Design Equation for Multiple-Fastener Wood Connections

By John J. Zahn,[1] Member, ASCE

Abstract: A compared design equation is presented for the design of multiple fastener connections of wood members. It was obtained by algebraic simplification of the Lantos analysis of the unequal load sharing among fasteners in series. This equation can replace the double-entry tables now used in wood design codes. Those tables are constructed using the Lantos analysis and for simplicity have to ignore certain effects and cover only a limited range of design parameters. In addition to the specification equation, two design aids are derived: (1) A simple expression for the maximum possible capacity of a serial row of fasteners: and (2) an expression for the number of fasteners required to achieve a given row capacity.

Introduction

There is a movement toward in all-equation format for the National Design Specification (NDS) for wood. Because computers are now commonplace, design by reference to tables is not necessarily simple, especially if multiple-entry tables are required. Therefore, I suggest replacing the tables of modification factors for multiple-fastener connections that currently appear in the NDS (*National Design* 1986) with a design equation.

The NDS tables provide reduction factors to account for the fact that fasteners arranged in a serial row do not share load equally. A serial row is one aligned parallel to the direction of the applied load (Fig. 1). In such an arrangement, the outermost fasteners carry the most load while the innermost ones carry little or no load. The NDS tables are based upon an elementary analysis by Lantos (1969), in which he assumed that the normal stresses in the joined members are uniformly distributed on their cross sections and that a linear relationship exists between fastener load and fastener deformation. Wilkinson (1980) assessed all available analyses arid concluded that the Lantos analysis differed from other more sophisticated analyses by far less than the difference between theory arid experiment (Fig. 2). Most of the experimental discrepancy is attributable to inaccurate hole alignment during joint manufacture. Therefore, Wilkinson endorsed the use of the Lantos analysis as the basis for design, its in the current NDS tables. A more accurate and convenient method for computerized design would be to present the design specification in the form of an equation. Unfortunately. Lantos did not reduce his analysis to a single equation.

Such a reduction is possible, however, and the result of the reduction is shown in this paper in a form suitable for use in a code specification. Furthermore, it is a simple matter to take the inverse, that is. to solve for the required number of fasteners to meet a given load capacity. This greatly simplifies the design process, which is currently a "cut arid try" procedure using double-entry table interpolation.

---

[1]Engr., USDA Forest Service, Forest Products Lab.,. One Gifford Pinchot Drive, Madison, WI 53705-2398.

Note. Discussion open until April 1, 1992. To extend the closing date one month. a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted lor review and possible publication on September 25, 1990. This paper is part of the ***Journal of Structural Engineering,*** Vol. 117. No. 11, November, 1991.

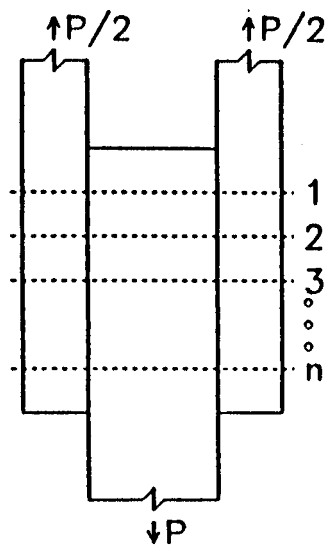

FIG. 1. Three-Member Connection with Serial Row of Fasteners. Each Dashed Line Represents Fastener

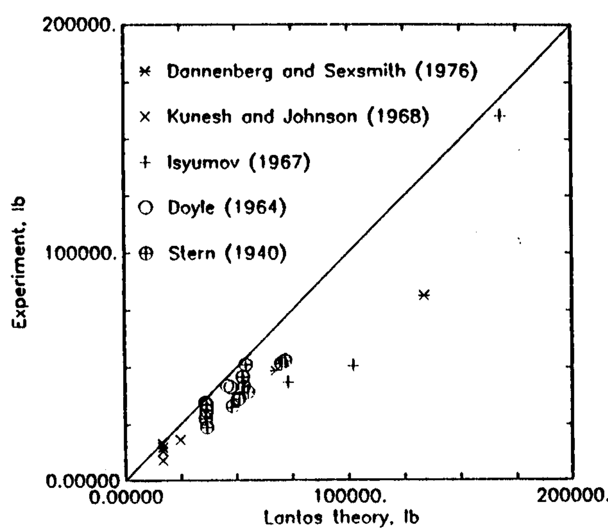

FIG. 2. Comparison of Experimental Connection Capacities with Theoretical Capacity Obtained from Lantos Theory (Axes Range from 0 to 190 kN)

LIMITATIONS OF CURRENT DESIGN FORMAT

The current design tables have limitations that partly account for the current interest in converting the design specification to an all-equation format:

1. Interpolation in a double-entry table is inconvenient. Furthermore, the table is constructed to calculate the capacity of a given row of fasteners. To solve for the required number of fasteners given the desired capacity, the user must iteratively interpolate by trial. It would be much easier to solve for the number of fasteners if the specification expressed the capacity as an invertible function of the number of fasteners. Unfortunately, Lantos left his result in a form that could not be easily inverted.
2. The tables cover a limited range of member sizes and number of fasteners, making it impossible to design outside this range.
3. The tables ignore two important effects: (1) Stiffening the fastener with shear plates or split rings; and (2) reducing member stiffness in cases where the fastener bearing load is perpendicular to grain.

In regard to the third limitation, the table constructors assumed that the modulus of elasticity of wood is 1.8×10^6 lb/sin.[8] (12.4 GPa) and that the load/slip constant of a single fastener is 220.000 lb/in. (38.5 N/m). For most bolts and lag screws. this value is conservative; however, when shear plates or split rings are employed, the value is *un*conservative by roughly a factor of two. Likewise, the modulus of elasticity is *un*conservative by a factor of approximately 20 whenever the fasteners bear upon the wood in a direction perpendicular to the grain. To keep the tables compact, the effect of these two variables had to be suppressed, leaving a large gap in the design criteria. With no guidance in the specification and no proscription against doing so, designers are probably using the current tables to design connections with bearing perpendicular to grain, or with timber connectors, or both. If for no other reason than to remedy this important omission, a conversion to equation format is highly desirable.

However, it is not an easy task to write a “simple” equation for the capacity in terms of all the relevant variables, namely the moduli of elasticity of the members, cross-sectional areas, load/slip constant, number of fasteners, and fastener spacing. The Lantos analysis offers the greatest hope of achieving such an equation.

Reduction of Lantos Analysis to A Single Equation

The connection geometry studied by Lantos consists of a main member and two side members, as shown in Fig. 1. Each fastener is loaded in double shear. The fasteners are numbered from 1 to *n*, with fastener 1 being closest to the free end of the main member. Lantos reported his result in the following form:

$$P_{f1} = C_1 P \quad (1)$$

$$P_{fn} = C_2 P \quad (2)$$

in which P = load applied to connection: P_{f1} = load carried by fastener number 1, and P_{fn} = load carried by fastener number *n*. Parameters C_1 and C_2 are given as follows:

$$C_1 = 1 - m_1(1 + \mu) + \mu + (m_1 - m_2)\frac{m_1''(1 + \mu) - \mu}{m_1'' - m_2''} \quad (3)$$

$$C_2 = -\mu + m_1^{n-1}(1 + \mu) - (m_1^{n-1} - m_2^{n-1}) \frac{m_1^n(1 + \mu) - \mu}{m_1^n - m_2^n} \quad \ldots\ldots (4)$$

in which

$$\mu \equiv \frac{-1}{1 + \dfrac{(EA)_{\text{main}}}{(EA)_{\text{sides}}}} \quad \ldots\ldots (5)$$

$$m_1 \equiv \frac{\omega + \sqrt{\omega^2 - 4}}{2} \quad \ldots\ldots (6)$$

$$m_2 \equiv \frac{\omega - \sqrt{\omega^2 - 4}}{2} \quad \ldots\ldots (7)$$

$$\omega \equiv 2 + \gamma S\left[\frac{1}{(EA)_{\text{main}}} + \frac{1}{(EA)_{\text{sides}}}\right] \quad \ldots\ldots (8)$$

and $(EA)_{\text{main}}$ = axial stiffness of main member (modulus of elasticity multiplied by cross-sectional area): $(EA)_{\text{sides}}$ = axial stiffness of side members (modulus of elasticity multiplied by sum of cross-sectional areas); g = load/slip constant of single fastener; and S = fastener spacing.

Because of unequal load sharing, the largest fastener load will be either P_{f1} or P_{fn}. The user is instructed to calculate both and take the larger of the two.

Result of Algebraic Reduction

First note that $m_1 m_2 \equiv 1$. Therefore, define $m \equiv m_2$ and write $1/m$ for m_1. This ensures that $m < 1$ and that $m^n \to 0$ as $n \to \infty$, a limit that will prove useful later.

Further, to keep the notation compact, define t ≡ w/2 so that

$$m \equiv \tau - \sqrt{\tau^2 - 1} \quad \ldots\ldots (9)$$

and define r_1 ° $(EA)_{\text{side}}/(EA)_{\text{main}}$ so that

$$\mu \equiv \frac{-r_1}{1 + r_1} \quad \ldots\ldots (10)$$

Then, regarding C_1 and C_2 as functions of r_1, it can be shown algebraically that

$$C_2(r_1) \equiv C_1\left(\frac{1}{r_1}\right) \quad \ldots\ldots (11)$$

so that C_1 governs when $r_1 < 1$ and C_2 governs when $r_1 > 1$. Therefore, we may make it a rule to always use C_1 and to replace r_1 with its reciprocal whenever $r_1 > 1$. In fact, let us define

$$r \equiv \min\left[\frac{(EA)_{\text{main}}}{(EA)_{\text{sides}}}, \frac{(EA)_{\text{sides}}}{(EA)_{\text{main}}}\right] \quad \ldots\ldots (12)$$

and write

$$P_{f,\max} = CP \quad (13)$$

$$C = \frac{1 - m}{1 + r}\left[\frac{(1 + rm^n)(1 + m) - 1 + m^{2n}}{m(1 - m^{2n})}\right] \quad (14)$$

where C was obtained from C_1 by substituting for μ from (10) and dropping the subscript on r. This completes the reduction to a single equation.

For purposes of code writing. we are more interested in the reciprocal of C, which may be called the "effective number of fasteners," a. Then the row capacity would be a times the capacity of a single fastener. Let us therefore define the effective number of fasteners a as the reciprocal of C and write

$$P = aP_{f,\max} \quad (15)$$

in which

$$a = \frac{1 + r}{1 - m}\left[\frac{m(1 - m^{2n})}{(1 + rm^n)(1 + m) - 1 + m^{2n}}\right] \quad (16)$$

Limiting Capacity of a Serial Row

If there were equal load sharing, a would be simply n. As it is, the effective number of fasteners a is always a number between 1 and n. It is interesting to note that a does not increase indefinitely as the number of fasteners is increased. Instead. it approaches an upper limit that cannot be exceeded regardless of how many fasteners are employed (Fig. 3). As $n \rightarrow \infty$, $a \rightarrow a_\infty$:

$$a_\infty = \frac{1 + r}{1 - m} \quad (17)$$

Eq. (17) is obtained from (16) with $m^n = 0$. This quantity is useful to the designer, as it helps in deciding the number of serial rows required to carry a given load. As a practical limit, $0.8a_\infty$ is more realistic. Beyond that. a will increase by less than one-third when n is increased by one; that is, an additional fastener will only add less than one-third of its capacity to the total capacity of the row.

Inverse Function

Having obtained a single expression for the effective number of fasteners a as a function of the number of fasteners n, it is now possible to solve for the inverse function. Solving (16) for n yields

$$n = \frac{\ln[\sqrt{1 - 2Q + (rQ)^2} - rQ]}{\ln m} \quad (18)$$

in which

$$Q \equiv \frac{ma + a}{2(ma_\infty + a)} \quad (19)$$

and in which it is understood that a must be less than a_∞.

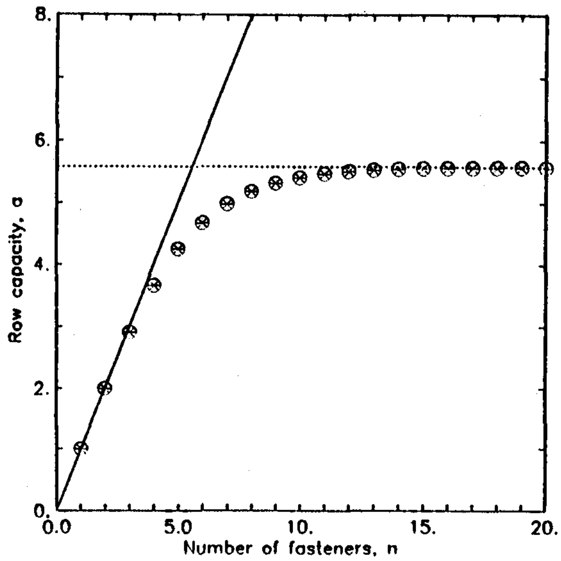

FIG. 3. Increase of Effective Number of Fasteners a with Increase in Number of Fasteners *n*. If Fasteners Could Share Load Equally, a Would Equal *n*, as Shown by Solid Line. Instead a Approaches Upper Limit a_∞ (Dashed Line)

Eq.(18) does not, of course, produce an integer result. The required number of fasteners would be the smallest whole number that exceeds the calculated *"n."*

RESULTS

By algebraically reducing the Lantos analysis to a single equation, we have achieved a suitable criterion for inclusion in a design specification. A "row modification factor" *K* could be defined as in the current NDS. In that case, *K* would simply be *a/n*. Better yet, we could define a "multiple fastener modification factor" *K* as follows:

$$K = \frac{1}{n_t}\sum_{i=1}^{n_r} a_i \qquad (20)$$

in which n_r = number of parallel serial rows that make up the connection; i = row number; and n_t = total number of fasteners in the connection. Here, each effective number of fasteners a_i would be calculated from (16) using the number of fasteners n_i in that row. Then the total connection capacity *P* would be simply the sum of the row capacities. Therefore

$$P = n_t K F \qquad (21)$$

in which F = load capacity of a single fastener. Eqs. (17), (18), and (19) could also be provided as design aids in the commentary.

The specification for a_i would be

$$a_i = \frac{1 + r}{1 - m} \frac{m(1 - m^{2n_i})}{(1 + rm^{n_i})(1 + m) - 1 + m^{2n_i}}$$

in which

$$m = \tau - \sqrt{\tau^2 - 1}$$

and t = 1 + [1/$(EA)_{main}$ + l/$(EA)_{sides}$] gS/2: g = load/slip constant for a single fastener; S = pitch spacing; $(EA)_{main}$ = axial stiffness (modulus of elasticity of main member multiplied by gross cross-sectiona1 area before boring or grooving); $(EA)_{sides}$ = axial stiffness (modulus of elasticity of side member multiplied by sum of gross cross-sectional areas before boring or moving; and $r \leq 1$ = ratio of smaller to larger axial stiffness value.

The formula contains a formidable number of parameters. On the other hand, it permits a wide range of connection designs. An examination of the references listed at the end of this paper reveals something about the values that should be used for these parameters. Wilkinson (1980) searched bolt and timber connector data from the Forest Products Laboratory and presented typical results in several figures and tables. Also, the references cited in Fig. 2 (Dannenberg and Sexsmith 1976: Kunesh and Johnson 1968; Isyumov 1967; Doyle 1964; Stern 1940) each contain values of the load/slip constant of single fasteners.

On the basis of the data in these references, g is apparently independent of bolt length and wood species. Conservative values of y are 2×10^5 lb/in. (35 MN/m) for bolts, 4×10^5 lb/in. (70 MN/m) for small timber connectors [diameter < 4 in. (0.1 m)], and 5×10^5 lb/in. (88 MN/m) for large timber connectors [diameter ≥ 4 in. (0.1 m)]. These values are for wood side plates. The use of metal side plates increases g by 1.5 but has no effect when timber connectors are employed.

More recent data (Soltis et al. 1986) for bolted connections in Douglas fir seem to indicate a dependence of y on bolt diameter (Fig. 4). A simple formula for this dependence is

$$\gamma = 180{,}000 d^{1.5} \quad \text{(22)}$$

for bearing parallel to grain. Bearing perpendicular to grain appears to cut g approximately in half. Note that each point on Fig. 4 is the average of 15 tests, making this the largest data set available at this time. Unpublished data (Wilkinson, unpublished report) for bolted connections with steel sideplates show a similar dependence on bolt diameter (Fig. 5). Each point on Fig. 5 is the average of 20 tests. Comparison with the data on Fig. 4 upholds the conclusion that steel sideplates increase y by a factor of 1.5 and bearing perpendicular to grain decreases g by one-half.

The code could prescribe conservative values of y and mandate their use unless more accurate supporting data were available. (The current NDS tables were constructed assuming that g = 2.2×10^5 lb/in. for wood sideplates and 3.3×10^5 lb/in. for metal sideplates. There is no provision for the use of timber connectors or for bearing perpendicular to grain.)

The values of the moduli of elasticity should be those of the materials actually employed, except that the modulus of elasticity of a wood member shall be divided by 20.0 whenever bearing is perpendicular to the grain. This is an important omission in the current NDS. The value 20.0 is supported by data from the "Wood Handbook" (1987).

Such a specification would have the following advantages:

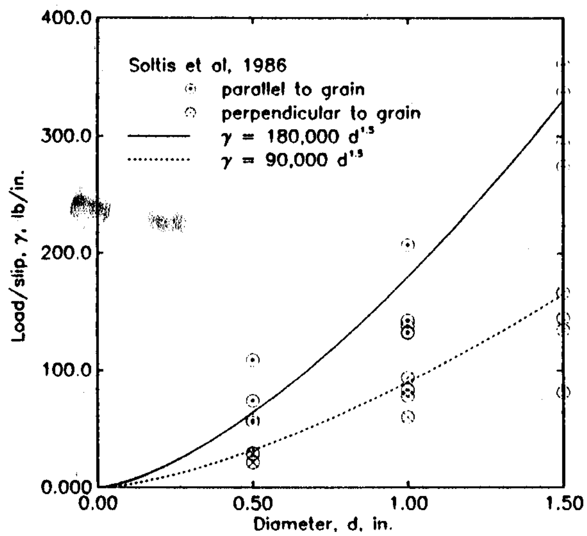

FIG. 4. Dependence of Load/Slip Constant g on Bolt Diameter. Connections with Wood Sideplates (Axis for Load/Slip Ranges from 0 to 7 × 10^4 N/m; Axis for Diameter Ranges from 0 to 3.81 cm)

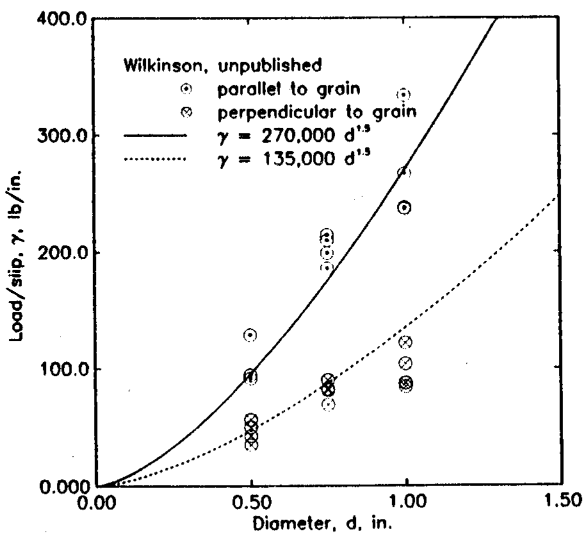

FIG. 5. Dependence of Load/Slip Constant g on Bolt Diameter. Connections with Steel Sideplates (Axis for Load/Silp Ranges from 0 to 7 × 10^4 N/m; Axis for Diameter Ranges from 0 to 3.81 cm)

1. The specification fills the gap in the current specification regarding timber connectors and bearing perpendicular to grain.

2. It is unlimited in the range of parameter values that may he employed.

3. It is compact in form, is easily to programmable, and can be inverted in closed form thereby eliminating the need for iterative interpolation in a double-entry tabe. Furthermore, it provides a simple formula for a_∞ the maximum possible effective number of fasteners in a serial row. With a specification of this kind, it is very easy to program a computer to design multirow connections.

ACKNOWLEDGMENTS

The Forest Products Laboratory is maintained in cooperation with the University of Wisconsin. This article was written and prepared by U.S. Government employees on official time, and is therefore in the public domain arid not subject to copyright.

APPENDIX I. REFERENCES

Dannenberg, L. J., and Sexsmith, R. G. (1976). "Shear-plate load distribution in laminated timber joints." *Report No. 361*, Dept. Struct. Engrg., School of Civ. and Envir. Engrg., Cornell Univ., Ithaca, N.Y.
Doyle, D. V. (1964). "Performance of joints with eight bolts in laminated Douglas-fir." *Res. Paper FPL 10*, USDA Forest Service. Forest Products Lab., Madison, Wisc.
Isyumov, N. (1967). "Load distribution in multiple shear plate joints in timber." *Pub. No. 1203*, Dept. of Forestry. Ottawa, Ontario, Canada.
Kunesh, R. H., and Johnson, J. W. (1968). "Strength of multiple-bolt joints: influence of spacing and other variables." *Report T-24*, Forest Research Lab., School of Forestry, Oregon State Univ., Corvallis, Oreg.
Lantos, G. (1969). "Load distribution in a row of fasteners subjected to lateral load." *Wood Sci.*, 1(3), 129–136.
National design specification for wood construction. (1986). National Forest Products Association. Washington D.C.
Soltis, L. A., Hubbard, F. K., and Wilkinson, T. L. (1986). "Bearing strength of bolted timber joints." *J. Struct. Engrg.*, ASCE, 112(9), 2141–2154.
Stern, E. G. (1940). "A study of lumber and plywood joints with metal split-ring connectors." *Bull. No. 53*, Pennsylvania State Univ., Engrg. Experiment Station, University Park, Pa.
Wilkinson, T. L. (1980). "Assessment of modification factors for a row of bolts or timber connectors." *Research Paper FPL 376*, U.S. Dept. of Agric., Forest Service, Forest Products Lab., Madison, Wisc.
Wood handbook: Wood as an engineering material. (1987). *Agricultural handbook 72*, Forest Products Lab., U.S. Dept. of Agric., Washington, D.C., 4–6.

APPENDIX II. NOTATION

The following symbols are used in this paper:

a = effective number of fasteners in a serial row [(16)]
a_i = effective number of fasteners for row i:
a_∞ = upper limit of a as n increases:
C, C_1, C_2 = Lantos parameters [(3), (4), and (14)]
$(EA)_{main}$ = axial stiffness of main member;
$(EA)_{sides}$ = axial stiffness of both side members;
F = load capacity of single fastener;

i = row number;
K = multiple fastener modification factor [(20)];
m, m_1, m_2 = Lantos parameters [(6), (7), and (9)];
n, n_i = number of fasteners in serial row;
n_r = number of serial rows in connection;
n_t = total number of fasteners in connection;
P = load applied to connection:
P_{f1} = load carried by fastener number 1;
$P_{f,max}$ = load carried by most heavily loaded fastener;
P_{fn} = load carried by fastener number *n;*
r, r_1 = axial stiffness ratio [(10) and (12)];
S = fastener spacing; and
g = load/slip constant of single fastener.