# Wood Mechanical Property Variation with Grain Slope

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## Abstract

This paper presents the application of the Arcan shear test developed for composite materials to determine the shear modulus in the longitudinal-radial plane and the elasticity modulus in the radial direction of wood. Variations of some transformed engineering constants of Sitka spruce with grain slope are then analyzed.

## Introduction

Wood may be described as an orthotropic material with independent mechanical properties in the directions of three mutually perpendicular axes-longitudinal ($L$), radial ($R$), and tangential ($T$). These are called the principal material axes, and the related mechanical properties are the engineering constants. The material axes and the geometrical axes used to describe a rectangular structural member do not usually coincide. The mechanical properties referred to the geometrical axes are called the transformed engineering constants. In a 2D situation, the relations between transformed engineering constants and engineering constants are well documented (Jones 1975, Daniel and Ishai 1994). This study discusses some problems in determining the engineering constants of Sitka spruce (*Picea sitchensis* (Bong.) Carr.), suggests appropriate procedures to solve these problems, and then demonstrates the numerical variations of some transformed engineering constants with grain slope.

### Transformation Relations for Engineering Constants

Let the 1-2 coordinate system represent the principal material axes and the *x-y* coordinate system the geometrical axes with angle q from the *x* axis to the 1 axis as shown in Fig. 1. The transformed engineering constants can be expressed in terms of

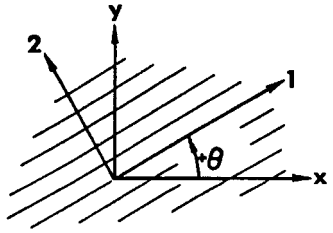


**Figure 1. Principal axes of wood.**

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the four basic engineering constants: $E_1$ and $E_2$, elasticity moduli in the 1 and 2 axes; $G_{12}$, shear modulus in the l-2 plane; and $v_{12}$, Poisson's ratio with 1 referring to direction of applied stress and 2 referring to direction of strain. Identifying the 1 axis with the $L$ axis and the 2 axis with the $R$ axis of Sitka spruce, we have found that the conventional methods used to generate data for $G_{12}$ and $E_2$ cannot yield correct results, while the data for $E_1$ and $v_{12}$ are relatively consistent (Kollmann and Côté 1968).

***Shear Modulus***—The transformed shear modulus in the x-y plane is (Jones 1975)

$$\frac{1}{G_{xy}} = 2\left(\frac{2}{E_1} + \frac{2}{E_2} + \frac{4\nu_{12}}{E_1} - \frac{1}{G_{12}}\right)\sin^2\theta\cos^2\theta + \frac{1}{G_{12}}(\sin^4\theta + \cos^4\theta) \tag{1}$$

which can be reduced to the following form with $G_{xy}$ replaced by $G(\theta)$:

$$\frac{1}{G(\theta)} = \frac{\phi}{E_2}\sin^2 2\theta + \frac{1}{G_{12}}\cos^2 2\theta \tag{2}$$

in which

$$\phi = 1 + \frac{E_2}{E_1}\left(1 + 2\nu_{12}\right) \tag{3}$$

At $\theta = 0°$ in Eq. (2),

$$1/G(0°) = 1/G_{12} \tag{4}$$

and at $\theta = 45°$,

$$1/G(45°) = \phi/E_2 \tag{5}$$

Substituting Eqs. (4) and (5) into Eq. (2) and inverting both sides yields

$$G(\theta) = \frac{G(0°)G(45°)}{G(0°)\sin^2 2\theta + G(45°)\cos^2 2\theta} \tag{6}$$

Applying Arcan shear test (Arcan et al. 1978, Liu and Ross 1997) on Sitka spruce specimens, we obtained the results in Table 1. Using the G(0°) and G(45°) values in Eq. (6) for $\theta = 22.5°$, we obtained $G(22.5°) = 1{,}179$ MPa.

Recently, Kubojima et al. (1996) performed torsion tests on Sitka spruce specimens and obtained an average value of 884 MPa for G(0°). The G(0°) value listed in Kollmann and Côté (1968) is 745 MPa, which was obtained using the method by March et al. (1942) or ASTM Standard D3044-76 (ASTM 1976). The method is called the plate twist test. For solid wood, it is very difficult to make a plate specimen of practical size with uniform properties. Also, the plate twist test cannot consider the effect of grain slope.

***Elasticity Moduli***—The transformed modulus of elasticity in the $x$ direction is (Jones 1975)

$$\frac{1}{E_x} = \frac{1}{E_1}\cos^4\theta + \left(\frac{1}{G_{12}} - \frac{2\nu_{12}}{E_1}\right)\sin^2\theta\cos^2\theta + \frac{1}{E_2}\sin^4\theta \tag{7}$$

**Table 1. Summary of shear modulus test data for Sitka spruce[a]**

| Slope of grain (deg) | Number of tests | Avg. shear modulus (MPa) | Coefficient of variation (%) | Avg. moisture content (%) | Avg. specific gravity |
|---|---|---|---|---|---|
| 0 | 5 | 910 | 8.96 | 9.4 | 0.33 |
| 22.5 | 5 | 1,194 | 5.16 | 9.4 | 0.33 |
| 45 | 5 | 1,670 | 2.72 | 9.4 | 0.33 |

[a]Specimens were stabilized in conditioning room at 20°C and 50% RH.

which can be reduced to the form

$$E_x = \frac{E_1 E_2}{E_2 \cos^2\theta(\cos^2\theta + \psi \sin^2\theta) + E_1 \sin^4\theta} \tag{8}$$

with

$$\psi = E_1/G_{12} - 2\nu_{12} \tag{9}$$

Kubojima et al. (1996) has $E_1$ = 11,800 MPa. Kollmann and Côté (1968) give $E_1$ = 11,600 MPa, $E_2$ = 902 MPa, and $\nu_{12}$ = 0.37.

From Eqs. (3) and (5), we obtain

$$\nu_{12} = \frac{1}{2}\left[\left(\frac{E_2}{G(45°)} - 1\right)\frac{E_1}{E_2} - 1\right] \tag{10}$$

which requires that $E_2 > G(45°)$ since $\nu_{12} > 0$. This condition is not satisfied with $E_2$ = 902 MPa and G(45°) = 1,670 MPa. We rewrite Eq. (10) in the form

$$E_2 = \frac{E_1 G(45°)}{E_1 - G(45°)(1 + 2\nu_{12})} \tag{11}$$

With G(45°) = 1,670 MPa, $E_1$ = 11,800 MPa, and $\nu_{12}$= 0.37, Eq. (11) gives $E_2$ = 2,216 MPa. The data for $E_2$ in Kollmann and Côté (1968) were obtained using a compression test (Doyle et al. 1945). According to Pellicane et al. (1994), compression test in the 2 axis as described in ASTM D143 (ASTM 1952) produces nonuniform stress distribution and nonlinear stress-strain relation. Clearly, it can significantly underestimate the $E_2$ values.

***Shear Coupling Coefficient***—The shear coupling coefficient $\eta_{xs}$ is the ratio of shear strain in the *x-y* plane to axial strain in the *x* direction with *x* referring to the direction of applied stress and *s* to the induced shear strain. $\eta_{sx}$, $\eta_{ys}$, and $\eta_{sy}$ are defined likewise (Daniel and Ishai 1994). Here we show only the shear coupling coefftcient $\eta_{xs}$ as a ratio to $E_x$ :

$$\frac{\eta_{xs}}{E_x} = \frac{\eta_{sx}}{G_{xy}} = \left[\left(\frac{1}{E_1} + \frac{\nu_{12}}{E_1} - \frac{1}{2G_{12}}\right)\cos^2\theta - \left(\frac{1}{E_2} + \frac{\nu_{12}}{E_1} - \frac{1}{2G_{12}}\right)\sin^2\theta\right]\sin 2\theta \tag{12}$$

***Poisson's Ratio***-The transformed Poisson's ratio in the *x-y* plane is put as a ratio to $E_x$ (Daniel and Ishai 1994) as

$$\frac{\nu_{xy}}{E_x} = \frac{\nu_{12}}{E_1}\left(\sin^4\theta + \cos^4\theta\right) - \left(\frac{1}{E_1} + \frac{1}{E_2} - \frac{1}{G_{12}}\right)\sin^2\theta\cos^2\theta \tag{13}$$

At θ = 90°, $\nu_{xy} = \nu_{21}$, and $E_x = E_2$, Eq. (13) becomes $\nu_{21}/E_2 = \nu_{12}/E_1$, which agrees with Betti's reciprocal law.

## Results and Discussion

We have obtained numerical results for the transformed engineering constants for Sitka spruce with the input $E_1$ = 11,800 MPa, $E_2$ = 2,216 MPa, $G_{12}$= 910 MPa, and $\nu_{12}$ = 0.37. Figure 2 shows $G_{xy}$ increases from θ = 0° to its maximum value at θ = 45°. The ratio of its maximum to minimum is 1.84. For $E_x$, the maximum value occurs at θ = 0°. It decreases drastically to θ = 30° and reaches a constant value of $E_2$ at about θ = 55°. For θ = 10°, the decrease in $E_x$ is about 23% of $E_1$, which is significant.

Fig. 3 shows that $\eta_{xs}$ is negative at 0° ≤ θ ≤ 69° with a minimum value of -1.42 at θ = 15°. At 69° < θ ≤ 90°, $\eta_{xs}$ is positive with a maximum value of 0.04 at θ = 79°. The Poisson's ratio starts from 0.37 at θ = 0°, reaches a maximum value of 0.47 at θ = 25°, and drops to 0.07 at θ = 90°.

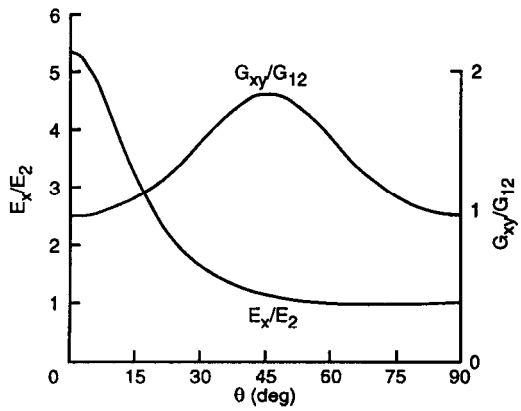


**Figure 2. Elasticity and shear moduli of Sitka spruce as functions of grain slope.**

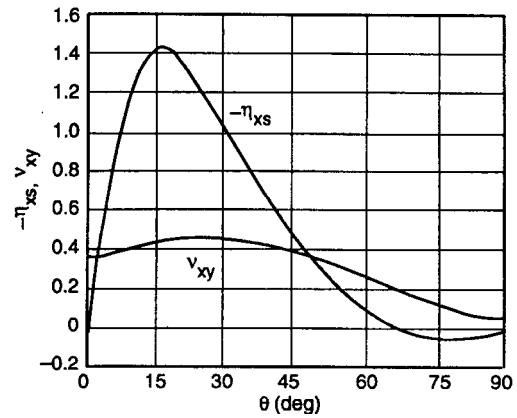


**Figure 3. Poisson's ratio and shear coupling coefficient of Sitka spruce as functions of grain slope.**

## Conclusions

Wood in its clear state may be described as an orthotropic material. However, $G_{12}$ in the *L-R* plane and $E_2$ in *R* direction of Sitka spruce cannot be correctly estimated using the conventional test methods. The Arcan shear test developed for composite materials has been shown to be adequate to meet the needs. Since grain slope often exists in structural members of wood, the transformation relations for engineering constants in this study should be of practical significance in the design of wood structures.

## References


Arcan, M., Hashin, Z. and Voloshin, A. (1978). "A method to produce uniform plane-stress states with application to fiber-reinforced materials." *Experimental Mechanics.* 18(4):141–146.

ASTM. (1952). "Standard methods of testing small clear specimens of timber." ASTM D143. American Society for Testing and Materials, Philadelphia, PA.

ASTM. (1976). "Standard method of testing for shear modulus of plywood." ASTM D3044-76. American Society for Testing and Materials, Philadelphia, PA.

Daniel, I.M. and Ishai, O. (1994). *Engineering mechanics of composite materials.* Oxford, 395 p.

Doyle, D.V., Drow, J.T. and McBurney, R.S. (1945). "Elastic properties of wood." Rep. No. 1528. USDA Forest Service, Forest Products Laboratory, Madison, WI.

Jones, R.M. (1975). *Mechanics of composite materials.* Scripta Book Co., Washington, DC. 344 p.

Kollmann, F.F.P. and Côté, W.A. Jr. (1968). *Priciples of wood science and technology I. Solid wood.* Springer–Verlag, Inc., New York. 592 p.

Kubojima,Y., Yoshihara, H., Ohta, M. and Okano, T. (1996). "Examination of the method of measuring the shear modulus of wood based on the Timoshenko theory of bending." *Mokuzai Gakkaishi* 42(12):1170–1176.

Liu, J.Y. and Ross, R.J. (1997). "Shear modulus variation with grain slope." In: Perkins, R., ed. Mechanics of Cellulosic Materials. American Society of Mechanical Engineers. AMD–Vol. 221/MD–Vol.77:107–111.

March, H.W., Kuenzi, E.W. and Kommers, W.J. (1942). "Method of measuring the shearing moduli in wood." Rep. No. 1301. USDA Forest Service, Forest Products Laboratory, Madison, WI.

Pellicane, P.J., Bodig, J. and Mrema, A.L. (1994). "Behavior of wood in transverse compression." *Journal of Testing and Evaluation.* 22(4): 383–387.