

## **Determination and use of moisture diffusion coefficient to characterize drying of northern red oak (*Quercus rubra*) \***

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**Summary.** Diffusion analysis can be used to estimate the time required to dry lumber. However, more accurate calculations require additional information on the relationship between the diffusion coefficient as a function of moisture content and on the effects of temperature, equilibrium moisture content, board thickness, and air velocity on drying time and moisture gradients. The primary objectives of this study were (1) to determine the diffusion coefficient of northern red oak (*Quercus rubra*) as a function of moisture content and (2) to compare experimentally determined sorption times and moisture content gradients with those calculated by the diffusion model. The diffusion coefficient was found to increase approximately exponentially with moisture content over a range of 6 to 30 percent at 43 °C. This relationship was similar in both adsorption and desorption tests. Experimental adsorption and desorption times and desorption moisture content gradients were compared with finite difference solutions to the diffusion equation. Practical uses of solutions to the diffusion equation are illustrated for kiln drying wood that has first been predried to below the fiber saturation point. Drying time is also related to variable kiln conditions and board thickness.

### **Introduction**

Estimates of the time required to dry lumber and the effect of material and process variables on drying time are valuable information for kiln operators. There are several approaches to mathematical models of drying for these estimates. The diffusion approach offers one generalized way that works well for boards with moisture content below the fiber saturation point. This approach also allows estimation of moisture content gradients at any time during drying as well as estimation of the effects of temperature, equilibrium moisture content conditions, board thickness, and air velocity on both drying time and moisture gradients. However, not all the information and relationships necessary to make these calculations have been determined. The general objectives of this report are to provide some of this information and to illustrate how it can be used.

The primary objectives of our research were (1) to determine the diffusion coefficient of northern red oak (*Quercus rubra*) as a function of moisture content and (2) to compare experimentally determined sorption times and moisture content gradients

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with those calculated by the diffusion model. Further objectives were to illustrate how the diffusion model can describe kiln drying after predrying to below the fiber saturation point and how sensitive the model predictions are to variations in kiln conditions and board characteristics.

### Analytical methods

The analytical methods were described by Simpson and Liu (1991). In brief, the method requires a series of adsorption or desorption experiments from the same initial moisture content to a series of different final moisture contents. This information is used with a finite difference solution to Fick's diffusion equation

$$\partial m / \partial t = \partial (D \partial m / \partial x) / \partial x \quad (1)$$

where  $m$  is moisture content,  $t$  time,  $D$  diffusion coefficient, and  $x$  thickness direction. The final result is an expression relating the diffusion coefficient to moisture content. The finite difference solution also includes variable boundary conditions so that the effect of surface equilibrium moisture content can be examined. These variable boundary conditions also have the potential to include the effect of air velocity on sorption time and moisture gradients. With the exception of work by Rosen (1978) little has been done to include the effect of air velocity in the boundary conditions.

The diffusion coefficient describes the rate at which water moves from the interior to the surface, and the surface emission coefficient  $S$  describes the rate at which water evaporates from the surface. The ratio of the surface emission coefficient to the diffusion coefficient is included in the boundary condition for the finite difference solution. This ratio is written as

$$-\partial m / \partial x|_{x=0} = (S/D)(m_e - m_s) \quad (2)$$

where  $m_e$  is equilibrium moisture content of the surface and  $m_s$  is surface moisture content at any time.

Equation (2) shows how the EMC conditions  $m_e$  can affect calculations. If the relationship between air velocity and  $S/D$  is known, the effects of air velocity can also be included in the calculating of drying times and moisture gradients.

In the experiment and the analysis to determine the diffusion coefficient, immediate surface equilibrium is assumed (Liu, 1989). Rosen (1978) found that the effect of air velocity on drying time decreases rapidly above 3 m/s. Therefore, we assumed that  $S/D$  is very large for the air velocity used in the experiment (5 m/s).

### Experimental procedure

Northern red oak (*Quercus rubra*) was obtained in log form and machined into flat-sawn boards 76 mm wide by 25 mm thick by 900 mm long. As shown in Figure 1, a series of two end-matched test specimens were cut from these boards. One of each pair was 102 mm along the grain, and the other was 13 mm along the grain. All specimens used in the tests were equilibrated at 43 °C and 96 percent relative humidity

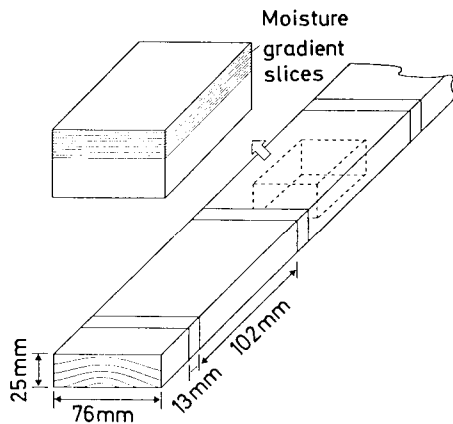


Fig. 1. Preparation of test specimen

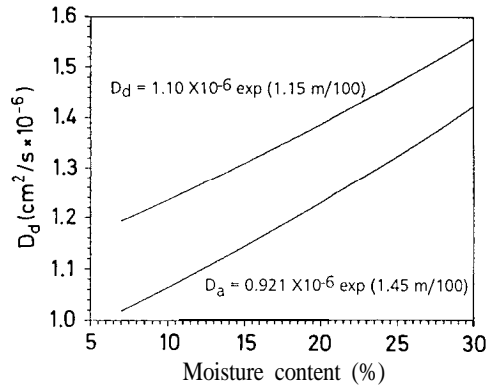


Fig. 2. Relationship between coefficient  $D$  and moisture content of red oak

(23.3 percent EMC). After equilibration, the 102-mm specimens were painted on all edges with two coats of heavily pigmented aluminum paint to restrict diffusion to the 25 mm thickness (radial direction). The 13-mm specimens were not coated and were used in the tests to determine the final EMC after adsorption or desorption. In the 13-mm specimens, the diffusion along the grain was much faster than the diffusion in the radial direction in the 102-mm specimens. The 13-mm specimens were thus used to estimate the final EMC of the 102-mm specimens rather than waiting for the longer specimens to reach equilibrium.

Two series of sorption tests were conducted. After equilibration at 43°C and 96 percent relative humidity, four sets of five specimens each were desorbed to four relative humidities at 43°C – 83, 70, 53, and 33 percent (15.9, 11.8, 8.3, and 5.5 percent EMC). Ten additional 76- by 102-mm specimens were included in this series to determine moisture content gradients; these specimens were removed one at a time periodically and sectioned as follows. A 50- by 50- by 25-mm piece was cut from the center of the specimens (Fig. 1). This piece was sliced into 10 sections up to one half the thickness of the specimen, so that each gradient section was about 1.25-mm thick. The piece was sliced into sections with a knife mounted in a heavy fly wheel. Immediately after slicing, each section was sealed in a polyethylene bag. The sections were weighed after all sections were cut.

After desorption, the four sets of five specimens were equilibrated at 30 percent relative humidity. The second series of sorption tests were adsorption from this EMC to four relative humidities at 43 °C – 53, 70, 83, and 92 percent (7.5, 10.3, 13.5, and 16.8 percent EMC). No gradient specimens were included in the adsorption tests.

The desorption and adsorption tests were done in a chamber that was attached to a generator unit for temperature and relative humidity conditioning. The chamber was equipped with an adjustable blower for air flow. A perforated plate was installed just in front of the specimens to help create uniform air flow. In these tests, the air speed was 5 m/s.

Dry bulb and wet bulb temperatures were measured with electrical resistance sensors calibrated with glass-stem thermometers calibrated to U. S. National Bureau

of Standards reference thermometers. Temperatures were recorded periodically during each test. The average dry bulb temperature was 43.4°C, with a standard deviation of 0.26°C. The average standard deviation of the relative humidity was 0.73 percent.

## Results

The moisture content dependence of the diffusion coefficient is shown in Fig. 2. The relationship was slightly different for adsorption and desorption; the values from the desorption tests were about 6 percent higher than those from the adsorption tests. The relationships were determined as described previously (Simpson and Liu, 1991):

$$D_d = 1.10 \times 10^{-6} \exp(1.15 m/100) \text{ cm}^2/\text{s} \quad (3)$$

$$D_a = 0.921 \times 10^{-6} \exp(1.45 m/100) \text{ cm}^2/\text{s} \quad (4)$$

where  $D_d$  and  $D_a$  are the diffusion coefficients from desorption and adsorption, respectively, and  $m$  is moisture content (percent).

Figures 3 and 4 show the average moisture content-time curves for desorption and adsorption. The figures show both experimental data points and curves calculated by the diffusion model. The average deviation between experimental moisture contents and moisture contents calculated for the same time was 0.5 and 0.2 percent for desorption and adsorption, respectively. The worst deviations were 1.3 and 0.5 percent.

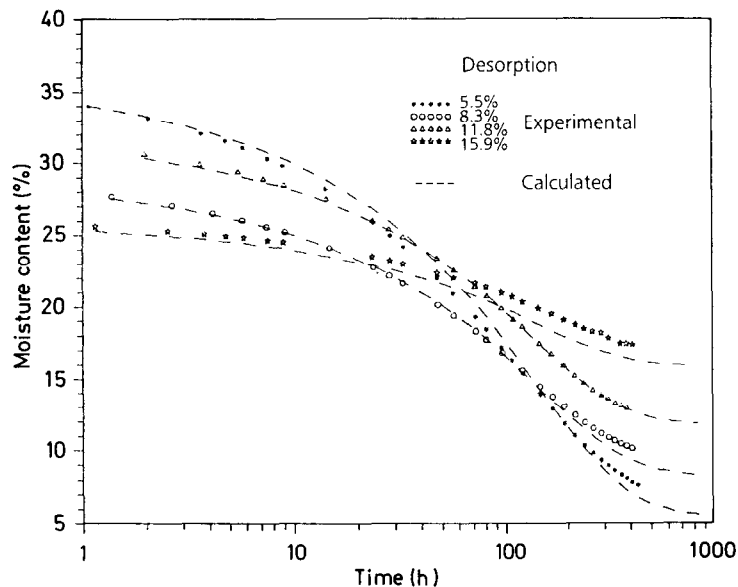


Fig. 3. Experimental and calculated drying of red oak

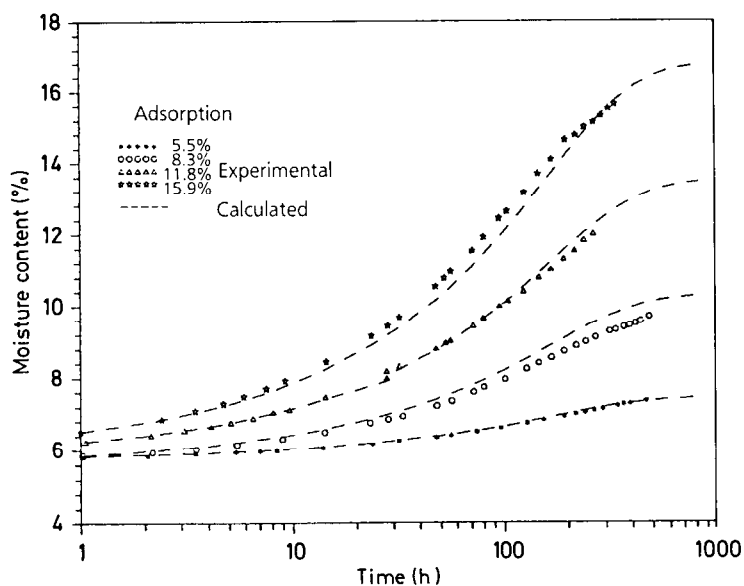


Fig. 4. Experimental and calculated adsorption times of red oak

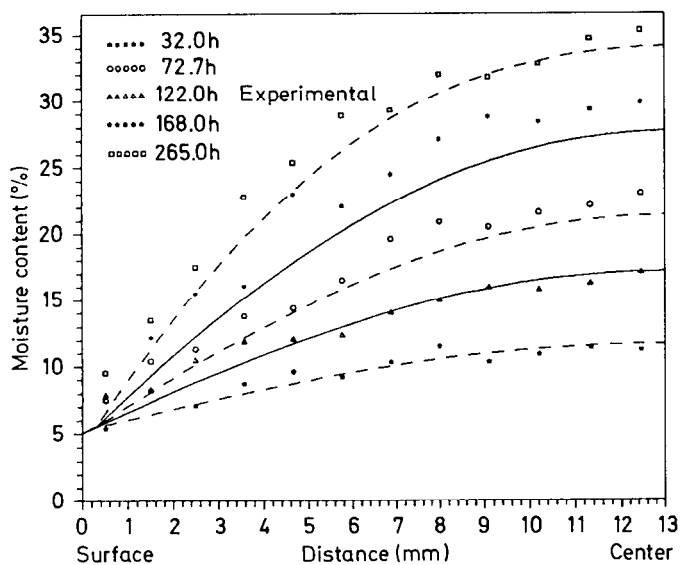


Fig. 5. Experimental and calculated (continuous curves) moisture content gradients of red oak dried at 5.5 percent EMD

The experimentally determined moisture contents of the moisture gradient sections and the calculated moisture content gradients are shown in Fig. 5-8 for the four levels of EMC. There is general agreement among the curves, but some deviations are large. The worst deviation was 5.6 percent moisture content, and the average deviation of all comparisons was 1.1 percent.

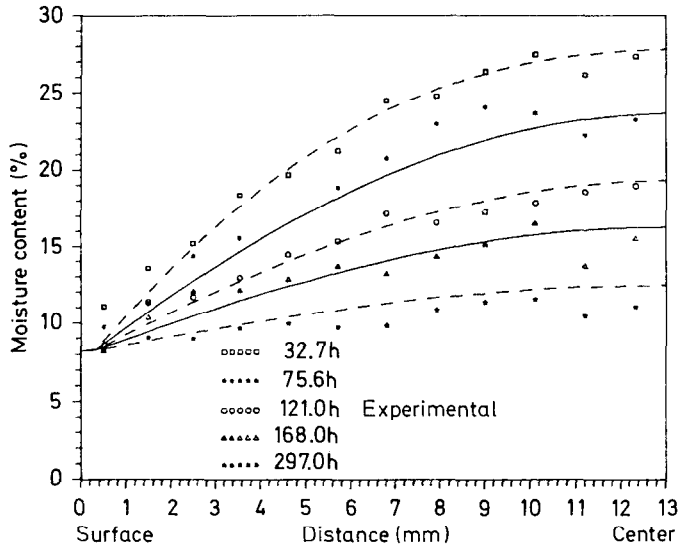


Fig. 6. Experimental and calculated moisture content gradients of red oak dried at 8.3 percent EMC

### Practical uses of diffusion analysis

#### *Simulated kiln schedule*

Diffusion analysis has had limited application to wood drying, mainly because moisture movement above the fiber saturation point is a combination of diffusion driven by a partial pressure gradient and mass flow driven by a total pressure gradient. However, diffusion analysis can be used when wood has first been air- or pre-dried to below the fiber saturation point. In the following illustration, we will analyze kiln drying 29-mm-thick red oak from approx. 25 percent to 6 percent average moisture content by the kiln schedule in Table 1.

This analysis requires knowledge about the temperature dependence of the diffusion coefficient. Skaar (1958), Stamm (1964), and Choong (1965) showed that this temperature dependence can be expressed in the form

$$D = A \exp(B/T) \quad (5)$$

where  $A$  and  $B$  are coefficients and  $T$  is temperature in kelvin.

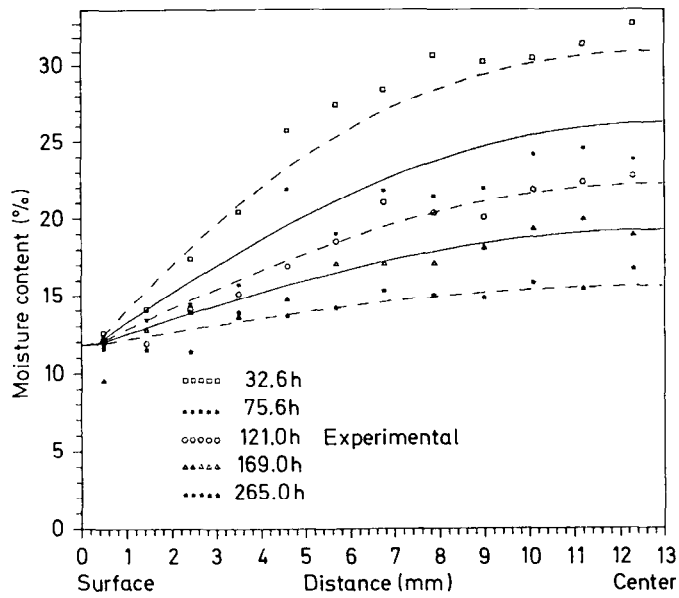
Stamm's review (1964) showed that the coefficient  $B$  of Eq. (5) is similar for many species and is approx. 5,280. Skaar (1958) and Choong (1965) also showed that the combined effects of temperature and moisture content on  $D$  can be described by

$$D = A \exp(B/T) \exp(C m/100) \quad (6)$$

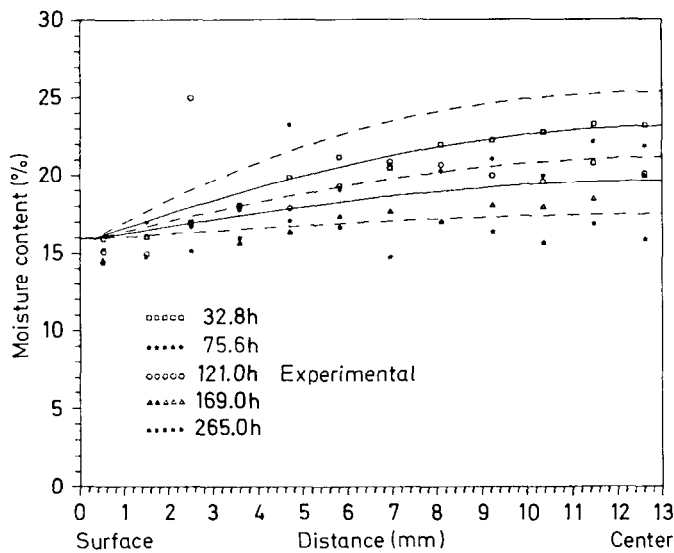
where  $C$  is another coefficient. Knowing  $D$  at 43 °C, we can solve Eq. (6) for  $A$  and arrive at the following equations for  $D$ :

$$D_d = 19.2 \exp(-5,280/T) \exp(1.15 m/100) \quad (7)$$

$$D_a = 16.0 \exp(-5,280/T) \exp(1.45 m/100) \quad (8)$$



**Fig. 7.** Experimental and calculated moisture content gradients of red oak dried at 11.8 percent EMC



**Fig. 8.** Experimental and calculated moisture gradients of red oak dried at 15.9 percent EMC

The diffusion analysis assumes isothermal conditions, which do not apply because of the temperature changes in the kiln schedule. However, when drying is as slow as diffusion-controlled drying in oak, heat flow is much faster than moisture flow and the assumption of isothermal conditions within each schedule step is a valid approximation. For example, using the heating time analysis methods described by Kollman and Côté (1968) and Gu and Garrahan (1984) the estimated time required for the

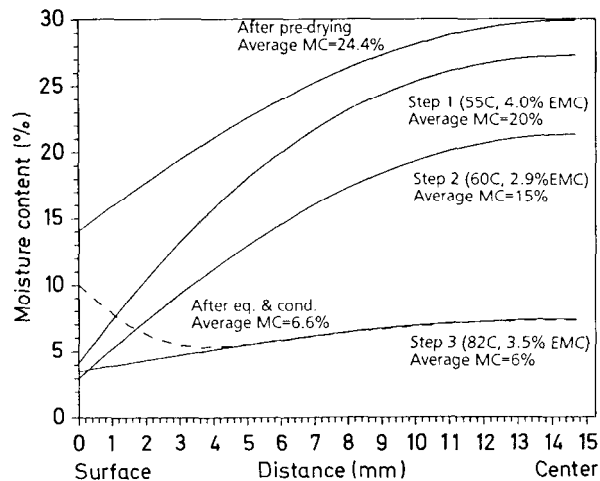
**Table 1.** Kiln schedules for northern red oak (*Quercus rubra*)

Specimen thickness (mm)	Moisture content (percent)	Dry-bulb temperature (°C)	Wet-bulb temperature (°C)	EMC (percent)
29–43	25–20	55	32	4.0
	20–15	60	32	2.9
	15–6	82	55	3.5
	(Equalize)	82	55	3.5
	(Condition)	82	76	10.1
57	25–20	55	32	4.0
	20–15	60	32	2.9
	15–6	71	44	3.4
	(Equalize)	71	44	3.4
70–83	25–20	55	41	6.7
	20–15	60	32	2.9
	15–6	71	44	3.4
	(Equalize)	71	44	3.4

center of the boards to heat to within 1 °C of the schedule temperatures ranges from about 20 to 40 min, which is < 2 percent of the drying time for the schedule steps.

The results of the kiln schedule stimulation are shown in the moisture content plots of Fig. 9. A gradient typical of what might be expected after pre-drying was assumed, which is 14 percent EMC at the surface, 30 percent moisture content in the center, and an average moisture content of about 25 percent. This gradient was the starting point for the diffusion analysis. Figure 9 shows the gradients after each of the three schedule steps and after conditioning. If an equalizing period is included to dry wet boards (assumed to be at 10 percent moisture content when the average moisture content was 6 percent), the total time for drying from 24.4 percent, equalizing, and conditioning is 5.6 days.

One factor not considered in the kiln schedule simulation was the effect of the width of lumber stacks. The diffusion coefficients were determined in an experiment where air flow was across only one board. In a kiln, the air flow is across a stack of lumber that may be  $\geq 3$  m wide. Depending on air velocity and the rate of surface evaporation from the wood, the EMC conditions across the stack width may increase because of water evaporated into the air stream. For example, if the EMC above the first board is 4 percent, the EMC may be higher further downstream in the stack. Where the EMC is higher, drying is slower. Hardwood drying is usually scheduled by the progress of sample boards located on the edges of the stacks, and thus the moisture content of the slower drying center boards is not used for kiln control. However, air flow direction is usually reversed, so any edge sample board is on the entering-air-side of the stack only one-half the time. Consequently, we might expect that the 5.6-day estimate of the diffusion analysis is shorter than what would occur in a kiln. This error, however, does not seriously affect the ability of the model to perform sensitivity analyses when the observations in comparisons are affected the same by the error. Furthermore, with psychrometric relationships and the variable boundary conditions of Eq. (2), the effect of stack width can be included in the model.



**Fig. 9.** Calculated moisture content gradients at several stages during simulated kiln drying of 29-mm-thick red oak

#### *Drying time and S/D ratio*

Equation (2) describes the boundary conditions through which the S/D ratio affects drying time. This effect is illustrated in Fig. 10. The drying plus equalizing time varies from 5.3 days at high S/D to about 10 days at low S/D. If complete information on how wood and process variables affect S/D were available, this would provide a useful tool for modeling drying below the fiber saturation point. Rosen (1978) found that S/D of black walnut (*Juglans nigra*) increased with increasing air velocity. His results can be represented by

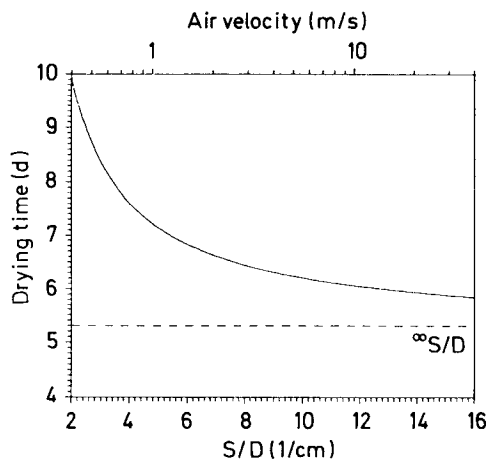
$$S/D = 4.66 + 3.12 \ln V \text{ cm}^{-1} \quad (9)$$

where V is air velocity in m/s.

The top horizontal axis in Fig. 10 is air velocity as related to S/D by Eq. (9). The figure illustrates how air velocity could be related to drying time if we knew the relationship between S/D and air velocity for red oak. We do not know if the relationships are the same for walnut and oak, so Fig. 10 should be interpreted accordingly.

#### *Drying time and variable kiln conditions*

Control of dry- and wet-bulb temperatures in a dry kiln is seldom perfect. Diffusion analysis offers a way to estimate the effect of poor control on drying and equalizing time. As an example, assume that the dry- and wet-bulb temperatures in the kiln schedule for 29-mm-thick oak can vary by  $\pm 2^\circ\text{C}$ . This causes both the temperature and EMC, and thus the drying time, to vary. This example of poor control consists of nine combinations of kiln conditions; the results are shown in Table 2. Drying times range from 4.4 days with a high dry-bulb temperature and a low wet-bulb tempera-



**Fig. 10.** Relationship between drying time and ratio of surface emission coefficient to diffusion coefficient ( $S/D$ )

**Table 2.** Sensitivity of kiln drying time to variations in dry- and wet-bulb temperatures<sup>a</sup>

Wet-bulb temperature	Dry-bulb temperature (days)		
	Normal	High	Low
Normal	5.3	4.6	6.4
High	5.9	5.0	7.1
Low	5.0	4.4	5.9

<sup>a</sup> Normal: dry- and wet-bulb temperature as designated in Table 1; High: temperatures 2°C higher than those in Table 1; Low: temperatures 2°C lower than those in Table 1

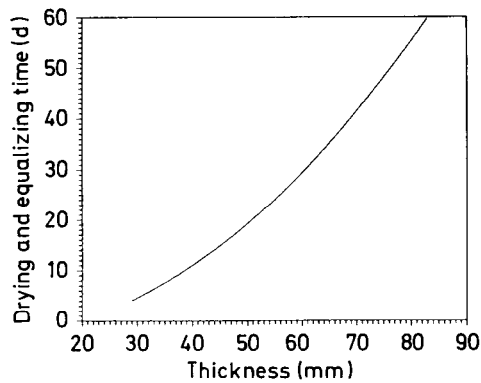
**Table 3.** Effect of thickness on drying plus equalizing time

Specimen thickness (mm)	Drying and equalizing time (days)
29	5.3
37	8.4
43	11.4
51	27.4
70	42.1
83	59.1

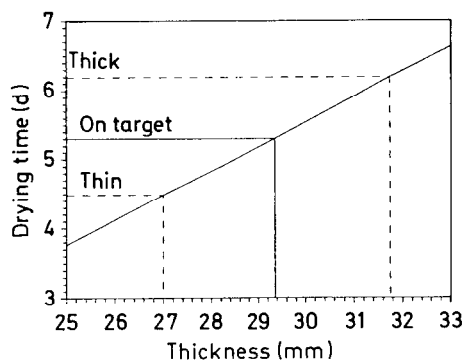
ture, to 5.3 days with both temperatures on set point, to 7.1 days with a low dry-bulb temperature and a high wet-bulb temperature. If the relationship between  $S/D$  and air velocity were known for red oak, variable air velocity could be added to the estimates in Table 2.

#### *Drying time and board thickness*

Diffusion analysis can also be used to estimate the effect of board thickness on drying and equalizing time (Table 3). As shown in Fig. 11, the times range from 5.3 days for 29-mm-thick boards to 59 days for 83-mm-thick boards.



**Fig. 11.** Effect of thickness on drying time of 29-mm-thick red oak



**Fig. 12.** Effect of thickness variation in sawing on drying time of 29-mm-thick red oak

We can also estimate the effect of thickness variations in sawing on drying and equalizing time, as illustrated in Fig. 12. Boards at the exact target thickness of 29.4 mm require 5.3 days for drying and equalizing. Boards below target thickness, for example 27-mm-thick boards, require only 4.5 days, and oversize boards (31.8-mm-thick) require 6.2 days.

### Concluding remarks

The experimentation and analysis reported here resulted in a relationship between the diffusion coefficient of northern red oak (*Quercus rubra*) and moisture content in the range of 5 to 30 percent. This relationship can be characterized by an exponential function. When the relationship is used with a finite difference solution to the diffusion equation with variable boundary conditions, there is reasonably good agreement between experimental data and calculated results.

The practical value of the results is that they illustrate the potential for modeling kiln drying of wood that has first been predried to below the fiber saturation point. The relationship between the diffusion coefficient and moisture content is just one of the necessary items of information needed for a complete modeling system. Additional information that requires research is further confirmation of the relationship between the diffusion coefficient and temperature, the dependence of the diffusion

coefficient on species or specific gravity, and the effect of growth-ring orientation of the diffusion coefficient. Information is also needed on how air velocity, temperature, moisture content, and species affect the ratio of the surface emission coefficient to diffusion coefficient. Some estimate of the statistical variability of diffusion coefficients and surface emission coefficients would also be helpful. This information will lead to a conceptually valid model for drying wood with moisture content below the fiber saturation point.

## References

- Avramidis, S. T.; Siau, J. F. 1987: Investigation of the external and internal resistance to moisture diffusion in wood. *Wood Sci. Technol.* 21:249–256
- Choong, E. T. 1965: Diffusion coefficients of softwoods by steady-state and theoretical methods. *Forest Prod. J.* 15(1):21–27
- Choong, E. T.; Skaar, C. 1972: Diffusivity and surface emissivity in wood drying. *Wood Fiber* 4(2):80–86
- Gu, L. B.; Garrahan, P. 1984: The temperature and moisture content in lumber during preheating and drying. *Wood Sci. Technol.* 18:121–135
- Kollman, F. F. P.; Côté, W. A. 1968: *Principles of Wood Sciences and Technology*. New York: Springer-Verlag
- Liu, J. Y. 1989: A new method for separating diffusion coefficient and surface emission coefficient. *Wood Fiber Sci.* 21(2):133–141
- Rosen, H. N. 1978: The influence of external resistance on moisture adsorption rates in wood. *Wood Fiber* 10(3):218–228
- Simpson, W. T.; Liu, J. Y. 1991: Dependence of the water vapor diffusion coefficient of aspen (*Populus spp.*) wood on moisture content. *Wood Sci. Technol.* 26(1):9–22
- Skaar, C. 1958: Moisture movement in beech below the fiber saturation point. *Forest Prod. J.* 8(12):352–357
- Stamm, A. J. 1964: *Wood and Cellulose Science*. New York: The Ronald Press Company

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