SOME RELATIONS AMONG ENGINEERING CONSTANTS OF WOOD

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Wood may be described as an orthotropic material with unique and independent mechanical properties in the directions of three mutually perpendicular axes– longitudinal (*L*), radial (*R*), and tangential (*T*). These mechanical properties are also called engineering constants. Orthotropic materials are of special relevance to composite materials. Therefore, mathematical relations and experimental procedures for engineering constants of orthotropic composites can be conveniently applied to wood when considered as an engineering material. In this paper, we present how variation of shear modulus with grain slope of wood can be studied using methods of engineering mechanics of composite materials and how the results can be used to obtain some useful relations among shear modulus, moduli of elasticity, and Poisson's ratio of wood in a two-dimensional analysis.

Formulas for Shear Modulus

Let the 12 coordinate system represent the principal material directions and the *xy* coordinate system represent the geometrical directions of a structural member with angle θ from the *x* axis to the 1 axis, as shown in Figure 1. Transforming stress-strain relations from one coordinate system to the other [1], the shear modulus for an orthotropic lamina that is stressed in the xy coordinates is

$$\frac{1}{G_{xy}} = 2\left(\frac{2}{E_1}+\frac{2}{E_2}+\frac{4\nu_{12}}{E_1}-\frac{1}{G_{12}}\right)\sin^2\theta\cos^2\theta + \frac{1}{G_{12}}(\sin^4\theta+\cos^4\theta) \quad (1)$$

where G_{xy} and G_{12} are shear moduli in xy and 12 planes, respectively; E_1 and E_2 are elasticity moduli in axes 1 and 2, respectively; and v_{12} is Poisson's ratio, with 1 referring to direction of applied stress and 2 referring to direction of lateral strain.

Equation (1) can be reduced to the following form with G_{xy} replaced by G(θ):

$$\frac{1}{G(\theta)} = \frac{\phi}{E_2}\sin^2 2\theta + \frac{1}{G_{12}}\cos^2 2\theta \quad (2)$$

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in which

$$\phi = 1+\frac{E_2}{E_1}\left(1+2\nu_{12}\right) \quad (3)$$

At θ = 0 in Equation (2),

$$\frac{1}{G(0)} = \frac{1}{G_{12}} \quad (4)$$

and at θ = π/4,

$$\frac{1}{G(\pi/4)} = \frac{\phi}{E_2} \quad (5)$$

Substituting Equations (4) and (5) into Equation (2) yields

$$\frac{1}{G(\theta)} = \frac{1}{G(\pi/4)}\sin^2 2\theta + \frac{1}{G(0)}\cos^2 2\theta \quad (6)$$

or

$$G(\theta) = \frac{G(0)G(\pi/4)}{G(0)\sin^2 2\theta + G(\pi/4)\cos^2 2\theta} \quad (7)$$

Therefore, given G(0) and G(π/4), G(q) for any value of q can be evaluated from Equation (7).

Evaluation of Shear Modulus

The Arcan shear test [2,3] was used to evaluate shear modulus G(q) at θ = 0, π/8, and p/4 to be used in Equation (7). Although this method is straightforward, to our knowledge it is the only one that has been successfully applied on wood.

The butterfly-shaped test specimen is oriented horizontally rather than vertically in compliance with the specified coordinate systems (Fig 1). The critical section is along the x axis on which pure shear stresses act. The strain gages are on the *x'* and *y'* axes with angle θ' = π/4 from the x axis to the x1 axis. The *x'y'* coordinates are independent of the 12 coordinates.

The strain components referred to the *x'y'* coordinates can be expressed in terms of those referred to the *xy* coordinates as

$$\varepsilon_{x'} = \varepsilon_x\cos^2\theta' + \varepsilon_y\sin^2\theta' + \tau_x\cos\theta'\sin\theta' \quad (8)$$

$$\varepsilon_{y'} = \varepsilon_x\sin^2\theta' + \varepsilon_y\cos^2\theta' - \tau_x\cos\theta'\sin\theta' \quad (9)$$

With θ' = π/4, we have from Equations (8) and (9)

$$\tau_x = \varepsilon_{x'} - \varepsilon_{y'} \quad (10)$$

which is the shear strain referred to the *xy* plane. The shear gage reported by Ifju [4] is based on Equation (10) and was used by us previously [3].

Test data of Sitka spruce (*Picea sitchensis*) specimens with the 1 axis identified with the *L* axis and the 2 axis with the *R* axis are shown in Table 1. Using the average values of G(θ) at θ = 0 and π/4 in Table 1, Equation (7)

Table 1. Summary of shear modulus test data for Sitka spruce[a]

Slope of grain (rad)	Tests (no.)	Avg shear modulus (MPa)	COV (%)	Avg MC (%)	Avg SG
0	5	910	8.96	9.4	0.33
p/8	5	1,194	5.16	9.4	0.33
p/4	5	1,670	2.72	9.4	0.33

[a]Specimens stabilized in conditioning room at 20°C and 50% relative humidity. COV is coefficient of variation; MC, moisture content; SG, specific gravity.

gives shear modulus of 1,179 MPa at $\theta = \pi/8$, which compares with 1,194 MPa in Table 1.

In recent torsion tests on Sitka spruce specimens, Kubojima et al. [5] obtained an average value of 884 MPa for $G(0)$. They verified this result using the Timoshenko theory of bending, which strongly supports that our data (Table 1) are all within reasonable ranges.

Relations Among Engineering Constants

From Equations (3) and (5), we obtain

$$\nu_{12} = \frac{1}{2}\left[\left(\frac{E_2}{G(\pi/4)} - 1\right)\frac{E_1}{E_2} - 1\right] \tag{11}$$

For $0 < \nu_{12} < 1$, we must have $E_2 > G(\pi/4)$. In reference [6], $E_{2 \text{ or } R}$ is 902 MPa for Sitka spruce, which is much less than our result of $G(\pi/4) = 1{,}670$ MPa (Table 1).

Rewrite Equation (11) in the form

$$E_2 = \frac{G(\pi/4)E_1}{E_1 - G(\pi/4)(1 + 2\nu_{12})} \tag{12}$$

Kubojima et al. [5] give $E_1 = 11{,}800$ MPa. In reference [6], $E_1 = 11{,}600$ MPa and $\nu_{12} = 0.37$. Using $E_1 = 11{,}800$ Mpa, $\nu_{12} = 0.37$, and $G(\pi/4) = 1{,}670$ Mpa in Equation (12) gives $E_2 = 2{,}216$ MPa, which is 2.46 times the E_2 value in reference [6]. Since E_2 is relatively insensitive to ν_{12} in Equation (12), we can take ν_{12} to be valid.

We note that in reference [6], when using prismatic specimens the tensile strength in the 2 or R axis is about 2.2 times that using specimens according to ASTM D143–52.This huge difference was attributed to the high stress concentration in the specimen in ASTM D143–52 [6]. We therefore suspect the accuracy of the method used to obtain the data for E_2 cited in reference [6]. For wood to be an orthotropic material, it is necessary for its engineering constants to satisfy, at least approximately, Equation (11) or (12).

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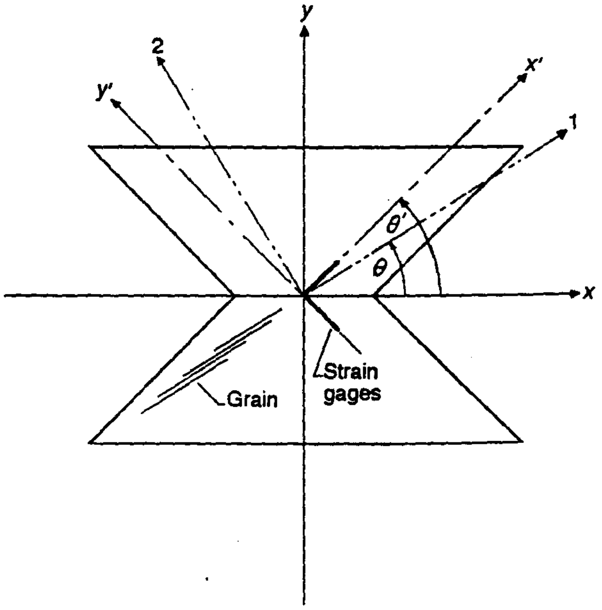

Figure 1. Schematic of specimen with wood grain and strain gage orientations ($\theta' = \pi/4$).

ICCE/5

FIFTH INTERNATIONAL CONFERENCE ON COMPOSITES ENGINEERING

Edited by

David Hui

July 5-11, 1998

Las Vegas, Nevada

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