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Energy Criterion for Load Duration Problem in Wood

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Abstract

This paper presents an analysis of time-dependent strength of cellulosic and polymeric materials under ramp loading. The analysis is based on the Reiner—Weissenberg strength theory in conjunction with an Eyring's three-element model. Parameters of the strength model system are evaluated based on existing constant and ramp loading test data of Douglas-fir beams.

Introduction

Bach (1973) used the Reiner-Weissenberg strength theory (Reiner 1964) in combination with a linear mechanical model to analyze the time-dependent strength of wood. However, he did not include any numerical work in his analysis. With the linear model replaced by an Eyring's three-element model (Krausz and Eyring 1975). Teoh et al. (1987) and others succeeded to predict the creep rupture time of polymers and tropical wood polymer composites. The present study further extends the work of creep rupture (i.e., rupture due to constant loading) to include rupture due to ramp loading with the same Eyring's model.

Energy Criterion of Strength and Rheological Model

Reiner and Weissenberg (Reiner 1964) postulate that failure. of a viscoelastic material depends on a maximum value of the intrinsic free energy, which can be stored elastically in the volume element of the material. The intrinsic free energy may be called strain work w_c, "strain" denoting the recoverable part of deformation. Failure will occur at the time t_f, when

$$w_c = \int_0^{t_f} (\dot{w} - \dot{D})\, dt = R_1 \qquad (1)$$

where $\dot{w}$ is rate of stress work done on the material, $\dot{D}$ is rate of dissipation of nonelastic energy, and R_1 is a material constant, which may be called the resilience of the material. Equation (1) may be called the failure condition.

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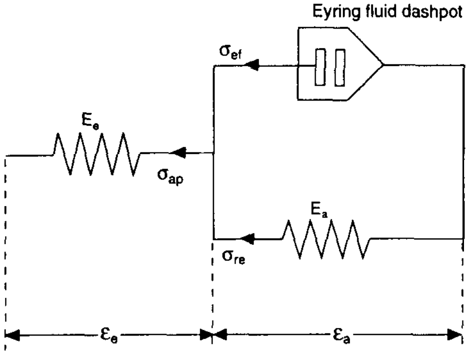

FIG. 1. Three-Element Eyring's Model.

In conjunction with the failure condition, an Eyring's model as described in Fig. 1 was successfully applied to predict creep rupture of polymers and tropical wood polymer composites.

Rupture Under Ramp Loading

Referring to Eq. (1) and Fig. 1, we have

$$\int_0^{\bar{\varepsilon}_a} \sigma_{re}\, d\varepsilon_a + \int_0^{\bar{\varepsilon}_e} \sigma_r\, d\varepsilon_e = R_1 \tag{2}$$

where $\bar{\varepsilon}_a$ is anelastic strain at rupture, $\bar{\varepsilon}_e$ is elastic strain at rupture, σ_{re} is recovery stress, $\sigma_r \equiv \sigma_{ap}$ is applied stress, and R_1 is material resilience.

With $\sigma_{re} = e_a E_a$, where e_a and E_a are the anelastic strain and anelastic modulus, respectively, the first integral in Eq. (2) becomes $\bar{\varepsilon}_a^2 E_a/2$. Under ramp loading, $\sigma_r = \alpha t = \varepsilon_e E_e$, where α is a constant denoting the rate of loading and e_e and E_e are the elastic strain and elastic modulus, respectively, the second integral becomes $\bar{\sigma}_r^2/2E_e$. Thus, from Eq. (2)

$$\bar{\varepsilon}_a = \left[(2/E_a)\left(R_1 - \bar{\sigma}_r^2/2E_e\right)\right]^{1/2} \tag{3}$$

where $\bar{\sigma}_\rho = \alpha t_r$ and $t_r \equiv t_f$ is time of rupture.

The anelastic strain rate $\dot{\varepsilon}_a$ can be expressed in terms of the effective stress σ_{ef} as (Krausz and Eyring 1975)

$$\dot{\varepsilon}_a = K_1 \sinh(\beta \sigma_{ef}) \tag{4}$$

where K_1 is a function of the activation energy and b is a stress coefficient (Liu et al. 1994). The anelastic strain rate can also be expressed as

$$\dot{\varepsilon}_a = \dot{\sigma}_{re}/E_a \tag{5}$$

in which the recovery stress rate can be written as

$$\dot{\sigma}_{re} = \dot{\sigma}_r - \dot{\sigma}_{ef} \tag{6}$$

and, with $\dot{\sigma}_r = \alpha$, we obtain

$$\dot{\varepsilon}_a = (1/E_a)(\alpha - \dot{\sigma}_{ef}) \tag{7}$$

From Eqs. (4) and (7), it follows that

(8)

Equation (8) can be nut in closed form as

$$t_r = \frac{1}{\beta(\alpha^2 + E_a^2 K_1^2)^{1/2}} \ln\left\{ \frac{[E_a K_1 + \alpha \sinh(\beta\bar{\sigma}_{ef}) + (\alpha^2 + E_a^2 K_1^2)^{1/2} \cosh(\beta\bar{\sigma}_{ef})][(\alpha - E_a K_1 \sinh(\beta\bar{\sigma}_r))]}{[E_a K_1 + \alpha \sinh(\beta\bar{\sigma}_r) + (\alpha^2 + E_a^2 K_1^2)^{1/2} \cosh(\beta\bar{\sigma}_r)][(\alpha - E_a K_1 \sinh(\beta\bar{\sigma}_{ef}))]} \right\} \quad (9)$$

where the effective stress $\bar{\sigma}_{ef}$ can be expressed by

$$\bar{\sigma}_{ef} = \bar{\sigma}_r - \bar{\sigma}_{re} = \bar{\sigma}_r - \bar{\varepsilon}_a E_a = \bar{\sigma}_r - \left[2E_a(R_1 - \bar{\sigma}_r^2 / 2E_e)\right]^{1/2} \quad (10)$$

If the effective stress $\bar{\sigma}_{ef}$ and the applied stress $\bar{\sigma}_r$ are equal, we obtain from Eqs. (9) and (10) the upper stress limit at $t_r = 0$ to be

$$\bar{\sigma}_r = (2E_e R_1)^{1/2} \quad (11)$$

If, however, the effective stress $\bar{\sigma}_{ef}$ remains at zero, the stressed material should last forever. We can likewise find from Eqs. (9) and (10) the lower stress limit at $t_r = \infty$, which is

$$\bar{\sigma}_r = [2E_a E_e R_1 / (E_a + E_e)]^{1/2} \quad (12)$$

Rupture Under Constant Loading

The expression for the time of failure $t_c \equiv t_f$ can be found in the literature (e.g., Teoh et al. 1987) as

$$t_c = \frac{1}{\beta E_a K_1} \ln\left[\frac{\tanh(\beta\bar{\sigma}_c/2)}{\tanh(\beta\bar{\sigma}_{ef'}/2)}\right] \quad (13)$$

where $\bar{\sigma}_c \equiv \bar{\sigma}_{ap}$ is applied stress at rupture, and the effective stress $\bar{\sigma}'_{ef}$ is

$$\bar{\sigma}'_{ef} = \bar{\sigma}_c - \left[2E_a\left(R_1 - \bar{\sigma}_c^2 / E_e\right)\right]^{1/2} \quad (14)$$

From Eqs. (13) and (14). we also obtain the upper stress limit at $t_c = 0$

$$\bar{\sigma}_c = (E_e R_1)^{1/2} \quad (15)$$

and the lower stress limit as $t_c \to \infty$

$$\bar{\sigma}_c = [2E_a E_e R_1 / (2E_a + E_e)]^{1/2} \quad (16)$$

Model Correlation

The test data on bending strength of small, clear Douglas-fir beams under rapid loading were reported by Liska (1950), covering a time span from 1 to 150 s. The data contain considerable scatter and are represented by

$$(\bar{\sigma}_r / \sigma_u)100 = 124 - 3.8\ln(t_r) \quad (17)$$

where σ_u = 53.1 MPa, defined as the ultimate or short-term strength of Douglas-fir, and t_r is in seconds. The test data on bending strength of small beams of the same material under constant loading reported by Wood (1951) cover a time span from 0.1 h to 10 years and are represented by

$$(\bar{\sigma}_c / \sigma_u)100 = 112.8 - 2.73\ln(t_c) \quad (18)$$

where t_c is in seconds. We use Eq. (13) to fit Wood's data represented by Eq. (18) to evaluate the model parameters. Results are as follows: E_e = 11,200 MPa, E_a = 150 MPa, R_1 = 0.34 MJm^{-3}, b = 0.5984 MPa^{-1}, and K_1 = 8.8779 E-17 s^{-1}. Equations (13) and (18) and Wood's data are plotted in Fig. 2.

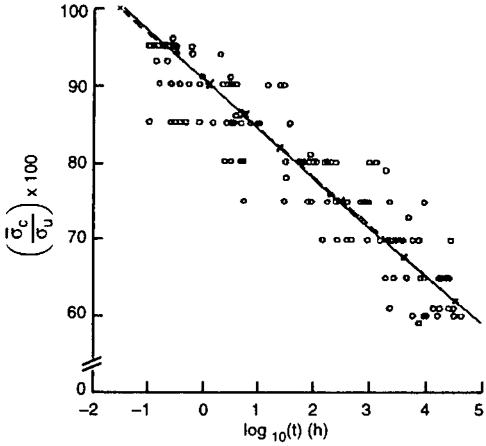

FIG. 2. Variation of Constant Stress Level at Failure with Logarithm of Time (--x-- Eq.(13); ---- Eq.(18); o Wood (1951); σ_u = 53.1 MPa)

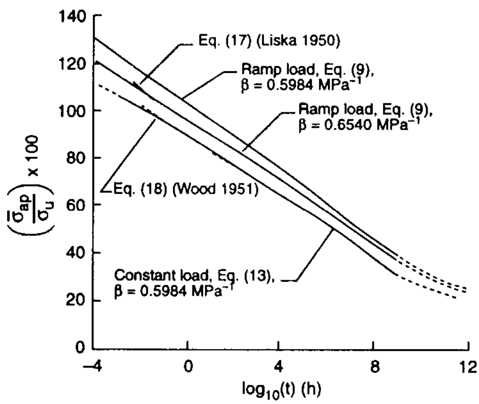

FIG. 3. Variations of Stress Level at Failure with Logarithm of Time for Constant and Ramp Loading

Results and Discussion

Using the obtained parameter values, we compare results from Eq. (9) with those from Eq. (17) in Fig. 3. The comparison clearly shows that Eq. (9) tends to overestimate the rupture load based on Eq. (17). In the literature (e.g., Liu et al. 1994), it has been reported that the stress coefficient ß is dependent on the loading condition. By setting ß = 0.6540 MPa^{-1}, we see that Eq. (9) can give a very good fit in Fig. 3.

We note Eq. (9) can conveniently be put in the form

$$\alpha - E_a K_1 \sinh(\beta \overline{\sigma}_{ef}) = \phi \qquad (19)$$

With the same input, results obtained from Eq. (19) with ϕ approaching 0 from the negative side are essentially the same as those obtained from Eq. (9).

Conclusions

The Reiner-Weissenberg strength theory in association with an Eyring's three-element model possesses the peculiar features of predicting (1) upper stress limit, at which material rupture occurs immediately, and (2) lower stress limit, at which material rupture will never occur. The strength model system provides more reliable estimates of the so-called short-term strength and the allowable design load in any current design practice.

Appendix 1. References

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