In: Gopu, Vijaya K.A., ed. Proceedings of the international wood engineering conference 1996, October 28-31; New Orleans, LA. Baton Rouge, LA: Louisiana State University: Vol. 4: 288-294

Fracture Modeling of Lumber Containing Multiple Knots

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Abstract

A finite element/fracture mechanics model is presented for assessing the influence of closely-spaced knots on lumber tensile strength. The important role of the model is illustrated in that strength interactions of closely-spaced knots cannot readily be assessed by other means. The two-dimensional model uses nondestructively-gathered grain angle and density data to provide a detailed description of each lumber member considered. The analysis simulates the fracture process of lumber during failure and yields a prediction of tensile strength. Predicted lumber tensile strengths correlated with measured strengths with an R^2 of 0.76, an average error of -2% and an average absolute error of 16%. Predicted fracture patterns matched approximately one half the fracture patterns observed from tension tests of lumber. The model was applied to establish the strength reducing influence of closely spaced knots. Knots spaced as far as apart as 200 mm or more can interact to produce strengths considerably less than that based on the worst individual knot. Simple rules applicable to grading for distinguishing multiple knots that cause a significant strength reduction are not identified here, but it is shown that the finite element/fracture mechanics model provides a tool for establishing such rules.

Keywords: Lumber, Knots, Fracture, Tensile Strength

Introduction

Visual stress grading of lumber has been based on identification of the single most severe strength-reducing situation in a board. There are limitations in grading rules for closely-spaced knots, but little information has been published on quantifying the combined effects of growth characteristics such as closely-spaced knots occurring in an individual lumber board. Considerable amounts of dimension lumber possess the pith and as a result many small and closely-spaced knots commonly occur. It is not uncommon in strength tests of lumber to witness cracks extending from one individual knot to another during failure. Reported here is an investigation that examined the effect of closely-spaced knots on lumber tensile strength. The study focussed on examining the effect of two knots located on the wide face of lumber and generally within 200 mm center-to-center of each other.

Over the years, knots have been acknowledged as the most severe of several strength-reducing situations occurring in lumber. The presence of a knot inflicts three major strength reducing effects. First, it replaces part of a board cross-section with harder and denser, but weaker, knotwood. Secondly, the material inhomgeneity associated with the knot surrounded by the rest of the board induces a large stress concentration further reducing capacity. Finally, the branch disrupts the growth pattern of the trunk, resulting in significant grain angle distortion surrounding the knot and providing the opportunity for tensile stresses perpendicular to grain to develop and cracks to form.

Recently, we examined over sixty tension poof-load failures of dimension lumber collected by the Southern Pine Inspection Bureau. An attempt was made to identify the main contributing factor to the premature failure through careful visual examination of the fractures. It was observed that:

* 44% of the failure sections contained the pith,
* 33% of the failures were attributed to wide-face edge knots,
* 30% of the failures were attributed to single spike knots,
* 25% of the failures were attributed to multiple knots where a single knot instigating failure could not be determined, and
* 10% of the failures were attributed to center knots.

This admittedly subjective assessment provided several insights. Multiple knot failures were more common than expected given current stress grading rules that focus only on the most severe strength-reducing condition. It was nearly impossible to visually determine the different roles in the failure process of knot position, knot size, grain distortions, and relative distance between knots when assessing a cause of failure. It is likely that multiple knot conditions influenced to varying degrees a larger number of the failures than initially attributed to multiple knots in the above analysis. It would be very difficult, if not impossible, to identify the different roles of strength-reducing characteristics from carefully instrumented tension tests alone.

With a more through knowledge of the failure processes of lumber and an understanding of the individual roles of strength-reducing characteristics, lumber could be more accurately graded, allowable stresses would be more rationally established and wood resources would be better utilized. A numerical tool was created in this research to assess the individual roles of strength-reducing characteristics.

Finite Element/Fracture Mechanics Model for Lumber with Multiple Knots

Background

Finite element analysis with fracture mechanics capability has been applied to wood situations for at least 20 years (Pearson 1974). Most relevant to the development of this model was early work on tensile strength modeling that included development of the flow grain analogy to model grain distortion as it occurs around knots (Phillips et al. 1981). With the ability to model grain distortions, representative finite element meshes were achievable for single wide-face knot situations and methodologies developed to model the fracture process (Cramer and Goodman 1986). More recently, methods and related equipment have become available to measure local grain angles as they occur in boards allowing the more accurate characterization of grain angles around knots (McDonaid and Bendtsen 1986; Cramer and McDonald 1989). Tensile strength analysis has primarily been pursued because of its applicability to both tension members and the tension side of beams. It also offers for a more simple numerical analysis since wood material tends to be brittle and linear, elastic in tension. Linear elastic stress analysis and fracture mechanics was used in this study.

Board Characterization

One disadvantage of finite element analysis is the large amount of geometric and material property data needed to conduct a meaningful analysis. To overcome this disadvantage, we have sought methods to rapidly and nondestructively characterize the internal board structure. A commercially-produced electrical capacitance scanner was used to establish a map of grain angles on the wide face of each board under consideration. The map typically consisted of grain angle readings on a grid of 3-mm by 3-mm. This technology is limited to measuring the in-plane component of the total grain angle. The out-of-plane component is not measured, but is considered equally important for fracture analysis. Lacking a means to measure this component we found out-of-plane grain angles (dive angles) to be a predictable function of knot diameter, and more localized around the knot than in-plane angles (Shi 1996).

For tension loads, stresses within boards are dominated by the in-plane wide-face component of the total stress. For this reason and reasons of computational practicality a two-dimensional analysis with three-dimensional considerations has been developed to-date. The limitation to two-dimensions creates a considerable challenge to establishing a representative single plane model of a board. In multiple-knot situations it is common for the pith to be within the board or located outside but near a board surface. This proximity of the pith can readily result in the two wide faces being considerably different and confounding attempts to define a representative average midplane surface of the board for analysis. A method has been developed that locates the pith using grain angle data and knot locations (Shi 1996). With the pith established, a cylindrical coordinate system is used to establish an average of the opposing wide faces of the board under consideration.

In addition to grain angles, mechanical properties parallel and perpendicular to grain vary within a board. To aid in identification of within-board property variations, full-field x-ray scanning was employed as another descriptor of board structure. Density, as computed from x-ray attenuation, was established on a 0.2-mm by 0.2-mm grid that was later reduced to 3-mm by 3-mm grid The methodology used a medical x-ray tube with the image captured on a phosphor plate. The phosphor plate was subsequently read with a laser-based system to yield a digital record of the x-ray intensity. The x-ray intensity was then used to compute density (Suryoatmono et al. 1994).

We found that density in and around the knot region in Southern Pine could be as high as twice the average board density. This high variation was important because the density readings were intended for use as predictors of mechanical properties and the unusually high readings would have led to erroneously high predictions of mechanical properties. It was quickly established that the high density readings in and around knotwood are a complex combination of resin deposits and higher density knotwood. It could not be established that these high density readings lead to predictable increases in mechanical properties (USDA Forest Products Laboratory 1955). As a result, the density readings were largely ignored close to knots but did provide information on average board density, density variations away from knots and knot location.

The density readings were converted to specific gravity assuming uniform board moisture content and used with regression equations developed at the USDA Forest Products Laboratory to predict mechanical properties of Southern Pine lumber specimens (Green and Kretschmann 1994). Properties were assigned to each finite element (Fig. 1) and varied according to specific gravity within the finite element model of each specimen. Properties needed for the finite

element/fracture analysis include:
1) moduli of elasticity parallel and perpendicular to grain (from Green and Kretschmann 1994),
2) shear moduli (from Bodig and Jayne 1982),
3) Poisson's ratios (from Green and Kretschmann 1994),
4) elastic properties of knotwood (from Pugel 1980),
5) tensile strengths parallel and perpendicular to grain (from Green and Kretschmann 1994),
6) shear strength parallel to grain (from Green and Kretschmann 1994),
7) Fracture toughnesses, $K_{Ic\text{-}TL}$ (from Green and Kretschmann 1994).

Although fracture analyses were conducted on a board-by-board basis, no direct measurements of individual board properties were made other than those associated with grain angle and x-ray scanning. This ability to rapidly characterize lumber from nondestructive measures is critical to the eventual practical application of this research.

The grain angle and property information are used to construct the finite element model (mesh) of the individual board. The finite element meshes consist of isoparametric eight-node quadrilaterals, six-node triangles, and six-node quarter-point singular elements. To faciltiate fracture modeling, the mesh reflects the actual grain direction of the specimen. An example of such a finite element model is shown in Figure 1.

Computational Aspects of Failure Modeling

As previously, discussed, the fiber structure of a typical board involves all three dimensions of the board, and, as a result, even uniaxial tension loading will produce deformations involving all three dimensions. The most significant deformations and stress components for defining structural performance tend to lie in the wide face plane. Element stiffness matrices are based on the classical three-dimensional orthotropic constitutive relationship for linear elasticity (Hooke's Law) (Hearmon 1948). The material compliance matrix becomes full (all nonzero terms) when transformed for surface and dive grain angles. This three-dimensional material compliance matrix is then reduced to satisfy plane stress conditions in a way that yields a result equivalent to static condensation (Cook et al. 1989).

The plane stress compliance matrix for each element is inverted and then assembled into a structure stiffness matrix. A uniform displacement is applied as a load to simulate the conditions produced by most lumber tension machines. The resulting system of equations is solved to yield nodal displacements and then element stresses

Initial fracture is predicted when any element stresses are found to satisfy any of the criteria associated with a maximum stress failure theory as shown in Eq. 1 through Eq. 3.

$$\frac{\sigma_{\perp}}{\sigma_{\perp\text{-}ult}} = 1 \qquad (1)$$

$$\frac{\tau}{\tau_{ult}} = 1 \qquad (2)$$

$$\frac{\sigma_{\parallel}}{f} = 1 \qquad (3)$$

$$f_{\sigma\parallel} = \frac{2\sigma_{\parallel\text{-}ult}}{1+\cos(2\theta_D)}$$
$$f_{\sigma\perp} = \frac{2\sigma_{\perp\text{-}ult}}{1-\cos(2\theta_D)} \qquad (4)$$
$$f_{\tau} = \frac{2\tau_{ult}}{\sin(2\theta_D)}$$

$\sigma_{\perp}$ is the normal stress perpendicular to grain, $\sigma_{\parallel}$ is the normal stress parallel to grain and τ is the shear stress. The "ult" subscript identifies strength values. *f (Eq.* 3) is a function that depends on the dive angle, θ_D. The lowest *f* as defined in Eq. 4 is used in Eq. 3. More sophisticated theories such as the tensor polynomial strength theory have been proposed for wood but compelling evidence that such theories accurately predict wood failure has not been presented (Liu 1984).

Once the failure criteria in Eqs. 1-3 are satisfied a small crack is inserted into the finite element mesh with crack tips surrounded by six node singular elements. The applied displacement is increased and the modified finite element mesh is analyzed. Linear elastic, orthotropic fracture mechanics is used to predict crack extension. stress intensities in the opening (K_I) and shear (K_{II}) modes are computed.

The computed stress intensities are evaluated for possible crack extension using the maximum tangential stress theory. The isotropic form of this theory was extended to orthotropy by Saouma and others (1987). The derivations of this theory are tedious and only a summary of the concept will be provided here. The tangential stress is computed from K_I and

K_{II} and then evaluated against the opening mode fracture toughness. Cracks are assumed to always initiate along the fiber direction and typically they will propagate along the fiber direction. In most lumber failures, however, cracks will occasionally make small jumps across the grain. The primary advantage of the maximum tangential stress theory is that it provides a prediction for the direction of crack growth and for predicting the observed crack jumps across the grain. In our finite element mesh, the crack growth directions are simplified to either parallel to grain or perpendicular to grain. The perpendicular to grain crack growth is modeled as a small transverse separation from one grain line in the mesh to another. The primary disadvantage of the maximum tangential stress theory is that it is not a mixed mode theory and thus ignores the material mode II fracture toughness, K_{IIc}.

The process of failure modeling using this finite element model is one of step-wise fracture modeling. The mesh is analyzed and a crack is introduced. The mesh is subject to increased load through an increased applied displacement, and existing cracks extend and new cracks form. This process continues until a peak load is realized and the cross section becomes so damaged that increasing load cannot be sustained.

Strength Prediction Verification

Southern Pine lumber containing multiple critical knots were selected, scanned, analyzed and tested in tension at the USDA Forest Service Forest Products Laboratory. A total of 44 specimens including 2 by 4's and 2 by 6's containing single or double critical wide face knots were included in the verification. Predicted strengths correlated with measured strengths with an R^2 of 0.76, an average error of -2% and an average absolute error of 16%. Generally, predicted strengths were within ± 20% of the actual tensile strength. Strengths were predicted using only grain angle and density information for each board with the finite element/fracture mechanics model. The gretaest source of error was beleived to be the loss of board three-dimensional detail in the two-dimensional representation and the use of predicted material properties from regressions depending only on specific gravity.

Final fracture patterns were recorded during the tension tests of the 44 specimens. These fracture patterns were compared to the final fracture patterns predicted with the model. This comparison by its nature is subjective and cannot be rigorously quantified. Almost half the predicted fracture patterns closely resembled the fracture patterns observed in the tests. The remaining predicted fracture patterns varied from some to little resemblance with fracture pattern observed in the tests. An ability to match actual fracture patterns as much as half the time is quite good considering that the full three-dimensional structure of the board is neglected as well as other features such as the earlywood and late wood bands.

Model Application: Strength Interaction of Two Closely-Spaced Knots

The verified finite element/fracture mechanics model provides a tool to establish and study the parameters controlling lumber tensile strength. In the case of multiple knots many parameters potentially influence strength including the size of each individual knot and the position of the knots with respect to the board edges and with respect to one another. These parameters are not independent in controlling strength but interact in a seemingly endless array of unique combinations. To comprehensively establish the influence of multiple knots on strength by experimental means alone would be extremely difficult.

Once a digital representation of a lumber board is established and the finite element/fracture mechanics model is accepted as a reliable predictor of actual behavior, options for analysis exist that cannot be duplicated with physical specimens and testing. The multiple knot problem was studied by conducting analyses of an actual multiple knot condition and then one-by-one digitally removing the knots and the associated grain deviation to separate out the individual strength-reducing effect of each knot as shown m Fig. 1. When a knot is digitally removed it is replaced in the finite element mesh with a region of clear wood possessing grain angles and densities corresponding to the board average. This morphing is subjective and there is no guarantee that trees actually grow and yield lumber exactly as morphed. This process, however, is consistent with the level of detail and accuracy of the model.

Table 1 contains a listing of 28 two-knot specimens that were analyzed with the finite element/fracture mechanics model. The position of each knot is identified following the rules of ASTM D245 (1995). The first knot is distinguished from the second as the one having the greatest impact on strength. The distance between the knots is identified in the fourth column of Table 1 and the average diameter of the critical knot is shown. The predicted strength is listed in the sixth column and the relative strengths of the specimen with only one knot present are shown in the next two columns of Table 1.

Comparison of these columns of Table 1 reveals the predicted interaction of the knots on strength. The final column (ninth column) reveals the multiple knot effect. For example, in the first row with Specimen MK-22, the most critical knot yields a specimen strength of 24.00 MPa. This value is 255% of the two-knot strength of 9.41 MPa. The multiple knot factor in Table 1 is the value multiplied by the controlling single knot strength to yield the two-knot strength.

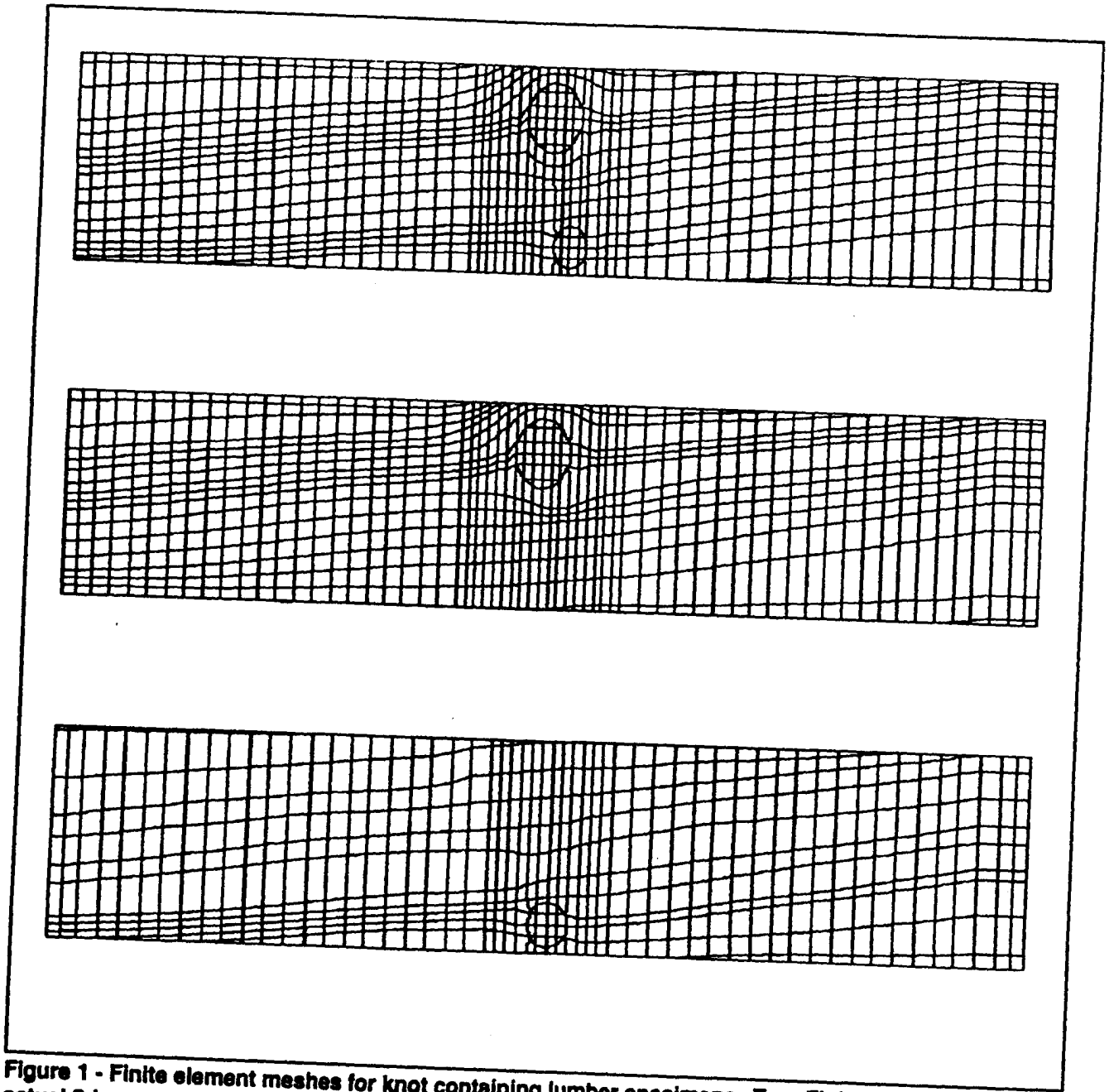

Figure 1 - Finite element meshes for knot containing lumber specimens. Top: Finite element mesh for actual 2-knot specimen. Middle and Bottom: Finite element meshes where one of the knots is digitally removed and replaced with clear wood.

The lower the multiple knot factor the greater amount of interaction that occurs between the two knots in controlling the specimen tensile strength. A multiple knot factor of 100% means that the two-knot combination induced the same tensile strength as that of the most critical single knot. For MK-22 the multiple knot factor is equal to 9.41 divided by 24.00 equaling 39%. The two-knot situation of MK-22 has Only 39% of the tensile strength corresponding to the most critial single knot. The specimens are listed in rank order according to the multiple knot factor. Occasionally the multiple knots appear to have a slight strengthening effect compared to the single most critical knot acting by itself. It is not clear if this result is valid or only error resulting from approximation within the methodology. Clearly, closely spaced knots cart result in a range of strength behavior ranging from strong interaction to no strength interaction.

Table 1 - Results of the Multiple Knot Analyses

Specimen ID 1	Knot Location Knot-1 2	Knot Location Knot-2 3	Distance Between Knots, mm 4	Knot-1 Diameter, mm 5	2-Knot Strength, Mpa 6	Strength Knot 1 Only , % 7	Strength Knot 2 Only, % 8	Multiple Knot Factor 9
MK-22	edge	center	85	55	9.41	255%	405%	39%
01-2	center	edge	175	48	9.56	193%	338%	52%
SP-15	edge	center	92	29	19.48	170%	202%	59%
MK-23	center	center	68	40	18.48	166%	196%	60%
SP-18	edge	edge	95	27	8.67	161%	174%	62%
MK-19	edge	center	133	49	11.38	145%	410%	69%
MK-20	edge	center	226	45	17.10	138%	340%	72%
MK-12	edge	center	169	35	19.51	133%	226%	75%
09-2	center	center	127	22	24.64	130%	137%	77%
SP-04	edge	center	51	29	16.49	130%	264%	77%
10-2	center	center	201	37	17.97	126%	147%	79%
MK-05	center	center	129	84	21.55	126%	182%	79%
SP-05	edge	edge	58	34	14.91	118%	151%	85%
MK-11	edge	center	80	31	13.56	118%	320%	85%
SP-02	edge	edge	66	28	17.19	115%	124%	87%
SP-17	edge	edge	167	36	11.96	113%	122%	88%
SP-14	edge	center	68	39	20.38	113%	154%	88%
SP-03	edge	edge	73	36	10.63	111%	203%	90%
SP-19	edge	edge	70	41	9.74	111%	142%	90%
MK-10	center	center	180	35	40.76	110%	143%	91%
SP-09	edge	center	56	26	14.45	109%	242%	92%
SP-07	center	edge	146	40	18.62	107%	182%	93%
06-1	edge	center	153	53	11.85	106%	232%	94%
SP-13	edge	center	64	29	19.51	100%	228%	100%
SP-20	edge	center	204	50	11.95	96%	151%	104%
MK-16	edge	center	170	53	13.38	95%	345%	105%
05-1	edge	edge	115	52	8.64	92%	220%	109%
SP-10	center	center	99	29	28.84	90%	136%	111%

There are no obvious correlations between critical knot size or distance between knots and the multiple knot factor. The resulting multiple knot factors appear to be a synthesis of a variety of parameters and conditions unique to each board. Knots as far apart as 200 mm or more can interact to cause significant reductions in tensile strength.

Summary and Conclusions

There is currently no accepted method for quantifying the strength reducing effect of closely spaced knots in lumber. Visual grading procedures have some restrictions on multiple knots within the same 150-mm length of lumber, but a means of quantifying the effect has not been developed and implemented. Quantifying the effect of multiple knots is fraught with difficulty because of the many parameters such as individual knot size, knot position, and relative position of the knots.

A finite element/fracture mechanics computer model has been developed to quantify the effect of single and multiple knots on lumber tensile strength. The two dimensional model requires only grain angle and density data gathered nondestructively using electrical capacitance and x-ray scanning. Predicted strengths correlated with measured strengths with an R^2 of 0.76, an average error of -2% and an average absolute error of 16%. Generally, predicted strengths were within ± 20% of the actual tensile strength.

The effect of closely spaced multiple knots was investigated using the model by individual removing one knot through a morphing process and comparing the strength predicted with two knots versus that with only one knot. In this study, the combined effect of two closely spaced knots can reduce the predicted tensile strength based only on the single most critical knot by as much as 60%. In other situations, the single most critical knot controls and the combined effect of two knots causes no further strength reduction. In this study, some two-knot situations with knots spaced as far apart as 200 mm or more still caused significant reduction in strength compared to strength based on only the single most critical knot. A simple method to distinguish those situations where multiple knots cause a severe strength reducing effect is not defined here. A finite element/fracture mechanics tool is presented, however, that will allow the investigation of failure and fracture mechanisms controlling the tensile strength of lumber containing multiple knots. Application of this model will yield information on the important parameters controlling lumber strength that can be implemented for grading.

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Acknowledgements

This research was funded by the United States Dept. of Agriculture National Research Initiative under Grant No. 93-37103-9323. The contributions of Dr. William Fohrell of Olympic Technical Services; Dr. Walter Peppler of the Dept. of Radiology and Medical Physics at the Univ. of Wisconsin-Madison; Melvin Travis, Ray Lockhart, and Linda Brown of the Southern Pine Inspection Bureau; and William Snyder of Weyerhaeuser are gratefully acknowledged.