VISCOELASTICITY

Constitutive modeling of paper account for rate of load and transient relative humidity effects*

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ABSTRACT

A nonlinear viscoelastic model was developed for predicting the strain response of paper subjected to compressive stress. The model can be calibrated to incorporate the hygroexpansive companent of strain that occurs in transient humidity environments. In this study, data gathered from tests of virgin 205-glm2 linerboard paper were used to calibrate and verify the model. Calibration was accomplished through long-term (16-h) experiments. The results showed that the linear superpostition of strains that resulting from load and strain that resulted from hygroexpansion does not fully account for the total response of the material. The model was also extended to account for strains resulting from creep loads.

Application:

A constitutive model of paper can be constructed from a machanistic perspective. The creation of such models for various furnishes of paper can impart engineering significance ot the various terms of the model.

THE PURPOSE OF THIS STUDY WAS TO create a mechanistic constitutive model that could predict the strain response of paper subjected to compressive loads in either a static relative humidity (RH) environment or a single half-cycle of transient RH. A mechanistic model such as this can be useful in a number of ways. First, it can impart meaning to a constitutive material response in a more significant manner than simply curve-fitting the results. For example, attributing such engineering significance to the terms of the model can form the basis for comparing various paper furnishes. Second, such a model can be solved for various types of loading histories. The fundamental differential equation of motion for the system proposed herein has been solved for creep loading and for mo notonically increasing loading histories. Another application could be the solution for arbitrary loading histories, which would require numerical integration. Finally, such a model could be extended to cover biaxial stress states.

However, there are some inherent limitations to this type of model. One limitation is that the nonlinearity of strains resulting from the mechanosorptive effect cannot be fully accounted for by a model thats sumes the validity of strain superposition. Attempts were made to account for the unique strain response of the first half-cycle of RH change by subjecting unloaded paper specimens to these RH environments and measuring the response. We suspected that the nonadditivity of strain responses was probably most severe in the first half-cycle. This pure hygroexpansive component was subtracted from the strain response of the loaded paper (in transient RH environments) to develop the constitutive model. The model was used to test the validity of strain additivity when hygroexpansive strain deformation and compressive strain deformation occur in the same direction or in opposite directions. We investigated whether the mechanosorptive effect in compression is more serious during an adsorp tion or resorption half-cycle.

Calibration was performed using long-term (16-h) compressive tests. The calibrated model was then used to predict the response of short-term compressive tests in static and transient RH environments (5-min and 30-min tests, respectively). The results showed that the model can account for rate of loading, but the assumption of strain superposition does not fully account for strain response in a transient humidity environment.

BACKGROUND

A suitable one-dimensional model for studying viscoelastic materials incorporates instantaneous elasticity, retarded elasticity, and plastic flow. The simplest such model to account for these three main components is the configuration of springs and dashpots that is shown in **Fig. l.**

In this viscoelastic model, a Hookean spring (elastic component) is added in series to a Hookean spring in parallel with a Newtonian dashpot (viscoelastic component), which in turn is added in series to a Newtonian dashpot (viscopiastic component). Many reseaders *(1,2)* have used this model to describe viscoelastic materials. A similar configuration (three-element model) was proposed by Tobolsky and Eyring *(3)* to predict primary creep. Mason *(4)* laid the groundwork for using such an element in the rheology of paper. Fridley *et al. (5)* proposed that in general, the total strain ε of a viscoelastic material at any given time *t* can be expressed as follows:

$$\varepsilon(t) = \int_0^t \Psi\,[\sigma(t)]dt \qquad (1)$$

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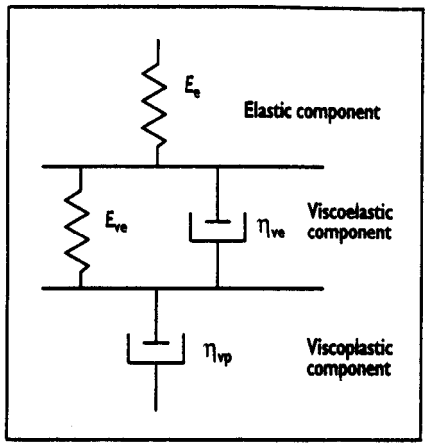
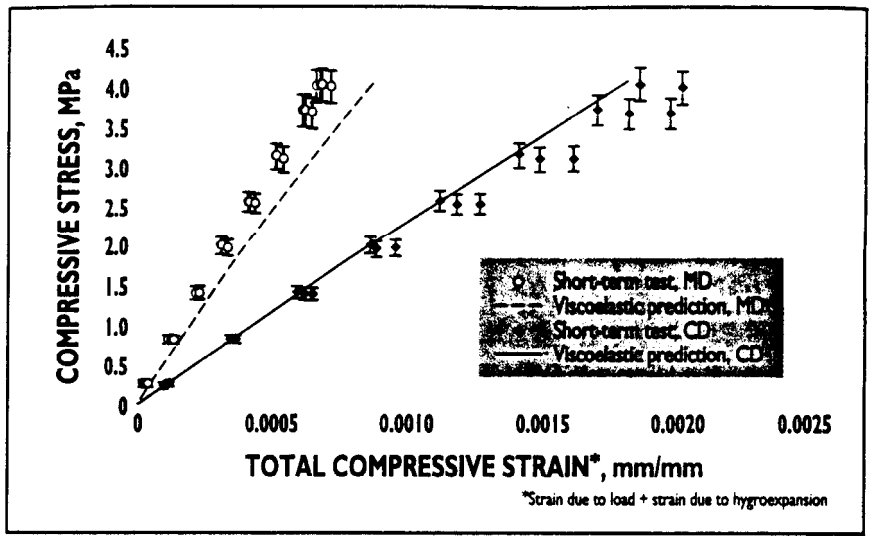


1. Viscoelastic model



2. Comparison of pure hygroexpansive component, virgin CD

where Ψ is the governing constitutive functional for the material and $\sigma(t)$ is the applied stress history.

Although humidity changes in the testing environment will effect the material properties of the test specimen, it is actually the moisture content of the material that changes its strength. However, in this study, it was not pos sible to measure the moisture content of the specimen as it was being tested. For this reason, we decided to use the RH profile as the state variable rather than the moisture content. Following this line of reasoning, if the functional Ψ is to include the response of the material caused by changes in the RH profile [RH (t)],the expression for total strain can be rewritten as follows:

$$\varepsilon(t) = \int_0^t \Psi\,[\sigma(t), RH(t)]\,dt \qquad (2)$$

If an appropriate functional Ψ is found, then the strain history will be fully defined as a function of the stress history $\sigma(t)$ and the RH profile history RH (t). To quantify the functional Ψ , it is necessary to consider the three independent components of the model shown in Fig. 1. Plevris and Traintafillou *(6)* defined such a functional as follows:

$$\Psi = \frac{d}{dt}\varepsilon = \frac{d}{dt}\varepsilon_e + \frac{d}{dt}\varepsilon_{ve} + \frac{d}{dt}\varepsilon_{vp} \qquad (3)$$

where

$d(\varepsilon)/dt$ = total time rate of strain
$d(\varepsilon_e)/dt$ = elastic time rate of strain
$d(\varepsilon_{ve})/dt$ = viscoelastic time rate of strain
$d(\varepsilon_{vp})/dt$ = viscoplastic time rate of strain.

Equation 3 can be solved for different loading histories, and the resulting solutions of these differential equations can be fitted to experimental data. Two cases have been solved for (a) a constant load ramp (linearly increasing load) and (b) the same case as (a) but with a nonlinear spring in the elastic element. An assumption was made in the solution of case (a), the linearly increasing load romp. We assumed an engineering stress in our calculation, i.e., the stress in the specimen is defined as the load divided by the original cross-sectional area. This assumption is not technically correct because of the swelling (or shrinking) that occurs in a transient humidity environment. Also, the Poisson effect would lead to a changing cross-sectional area as the load increases. However, based on studies of thickness changes in transient humidity environments *(7)* and reasonable Poisson ratios ($v_{MD\,CD}$ = 0.39 and $v_{CD\,MD}$ = 0.14), we calculate the error to be 8.1%. This is based on Crook and Bennet's calculations *(7)* of an approximate 2% change in thickness for every 10% change in RH. This accounts for an 8% change, plus approximately 0.1 % for the Poisson effect. Given this assumption of engineering stress (stress ramp = load ramp / original cross-sectional area), the following calculations can be made in closed form.

***Case** (a): Linearly increasing load*

The elastic term is defined as follows:

$$\frac{d}{dt}\varepsilon_e(t) = \frac{1}{E_e}\cdot\frac{d}{dt}\sigma(t) \qquad (4a)$$

or

$$\varepsilon_e(t) = \frac{1}{E_e}\sigma(t) \qquad (4b)$$

and making use of our engineering stress assumption.

$$\varepsilon_e(t) = \frac{1}{E_e}(kt) \qquad (4c)$$

where k : = the stress camp.

The viscoelastic term is defined as follows:

$$\sigma(t) = \varepsilon_{ve}(t)\cdot E_{ve} + \eta_{ve}\,\frac{d\varepsilon_{ve}(t)}{dt} \qquad (5a)$$

but $\sigma(t) = kt$; therefore the following equations apply:

$$\frac{d\varepsilon_{ve}(t)}{dt} + \frac{E_{ve}}{\eta_{ve}}\varepsilon(t) = \frac{kt}{\eta_{ve}} \quad (5b)$$

$$\varepsilon_{ve}(t) = \frac{k\tau^2}{\eta_{ve}}\left(-1 + \frac{t}{\tau} + e^{-t/\tau}\right) \quad (5c)$$

The viscoplastic terms are defined as follows:

$$\sigma(t) = \eta_{vp} \cdot \frac{d\varepsilon_{vp}}{dt} \quad (6a)$$

$$\int_0^t \sigma(t)dt = \eta_{vp} \cdot \varepsilon_{vp}(t) \quad (6b)$$

$$\int_0^t kt\, dt = \eta_{vp} \cdot \varepsilon_{vp}(t) \quad (6c)$$

$$k\frac{t^2}{2} = \eta_{vp} \cdot \varepsilon_{vp}(t) \quad (6d)$$

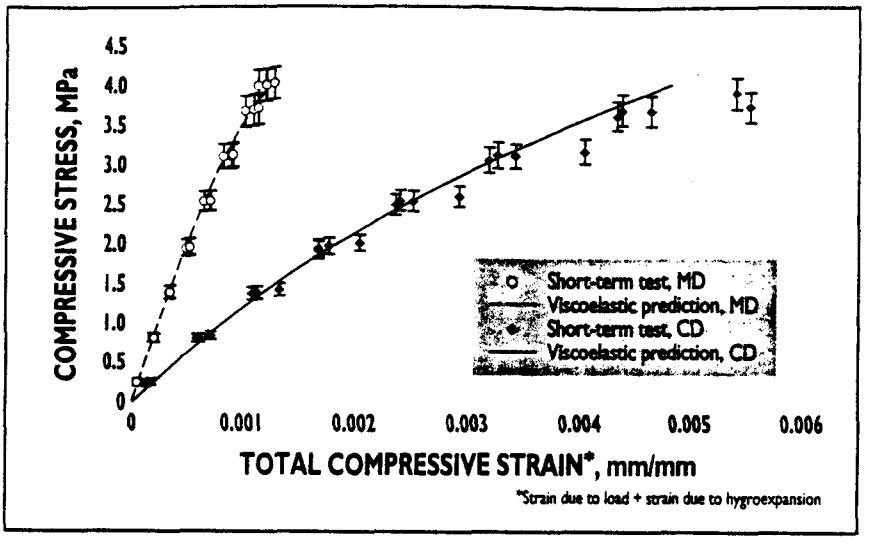
$$\varepsilon_{vp}(t) = \frac{kt^2}{2\eta_{vp}} \quad (6e)$$

The total strain can be described by the summation of these three components, as follows:

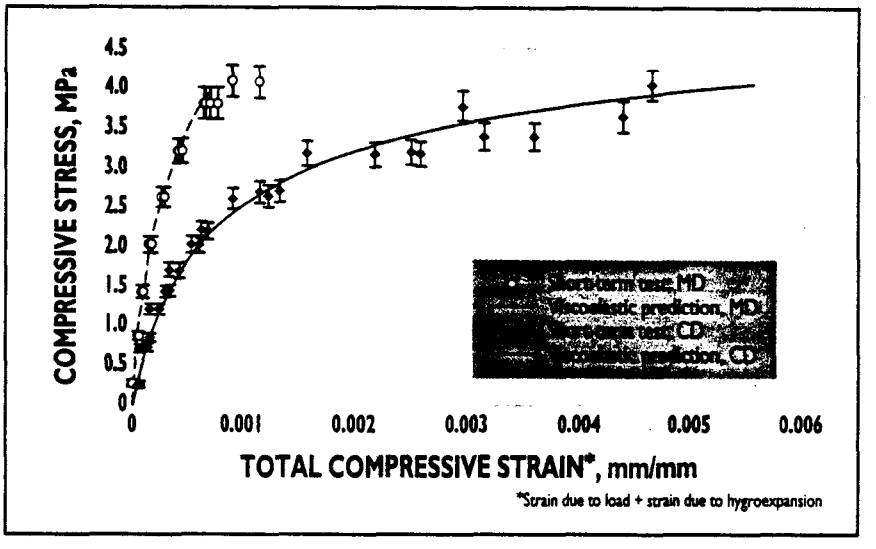
$$\varepsilon(t) = \frac{kt}{E_e} + kD\tau\left(-1 + \frac{t}{\tau} + e^{-t/\tau}\right) + \frac{kt^2}{2\eta_{vp}} \quad (7)$$

The four empirical parameters in this model (E_e , D, τ , and η_{vp}) must be found experimentally. It is helpful to consider the physical significance of these parameters. The parameter E_e is analogous to the stiffness of the elastic spring, and it is considered to be the elastic modulus of the material. This parameter is easily found from simple shon-terrn uniaxial compressive tests. There is no need to solve for it in a least-squares minimization scheme as one of four simultaneously solved-for parameters. Thus, all of the E_e values in this study were determined from fitting a hyperbolic tangent to uniaxial tests and finding the initial slope of the fitted curve. The hyperbolic tangent method is a popular means of estimating the elastic modulus of the material *(8).* These E_e values agree well with other data describing this batch of paper *(9).* The terms D and η refer to the compliance and relaxation time of the viscoelastic element, respectively. Compliance *(D)* is simply $1/E_{ve}$ of the viscoelastic component, and relaxation time (τ) can be stipulated as a proportionality constant relating E_{ve} to η_{ve}, the viscosity of this component. The fourth parameter, η_{vp}, is the viscosity of the viscoplastic element.

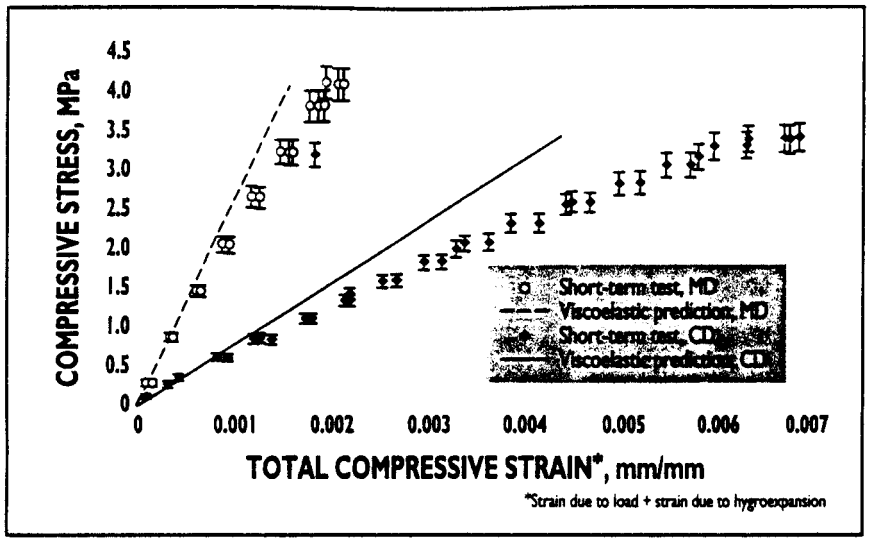
While this model was adequate to describe long-term (16-h) compressive tests, its application to short-term (5-min static RH) compressive tests was inadequate. The major weakness in modeling the short-term tests was the overwhelming contribution of the first term (the elastic component). The viscoelastic and viscoplastic



3. Comparison of pure hygroexpansive component, virgin MD



4. Static 50% RH results



5. Static 90% RH results

RH,%	Time	VIRGIN MD A	VIRGIN MD B	VIRGIN CD A	VIRGIN CD B
50-90	30 min	3.714e08	90,786	6.926e07	37,895
50-90	16h	2.90e10	57,649	1.717e09	54,771
90-50	30 min	5.546e08	59,667	2.340e08	20,173
90-50	16h	5.786e09	121,619	3.830e09	24,328

I. Parameters for pure hygroexpansive component

RH, %	E_e, MPa	D, I/MPa	τ, s	η_{vp}, N•s/mm²	c
50	2417	8.700e05	1.442e07	1.598e08	-0.020
90	1397	1.438e06	2.499e07	7.970e07	-0.054
50→90 90	2417 Use E_{50}	3.149e07	4.528e06	5.897e07	0.176
90→50 50	1397 Use E_{90}	6.457e04	6.161e04	7.408e07	+0.071

II. Virgin CD parameters for Eqs. 9 and 15

terms did not have sufficient time to create a substantial contribution. Thus, the model predicts an essentially linear response, even though experiments showed that the short-term tests are in that nonlinear. This weakness was corrected by a modification of the first term to include a nonlinear spring.

Case (b): Inclusion of a nonlinear spring

To incorporate a nonlinear constitutive response for short-term tests, the spring in the elastic component of the model was modified by allowing the stiffness of the spring to be a function of the stress in the spring. The strain of this component, previously deseribed by Eq. 4, is now defined as follows:

$$\frac{d}{dt}\varepsilon_e(t) = \frac{d}{dt}\left(\frac{\sigma(t)}{E_e(1 - ckt)}\right) \tag{8a}$$

where

k = stress ramp or rate of stress (i.e., stress/time)

t = time

kt = stress

c = empirical parameter.

The following equations also apply:

$$\varepsilon_e(t) = \frac{\sigma(t)}{E_e(1 - ckt)} \tag{8b}$$

or

$$\varepsilon_e(t) = \frac{kt}{E_e(1 - ckt)} \tag{8c}$$

The total stain for the newly modified viscoelastic system is now defined as follows:

$$\varepsilon(t) = \frac{kt}{E_e(1 - ckt)} + kD\tau\left(-1 + \frac{t}{\tau} + e^{-t/\tau}\right) + \frac{kt^2}{2\eta_{vp}} \tag{9}$$

The increased level of modeling control allowed by this five-parameter model (E_e, c, D, τ and η) more than offsets the numerical burden of solving for the additional parameter c.

For completeness, the same model was applied to the creep test, or constant load case. Note that the spring in the elastic element remains linear in this case, because its stiffness was previously made to be a function of the stress ramp k and time t. Here in the creep test, $k = 0$; thus, the spring remains at its initial stiffness E_e.

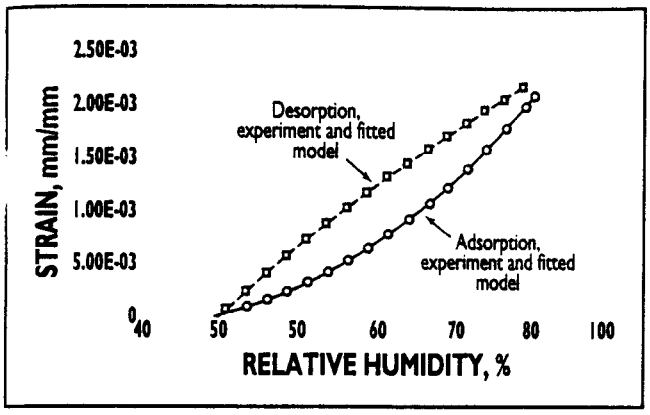
The elastic term is defied as follows:

$$\frac{d}{dt}\varepsilon_e(t) = \frac{1}{E_e}\cdot\frac{d}{dt}\sigma(t) \tag{10a}$$

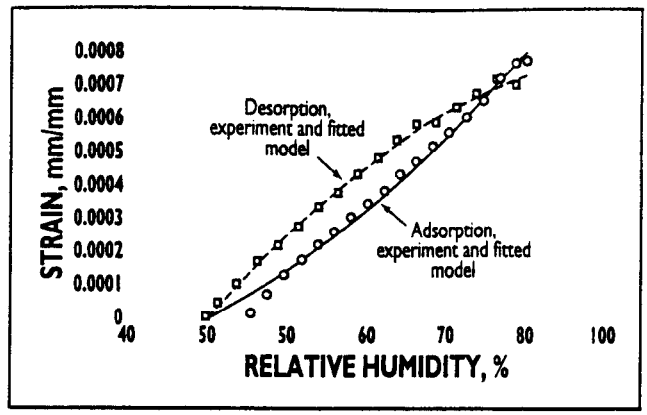
or

$$\varepsilon_e(t) = \frac{1}{E_e}\sigma(t) \tag{10b}$$

The viscoelastic term is defined as follows:



6. Transient 50% → 90% RH results



7. Transient 90% → 50% RH results

$$\frac{d}{dt}\varepsilon_{ve}(t) + \frac{\varepsilon_{ve}(t)}{\tau} = \frac{\sigma}{\eta_{ve}} \quad (11a)$$

where

$$\varepsilon_{ve}(t) = \sigma \cdot D\,(e^{t/\tau}-1) \quad (11b)$$

or

$$\frac{\varepsilon_{ve}(t)}{\sigma} = D\,(1-e^{-t/\tau}) \quad (11c)$$

where D = the compliance of the viscoelastic component.

The viscoplastic terms are defined as follows (again, σ is a constant):

$$\frac{d}{dt}\varepsilon_{vp} = \frac{\sigma}{\eta_{vp}} \quad (12a)$$

$$\varepsilon_{vp} = \frac{\sigma(t)}{\eta_{vp}} \quad (12b)$$

Thus, the total strain for the creep test can be described by the summation of these three components, as follows:

$$\varepsilon(t) = \sigma_0\left(\frac{1}{E_e} + D\,(1-e^{-t/\tau}) + \frac{t}{\eta_{vp}}\right) \quad (13)$$

RH, %	E_e, MPa	D, I/MPa	τ, s	η_{vp}, N•s/mm^2	c
50	6655	3.348e-05	1.010e03	2.037e09	-0.041
90	4572	1.164e-04	1.234e04	1.579e09	-0.019
50→90	6655 Use E_{50}	2622e-08	3.443e05	9.951e07	-0.019
90→50	4572 Use E_{50}	31.135e-04	1.486e04	1.610e21	+0.085

III. Virgin MD Parameters for Eqs. 9 and 15

ε_e	ε_{ve}	ε_{vp}	ε_{hygro}	ε_{total}	Time, s	Strain	Stress, MPaa
–0.00011	–2.3E–05	–8.4E–09	0	–0.00013	50	–0.00013	–0.681
–0.00022	–4.6E–05	–3.3E–08	0	–0.00026	100	–0.00026	–1.362
–0.00034	–6.8E–05	–7.5E–08	0	–0.00040	150	–0.00040	–2.043
–0.00046	–9.1E–05	–1.3E–07	0	–0.00055	200	–0.00055	–2.724
–0.00059	–0.00011	–2.1E–07	0	–0.00071	250	–0.00071	–3.405
–0.00074	–0.00014	–3.0E–07	0	–0.00087	300	–0.00087	–4.086

IV. Virgin MD (static 50% RH)—5-min prediction

ε_e	ε_{ve}	ε_{vp}	ε_{hygro}	ε_{total}	Time, s	Strain	Stress, MPaa
–0.0003	–1E–10	–1E–07	0	–0.0003	50	–0.0003	–0.681
–0.0006	–4E–10	–4E–07	0	–0.0006	100	–0.0006	–1.362
–0.0009	–9E–10	–1E–06	0	–0.0009	150	–0.0009	–2.043
–0.0012	–2E–09	–2E–06	0	–0.0012	200	–0.0012	–2.724
–0.0015	–3E–09	–3E–06	0	–0.0015	250	–0.0015	–3.405
–0.0018	–4E–09	–4E–06	0	–0.0018	300	–0.0018	–4.086

CALIBRATION OF CONSTITUTIVE MODEL

To determine the values of the five necessary parameters of the viscoelas- compressive experiments was undertaken. The mason for conducting such long tests was to include plastic flow in the material response. The paper

ε_e	ε_{ve}	ε_{vp}	ε_{hygro}	ε_{total}	Time, s	Strain	Stress, MPa
–0.0002	–2E–07	–1E–08	0	–0.0002	50	–0.0002	–0.681
–0.0003	–6E–07	–4E–08	0	–0.0003	100	–0.0003	–1.362
–0.0005	–1E–06	–1E–07	0	–0.0005	150	–0.0005	–2.043
–0.0007	–3E–06	–2E–07	0	–0.0007	200	–0.0007	–2.724
–0.0009	–4E–06	–3E–07	0	–0.0009	250	–0.0009	–3.405
–0.0011	–6E–06	–4E–07	0	–0.0011	300	–0.0011	–4.086

VI. Virgin MD (90% static RH)—5-min prediction

ε_e	ε_{ve}	ε_{vp}	ε_{hygro}	ε_{total}	Time, s	Strain	Stress, MPa
–0.0005	–1E–12	–2E–07	0	–0.0005	50	–0.0005	–0.681
–0.0011	–4E–12	–9E–07	0	–0.0011	100	–0.0011	–1.362
–0.0018	–9E–12	–2E–06	0	–0.0018	150	–0.0018	–2.043
–0.0026	–2E–11	–3E–06	0	–0.0026	200	–0.0026	–2.724
–0.0036	–2E–11	–5E–06	0	–0.0036	250	–0.0036	–3.405
–0.0048	–4E–11	–8E–06	0	–0.0048	300	–0.0048	–4.086

VII. Virgin CD (90% static RH)—5-min prediction

ε_e	ε_{ve}	ε_{vp}	ε_{hygro}	ε_{total}	Time, s	Strain	Stress, MPa
–0.0001	–2E–11	–1E–06	9.6E–05	–4E–05	360	–4E–05	–0.8172
–0.0006	–1E–10	–1E–05	0.00033	–0.0002	1080	–0.0002	–2.4516
–0.0006	–2E–10	–2E–05	0.00038	–0.0003	1200	–0.0003	–2.7240
–0.0009	–2E–10	–2E–05	0.00048	–0.0004	1440	–0.0004	–3.2688
–0.0012	–3E–10	–3E–05	0.00058	–0.0006	1680	–0.0006	–3.8136
–0.0014	–4E–10	–4E–05	0.00063	–0.0008	1800	–0.0008	–4.0860

VIII. Virgin MD (50% → 90% RH)—30-min prediction

basis weight of 205 g/m^2.

Compressive load was applied at a constant rate of 6.6 x 10^{-4} N/s until a prescribed peak load of 38 N was reached. The peak load was arrived at in 57,600 s (16 h). The original cross-sectional area of the paper specimens was 0.31 mm (thick) x 30 mm (wide), resulting in a net compressive stress of 4.08 MPa. The load was increased during this 16-h period, while the RH profile was controlled. The RH prokes were as follows:

- static 50% RH
- static 90% RH
- Transient 50% → 90% RH (2.5% RH/h)
- Transient 90 → 50% RH (-2.5% RH/h).

Each testing configuration was repeated four times. Repeatability of the tests is depicted with error bars in **Figs. 2-5.** It was necessary to calculate the five parameters of the viscoelastic model in each humidity environment this resulted in four sets of parameters, one for each humidity environment. A given set of parameters was sufficient to characterize the constitutive behavior of a given test. Thus, the parameters remained constant during a given test simulation only the stress level and time were incremented.

As mentioned previously, the first parameter (E_e) was calculated from the initial slope of the stress vs. strain history of 60-s load ramp tests. The other terms (c, D, τ, and η_{vp}) were determined by fitting Eq. 9 to the 16-h tests. A commercial curve-fitting package *(10)* that featured a Levenburg-Marquardt algorithm was used to minimize the residual errors between experimental data points and Eq. 9. Note that such a curve-fitting technique does not guarantee the minimum least-squares solution. Sometimes the curve, consisting of n parameters, will converge to a local minimum in n-dimensional space *(10).* Thus, good starting estimates are of utmost importance, giving further credence to attaching physical engineering significance to the parameters of the model, as was disscused earlier.

The constitutive model was developed with the pure hygroexpansive component of the paper response removed from the data. Doing so permitted separate calculations of the main resulting from load and from adsorption/desorption. A fundamental experimental assumption here is that the hygroexpansive strain measured from unloaded specimens can be applied to loaded specimens. This assumption could have been a source of experimental error in this study, because it may not be possible to simply add the deformations resulting from load to deformations resulting from pure hygroexpansion. Yet, when analyzing experimentally gathered data on loaded specimens, one cannot distinguish which portion of the strain is due to load and which portion is due to hygroexpansion. To separate these effects, we chose to adopt the method described here. This method is not without precedence. Other studies *(5, 6, 11)* have added the hygroexpansive

KEYWORDS

Calibration, compression tests, equations, humididty, hygroexpansivity, liner boards, loads, mechanical properties, models, paper parameters, stress strain properties, tests methods, viscoelasticity.

deformation to the material deformation resulting from load, for a net deformation.

Although the final short-term static RH tests were **5** min long, the final transient RH tests lasted 30 min. These tests were longer because it was suspected that 5 min would be insufficient for moisture equilibrium to occur. The tests needed to obtain the hygroexpansive components were (a) 50% → 90% RH, but no load applied and (b) 90% → 50% RH, but no load applied. A Maxwell model (spring and dashpot in series), described by Eq. 14, was applied to these hygroexpansive data.

$$\varepsilon(t) = \frac{(\text{RH rate})t^2}{A} + \frac{(\text{RH rate})t}{B} \tag{14}$$

Equation 14 was subtracted from the original data obtained from loaded test specimens. This removed the hygroexpansive component of strain; the remaining data could be used to create a constitutive model for paper. Different values for parameters A and B were used to characterize adsorption and resorption behavior using this one hygroexpansive model. The model described by Eq. 14 was calibrated for machine direction (MD) and cross-machine direction (CD) response **(Table 1).**

Figures 6 and 7 show the hygroexpansive data for virgin CD and MD paper fix the 16-h calibration tests, as well as Eq. 14 fitted to these data. **Tables II and III** show the values of the parameters in Eqs. 9 and 15 (to follow).

Combining the strain response resulting from load (Eq. 9) and the strain response resulting from hygroexpansivity (Eq. 14) yields an expression for the nonlinear viscoelastic response of paper. It is this combined response (Eq. 15) that will be depicted in the following section.

ε_e	ε_{ve}	ε_{vp}	ε_{hygro}	ε_{total}	Time, s	Strain	Stress, MPa
−0.0004	−2E−11	−2E−06	0.00025	−0.0002	360	−0.0002	−0.8172
−0.0019	−2E−10	−2E−05	0.00101	−0.0009	1080	−0.0009	−2.4516
−0.0023	−2E−10	−3E−05	0.00117	−0.0012	1200	−0.0012	−2.7240
−0.0036	−3E−10	−4E−05	0.00151	−0.0021	1440	−0.0021	−3.2688
−0.0057	−4E−10	−5E−05	0.00189	−0.0039	1680	−0.0039	−3.8136
−0.0076	−4E−10	−6E−05	0.00210	−0.0055	1800	−0.0055	−4.0860

IX. Virgin CD (50% → 90% RH)—30-min prediction

ε_e	ε_{ve}	ε_{vp}	ε_{hygro}	ε_{total}	Time, s	Stain	Stress, MPa
−0.0002	−3E−06	−9E−08	−0.0001	−0.0003	360	−0.0003	−0.8172
−0.0004	−3E−05	−8E−07	−0.0004	−0.0009	1080	−0.0009	−2.4516
−0.0005	−3E−05	−1E−06	−0.0005	−0.0010	1200	−0.0010	−2.7240
−0.0006	−5E−05	−1E−06	−0.0006	−0.0012	1440	−0.0012	−3.2688
−0.0006	−7E−05	−2E−06	−0.0007	−0.0014	1680	−0.0014	−3.8136
−0.0007	−7E−05	−2E−06	−0.0008	−0.0015	1800	−0.0015	−4.0860

X. Virgin MD (90% → 50% RH)—30-min prediction

ε_e	ε_{ve}	ε_{vp}	ε_{hygro}	ε_{total}	Time, s	Strain	Stress, MPa
−0.0013	−1E−05	−2E−05	−0.0013	−0.0026	1080	−0.0026	−2.0628
−0.0014	−1E−05	−2E−05	−0.0015	−0.0029	1200	−0.0029	−2.2920
−0.0016	−2E−05	−3E−05	−0.0018	−0.0035	1440	−0.0035	−2.7504
−0.0019	−3E−05	−4E−05	−0.0021	−0.0041	1680	−0.0041	−3.2088
−0.0020	−3E−05	−4E−05	−0.0023	−0.0043	1800	−0.0043	−3.4380

XI. Virgin CD (90% → 50% RH)—30-min prediction

$$\varepsilon(t) = \frac{kt}{E_e(1 - ckt)} + kD\tau\left(-1 + \frac{t}{\tau} + e^{-t/\tau}\right) + \frac{kt^2}{2\eta_{vp}} + \frac{(\text{RH rate})\,t^2}{A} + \frac{(\text{RH rate})\,t}{B} \tag{15}$$

USE OF CONSTITUTIVE MODEL

As described in the previous section, the model was calibrated from the long-term test data. This model was then used to predict the response of the short-term tests. The 205-g/m^2 basis weight virgin linerboard was investigated in the machine and cress directions. The viscoelastic model was able to predict the response of these tests and account fix the rate of loading as well as the RH profile. The following section shows the agreement between theoretical predicted viscoelastic response and experimental results. The contributions of the terms in the model are shown in **Tables IV through XI.** Some contributions were negligible in the short-term tests but were nevertheless included, because their effect was more pronounced in the 16-h calibration tests.

When viewing the graphical representation of the results (Figs. 2-5), note that the strain shown is the total strain of the specimen, i.e., strain due to load plus strain due to hygroexpansion. The figures also show error bars depicting a 5% standard error.

The figures show good agreement between the experimental results and the predicted viscoelastic response.

The range of peak strains is broad—from 0.0007 in the case of MD response in a static 50% RH environment (Fig. 2) to approximately 0.007 in the case of CD response in a transient 90% to 50% RH environment (Fig. 5). This highlights the robustness of the viscoelastic model. Agreement is least satisfactory in the transient 90% to 50% RH environment. It is suspected that the paper did not sufficiently dry during the experiment and did not math an equilibrium moisture content at the conclusion of the test. An even mose likely cause is the assumption of superposition of strains that we made at the onset of the work. If the hygroexpansive strain profile were different in the loaded specimen than in the unloaded specimen, then it would not be possible to add these for a resultant total strain.

As stated at the outset of this paper, we aimed to test the validity of superposing strains resulting from hygroexpansion and those resulting from load. We suspected that effects would be most severe in the first half-cycle of adsorption or resorption. As Fig. 4 (adsorption) and Fig. 5 (resorption) show, markedly different responses occurred. A study conducted by Back *et al. (12)* concluded that such nonlinearity of responses can be expected. These authors stated that the transient effects of sorption conditions may be related to the dimensional changes occurring with time within the material. Based on our results (Figs. 4 and 5), we conclude that specimens can swell at different rates and to different extents in the three orthogonal directions. It is likely that these differences in constitutive response would require the creation of separate constitutive models for the first cycle of adsorption / desorption. Hunt *(23)* came to a similar conclusion and found it necessary to classify different types of mechanical response after the first sorption cycle because of the large increase in the compliance of the material during the first humidity cycle.

CONCLUSIONS

The strength of the nonlinear viscoelastic model is that it is able to accommodate a wide variety of strain responses. Further work is needed on applying the model to a wider range of paper types. The model may also be applied to different orthotropic materials that are susceptible to long-term plastic strains. **TJ**

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