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BENDING AND TWISTING TESTS FOR MEASUREMENT OF THE STIFFNESSES OF CORRUGATED BOARD

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ABSTRACT

In this work, an experimental study on the bending stiffnesses of corrugated board has been performed. An experimental procedure to measure the bending stiffnesses D_{11}, D_{22}, D_{12}, and D_{66} of an equivalent single layer orthotropic plate representing a corrugated board panel has bean proposed The technique includes standard four-point bending tests of beam specimens to assist in the evaluation of the stiffnesses D_{11} and D_{22} In addition, special plate twisting teats have been suggested for finding the stiffnesses D_{12} and D_{66} and the effective Poisson's ratios of the combined board For demonstration purposes, the proposed procedure has bean applied to a commercially available corrugated board

INTRODUCTION

Unlike fiber-reinforced composite materials, paper (paperboard) is a multiphase composite composed of moisture, fibers, voids, and chemical additives. Its three dimensional structure is basically an assembly of discrete fibers bonded together into a complex network. A single preferred orientation of the fibers results from the hydrodynamic forces present in the papermaking machine. This orientation is called the machine direction (MD). The in-plane direction perpendicular to the preferred direction is called the cross-machine direction (CD). These directions of material symmetry allow for paper to be modeled microscopically as an orthotropic solid.

The end-use performance characteristics of paperboard products are often predicted using models based upon the measured resistance of the utilized paper materials to burst, tear, fold, tension, compression, shear, and curl. However, because paperboard is used extensively as a packaging material, the flexural rigidity or bending stiffness of the product is an especially important quantity. In such applications, a stiff paperboard provides the necessary resistance to bulging of the panels of the container, ensuring a high degree of protection for its contents.

Corrugated containers, often referred to erroneously as cardboard boxes, are the most prominent structural application of paper. The corrugated board structural panels comprising such packaging are formed from a pair of flat face plates called liners which are separated by a periodic fluted core referred to as the corrugating medium or medium. Such composite sandwich plates are typically lightweight and inexpensive, and have high stiffness-to-weight and strength-to-weight ratios. Because it is composed of two primary elements, corrugated board is also referred to as combined board. Figure 1 illustrates the basic geometry and component materials of a corrugated board sample. The paper materials used for the liners are usually quite stiff and strong so that the bending stiffness of the combined board is high. The role of the medium material is to maintain separation between the two liners.

New shipping requirements have changed the paperboard market from one based on tonnage to one based on performance. Hence, the structural performance of a corrugated container is the major factor in its success in the marketplace. Such performance is a function of a number of factors including the quality of the input cellulose fibers, the mechanical properties of the utilized linerboard and medium, and the structural properties of the combined board. It is generally agreed that the most important attributes of a corrugated board are its bending stiffnesses or flexural rigidities. In order to improve the design and manufacture of corrugated board products, it is necessary to have a clear, technically sound understanding of the contributions of all of these factors to performance.

The primary engineering method for analyzing the mechanical behavior of composite sandwich plate structures such as corrugated board is the use of equivalent plate models. In such theories, the actual complicated sandwich plate geometry is replaced by an equivalent orthotropic slab of constant thickness having the same or similar bending stiffness characteristics. Subsequent mathematical analyses are then performed using the relatively simple small deflection or large deflection plate theories for orthotropic materials. This conceptual substitution of a single sheet of uniform material for the true composite stuctural panel is by its nature an approximation. However, the equivalent plate technique has proved to be a useful, expedient, and accurate method for predicting the transverse deformations of stiffened plates such as corrugated board panels. The utility of the equivalent plate method can be appreciated further by considering the available alternatives. These include trial and error procedures which lack predictive qualities, and costly time-consuming three-dimensional finite element analyses.

Equivalent plate models have been used for a variety of stiffened structures including ribbed plates, plate girders, reinforced or prestressed elements, and composite beam gridworks. Such composite structural elements are often referred to as being “technically orthotropic” since they are modeled as a single orthotropic layer. Several extensive reviews of the published applications of equivalent plate models are available in the literature. These include the works of Troitsky [1,2], and Bares and Massonnet [3].

Prior investigations on particular stiffened plate configurations have adopted either analytical or experimental strategies to obtain the required bending stiffnesses. In the theoretical approach, analytical expressions are developed to predict the equivalent orthotropic bending stiffnesses of the stiffened plate. Input parameters to these formulas typically include the mechanical properties of the component materials and relevant geometrical dimensions and characteristics. In the experimental approach, measurements of the force versus deflection response of the actual stiffened plate are used to establish the equivalent flexural rigidities. The stiffnesses found using either procedure are then typically used as input to the governing equation of the classical small deflection orthotropic plate theory. This equation for the transverse deflection of the plate can be solved by a variety of relatively straight-forward techniques including analytical closed-form solution procedures and the finite element method.

There have numerous analytical investigations where the equivalent plate approach has been applied to the corrugated board panels used by the paper industry. These include theoretical investigations [4-9] where analytical models combined with suitable approximations have been utilized to estimate one or more of flexural rigidities of the board. In addition, the equivalent plate method has been applied to calculate corrugated board buckling loads and postbuckling behavior, as well as to estimate corrugated container failure loads [10-16].

In recent work of the authors [17-18], several new formulations to calculate the bending stiffnesses D_{11}, D_{22}, D_{12}, and D_{66} of an equivalent plate representing a corrugated board sheet were established and then compared to existing theories found in the literature. New methods for modeling the stiffnesses of the corrugated orthotropic medium material were introduced and an extension of the membrane analogy method was used to calculate the torsional rigidity D_{66} The developed plate bending stiffness formulas require the thicknesses and material properties of the component paperboards as input. Also, the shape of the corrugated core material must be precisely specified. In our previous studies, the shape of the fluted medium was modeled using sinusoidal, arc-and-tangent, and elliptical representations.

Experimental techniques for measuring the bending stiffnesses of corrugated board sheets have also been applied. In particular, four-point bending teats have been routinely used for several years [19-20] to evaluate the "beam" bending stiffness (EI) of corrugated board strips (beams). A Tappi standard (paper industry standard) has been adopted to define a conventional procedure for evaluation of such stiffnesses [21]. However, direct measurements of the plate bending stiffnesses (D_{ij}) of corrugated board have been rare. The only published work appears to be that of Pommier and Poustis [9] who used four-point bending tests combined with plate twisting experiments [22] to evaluate the four plate bending stiffnesses. In their work the twisting test used was the two point loading method originally proposed by Bergstrasser [23], and then subsequently implemented by several groups to study orthotropic plates [24-26]. In the Bergstrasser technique, equal transverse concentrated loads are applied at diagonally opposite corners of a square stiffened/sandwich plate which is supported by roller supports at the other two corners. The stiffnesses are evaluated by monitoring the applied load versus central deflection response of the plate.

In this work an experimental procedure to measure the bending stiffnesses D_{11}, D_{22}, D_{12}, and D_{66} of the equivalent single layer orthotropic plate representing a corrugated board panel is proposed and implemented. The technique includes standard four-point bending tests of beam specimens to assist in the evaluation of the stiffnesses D_{11} and D_{22} In addition, special plate twisting tests requiring only a single applied point load as originally proposed by Tsai [27] are utilized for finding the stiffnesses D_{12} and D_{66} and the effective Poisson's ratios of the combined board. These twisting measurements are easier to perform and more exact in theory than the Bergstrasser two point loading method [27]. For demonstration purposes, the proposed procedure has been applied to a commercially available corrugated board.

THEORETICAL PRELIMINARIES

Orthotropic Stress-Strain Relations

A thin single layer uniform thickness orthotropic plate is shown in Figure 2. The coordinate axes have been chosen so that they are aligned with the material symmetry directions of the material, and so that the x-y plane is the middle surface of the plate. If the material is linear elastic and in a state of plane stress, the stress-strain relations are

$$\begin{aligned} \epsilon_x &= S_{11}\sigma_x + S_{12}\sigma_y \\ \epsilon_y &= S_{12}\sigma_x + S_{22}\sigma_y \\ \gamma_{xy} &= S_{66}\tau_{xy} \end{aligned} \qquad (1)$$

The material compliances S_{ij} are defined by

$$S_{11} = \frac{1}{E_1}$$

$$S_{12} = -\frac{\nu_{12}}{E_1} = -\frac{\nu_{21}}{E_2}$$

$$S_{22} = \frac{1}{E_2} \qquad (2)$$

$$S_{66} = \frac{1}{G_{12}}$$

where E_1 and E_2 are the elastic moduli, ν_{12} and ν_{21} are the Poisson's ratios, and G_{1z} is the shear modulus. Equation (1) can be inverted to give

$$\sigma_x = Q_{11}\epsilon_x + Q_{12}\epsilon_y$$

$$\sigma_y = Q_{12}\epsilon_x + Q_{22}\epsilon_y \qquad (3)$$

$$\tau_{xy} = Q_{66}\gamma_{xy}$$

where the so-called reduced stiffnesses Q_{ij} are given by

$$Q_{11} = \frac{E_1}{1 - \nu_{12}\nu_{21}}$$

$$Q_{12} = \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}} = \frac{\nu_{21}E_1}{1 - \nu_{12}\nu_{21}} \qquad (4)$$

$$Q_{22} = \frac{E_2}{1 - \nu_{12}\nu_{21}}$$

$$Q_{66} = G_{12}$$

Orothotropic Plate Theory

The classical small deflection theory for linear elastic orthotropic plates (single layer, constant thickness) is a formulation describing the behavior of thin plates subjected to transverse loading. The transverse deflections of the plate are assumed to be small relative to its thickness. The governing equation for the transverse deflection is given by

$$D_{11}\frac{\partial^4 w}{\partial x^4} + 2(D_{12} + 2D_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22}\frac{\partial^4 w}{\partial y^4} = p(x,y) \qquad (5)$$

where w = w(x,y) is the transverse deflection of the middle surface of the plate, h is the plate thickness, p(x,y) is the transverse pressure loading on the plate, and

$$D_{11} = \frac{h^3}{12}Q_{11} = \frac{E_1 h^3}{12(1 - \nu_{12}\nu_{21})}$$

$$D_{22} = \frac{h^3}{12}Q_{22} = \frac{E_2 h^3}{12(1 - \nu_{12}\nu_{21})}$$

$$D_{12} = \frac{h^3}{12}Q_{12} = \frac{\nu_{21}E_1 h^3}{12(1 - \nu_{12}\nu_{21})} = \frac{\nu_{12}E_2 h^3}{12(1 - \nu_{12}\nu_{21})} \tag{6}$$

$$D_{66} = \frac{h^3}{12}Q_{66} = \frac{G_{12}h^3}{12}$$

are the bending stiffnesses or flexural rigidities of the anisotropic plate. The term

$$H = D_{12} + 2D_{66} \tag{7}$$

is often called the total torsional rigidity. The stiffness D_{66} is often referred to es the apparent torsional rigidity or simply the torsional rigidity.

The in-plane stresses for a plate subjected to transverse loading are given by

$$\sigma_x = \frac{12M_x}{h^3}z$$

$$\sigma_y = \frac{12M_y}{h^3}z \tag{8}$$

$$\tau_{xy} = \frac{12M_{xy}}{h^3}z$$

where the bending moment resultants (bending moments per unit length) are defined as

$$M_x = -\left(D_{11}\frac{\partial^2 w}{\partial x^2} + D_{12}\frac{\partial^2 w}{\partial y^2}\right)$$

$$M_y = -\left(D_{12}\frac{\partial^2 w}{\partial x^2} + D_{22}\frac{\partial^2 w}{\partial y^2}\right) \tag{9}$$

$$M_{xy} = -\ 2D_{66}\frac{\partial^2 w}{\partial x \partial y}$$

Equivalent Plate Models

The idea of applying the classical theory of orthotropic single layer constant thickness plates to slabs stiffened by ribs was proposed and developed by, Huber [28]. The basic assumption he proposed for estimating bending deflections in a stiffened slab, was to replace such a slab by an equivalent orthotropic plate of constant thickness having the same stiffness characteristics. Since corrugated boards are composed of two liners and one medium, the idea of an equivalent plate having the same stiffness characteristics can be introduced. However, it should be understood that the actual corrugated board obviously can't be modeled accurately by the equivalent plate in every respect.

Former theoretical investigations and experimental data have indicated that the equivalent plate concept is applicable to more complicated stiffened/sandwich plates such as corrugated board under the following provisions [1]:

1. The ratios of the periodic length of the medium to the plate boundary dimensions are small enough to insure approximate homogeneity of the stiffnesses.
2. The rigidities are uniformly distributed in both directions.
3. The flexural and twisting rigidities do not depend on the boundary conditions of the plate or on the distribution of the vertical load.
4. A perfect bond exists between the liners and the medium.

The substitution of an equivalent plate with the same stiffness characteristics as corrugated board has been called the method of elastic equivalence. By applying the principle of elastic equivalence, the discontinuous structure of the corrugated sandwich plate is represented by an idealized constant thickness orthotropic plate, which reflects the stiffnesses of the actual system. This substitution is illustrated in Figure 3. When using the equivalent plate method, structural orthotropy is replaced by natural orthotropy, and eq. (5) is used as the governing differential equation for the out-of plane displacements of the corrugated plate. It should be noted however, that stress distributions in the corrugated board cannot be found using eqs. (8) because of the discontinuous nature of the corrugated board structure when compared to its equivalent plate representation,

EXPERIMENTAL PROCEDURE

A common experimental method used to estimate the appropriate bending stiffnesses for complicated stiffened/sandwich plates requires recasting the expressions for the stiffnesses of a single layer orthotropic plate in eq. (6) as

$$
\begin{aligned}
D_{11} &= \frac{1}{1 - \nu_{12}\nu_{21}} \left(\frac{h^3}{12S_{11}} \right) \\
D_{22} &= \frac{1}{1 - \nu_{12}\nu_{21}} \left(\frac{h^3}{12S_{22}} \right) \\
D_{12} &= \frac{-\nu_{12}\nu_{21}}{1 - \nu_{12}\nu_{21}} \left(\frac{h^3}{12S_{12}} \right) \\
D_{66} &= \left(\frac{h^3}{12S_{66}} \right)
\end{aligned}
\qquad (10)
$$

Experimental configurations are then sought which can measure the effective values of the quantities $(h^3/12S_{11})$, $(h^3/12S_{22})$, $(h^3/12S_{12})$, and $(h^3/12S_{66})$ for the stiffened/sandwich plate. The desired bending stiffnesses of the equivalent plate can then be estimated by using the experimentally measured effective values of $(h^3/12S_{ij})$ in eq. (10) along with the effective Poisson's ratios calculated using

$$\nu_{12} = -\frac{S_{12}}{S_{11}} = -\frac{(h^3/12S_{11})}{(h^3/12S_{12})}$$
$$\nu_{21} = -\frac{S_{12}}{S_{22}} = -\frac{(h^3/12S_{22})}{(h^3/12S_{12})} \quad (11)$$

A pair of appropriate plate teat configurations which can evaluate all four of the required $(h^3/12S_{ij})$values include four-point bending of plate strips (beams) and the plate twisting test configuration proposed by Tsai [27]. These procedures are now discussed.

<u>Four-Point Bending Procedure</u>

The standard beam four-point bending configuration is shown in Figure 4. The central span in such a configuration is in a state of pure bending. Using classical beam theory, the relationship between the applied loading and the central deflection for a strip cut out of a constant thickness single layer orthotropic plate (which is essentially a beam with rectangular cross-sectional area) is

$$P = \left[\frac{16b}{dD^2}\right]\left(\frac{h^3}{12S_{xx}}\right)w_o \quad (12)$$

where P/2 is the loading on each end, D is the central span, w_o is the central deflection, d is the outer span between the load application point and the support point, and b is the width of the plate strip. Quantity S_{xx} is the material compliance along the length of the beam

$$S_{xx} = \frac{1}{E_x} \quad (13)$$

where E_x is the material stiffness along the length of the beam.

For corrugated board, two specimen configurations are useful. Figure 5 illustrates a MD plate strip (the machine direction of the liners is along the length of the strip). Using eq. (12), this specimen can be used to evaluate the quantity $(h^3/12S_{11})$. Similarly, Figure 6 illustrates a CD plate strip (the cross-machine direction of the liners is along the length of the strip). Using eq. (12), this specimen can be used to evaluate the quantity $(h^3/12S_{22})$.

<u>Twisting Test Procedure</u>

The twisting teat configuration proposed by Tsai [27] involves transversely loading a square plate at one comer while maintaining zero deflection through roller supports at the other three comers as illustrated in Figure 7. An analytical elasticity-based solution is available for the behavior of an orthotropic single layer constant thickness plate in this configuration. The load versus central deflection behavior is expressed as [27]

$$P = \frac{16}{a^2}\left(\frac{h^3}{12S_G}\right)w_o \quad (14)$$

where P is the applied comer load, a is the edge length of the plate, and W_o is the central transverse deflection. The parameter S_G in eq. (14) is a directionally dependent compliance parameter given by

$$S_G = 2mn(m + n)^2 S_{11} - 8m^2n^2 S_{12} - 2mn(m - n)^2 S_{22} + (m^2 - n^2)^2 S_{66} \quad (15)$$

where

$$m = \cos\alpha \qquad n = \sin\alpha \quad (16)$$

and a is the angle between the x-y axes parallel to the edges of the plate and the 1-2 axes along the material symmetry directions of the orthotropic medium (see Figure 7).

Typical choices of angle a which have been chosen by experimentalists in the past are 0°, 45°, and -45°. For these choices, the directional compliance parameter is found to be

$$\begin{aligned} S_G &= S_{66} & \alpha &= 0 \\ S_G &= 2(S_{11} - S_{12}) & \alpha &= 45 \\ S_G &= 2(S_{22} - S_{12}) & \alpha &= -45 \end{aligned} \quad (17)$$

Using both four-point bending and twisting test results, an experimental procedure can be designed to measure all four of the $(h^3/12S_{ij})$ needed in eqs. (10,11) to calculate the desired bending stiffnesses D_{ij} of the equivalent plate representation for corrugated board. As discussed above, values of $(h^3/12S_{11})$ and $(h^3/12S_{22})$ can be obtained using four-point bending tests on MD and CD corrugated board plate strips. In addition, values of $(h^3/12S_{66})$ and $(h^3/12[S_{22} - S_{12}])$ can be found using twisting tests with a equal to 0° and -45°, respectively. The value of $(h^3/12S_{12})$ can be easily extracted from the value of quantity $(h^3/12[S_{22} - S_{12}])$ measured in the twisting experiments once $(h^3/12S_{22})$ is known through the four-point bending experiments.

EXAMPLE APPLICATION

Corrugated Board and Component Materials

A commercially available"A" flute corrugated board was chosen as an example material for implementing the experimental bending stiffness measurement procedure. Referring to Figure 8, the characteristic dimensions of the chosen board were L = .167 inches, H = .083 inches, and t = .01 inches. Although not needed in this work the orthotropic material properties of the component materials are tabulated in Figure 9 for informational purposes.

Experimental Results

Both MD and CD corrugated board strips (beam specimens) were cut out of the obtained corrugated board panels. The MD specimens had a total length of 12 inches and four-point bending teat dimensions (see Figure 4) of D = 8 inches and d = 1.75 inches. The CD specimens had a total length of 11 inches and four-point bending teat dimensions of D = 7 inches and d = 1.75 inches. Figures 10 and 11 show measured load versus central deflection curves for typical MD and CD specimens, respectively. As predicted in eq. (12), the responses are very linear. For each specimen type, five test repetitions were performed. Figures 12 and 13 show the linear fits to the obtained load-deflection curves while Figures 14 and 15 present tables of the measured slopes of the linear fitting curves and the values of $(h^3/12S_{11})$ and $(h^3/12S_{22})$ extracted from the slopes using eq. (12).

Square twisting test specimens with a = 0° and -45° were also prepared. Both specimen types had edge dimensions (see Figure 7) of a = 6 inches. Figures 16 and 17 show measured load versus central deflection curves for typical a = 0° and a = -45° specimens, respectively. As predicted in eq. (14), the responses are again extremely linear. For each specimen type, five test repetitions were performed. Figures 18 and 19 show the linear fits to the obtained load-deflection curves while Figures 20 and 21 present tables of the measured slopes of the linear fitting curves and the values of $(h^3/12S_{66})$ and $(h^3/12[S_{22} - S_{12}])$ extracted from the slopes using eq. (14).

Stiffness Calculations

From the average four-point bending teat results in Figures 14 and 15

$$\frac{h^3}{12S_{11}} = 169.32 \text{ lb-in}$$
$$\frac{h^3}{12S_{22}} = 92.33 \text{ lb-in} \qquad (18)$$

Also, from the average a = -45° twisting test results in Figure 21

$$\left(\frac{h^3}{12(S_{22} - S_{12})}\right) = 68.32 \text{ lb-in} \qquad (19)$$

Thus, using eqs. (18, 19)

$$\frac{h^3}{12S_{12}} = -262.72 \text{ lb-in} \qquad (20)$$

and the equivalent Poisson's ratios are

$$\nu_{12} = -\frac{(h^3/12S_{11})}{(h^3/12S_{12})} = .644$$
$$\nu_{21} = -\frac{(h^3/12S_{22})}{(h^3/12S_{12})} = .351 \qquad (21)$$

Using eqs. (10,18,20,21) and the average value of $(h^3/12S_{66})$ in Figure 20, the bending stiffnesses of the equivalent plate representation are

$$D_{11} = \frac{1}{1 - \nu_{12}\nu_{21}}\left(\frac{h^3}{12S_{11}}\right) = 218.77 \text{ lb-in}$$
$$D_{22} = \frac{1}{1 - \nu_{12}\nu_{21}}\left(\frac{h^3}{12S_{22}}\right) = 119.30 \text{ lb-in}$$
$$D_{12} = \frac{-\nu_{12}\nu_{21}}{1 - \nu_{12}\nu_{21}}\left(\frac{h^3}{12S_{12}}\right) = 76.73 \text{ lb-in} \qquad (22)$$
$$D_{66} = \left(\frac{h^3}{12S_{66}}\right) = 36.18 \text{ lb-in}$$

SUMMARY AND CONCLUSIONS

In this work, an experimental procedure to measure the bending stiffnesses D_{11}, D_{22}, D_{12}, and D_{66} of an equivalent single layer orthotropic plate representing a corrugated board panel has been proposed and applied. The technique requires measurement of the effective values of quantities $(h^3/12S_{ij})$ for the equivalent plate representation of the corrugated board. Standard four-point bending tests of MD and CD plate strips (beam specimens) are utilized to measure effective values of $(h^3/12S_{11})$ and $(h^3/12S_{22})$. In addition, effective values of $(h^3/12S_{66})$ and $(h^3/12[S_{22} - S_{12}])$ can be found using the twisting test proposed by Tsai [27] with *a* equal to O° and -45°, respectively. The value of $(h^3/12S_{12})$ can be easily extracted from the value of quantity $(h^3/12[S_{22} - S_{12}])$ measured in the twisting experiments once $(h^3/12S_{22})$ is known through the four-point bending experiments. For demonstration purposes, the proposed procedure has been applied to a commercially available corrugated board.

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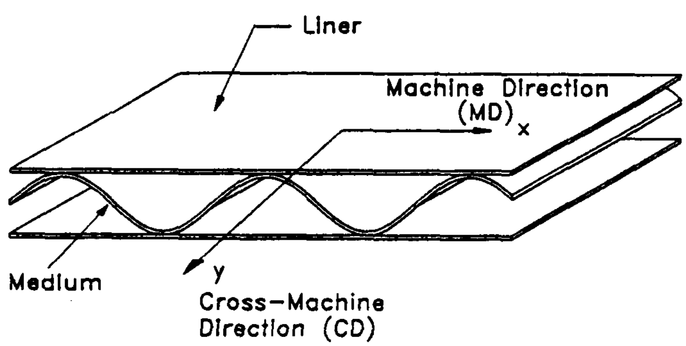

Figure 1- Basic Geometry and Component Materials of Corrugated Board

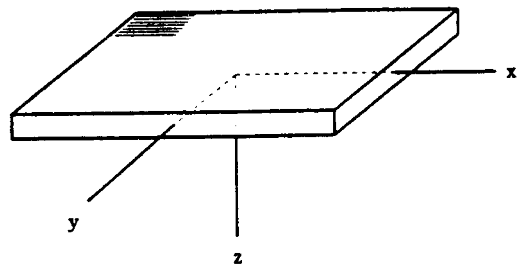

Figure 2- Thin Orthotropic Plate

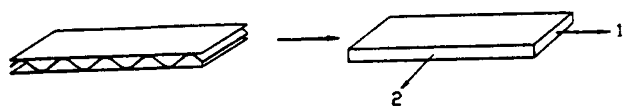

Figure 3- Orthotropic Equivalent Plate Model
for Corrugated Board

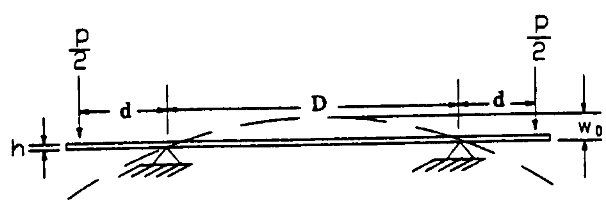

Figure 4- Geometry and Dimensions of the
Four-Point Bending Test

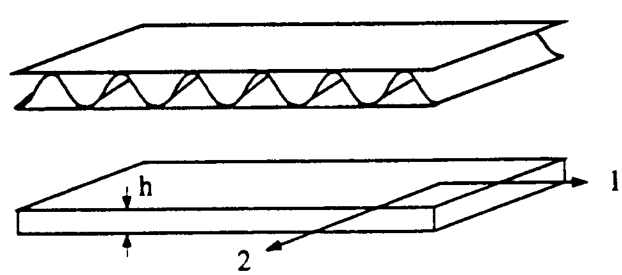

Figure 5- MD Corrugated Board Plate Strip for
the Four-Point Bending Experiments

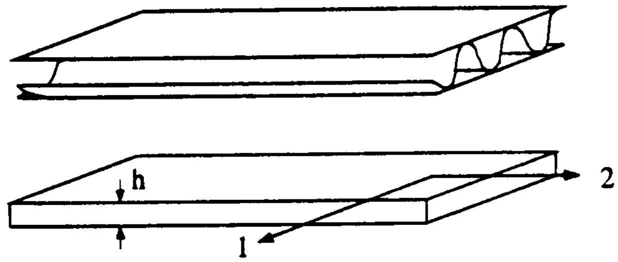

Figure 6- CD Corrugated Board Plate Strip for
the Four-Point Bending Experiments

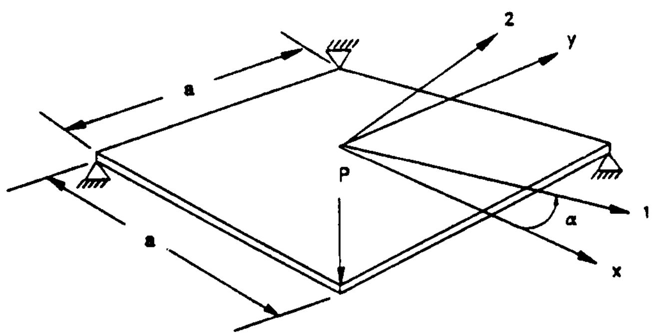

Figure 7- Loading Scheme for the Pure Twisting Tests [27]

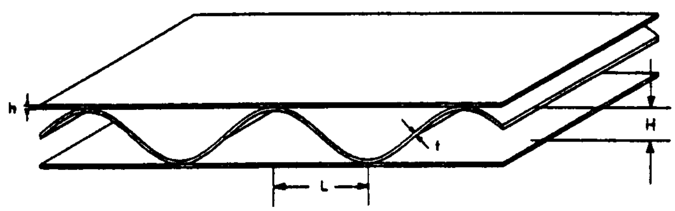

Figure 8- The Characteristic Geometry and
Dimensions of Corrugated Board

Property	Liner	Medium
E_1	$.837 \times 10^6$ psi	$.563 \times 10^6$ psi
E_2	$.331 \times 10^6$ psi	$.138 \times 10^6$ psi
ν_{12}	.614	.705
ν_{21}	.232	.185
G_{12}	$.242 \times 10^6$ psi	$.112 \times 10^6$ psi

Figure 9- Experimentally Measured Mechanical
Properties of the Liner and Medium Papers

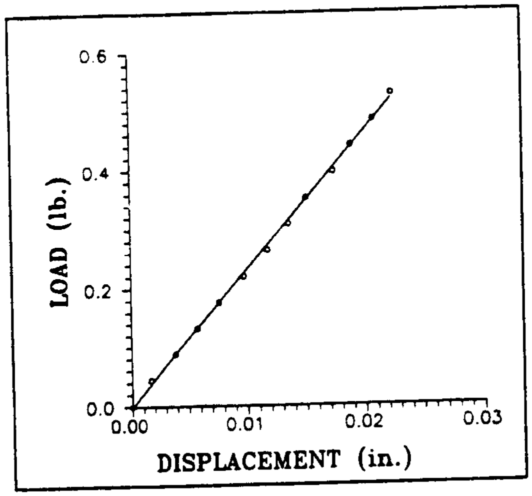

Figure 10- Typical Load-Deflection Curve for a MD Corrugated Board Plate Strip under Four-Point Bending

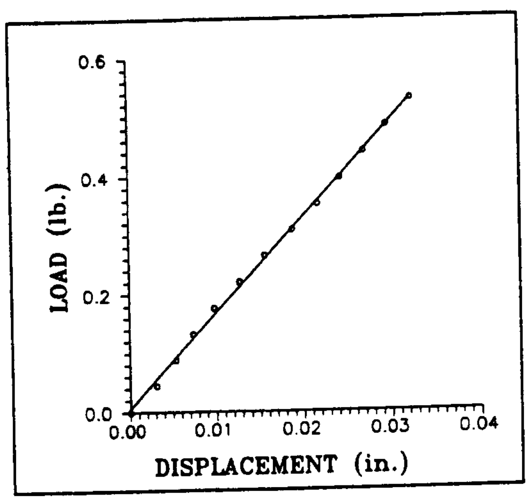

Figure 11- Typical Load-Deflection Curve for a CD Corrugated Board Plate Strip under Four-Point Bending

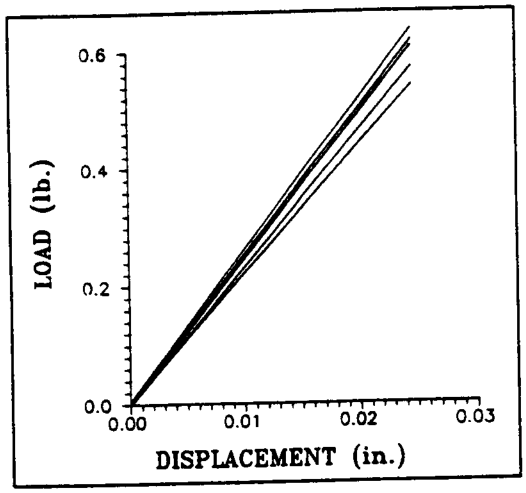

Figure 12- Load-Deflection Curves for MD Corrugated
Board Plate Strips under Four-Point Bending

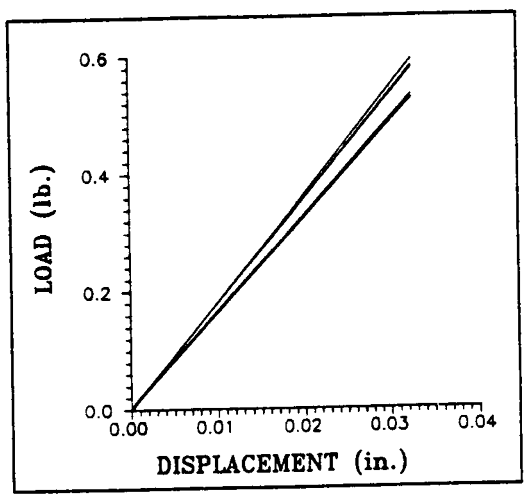

Figure 13- Load-Deflection Curves for CD Corrugated
Board Plate Strips under Four-Point Bending

Specimen	Slope (P vs. w_0) (lb/in)	$\frac{h^3}{12S_{11}}$ (lb-in)
1	23.35	163.45
2	24.92	174.44
3	21.89	153.23
4	25.14	175.98
5	25.64	179.48
Average	24.19	169.32

Figure 14- Four-Point Bending Test Results
for MD Strip Specimens

Specimen	Slope (P vs. w_0) (lb/in)	$\frac{h^3}{12S_{22}}$ (lb-in)
1	16.15	86.55
2	16.27	87.20
3	17.80	95.40
4	18.26	97.86
5	17.66	94.65
Average	17.23	92.33

Figure 15- Four-Point Bending Test Results
for CD Strip Specimens

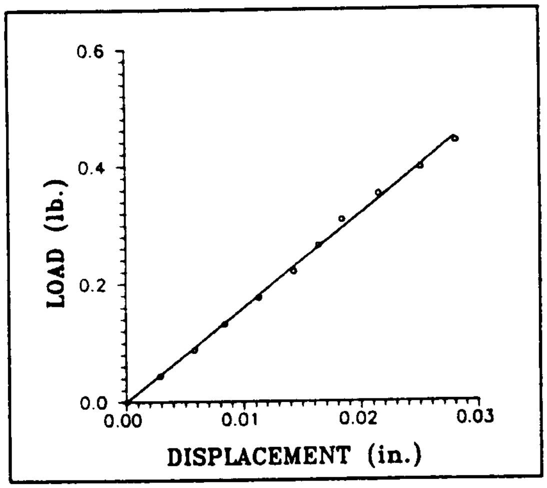

Figure 16- Typical Load-Deflection Curve for a Corrugated Board Specimen Subjected to Twisting *(a* = 0°)

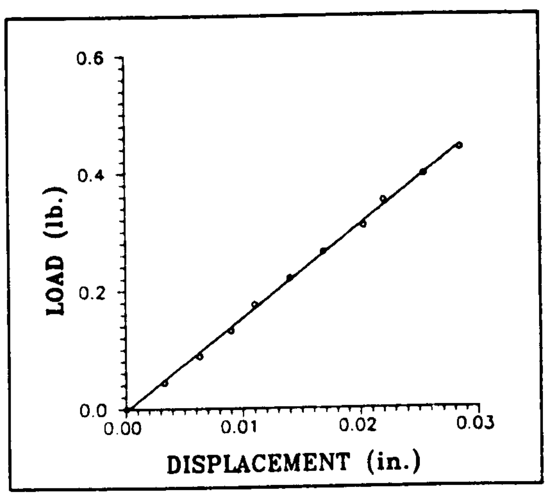

Figure 17- Typical Load-Deflection Curve for a Corrugated Board Specimen Subjected to Twisting *(a* = -45°)

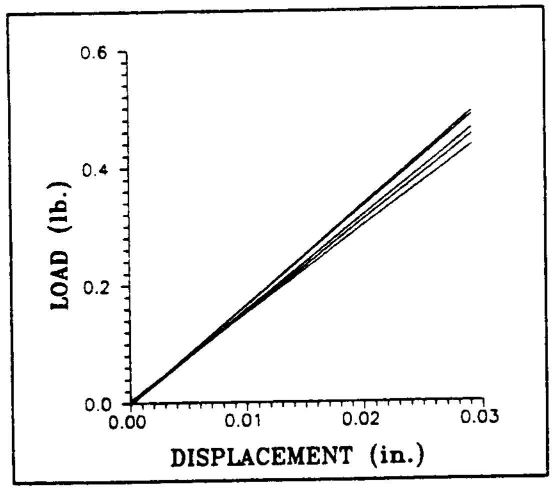

Figure 18- Load-Deflection Curves for Corrugated Board Specimens Subjected to Twisting $(a = 0°)$

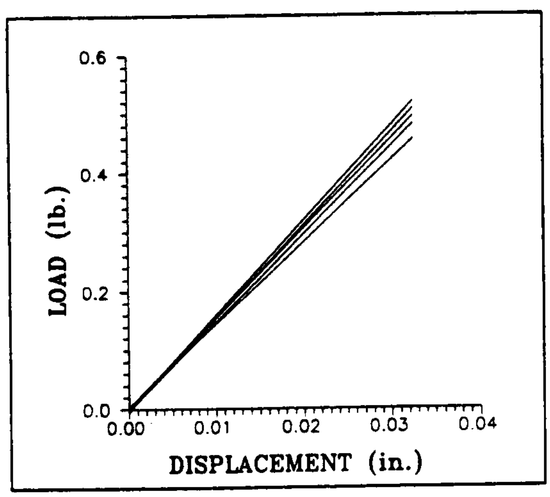

Figure 19- Load-Deflection Curves for Corrugated Board Specimens Subjected to Twisting $(a = -45°)$

Specimen	Slope (P vs. w_o) (lb/in)	$\frac{h^3}{12S_{66}}$ (lb-in)
1	15.93	35.84
2	16.97	38.18
3	16.69	37.55
4	16.03	36.07
5	14.78	33.26
Average	16.08	36.18

Figure 20- Twisting Test Results *(a* = 0°)

Specimen	Slope (P vs. w_o) (lb/in)	$\frac{h^3}{12(S_{22}-S_{12})}$ (lb-in)
1	13.95	62.78
2	16.07	72.32
3	15.17	68.27
4	15.79	71.06
5	14.93	67.19
Average	15.18	68.32

Figure 21- Twisting Test Results *(a* = -45°)

In: Perkins, Richard, ed. Mechanics of cellulosic materials -1995- Proceedings of 1995 joint ASME applied mechanics and materials meeting; 1995 June 28–30; Los Angeles. New York: The American Society of Mechanical Engineers: 91-109; 1995. AMD-Vol. 209/MD-Vol. 60.