AMD-Vol. 231/MD-Vol. 85
Mechanics of Cellulosic Materials — 1999
ASME 1999

NONLINEAR FINITE ELEMENT MODELING OF CORRUGATED BOARD

A. C. Gilchrist and J. C. Suhling
Department of Mechanical Engineering
Auburn University
Auburn, AL 36849-5341

T. J. Urbanik
U.S.D.A Forest Products Laboratory
One Gifford Pinchot Drive
Madison, WI 53705-2398

ABSTRACT

In this research, an investigation on the mechanical behavior of corrugated board has been performed using finite element analysis. Numerical finite element models for corrugated board geometries have been created and executed Both geometric (large deformation) and material nonlinearities were included in the models. The analyses were performed using the commercial nonlinear finite element code ABAQUS executed on Sun SparcStations and the State of Alabama Cray C90 Supercomputer. Both in-plane and transverse loadings of corrugated board configurations have been examined. The models considered the composite nature of the combined board structure in detail through the use of three-dimensional meshes (shell elements) where the liner and medium were each discretized.

The necessary input material properties (constitutive and failure properties) of the liner and medium materials were obtained from through uniaxial, biaxial, and shear tests performed as a part of this study. Particular combined board configurations analyzed include the four-point bending geometry, Edge Crush Test (ECT) geometry, and anticlastic bending test geometry used to evaluate board twisting stiffnesses. For evaluation purposes, results from the finite element simulations were correlated with the analogous experimental measurements performed using actual corrugated board specimens. Current work emphasizes prediction of moisture-induced warp, buckling, and creep behavior of corrugated panels.

INTRODUCTION

Corrugated containers are the most important structural application of paperboard The corrugated board panels used in these containers are sandwich structures consisting of two flat plates called liners, which are separated by a sine wave shaped fluted core referred to as the corrugating medium or simply the medium. Because it is composed of two different elements, corrugated board is a composite structure often referred to as combined board. Corrugated boards are typically lightweight and inexpensive, with high stiffness-to-weight and strength-to-weight ratios.

The basic geometry and component materials of corrugated board are illustrated in Figure 1. The paperboard used for the liners is stiff and strong so that a high bending stiffness of the combined board is achieved The medium serves as a low density core within the sandwich structure to separate the facings, prevent the facings from sliding relative to one another, and prohibit localized buckling. As shown in Figure 1, the machine directions of the linen and medium are in the direction of propagation of the sinusoidal medium. This is a result of the standing converting process where rolls of liner and medium paper are combined into corrugated board in a corrugator. The basic simplicity and versatility of the corrugated box has helped it maintain its position as the world's most common form of distribution package.

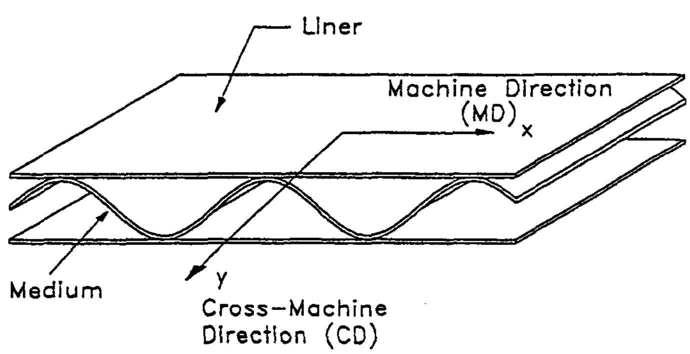

Figure 1- Corrugated Board Structure

The structural performance of a corrugated container is a function of numerous factors including the quality of the input cellulose fibers, the mechanical properties of the liner and medium, and the structural properties of the combined board. Various shipping and warehousing conditions should also be taken into consideration These include high long-term compressive forces from stacking, vibrations, impacts, and adverse atmospheric conditions. The complicated nonlinear behavior or paper makes modeling of the mechanical response of corrugated board and structures composed of corrugated board a difficult task

Numerous studies have been conducted on the bending stiffnesses of corrugated board. Most of these investigations have incorporated either theories or experiments to find the flexural rigidities of the board. In these studies, the bending stiffnesses were often determined using beam formulations. In this approach, the bending stiffness is equivalent to the product of an elastic modulus term and a moment of inertia term. Reviews of previous analytical or experimental work on the bending stiffnesses of combined board can be found in the work of Luo, et al. [1-2].

Several previous investigations have involved the application of finite element analysis and other energy-based solution methods to corrugated board panels and structures. Peterson [3] and Peterson and Fox [4] applied the von Karman large-deflection plate theory to paperboard The Rayleigh-Ritz method was employed to determine the theoretical displacement and stress fields of simplified rectangular panels. Several simple loadings were considered that are suitable for corrugated containers.

Peterson [5] developed a finite element model to study the stress fields generated in a machine-direction combined hoard beam under three-point loading. In this model, only short-term linear-elastic behavior was considered. Symmetry was used in the modeling procedure, and the liners and medium were discretized into straight-line beam finite elements. The medium was assumed to have a shape described by a single sine term. Stress fields for the liners and medium were predicted, and the maximum computed stress values were compared to allowable material strengths. This comparison revealed the controlling critical component of corrugated board is the medium under compression.

Pommier and Poustis [6] studied the bending stiffnesses of corrugated board structures using models based on the finite element method and a developed linear elastic analysis code referred to as SYSTUS. The corrugating medium defined in the calculation code incorporated trapezoidal mesh structure. The models assumed perfect bonding between the fluting and the liners. To accomplish this, the nodes of the fluting were merged with those of the liners at the points where contact occurred. The finite element code was executed to simulate the movements of the bending-stress test samples. These results were compared to the experimentally measured values for the bending stiffnesses. They concluded the proposed model was insufficient to determine the terms of the bending flexibility matrix of an equivalent orthotropic sheet

Pommier and Poustis [7l also employed the linear elastic finite element method to predict the top to bottom compression strength of a corrugated box. The bending stiffness and shear bending of the board were considered. The shear bending stiffness, was measured using an anticlastic bending test. Since all four faces of the corrugated board expand outward under vertical compression, free rotation about the folding ridges of the corrugated sleeve was modeled. The finite element calculations gave results in agreement with their experimental values.

Pommier, Poustis, Fourcade, and Morlier [8] applied the finite element method (linear elastic) to partly solve the optimization process of the components of a corrugated box subjected to vertical compression. The finite element model simulated the buckling of a flapless corrugated box from the determination of the stiffness of the board from which it was made. These stiffness values of the combined board were determined experimentally by conducting four-point and anticlastic bending tests. Symmetry allowed a quarter of the corrugated tube to be modeled where the liner and flute structure were discretized into shell quadratic interpolation rectangular finite elements. Each element received the membrane and the bending stiffnesses of the investigated board as input data Four different boundary conditions were imposed on the common edges during the modeling\ procedure. The results of the four different finite element models were evaluated and compared to experimental and analytical vertical compression strengths

McKinlay [9] demonstrated an analytical relationship between the machine direction' shear stiffness of a sandwich panel and the twisting stiffness of a long strip corrugated sample. This relationship allows correction for the effect of the liners on the twisting stiffness, which yields a true measure of the MD shear stiffness of corrugated panels. A finite element model was used to simulate the conditions of the analytical analysis. His work did not specify whether a linear or nonlinear analysis was performed. The finite element model consisted of four node shell elements, and two types of glue modeling were used to connect the liners to the medium. In one situation, multi point constraints (MPC's) were used, while in the other, the tips of the medium were joined to the liners using short shell elements. The finite element analysis confirmed the analytical results, concluding that the twist method provided a reliable and accurate manner to test the quality of corrugated fiberboard structures.

Nordstrand, Carlsson, and Allen [10] conducted an analysis of the transverse shear modulus for various corrugated medium configurations using curved beam theory. The reduction of the effective shear modulus across the corrugation due to deformations of the liners was quantified, and the results were compared with classical theory and finite elements. For the finite element analysis, the effective shear modulus was calculated for circular and triangular cores using ANSYS finite element code. Two-noded elastic beam elements were used to model one wavelength of the corrugated structure. The medium and the liners were rigidly connected to represent an ideal bond. The theoretical formula derived to approximate the transverse shear modulus based on deformation of the medium only was in good agreement with the finite element results.

Bronkhorst and Riedemann [11] and Nordstrand and Hagglund [12] have developed nonlinear finite element models for corrugated board configurations. In both cases, a single layer orthotropic sheet that obeyed a viscoelastic constitutive law was used to approximate the combined board structure of the corrugated board panels. These investigations generated predictions for compressive creep of a box and time dependent sagging of a corrugate board tray. Little published work is available on the use of plasticity or viscoplasticity constitutive models for paper.

In this work, the nonlinear mechanical response of corrugated board has been investigated using the finite element method. Experiments were conducted to characterize the nonlinear mechanical behavior of the liner and medium papers used to make the corrugated board panels. The orthotropic material properties obtained from these experiments were then utilized as input into to finite element models for several loading geometries including four-point bending of a corrugated board strip, the Edge Crush Test (ECT), and the anticlastic bending test geometry used to evaluate board twisting stiffnesses. The finite element predictions for the load versus deflection behavior of the corrugated board specimens were correlated with experimental results. In current studies, predictions are being made for moisture-induced warp, buckling, and viscoplastic behavior of corrugated panels.

SUMMARY OF RESULTS

Material Characterization

In order to determine the mechanical properties of the liner and medium papers used in the combined board samples tested in this work, uniaxial tensile extension tests and pure shear experiments were conducted. The materials tested in this study were a 42-lb (205 g/m^2) linerboard and a 26 lb (127 g/m^2) medium from Neutral Sulfite Semi-Chemical (NSSC) pulp. Environmental conditions were maintained as per TAPPI standard, T-402. A commercially produced "C" flute corrugated board constructed from these materials was obtained. Referring to Figure 2, the nominal thickness of the liner, t_L, was .01141 inches, and the nominal thickness of the medium, t_m, was .0077 inches. These values along with the remaining dimensions of the investigated corrugated board can be found in Table 1.

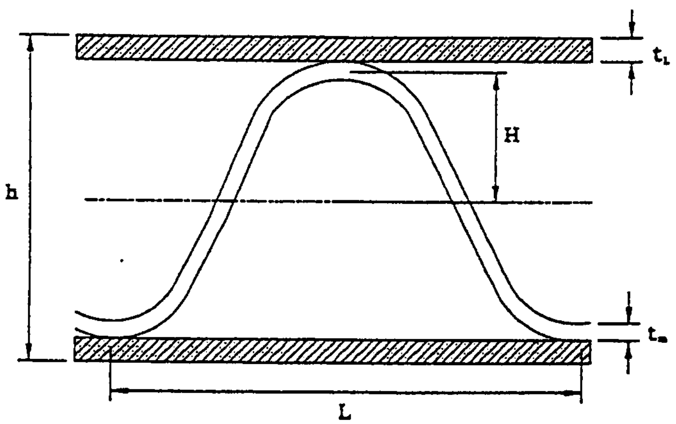

Figure 2 - Corrugated Board Key Dimensions

Property	Value (in)
t_L	.0114
t_m	.0077
H	.0710
h	.1725
L	.1430

Table 1 -Dimensions of Test Boards

Uniaxial tensile extension tests were perfomxd to evaluate the MD and CD elastic moduli (E1 and E2, Poisson's ratios (v_{12} and v_{21}), and the nonlinear shapes of the MD and CD stress-strain curves. Next, torsion tests (see Hung and Suhling [13]) were conducted in order to measure the shear modulus (G_{12}). These measured material properties (see Table 2) were then used as input parameters to the finite element models to be presented below. Typical experimental data from the uniaxial and shear testing are shown in Figures 3-5.

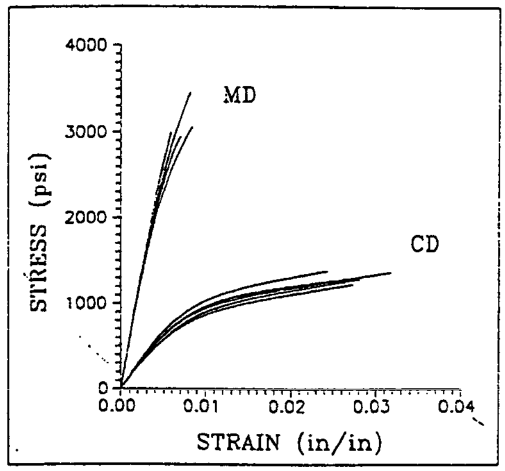

Figure 3 - Stress-Strain Curves (Tension, Liner)

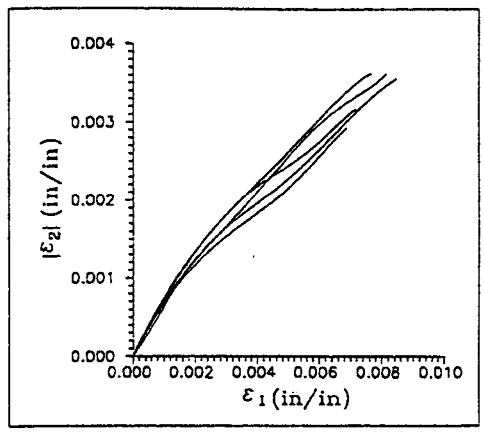

Figure 4 - Poisson's Ration Teat Data (MD Extension, Liner)

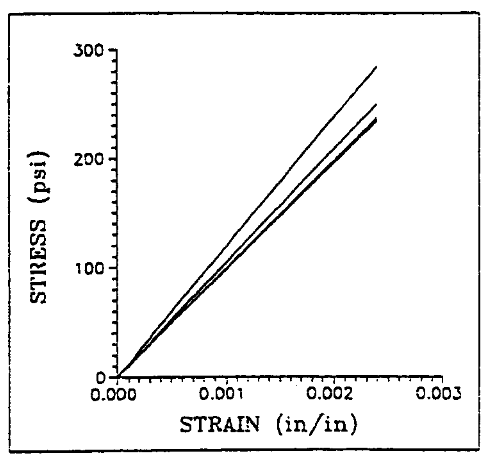

Figure 5 - Pure Shear Stress-Strain Curve Data (Liner)

Property	Liner	Medium
E_1	.91 x 10^6 psi	.78 x 10^6 psi
E_2	.34 x 10^6 psi	.24 x 10^6 psi
ν_{12}	.44	.41
G_{12}	.21 x 10^6 psi	.15 x 10^6 psi

Table 2 -Linear Elastic Material Constants

Finite Element Models

Each of the ABAQUS finite element models created for the different test configurations in this work were developed with a computer aided design pre-processor. The corrugated board finite element models had the same C-flute configuration as the samples used in the experimental tests. Three-dimensional (general orientation) plate/shell elements were used to discretize both the liner and the medium. The models created included:

1. CD Four-Point Bending Model
2. MD Four-Point Bending Model
3. Pure Twisting Model
4. Edge Crush Test (ECT)

The geometries of these models were chosen to be the same as those used in the corresponding experiments conducted on combined board samples.

The liner and medium were both discretized using four-node quadrilateral shear flexible shell elements (S4R5) in ABAQUS. These elements are first order quadrilateral elements where transverse shear deformation is allowed The S4R5 elements contain five degrees of freedom per node, which include three translations and two in-surface rotations. During analysis, these shell elements use reduced integration and hourglass control. In reduced integration, the stresses and strains are calculated at the locations that provide optimal accuracy. One disadvantage of the reduced integration procedure is the admission of deformation modes that cause no straining at the integration points. Therefore, "hour-glassing" can occur where the zero energy mode starts propagating through the mesh, leading to inaccurate solutions. For this reason, ABAQUS adds artificial stiffness is to the element in an hourglass control procedure so that excessive deformation is prevented:

The adhesive between the liner and medium was modeled in two different ways in this work. In one method, perfect bonding between the liner and medium was assumed. In these models, the extreme positions of the medium were connected directly to the liner by sharing the same node. In the second method, a multi-point constraint (MPC) technique was utilized. The MPC chosen acts as a beam element (MPC7) between the two nodes that connect the liner and medium and imposes the same rotations and translations on each node. In all of the models that were run both with and without MpC's, the results were approximately the same.

An elastic-plastic material model was specified for the paper materials in the ABAQUS input file. In this study, the elastic portion of the stress-strain response was governed by orthotropic Hooke's Law. The orthotropic linear elastic material properties from Table 2 were specified for the liner and medium.

The nonlinear portions of the average MD stress-strain curves for the liner and medium paperboards were characterized and entered into the ABAQUS input file in order to define the nonlinear plasticity behavior of the papers. Average curves for the measured data were obtained by using the empirical hyperbolic tangent model

$$\sigma = C_1 \tanh(C_2 \varepsilon) + C_3 \varepsilon \qquad (1)$$

to fit the data from all stress-main curves for a given material and testing orientation. Table 3 contains the coefficients obtained by performing nonlinear regression fits to the experimental stress-strain curve data. After defining the initial elastic regions of the average stress-strain Curves, the nonlinear regions were subdivided into small piecewise linear portions for the purpose of specifying the necessary stress versus plastic strain data in ABAQUS. Isotropic hardening and the Hill failure criteria were utilized in the models.

Material	C_1 (ksi)	C_2	C_3 (ksi)
Liner (MD)	5.15	150.43	273.98
Liner (CD)	2.10	157.89	58.44
Medium (MD)	3.42	168.69	278.81
Medium (CD)	1.46	145.33	29.68

Table 3 - Hyperbolic Tangent Coefficients for the Average Stress-Strain Curves

The developed models were also performed using large deflection options that incorporate geometric nonlinearities into the formulation were performed.

Typical Results

Figure 6 shows a schematic of the normal four point bending loading geometry. The four point bending finite element models in this work were designed to simulate the loading fixture shown in Figure 7. This device was designed to be placed into a uniaxial testing machine where the top is fixed and the bottom moves upward at a specified deformation rate. The anvils on the test set-up were designed in such a way as to produce a loading over a rectangular area of 1.5" x .75". The top and bottom anvils are designed for a self-adjusted rotation in a manner where no slippage of the corrugated board will occur. In order to measure the central deflection during the four-point bending tests, a Linear Variable Differential Transformer (LVDT) was placed in the center of the specimens. The load was measured using a load cell. Load and deflection data were collected using a PC-based data acquisition system.

The four-point bending specimens were prepared from the C-flute corrugated board material formed from the liner and medium materials discussed above. The samples were cut using a circular paper saw with a width of 1.5" while the length varied between the MD and CD samples. The length of the CD specimens was 10 inches while the length of the MD specimens was 11 inches. Referring to Figure 6, the outer span, d, was 2.25 inches while the inner span, D, was 4.5 inches. In the experiments, an applied stroke rate of .5 cm/min was utilized Again, TAPPI standard environmental conditions were maintained.

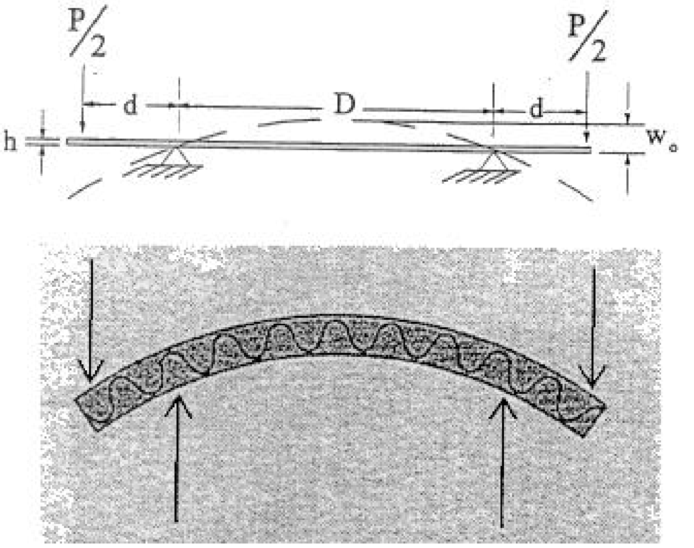

Figure 6 - Four Point Bending Geometry

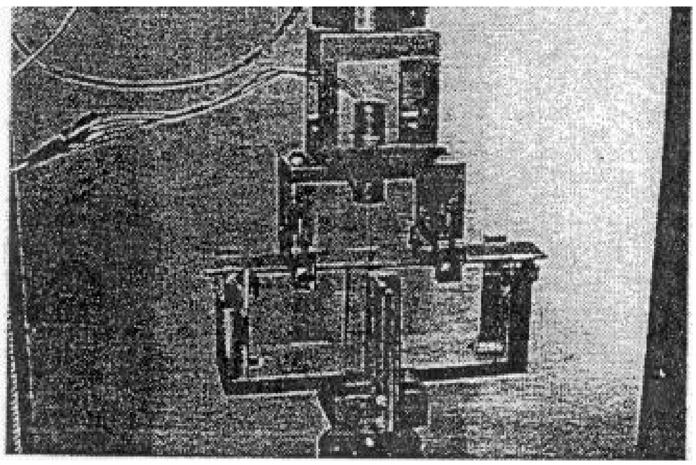

Figure 7 - Four Point Bending Test Fixture

Figure 7 illustrates the developed mesh for the MD specimen (1/4 model incorporating symmetry), and the correlation between the finite element predictions and the experimental load-displacement curves for the CD specimens are shown in Figure 8. Although the con-elation is reasonably good, it is seen that the FEA results underestimate the stiffness of the board due to the neglecting of any stiffness added by the glue bonding the liners to the medium. The experimental tests were all run until specimen failure, and nonlinearities can be seen in the predicted and measured responses.

Another loading geometry that was considered in this work was the twisting test procedure proposed by Tsai [14]. In his method, pure twisting tests are accomplished using zero and forty-five degree plates that contain three fixed points and one loading point The loading scheme for this test can be seen in Figure 10. This configuration insures that the necessary load distribution will be obtained while a fixed reference coordinate system is maintained. The test set-up used for the twisting tests is shown in Figure 11. It allowed for direct load-central deflection readings to be recorded easily using standard test equipment Pure twisting tests with 6 x 6 inch square corrugated board plates were performed The displacements at the center of the plate were measured using an LVDT in the same manner as in me four-point bending tests.

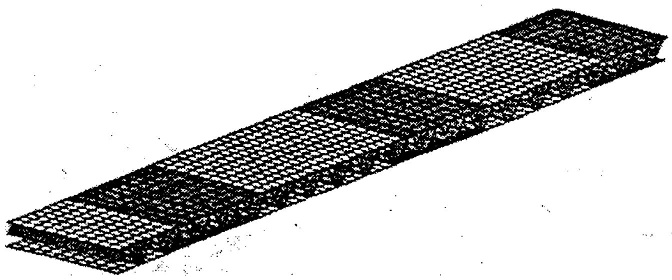

Figure 8 - Finite Element Mesh for MD Four Point Bending

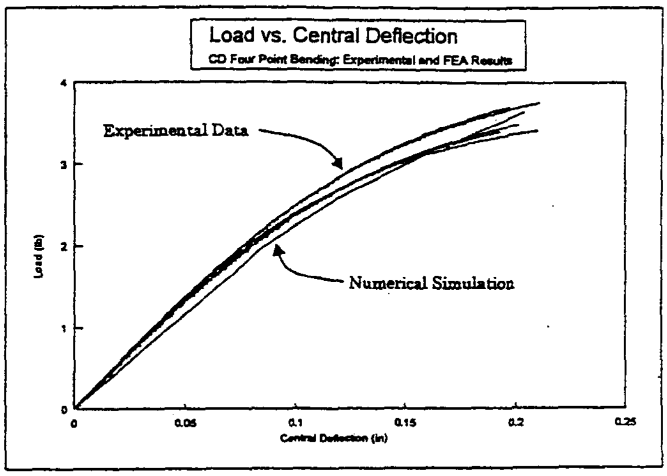

Figure 9 - Correlation of FEA Predictions with Experimental Data for the CD Four Point Bending Geometry

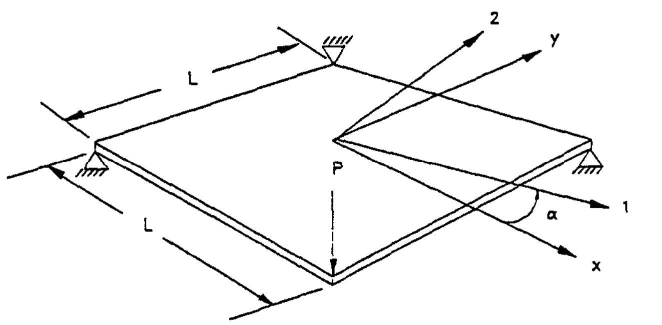

Figure 10 - Schematic of Pure Twisting Test Loading Geometry

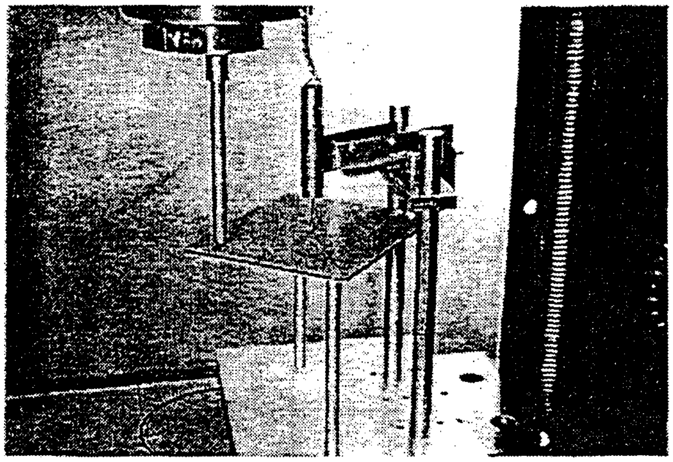

Figure 11- Plate Twisting Test Load Fixture

Figure 12 illustrates the developed mesh for the twisting test specimen (full model), and the correlation between the finite element prediction and the experimental load-displacement curves are shown in Figure 13. The correlation is again very good, with the models again being slightly under-stiffened as expected. Unlike the four-point bending results, the observed load vs. deflection response in the twisting test configuration was extremely linear all the way to failure.

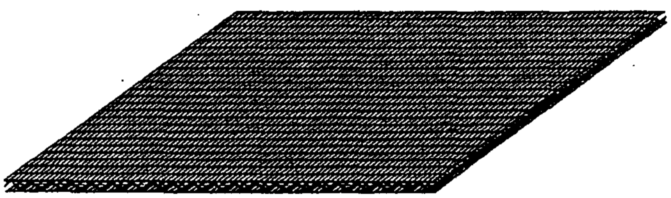

Figure 12 - Twisting Test Finite Element Mesh

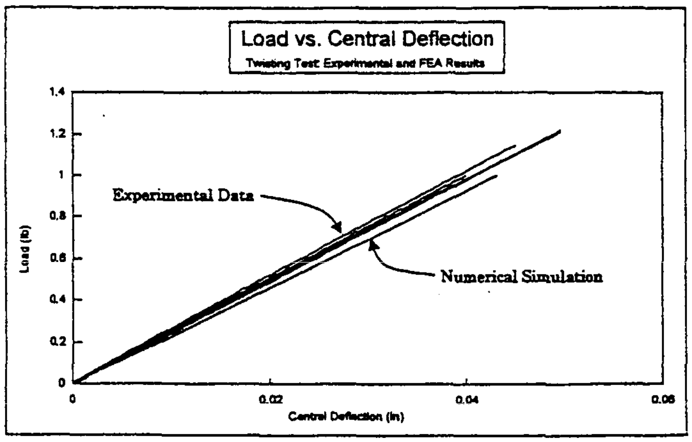

Figure 13 - Correlation of FEA Predictions with Experimental Data for the Anticlastic Plate Twisting Test Geometry

CONCLUSIONS

Numerical finite element models for corrugated board geometries have been created and executed. Both geometric (large deformation) and material nonlinearities (plasticity) were included in the models. Selected results have been presented including those for the four-point bending and plate twisting loading geometries. Results from the finite element simulations correlated reasonably well with the analogous experimental measurements performed using actual corrugated board specimens. Further work is necessary to accurately model the effects of the glue used to bond the liner and medium materials. This is our current task along with other projects dealing with prediction of moisture-induced warp, buckling, and creep behavior of corrugated panels.

ACKNOWLEDGMENTS

This work was supported by the USDA Forest Products Laboratory, Madison, WI and the Pulp and Paper Research and Education Center at Auburn University.

REFERENCES

1. Luo, S., Suhling, J. C., Considine, J. M. and Laufenberg, T. L., "The Bending Stiffnesses of Corrugated Board," in Mechanics of Cellulose Materials, Edited by R. W. Perkins, ASME AMD-Vol. 145, pp. 15-26, ASME Winter Annual Meeting, Anaheim, CA, November 8-13, 1992.
2. Luo, S., Suhling, J. C. and Laufenberg, T. L., "Bending and Twisting Tests for Measurement of the Stiffnesses of Corrugated Board," in Mechanics of Cellulose Materials - 1995, Edited by R. W. Perkins, ASME AMD-Vol. 209, pp. 91-109, ASME Applied Mechanics and Materials Summer Meeting, UCLA, Los Angeles, CA, June 28-30, 1995.
3. Peterson, W. S., "Unified Container Performance and Failure Theory I: Theoretical Development of Mathematical Model," TAPPI, Vol. 63(10), pp. 75-79, 1980.
4. Peterson, W. S. and Fox, T. S., "Unified Container Performance Theory," in Paper Science and Technology: The Cutting Edge, pp. 172-189, 1979.
5. Peterson, W. S., "Flute/Liner Interaction for Bending of Combined Board Beams," Paperboard Packaging, Vol. 68(8), pp. 37-41, 1983.
6. Pommier, J. C. and Poustis, J., "Bending Stiffness of Corrugated Board Prediction Using the Finite Element Method," in Mechanics of Wood and Paper Materials, ASME AMD-Vol. 112, Edited by R. W. Perkins, pp. 67-70, 1990.
7. Pommier, J. C. and Poustis, J., "Box Stacking Strength Prediction: Today McKee, Tomorrow?," Proceedings of the Third Joint ASCE/ASME Mechanics Conference, La-Jolla, California, pp. 209-213, July 9-12, 1989.
8. Pommier, J. C., Poustis, J., Fourcade, E. and Morlier, P., "Determination of the Critical Load of a Corrugated Box Subjected to Vertical Compression by Finite Element Methods," in Proceedings of the 1991 International Paper Physics Conference, pp. 437-447, Kona, HI, 1991.
9. McKinlay, P. R., "Analysis of the Strain Field in a Twisted Sandwich Panel with Application to Determining the Shear Stiffness of Corrugated Fibreboard," Products of Papermaking, Vol. 1, pp. 575-599, 1993.
10. Nordstrand, T., Carlsson, L. A. and Allen, H. G., "Transverse Shear Stiffness of Structural Core Sandwich," Composite Structures, Vol. 27, pp. 317-329, 1994.
11. Bronkhorst, C. A. and Riedemann, J. R., "The Creep Deformation Behavior of Corrugated Containers in a Cyclic Moisture Environment," Proceedings of the Symposium on Moisture Induced Creep Behavior of Paper and Board," pp. 249-273, Stockholm, Sweden, December 5-7, 1994.
12. Nordstrand, T. and Hagglund, R., Predicting Sagging of a Corrugated Board Tray Bottom Using Isochrones," Proceedings of the 3rd International Symposium on Moisture and Creep Effects on Paper, Board, and Containers, pp. 215-220, Rotorua, New Zealand, February 20-21, 1997.
13. Hung, S. C. and Suhling, J. C., "Rate Dependent Pure Shear Behavior of Linerboard, Proceedings of the 3rd International Symposium on Moisture and Creep Effects on Paper, Board, and Containers, pp. 69-84, Rotorua, New Zealand, February 20-21, 1997.
14. Tsai, S. W., "Experimental Determination of the Elastic Behavior of Orthotropic Plates, *Journal of Engineering for Industry*, Vol. 87(3), pp. 65-73, 1965.