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Viscoelastic Modeling of Paper

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Abstract

In this study, a viscoelastic model was used to predict the constitutive behavior of paper subjected to compressive stress. The same model was then used to solve for the response of paper subjected to creep loading. Results show that the model can account for both types of loading.

Introduction

To fully understand the effect that rate of loading has on the constitutive response of an engineering material, experimental measurements must be correlated to a mechanistic model. In this study, a viscoelastic model was used to predict the constitutive behavior of paper subjected to compressive stress. A suitable one-dimensional model for studying viscoelastic materials incorporates instantaneous elasticity, retarded elasticity, and plastic flow.

The model, shown in Figure 1, has been used by other researchers to describe a material's viscoelastic behavior (Alfrey 1948, Tobolsky and Eyring 1943). It has been applied to paper mechanics problems (Mason 1948) and used to study the creep response of wood structures (Fridley et al. 1992, Pelvris and Triantafillou 1995).

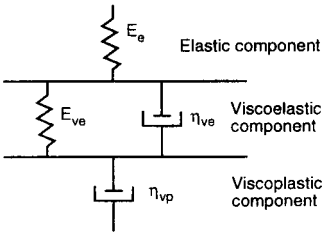

FIG. 1. The Viscoelastic Model

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Method

The study reported herein applied the model (Fig. 1) to predict the response of paper subjected to various loading histories. The fundamental differential equation of motion of this system was solved for monotonically increasing load and creep load. The elastic component was modified to allow for nonlinear behavior. The modification was such that the modulus of elasticity E no longer remained constant and decreased proportionately to the increase in stress. In other words, the elastic spring stiffness became a function of the stress in the element.

$$E \equiv E_e(1 - ckt) \tag{1}$$

where k = rate of applied stress and t = time; therefore, kt = stress and c is the constant creating nonlinearity in the spring. Solving the equation of motion for monotonically increasing loads resulted in the expression:

$$\varepsilon(t) = \frac{kt}{E_e(1 - ckt)} + kD\tau\left(-1 + \frac{t}{\tau} + e^{-t/\tau}\right) + \frac{kt^2}{2\eta_{vp}} \tag{2}$$

where E_e is the modulus of the elastic spring; D is $1/E_{ve}$, is the time constant relating E_{ve} to η_{ve}; and η_{vp} is the viscoplastic viscosity. This expression was able to model the constitutive behavior of paper loaded at three markedly different rates: 9.86×10^{0} MPa/s, 9.86×10^{-3} MPa/s, and 9.86×10^{-1} MPa/s. Calibration was performed by curve-fitting four parameters, (c, D, to the slowest rate of loading, i.e., 9.86×10^{-5} MPa/s. Curve fitting was accomplished with a commercially available package (Jandel Scientific 1994). The remaining parameter, E_e was found through fitting the data of a standard uniaxial test (Urbanik 1982). This same viscoelastic model was solved for creep loading, where the stress was previously represented by kt, it is now a constant σ_0. The solution for this case is the following:

$$\varepsilon(t) = \sigma_0\left(\frac{1}{E_e} + D(1 - e^{-t/\tau}) + \frac{t}{\eta_{vp}}\right) \tag{3}$$

In this solution, the constants have the same value as in Equation (2). Thus, having calibrated a mechanistic response for a monotonically increasing load (rate of load, ROL), you can automatically predict the response to creep loading, (duration of load, DOL). This results in an intrinsic coupling of ROL response to DOL response.

An alternative way to arrive at creep response is an isobaric response method. Here, the stress versus strain response of various rates of monotonically increasing load is plotted or tabulated such that for different constant stresses (isobars), the strain at a particular time can be collected (Fig. 2). Then, it is easy to fit a curve through the points (Fig. 2) (A, t_1), (A', t_2), and (A", t_3). Repeating this process for the B and C sets of points results in a generated three-dimensional map relating ROL response to DOL response (Fig. 3).

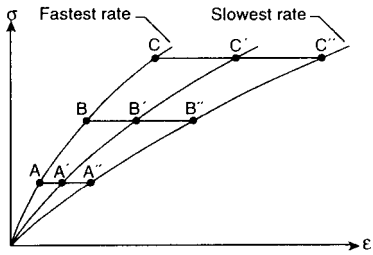

FIG. 2. Rate of Load Response Variation

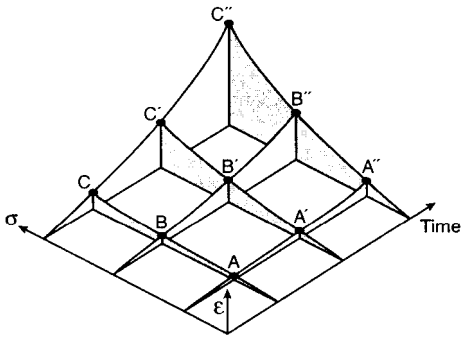

FIG. 3. Generation of isobaric Lines

A virgin paper (205 g/m^2 basis weight) was experimentally investigated by Gunderson et al. (1988) for ROL response in the machine direction (MD). The paper was tested at two levels of environmental relative humidity: 50% and 90%. Six replications were performed for each test. Newly acquired DOL data had three replications of each test. The viscoelastic model was able to account for both ROL and DOL. The DOL data are shown with both methods of prediction, i.e. the viscoelastic model prediction and the isobaric response prediction (Figs. 4 to 7).

Concluding Remarks

In this study, a viscoelastic model was applied to paper compressed at various rates of load. The same model was then used to solve for the response of paper subjected to creep loading. Results show that the model can account for both types of loading. In addition, a predictive isobaric response method was described and applied to the data.

Appendix. References

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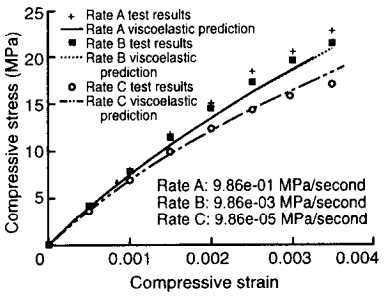

FIG. 4. Analysis of 6756 Paper, Machine Direction at 50% Relative Humidity

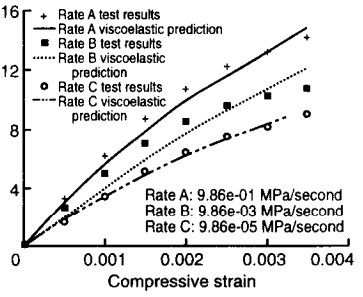

FIG. 6. Analysis of 6756 Paper, Machine Direction at 90% Relative Humidity

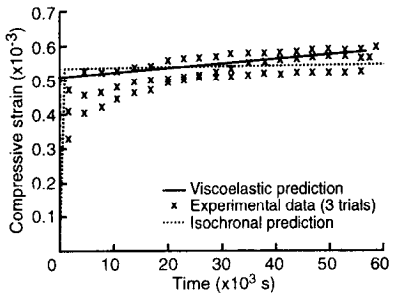

FIG. 5. Analysis of 6756 Paper, Machine Direction at 50% Relative Humidity

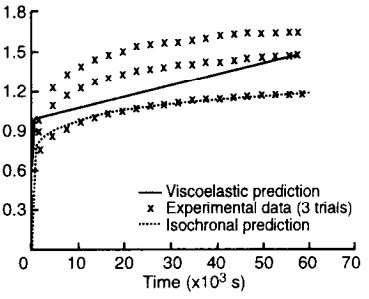

FIG. 7. Analysis of 6756 Paper, Machine Direction at 90% Relative Humidity

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