. 2 through 5 show that our proposed technique does capture behaviour that was previously not reported in the literature. We have demonstrated that the constitutive response on orthogonal axes and in shear are coupled to each other. We also present a technique for constructing a bilinear orthotropic model which has the ability to predict responses on the 1 and 12 axes based on the response on the 2 axis.

NOMENCLATURE

e_{break}	Strain associated with Y_i
e_i	Engineering normal strain
s_i	Engineering stress
g_{ij}	Engineering shear strain
v_{ij}	Poisson ratio
E_i	Young's modulus
A_i	First slope of bilinear curve *i*
B_i	Second slope of bilinear curve *i*
c	Shearing constant
C_i	y intercept of second line of bilinear curve *i*
e	Generalized strain
G_{ij}	Shear modulus
h	Plate thickness
$H = H(e)$	Strain energy density
$H'(e)$	Derivative of $H(e)$ with respect to e
N_{ij}	Force per unit width
O_R	Orthotropicity ratio
Yi	Yield stress

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