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DRYING OF POROUS MATERIALS IN A MEDIUM WITH POTENTIALS VARYING EXPONENTIALLY WITH TIME

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ABSTRACT

This paper presents an application of the Luikov system of heat and mass transfer equations to predict the temperature and moisture distributions in a slab of capillary-porous material during drying. The heat and mass transfer potentials of the external medium in the boundary conditions are assumed to vary exponentially with time. The method of solution should have a general application to this type of problem with variable boundary conditions. Numerical results based on thermophysical properties of spruce are presented.

Keywords: Boundary condition, capillary-porous material, drying rate, drying time, heat and mass transfer, heating rate, Luikov equations, lumber, moisture, temperature

NOMENCLATURE

The thermophysical parameters and dimensionless numbers used in the paper are as follows:

- a_m Moisture diffusion coefficient (m²/s)
- a_q Thermal diffusivity coefficient (m²/s)
- b Constant denoting rate of change of DBT (1/s)
- b' Constant denoting rate of change of EMTP (1/s)
- Bi_m Biot number of mass transfer $(\alpha_m \ell/\lambda_m)$
- Bi_q Biot number of heat transfer $(\alpha_q \ell/\lambda_q)$
- C_m Moisture capacity (1/°M)
- C_q Heat capacity (J/kg • K)
- Fo Fourier number; dimensionless time $(a_q\tau/\ell^2)$
- Ko Kossovich number $(\lambda C_m(\theta_0 - \theta_{p0})/C_q(t_{c0} - t_0))$
- ℓ Half the specimen thickness (m)
- Lu Lukomskii number (a_m/a_q)
- Pd'_m Predvoditelev number denoting exponential time dependence of equilibrium mass transfer potential $(Lu \cdot b'\ell^2/a_m)$
- Pd'_q Predvoditelev number denoting exponential time dependence of medium temperature $(b\ell^2/a_q)$
- Pn Posnov number $(\delta(t_{c0} - t_0)/(\theta_0 - \theta_{p0}))$
- t Temperature (K)
- T Dimensionless temperature $((t - t_0)/(t_{c0} - t_0))$
- W_m Dimensionless mass transfer potential parameter $(\theta_{p0}/(\theta_0 - \theta_{p0}))$
- W_q Dimensionless temperature parameter $(t_{c0}/(t_{c0} - t_0))$
- x Space coordinate (m)
- X Dimensionless space coordinate (x/ℓ)
- α_m Convective mass transfer coefficient (kg/m²·s·°M)
- α_q convective heat transfer coefficient (W/m²·K)
- δ Thermo-gradient coefficient (°M/K)
- ϵ Phase transformation number: ratio of vapor diffusion coefficient to coefficient of total moisture diffusion
- θ Mass transfer potential (°M)
- Θ Dimensionless mass transfer potential $((\theta_0 - \theta)/(\theta_0 - \theta_{p0}))$
- λ Heat of phase change (J/kg)
- λ_m Mass conductivity coefficient (kg/m·s·°M)
- λ_q Thermal conductivity coefficient (W/m·K)
- τ Time (s)

Subscripts

- c Surrounding medium
- m Mass transfer
- 0 Initial condition
- p Equilibrium value
- q Heat transfer

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INTRODUCTION

An application of the Luikov system of heat and mass transfer equations (Liukov, 1966) is presented to predict the temperature and moisture distributions in a slab of capillary-porous material during drying. The slab is subjected to boundary conditions of the third kind (Liukov and Mikhailov, 1965a,b), which relate the transfer potentials at the surfaces of the slab to the corresponding potentials of the external medium. The transfer potentials of the external medium are assumed to vary exponentially with time to represent the actual kinetics of the drying process (Liukov and Mikhailov, 1965a).

Analytical solutions for the Luikov system of equations with the heat transfer potential of the medium varying linearly or exponentially with time and the mass transfer potential of the medium remaining constant can be found in Luikov and Mikhailov (1965a,b). These authors used the Laplace transform technique to obtain their solutions without considering the possible existence of complex eigenvalues. As pointed our by Liu and Cheng (in press), if complex eigenvaluies do exist, their solutions can be grossly in error.

In a previous study (Liu, in press), both heat and mass transfer potentials of the medium were assumed to vary linearly with time. A numerical example mas used for the drying of lumber. in which the transfer potentials of the medium are the dry-bulb temperature (DBT) and the relative humidity of the drying air corresponding to the equilibrium mass transfer potential (EMTP) of lumber. Results showed that by increasing DBT only, the heat absorption of lumber was reduced in comparison with that in a constant drying environment, but drying time was, not reduced. However, by simultaneously increasing DBT and decreasing EMTP, the heat absorption of lumber and the drying time may both be reduced.

In this study, I extended my analysis (Liu, in press) to include the assumption that the transfer potentials of the medium vary exponentially with time. The complete solutions are composed of a homogeneous solution and a particular solution, the former being obtained by means of the same analytical technique developed previously (Liu and Cheng, in press; Liu, in press), the latter by the method of undetermined coefficients (Hildebrand, 1962). Unlike the particular solution presented previously (Liu, in press), here the solution is further separated into two parts, one associated with heat transfer potential of the medium and the other with mass transfer potential. The particular solutions are obtained by superimposing the two parts that satisfy the governing equations and the boundary conditions. This greatly simplifies an otherwise very complicated procedure to obtain the particular solutions. When the boundary conditions change, the homogeneous solutions remain the same, only the particular solutions need to be changed. The Laplace transform method (Liukov and Mikhailov, 1965a,b) cannot handle similar situations with equal ease because it can only yield the complete solutions directly. Therefore, the solution procedure presented in the study reported here is especially useful when the boundary conditions are complex or when they need to be changed in the process as in a zonal analysis (Liukov and Mikhailov, 1965b).

HEAT AND MASS TRANSFER EQUATIONS

For the one-dimensional case, as shown in Figure 1, heat and mass move along the x axis only. Under constant pressure condition, the governing equations (Liukov and Mikhailov, 1965a) are as follows:

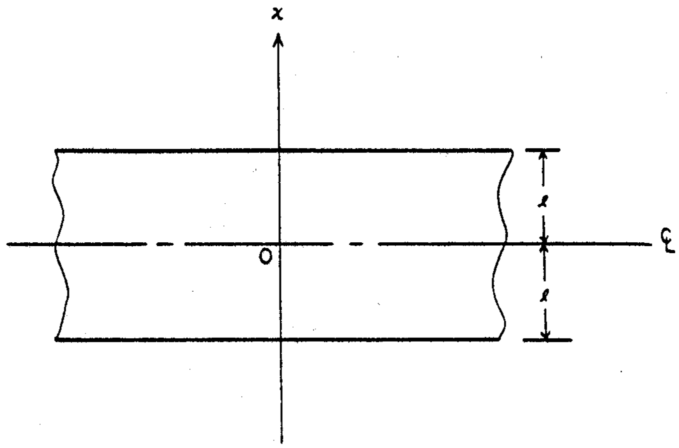

Figure 1–Schematic representation of specimen. (ML89 5580)

$$\frac{\partial T}{\partial \mathrm{Fo}} = \frac{\partial^2 T}{\partial X^2} - \epsilon \mathrm{Ko} \frac{\partial \Theta}{\partial \mathrm{Fo}} \qquad (-1 < X < 1; \mathrm{Fo} > 0) \quad (1)$$

$$\frac{\partial \Theta}{\partial \mathrm{Fo}} = \mathrm{Lu} \frac{\partial^2 \Theta}{\partial X^2} - \mathrm{LuPn} \frac{\partial^2 T}{\partial X^2} \qquad (-1 < X < 1; \mathrm{Fo} > 0) \quad (2)$$

where T is dimensionless temperature, Q dimensionless mass transfer potential, Fo dimensionless time or Fourier number, X dimensionless space coordinate, ϵ phase transformation number, Ko Kossovich number, Lu Lukomskii number, and Pn Posnov number (see Nomenclature).

Let DBT and EMTP represent heat and mass transfer potentials of the medium, respectively, for ease of presentation. For constant DBT and EMTP in the boundary conditions (Liu, in press),

$$T = \frac{t - t_0}{t_c - t_0} \quad (3)$$

$$\Theta = \frac{\theta_0 - \theta}{\theta_0 - \theta_p} \quad (4)$$

$$\mathrm{Ko} = \frac{\lambda C_m(\theta_0 - \theta_p)}{C_q(t_c - t_0)} \quad (5)$$

$$\mathrm{Pn} = \frac{\delta(t_c - t_0)}{\theta_0 - \theta_p} \quad (6)$$

where the notations on the right-hand sides are dimensional and t is temperature, t_c the DBT, t_0 initial temperature of slab, q moisture transfer potential, q_p the EMTP, q_0 initial mass transfer potential of slab, A heat of phase change, C_m and C_q are moisture capacity and heat capacity, respectively, and d is thermo-gradient coefficient. Assuming that DBT and EMTP are exponentially time dependent so that

$$t_c = t_{c0} e^{b\tau} \qquad (b \geq 0) \quad (7)$$

$$\theta_p = \theta_{p0} e^{-b'\tau} \qquad (b' \geq 0) \quad (8)$$

where t_{c0} is initial value of t_c, q_{p0} initial value of q_p, t time, and b and b' are constants denoting the rates of change of DBT and EMTP, respectively, t_c and q_p in Equations (3) to (6) must be replaced by t_{c0} and q_{p0}:

$$T = \frac{t - t_0}{t_{c0} - t_0} \tag{9}$$

$$\Theta = \frac{\theta_0 - \theta}{\theta_0 - \theta_{p0}} \tag{10}$$

$$\text{Ko} = \frac{\lambda C_m(\theta_0 - \theta_{p0})}{C_q(t_{c0} - t_0)} \tag{11}$$

$$\text{Pn} = \frac{\delta(t_{c0} - t_0)}{\theta_0 - \theta_{p0}} \tag{12}$$

Equations (9) to (12) are also true in Equations (1) and (2). The boundary conditions of the third kind based on Equations (7) and (8) are as follows:

$$\begin{aligned}\frac{\partial T}{\partial X} &+ \text{Bi}_q T - (1-\epsilon)\text{LuKoBi}_m\Theta \\ &= \text{Bi}_q(1 - W_q + W_q e^{\text{Pd}'_q\text{Fo}}) \\ &\quad - (1-\epsilon)\text{LuKoBi}_m(1 + W_m - W_m e^{-\text{Pd}'_m\text{Fo}}) \\ &\qquad (X = \pm 1; \text{Fo} > 0)\end{aligned} \tag{13}$$

$$\begin{aligned}\frac{\partial \Theta}{\partial X} - \text{Pn}\frac{\partial T}{\partial X} + \text{Bi}_m\Theta &= \text{Bi}_m(1 + W_m - W_m e^{-\text{Pd}'_m\text{Fo}}) \\ &\qquad (X = \pm 1; \text{Fo} > 0)\end{aligned} \tag{14}$$

where Bi_q and Bi_m are the Biot heat and mass transfer numbers, respectively, Pd'_q (the same as Pd' in Luikov and Mikhailov (1965a,b)) is the Predvoditelev number for the case of exponential time dependence of DBT, defined by

$$\text{Pd}'_q = \frac{b\ell^2}{a_q} \tag{15}$$

W_q, a dimensionless temperature parameter, defined by

$$W_q = \frac{t_{c0}}{t_{c0} - t_0} \tag{16}$$

Pd'_m a new dimensionless Prevoditelev number identified in the present study for the case of exponential time dependence of EMTP, defined by

$$\text{Pd}'_m = \frac{b'\ell^2}{a_m} \cdot \text{Lu} \tag{17}$$

and W_m a dimensionless mass transfer potential parameter. defined by

$$W_m = \frac{\theta_{p0}}{\theta_0 - \theta_{p0}} \tag{18}$$

In Equations (15) and (17), a_q and a_m are the thermal diffusivity coefficient and moisture diffusion coefficient, respectively, and ℓ is half the specimen thickness.

For $\text{Pd}'_m = 0$, the boundary conditions (13) and (14) agree with those of Luikov and Mikhailov (1965a) which considers the time dependence of DBT only.

Because of symmetry, at $X = 0$ we should have

$$\frac{\partial T}{\partial X} = 0 \qquad (X = 0; \text{Fo} \geq 0) \tag{19}$$

$$\frac{\partial \Theta}{\partial X} = 0 \qquad (X = 0; \text{Fo} \geq 0) \tag{20}$$

The initial conditions are assumed to be constant and are represented by

$$T = 0 \qquad (-1 \leq X \leq 1; \text{Fo} = 0) \tag{21}$$

$$\Theta = 0 \qquad (-1 \leq X \leq 1; \text{Fo} = 0) \tag{22}$$

METHOD OF SOLUTION

Because the boundary conditions (13) and (14) are nonhomogeneous, the solutions can be represented by two parts, a homogeneous solution and a particular solution.

Homogeneous Solution

For the homogeneous solutions of T and Q, we set the right-hand sides of equations (13) and (14) equal to zero. The detailed solution technique for this case was shown previously (Liu, in press). Here we list the resulting equations only.

The espressions for the homogeneous solutions T_H and Q_H are expressed in the form of infinite series:

$$\begin{aligned}T_H = \sum_{n=1}^{\infty} \mu_n^2 e^{-\mu_n^2\text{Fo}} A_n(&a_1 \cos \nu_1\mu_n X \\ &+ a_2 g(\mu_n)\cos \nu_2\mu_n X)\end{aligned} \tag{23}$$

$$\begin{aligned}\Theta_H = \sum_{n=1}^{\infty} \mu_n^2 e^{-\mu_n^2\text{Fo}} A_n(&b_1 \cos \nu_1\mu_n X \\ &+ b_2 g(\mu_n)\cos \nu_2\mu_n X)\end{aligned} \tag{24}$$

In Equations (33) and (34) the parameters ν_1 and ν_2 are

$$\begin{aligned}\nu_1^2 = \frac{1}{2}\left(1 + \frac{1}{\text{Lu}} + \epsilon\text{KoPn}\right) \\ - \left[\frac{1}{4}\left(1 + \frac{1}{\text{Lu}} + \epsilon\text{KoPn}\right)^2 - \frac{1}{\text{Lu}}\right]^{1/2}\end{aligned} \tag{25}$$

$$\begin{aligned}\nu_2^2 = \frac{1}{2}\left(1 + \frac{1}{\text{Lu}} + \epsilon\text{KoPn}\right) \\ + \left[\frac{1}{4}\left(1 + \frac{1}{\text{Lu}} + \epsilon\text{KoPn}\right)^2 - \frac{1}{\text{Lu}}\right]^{1/2}\end{aligned} \tag{26}$$

The eigenvalues μ_n are obtained from the characteristic equation

$$(\mu\nu_1 \tan \nu_1\mu + \psi_1)(\mu\nu_2 \tan \nu_2\mu + \psi_2) = \psi_3 \tag{27}$$

in which

$$\begin{aligned}\psi_1 &= [\text{Bi}_q - \text{Bi}_q\text{Lu}\nu_1^2 + (1-\epsilon)\text{Lu}^2\text{KoBi}_m\text{Pn}\nu_1^2 \\ &\quad - \text{Bi}_m\text{Lu}\nu_1^2(1 - \text{Lu}\nu_2^2)]/\text{Lu}(\nu_1^2 - \nu_2^2) \\ \psi_2 &= [\text{Bi}_q - \text{Bi}_q\text{Lu}\nu_2^2 + (1-\epsilon)\text{Lu}^2\text{KoBi}_m\text{Pn}\nu_2^2 \\ &\quad - \text{Bi}_m\text{Lu}\nu_2^2(1 - \text{Lu}\nu_1^2)]/\text{Lu}(\nu_2^2 - \nu_1^2) \\ \psi_3 &= -\text{Bi}_m\text{Bi}_q + \psi_1\psi_2\end{aligned}$$

The parameter μ in Equation (27) can take an infinite number of real values as well as some complex values and is therefore subscripted as μ_n.

The function $g(\mu)$ is

$$g(\mu) = \frac{-\mu\nu_1 \sin \nu_1\mu + \text{Bi}_m\text{Lu}\nu_1^2 \cos \nu_1\mu}{\mu\nu_2 \sin \nu_2\mu - \text{Bi}_m\text{Lu}\nu_2^2 \cos \nu_2\mu} \tag{28}$$

The constants a_1, a_2, b_1, and b_2 are

$$
\begin{aligned}
a_1 &= \mathrm{Lu}\nu_1^2 - 1 \\
a_2 &= \mathrm{Lu}\nu_2^2 - 1 \\
b_1 &= \mathrm{LuPn}\nu_1^2 \\
b_2 &= \mathrm{LuPn}\nu_2^2
\end{aligned}
\qquad (29)
$$

The constant coefficients A_n are to be evaluated from the initial conditions (21) and (22) later.

Particular Solution

For the particular solutions, T_p and Q_p of T and Q, it is convenient to consider the terms associated with Bi_q and Bi_m on the right-hand sides of Equations (13) mid (14) separately. Thus, we set

$$T_p = T_{p1} + T_{p2} \qquad (30)$$

$$\Theta_p = \Theta_{p1} + \Theta_{p2} \qquad (31)$$

with T_{p1} and Q_{p1} associated with Bi_q and T_{p2} and Q_{p2} with Bi_m. Note that Bi_q does not exist on the right-hand side of Equation (14).

(a) Rewrite Equations (13) and (14) as

$$
\begin{aligned}
&\frac{\partial T}{\partial X} + \mathrm{Bi}_q T - (1-\epsilon)\mathrm{LuKoBi}_m\Theta \\
&\quad = \mathrm{Bi}_q(1 - W_q + W_q e^{\mathrm{Pd}'_q\mathrm{Fo}}) \qquad (X = \pm 1; \mathrm{Fo} > 0)
\end{aligned}
\qquad (13')
$$

$$\frac{\partial \Theta}{\partial X} - \mathrm{Pn}\frac{\partial T}{\partial X} + \mathrm{Bi}_m\Theta = 0 \qquad (X = \pm 1; \mathrm{Fo} > 0) \qquad (14')$$

We want to solve Equations (1) and (2) under conditions (13'), (14'), (19), and (20). Replacing T with T_{p1} and Q with Q_{p1} in these equations and using the method of undetermined coefficients (Hildebrand, 1962), we assume

$$T_{p1} = c_1 + (c_2\cos c_3X + c_4\cos c_5X)e^{c_6\mathrm{Fo}} \qquad (32)$$

$$\Theta_{p1} = d_1 + (d_2\cos d_3X + d_4\cos d_5X)e^{d_6\mathrm{Fo}} \qquad (33)$$

It can readily be derived that in Equations (32) and (33)

$$
\begin{aligned}
c_1 &= 1 - W_q \\
c_2 &= \frac{\mathrm{Bi}_q W_q P_2}{P_2P_3 - P_1P_4} \\
c_3 &= i\sqrt{\mathrm{Pd}'_q}\nu_1 \\
c_4 &= \frac{\mathrm{Bi}_q W_q P_1}{P_1P_4 - P_2P_3} \\
c_5 &= i\sqrt{\mathrm{Pd}'_q}\nu_2 \\
c_6 &= \mathrm{Pd}'_q \\
d_1 &= 0 \\
d_2 &= \frac{\nu_1^2 - 1}{\epsilon\mathrm{Ko}}c_2 \\
d_3 &= c_3 \\
d_4 &= \frac{\nu_2^2 - 1}{\epsilon\mathrm{Ko}}c_4 \\
d_5 &= c_5 \\
d_6 &= c_6
\end{aligned}
\qquad (34)
$$

in which

$$
\begin{aligned}
P_1 &= \frac{(\nu_1^2 - 1)}{\epsilon\mathrm{Ko}}\left(\sqrt{\mathrm{Pd}'_q}\nu_1 \sinh\sqrt{\mathrm{Pd}'_q}\nu_1 + \mathrm{Bi}_m\cosh\sqrt{\mathrm{Pd}'_q}\nu_1\right) \\
&\quad - \mathrm{Pn}\sqrt{\mathrm{Pd}'_q}\nu_1 \sinh\sqrt{\mathrm{Pd}'_q}\nu_1 \\
P_2 &= \frac{(\nu_2^2 - 1)}{\epsilon\mathrm{Ko}}\left(\sqrt{\mathrm{Pd}'_q}\nu_2 \sinh\sqrt{\mathrm{Pd}'_q}\nu_2 + \mathrm{Bi}_m\cosh\sqrt{\mathrm{Pd}'_q}\nu_2\right) \\
&\quad - \mathrm{Pn}\sqrt{\mathrm{Pd}'_q}\nu_2 \sinh\sqrt{\mathrm{Pd}'_q}\nu_2 \\
P_3 &= \mathrm{Bi}_q\cosh\sqrt{\mathrm{Pd}'_q}\nu_1 + \sqrt{\mathrm{Pd}'_q}\nu_1\sinh\sqrt{\mathrm{Pd}'_q}\nu_1 \\
&\quad + \left(\frac{1}{\epsilon} - 1\right)(1 - \nu_1^2)\mathrm{LuBi}_m\cosh\sqrt{\mathrm{Pd}'_q}\nu_1 \\
P_4 &= \mathrm{Bi}_q\cosh\sqrt{\mathrm{Pd}'_q}\nu_2 + \sqrt{\mathrm{Pd}'_q}\nu_2\sinh\sqrt{\mathrm{Pd}'_q}\nu_2 \\
&\quad + \left(\frac{1}{\epsilon} - 1\right)(1 - \nu_2^2)\mathrm{LuBi}_m\cosh\sqrt{\mathrm{Pd}'_q}\nu_2
\end{aligned}
\qquad (35)
$$

(b) Rewrite Equations (13) and (14) as

$$
\begin{aligned}
&\frac{\partial T}{\partial X} + \mathrm{Bi}_q T - (1-\epsilon)\mathrm{LuKoBi}_m\Theta \\
&\quad = -(1-\epsilon)\mathrm{LuKoBi}_m(1 + W_m - W_m e^{-\mathrm{Pd}'_m\mathrm{Fo}}) \\
&\qquad\qquad (X = \pm 1; \mathrm{Fo} > 0)
\end{aligned}
\qquad (13'')
$$

$$
\begin{aligned}
&\frac{\partial \Theta}{\partial X} - \mathrm{Pn}\frac{\partial T}{\partial X} + \mathrm{Bi}_m\Theta = \mathrm{Bi}_m(1 + W_m - W_m e^{-\mathrm{Pd}'_m\mathrm{Fo}}) \\
&\qquad\qquad (X = \pm 1; \mathrm{Fo} > 0)
\end{aligned}
\qquad (14'')
$$

We want to solve Equations (1) and (2) under conditions (13''), (14''), (19), and (20). Replacing T with T_{p2} and Q with Q_{p2} in these equations and assuming likewise that

$$T_{p2} = e_1 + (e_2\cos e_3X + e_4\cos e_5X)e^{e_6\mathrm{Fo}} \qquad (36)$$

$$\Theta_{p2} = f_1 + (f_2\cos f_3X + f_4\cos f_5X)e^{f_6\mathrm{Fo}} \qquad (37)$$

we can obtain that in Equations (36) and (37)

$$
\begin{aligned}
e_1 &= 0 \\
e_2 &= \frac{[Q_2(1-\epsilon)\mathrm{LuKo} + Q_4]\mathrm{Bi}_m W_m}{Q_2Q_3 - Q_1Q_4} \\
e_3 &= \sqrt{\mathrm{Pd}'_m}\nu_1 \\
e_4 &= \frac{[Q_1(1-\epsilon)\mathrm{LuKo} + Q_3]\mathrm{Bi}_m W_m}{Q_1Q_4 - Q_2Q_3} \\
e_5 &= \sqrt{\mathrm{Pd}'_m}\nu_2 \\
e_6 &= \mathrm{Pd}'_m \\
f_1 &= 1 + W_m \\
f_2 &= \frac{\nu_1^2 - 1}{\epsilon\mathrm{Ko}}e_2 \\
f_3 &= e_3 \\
f_4 &= \frac{\nu_2^2 - 1}{\epsilon\mathrm{Ko}}e_4 \\
f_5 &= e_5 \\
f_6 &= e_6
\end{aligned}
\qquad (38)
$$

in which

$$Q_1 = \frac{(\nu_1^2 - 1)}{\epsilon \mathrm{Ko}}\left(\mathrm{Bi}_m \cos\sqrt{\mathrm{Pd}'_m}\nu_1 - \sqrt{\mathrm{Pd}'_m}\nu_1 \sin\sqrt{\mathrm{Pd}'_m}\nu_1\right) + \mathrm{Pn}\sqrt{\mathrm{Pd}'_m}\nu_1 \sin\sqrt{\mathrm{Pd}'_m}\nu_1$$

$$Q_2 = \frac{(\nu_2^2 - 1)}{\epsilon \mathrm{Ko}}\left(\mathrm{Bi}_m \cos\sqrt{\mathrm{Pd}'_m}\nu_2 - \sqrt{\mathrm{Pd}'_m}\nu_2 \sin\sqrt{\mathrm{Pd}'_m}\nu_2\right) + \mathrm{Pn}\sqrt{\mathrm{Pd}'_m}\nu_2 \sin\sqrt{\mathrm{Pd}'_m}\nu_2$$

$$Q_3 = \mathrm{Bi}_q \cos\sqrt{\mathrm{Pd}'_m}\nu_1 - \sqrt{\mathrm{Pd}'_m}\nu_1 \sin\sqrt{\mathrm{Pd}'_m}\nu_1 + \left(1 - \frac{1}{\epsilon}\right)(\nu_1^2 - 1)\mathrm{LuBi}_m \cos\sqrt{\mathrm{Pd}'_m}\nu_1$$

$$Q_4 = \mathrm{Bi}_q \cos\sqrt{\mathrm{Pd}'_m}\nu_2 - \sqrt{\mathrm{Pd}'_m}\nu_2 \sin\sqrt{\mathrm{Pd}'_m}\nu_2 + \left(1 - \frac{1}{\epsilon}\right)(\nu_2^2 - 1)\mathrm{LuBi}_m \cos\sqrt{\mathrm{Pd}'_m}\nu_2 \tag{39}$$

It is evident that Equations (30) and (31), with T_p replaced by T, Q_p by Q, and the right-hand sides of these equations expressed in terms of Equations (32), (33), (36), and (37), can satisfy Equations (1), (2), (13), (14), (19), and (20). Thus, from Equations (23), (24), (30), and (31)

$$T = T_H + T_p \tag{40}$$

$$\Theta = \Theta_H + \Theta_p \tag{41}$$

Now we must evaluate the coefficients A_n in Equations (23) and (24), which are the homogeneous solutions in Equations (40) and (41). By setting Fo = 0 in the homogeneous and particular solutions in Equations (40) and (41) and making use of the initial conditions (21) and (22), these coefficients can be evaluated using a least-squares technique (Cheng and Angsirikul, 1977; Hildebrand, 1974). First, we set up the following integral:

$$\Omega = \int_0^1 \Big\{ \Big[T_p(X,0) + \sum_{n=1}^{\infty} \mu_n^2 A_n (a_1 \cos\nu_1\mu_n X + a_2 g(\mu_n)\cos\nu_2\mu_n X)\Big] \times \Big[T_p(X,0) + \sum_{n=1}^{\infty} \bar{\mu}_n^2 \bar{A}_n (a_1 \cos\nu_1\bar{\mu}_n X + a_2 g(\bar{\mu}_n)\cos\nu_2\bar{\mu}_n X)\Big] + \Big[\Theta_p(X,0) + \sum_{n=1}^{\infty} \mu_n^2 A_n (b_1 \cos\nu_1\mu_n X + b_2 g(\mu_n)\cos\nu_2\mu_n X)\Big] \times \Big[\Theta_p(X,0) + \sum_{n=1}^{\infty} \bar{\mu}_n^2 \bar{A}_n (b_1 \cos\nu_1\bar{\mu}_n X + b_2 g(\bar{\mu}_n)\cos\nu_2\bar{\mu}_n X)\Big] \Big\} dX \tag{42}$$

which must be a minimum and in which

$$\begin{aligned} T_p(X,0) &= c_1 + c_2 \cos c_3 X + c_4 \cos c_5 X + e_1 + e_2 \cos e_3 X + e_4 \cos e_5 X \\ \Theta_p(X,0) &= d_1 + d_2 \cos d_3 X + d_4 \cos d_5 X + f_1 + f_2 \cos f_3 X + f_4 \cos f_5 X \end{aligned} \tag{43}$$

The a and b values are defined by Equation (29), the c and d values by Equation (34), and the e and f values by Equation (38). The parameters $\bar{\mu}_n$ and $\bar{A}_n$ are complex conjugates of μ_n and A_n, respectively.

The condition that W be a minimum requires that its partial derivatives with respect to A_m or A_m shall be zero. Therefore,

$$\frac{\partial\Omega}{\partial\bar{A}_m} = \int_0^1 \Big\{ \Big[T_p(X,0) + \sum_{n=1}^{\infty} \mu_n^2 A_n (a_1 \cos\nu_1\mu_n X + a_2 g(\mu_n)\cos\nu_2\mu_n X)\Big] \times \bar{\mu}_m^2 (a_1 \cos\nu_1\bar{\mu}_m X + a_2 g(\bar{\mu}_m)\cos\nu_2\bar{\mu}_m X) + \Big[\Theta_p(X,0) + \sum_{n=1}^{\infty} \mu_n^2 A_n (b_1 \cos\nu_1\mu_n X + b_2 g(\mu_n)\cos\nu_2\mu_n X)\Big] \times \bar{\mu}_m^2 (b_1 \cos\nu_1\bar{\mu}_m X + b_2 g(\bar{\mu}_m)\cos\nu_2\bar{\mu}_m X) \Big\} dX = 0 \quad (m = 1,2,3,\ldots) \tag{44}$$

Note that the same results are obtained if we set $\partial\Omega/\partial A_m = 0$. In matrix from Equation (44) a Hermitian matrix as

$$[C_{mn}]\{A_n\} = \{R_m\} \tag{45}$$

in which

$$C_{mn} = \int_0^1 \{[\bar{\mu}_m^2 (a_1 \cos\nu_1\bar{\mu}_m X + a_2 g(\bar{\mu}_m)\cos\nu_2\bar{\mu}_m X) \times \mu_n^2 (a_1 \cos\nu_1\mu_n X + a_2 g(\mu_n)\cos\nu_2\mu_n X)] + [\bar{\mu}_m^2 (b_1 \cos\nu_1\bar{\mu}_m X + b_2 g(\bar{\mu}_m)\cos\nu_2\bar{\mu}_m X) \times \mu_n^2 (b_1 \cos\nu_1\mu_n X + b_2 g(\mu_n)\cos\nu_2\mu_n X)]\} dX \tag{46}$$

$$R_m = -\int_0^1 \bar{\mu}_m^2 \{T_p(X,0)[a_1 \cos\nu_1\bar{\mu}_m X + a_2 g(\bar{\mu}_m)\cos\nu_2\bar{\mu}_m X] + \Theta_p(X,0)[b_1 \cos\nu_1\bar{\mu}_m X + b_2 g(\bar{\mu}_m)\cos\nu_2\bar{\mu}_m X]\} dX \tag{47}$$

The coefficients A_n can be determined from the system of linear Equations (45). We can then calculate T and Q from Equations (40) and (41) by means of Equations (23), (24), (30), and (31).

The espressions for T and Q, their averages $\bar{T}$ and $\bar{Q}$ across $0 \leq X \leq 1$, and the derivatives of T, Q, $\bar{T}$, and $\bar{Q}$ with respect to time Fo (that is, the heating and drying rates) are summarized as follows:

$$T = c_1 + (c_2 \cos c_3 X + c_4 \cos c_5 X)e^{c_6 \mathrm{Fo}} + e_1 + (e_2 \cos e_3 X + e_4 \cos e_5 X)e^{e_6 \mathrm{Fo}} + \sum_{n=1}^{\infty} \mu_n^2 e^{-\mu_n^2 \mathrm{Fo}} A_n (a_1 \cos \nu_1 \mu_n X + a_2 g(\mu_n) \cos \nu_2 \mu_n X) \tag{48}$$

$$\Theta = d_1 + (d_2 \cos d_3 X + d_4 \cos d_5 X)e^{d_6 \mathrm{Fo}} + f_1 + (f_2 \cos f_3 X + f_4 \cos f_5 X)e^{f_6 \mathrm{Fo}} + \sum_{n=1}^{\infty} \mu_n^2 e^{-\mu_n^2 \mathrm{Fo}} A_n (b_1 \cos \nu_1 \mu_n X + b_2 g(\mu_n) \cos \nu_2 \mu_n X) \tag{19}$$

$$\bar{T} = c_1 + \left(\frac{c_2}{c_3} \sin c_3 + \frac{c_4}{c_5} \sin c_5\right) e^{c_6 \mathrm{Fo}} + e_1 + \left(\frac{e_2}{e_3} \sin e_3 + \frac{e_4}{e_5} \sin e_5\right) e^{e_6 \mathrm{Fo}} + \sum_{n=1}^{\infty} \mu_n e^{-\mu_n^2 \mathrm{Fo}} A_n \left(\frac{a_1}{\nu_1} \sin \nu_1 \mu_n + \frac{a_2 g(\mu_n)}{\nu_2} \sin \nu_2 \mu_n\right) \tag{50}$$

$$\bar{\Theta} = d_1 + \left(\frac{d_2}{d_3} \sin d_3 + \frac{d_4}{d_5} \sin d_5\right) e^{d_6 \mathrm{Fo}} + f_1 + \left(\frac{f_2}{f_3} \sin f_3 + \frac{f_4}{f_5} \sin f_5\right) e^{f_6 \mathrm{Fo}} + \sum_{n=1}^{\infty} \mu_n e^{-\mu_n^2 \mathrm{Fo}} A_n \left(\frac{b_1}{\nu_1} \sin \nu_1 \mu_n + \frac{b_2 g(\mu_n)}{\nu_2} \sin \nu_2 \mu_n\right) \tag{51}$$

$$\frac{dT}{d\mathrm{Fo}} = c_6 (c_2 \cos c_3 X + c_4 \cos c_5 X)e^{c_6 \mathrm{Fo}} + e_6 (e_2 \cos e_3 X + e_4 \cos e_5 X)e^{e_6 \mathrm{Fo}} - \sum_{n=1}^{\infty} \mu_n^4 e^{-\mu_n^2 \mathrm{Fo}} A_n (a_1 \cos \nu_1 \mu_n X + a_2 g(\mu_n) \cos \nu_2 \mu_n X) \tag{52}$$

$$\frac{d\Theta}{d\mathrm{Fo}} = d_6 (d_2 \cos d_3 X + d_4 \cos d_5 X)e^{d_6 \mathrm{Fo}} + f_6 (f_2 \cos f_3 X + f_4 \cos f_5 X)e^{f_6 \mathrm{Fo}} - \sum_{n=1}^{\infty} \mu_n^4 e^{-\mu_n^2 \mathrm{Fo}} A_n (b_1 \cos \nu_1 \mu_n X + b_2 g(\mu_n) \cos \nu_2 \mu_n X) \tag{53}$$

$$\frac{d\bar{T}}{d\mathrm{Fo}} = c_6 \left(\frac{c_2}{c_3} \sin c_3 + \frac{c_4}{c_5} \sin c_5\right) e^{c_6 \mathrm{Fo}} + e_6 \left(\frac{e_2}{e_3} \sin e_3 + \frac{e_4}{e_5} \sin e_5\right) e^{e_6 \mathrm{Fo}} - \sum_{n=1}^{\infty} \mu_n^3 e^{-\mu_n^2 \mathrm{Fo}} A_n \left(\frac{a_1}{\nu_1} \sin \nu_1 \mu_n + \frac{a_2 g(\mu_n)}{\nu_2} \sin \nu_2 \mu_n\right) \tag{54}$$

$$\frac{d\bar{\Theta}}{d\mathrm{Fo}} = d_6 \left(\frac{d_2}{d_3} \sin d_3 + \frac{d_4}{d_5} \sin d_5\right) e^{d_6 \mathrm{Fo}} + f_6 \left(\frac{f_2}{f_3} \sin f_3 + \frac{f_4}{f_5} \sin f_5\right) e^{f_6 \mathrm{Fo}} - \sum_{n=1}^{\infty} \mu_n^3 e^{-\mu_n^2 \mathrm{Fo}} A_n \left(\frac{b_1}{\nu_1} \sin \nu_1 \mu_n + \frac{b_2 g(\mu_n)}{\nu_2} \sin \nu_2 \mu_n\right) \tag{55}$$

NUMERICAL RESULTS AND DISCUSSION

We consider three cases of drying environmental conditions for comparison (Liu, in press):

Case I–Both DBT and EMTP are constant; Pd'_q and Pd'_m defined in Equations (15) and (17), respectively, are then zero; DBT should maintain a high value while EMTP maintains a low value.

Case 2–DBT increases esponentially with time while EMTP remains constant; Pd'_q should then be positive and Pd'_m zero: initial DBT or t_{c0} should take a low value, as should the EMTP.

Case 3–DBT increases exponentially with time while EMTP decreases exponentially with time; Pd'_q and Pd'_m should both be positive; initial DBT or tc0 should take a low value. but initial EMTP or q_{p0} a high value.

To conveniently compare cases 1 and 2, we express t_{c0} in terms of t_0 , whereas q_{p0} need not be specified. To compare cases 2 and 3, t_{c0} is expressed in terms of t_0 , and q_{p0} in terms of q_0. Based on the estimated thermophysical properties of spruce (Thomas et al., 1980), the input and related data for the three cases are shown in Table 1: case 2 is denoted as case 2a for comparison with case 1 and as case 2b for comparison with case 3. The numbers Ko in Equation (11) and Pn in Equation (12) and the parameters W_q in Equation (16) and W_m in Equation (18) all contain t_{c0}, t_0, q_{p0}, and q_0. They hare been calculated for the three cases according to the assumed relationships between t_{c0} and t_0 and q_{p0} and q_0 in Table 1. For the numerical illustrations of this study, these data are also based on the assumption that the thermophysical properties remain constant during drying.

Note that in reality these properties may be functions of either temperature or moisture, requiring the application of zonal calculations (Luikov and Mikhailov, 1965b) for improved solutions. We can also apply numerical techniques, such as the finite difference methods employed by Plumb et ai. (1985) and Stanish et al. (1986), to account for variable thermophysical properties.

The real eigenvalues in Equation (27) were obtained by means of a bisection procedure. As pointed out by Liu and Cheng (in press), when complex eigenvalues also exist in Equation (27), they must be included in the calculations to satisfy the initial conditions. Using a method by Muller (1956), which was included in IMSL (1987), a pair of complex eigenvalues was obtained for each case in Table 1. The eigenvalues in Equation (27) are dependent on the product of Ko and Pn; therefore, the same complex eigenvalues were obtained for all cases. These values are $0.58311 \pm 0.0199078i$.

Equations (21) and (22) specify that initially the heat and mass transfer potentials are uniformly distributed in the specimen; however. as drying progresses, their distributions are likely to be approximately parabolic. For the heat potential or temperature distribution, the maximum value is at the surfaces ($X = \pm 1$) and the minimum value at the center ($X = 0$); for the mass or moisture potential distribution, the opposite is true. For comparison, it is convenient to consider their average values across the specimen thickness.

Figure 2 shows the variations of average dimensionless temperature $\bar{T}$ and mass transfer potential $\bar{Q}$ as functions of Fo for several values of Pd'_q with $\mathrm{Pd}'_m = 0$. For $\mathrm{Pd}'_q = 0$, we have case 1 in Table 1. The value $\bar{T}$ increases with Fo and approaches 1 when Fo > 200. For $\mathrm{Pd}'_q > 0$, we have case 2a in Table 1. The value $\bar{T}$ decreases initially with time until Fo = 6, when it starts to increase. This same phenomenon was also reported in the drying of food stuffs (Robbins and Özisik, 1988). The value $\bar{T}$ also increases with an increasing Pd'_q. The combined effects of Pd'_q, Ko, and Pn on Q are seen

Table 1–Input and related data for three cases of environmental drying conditions

Case	Lu	Bi_q	Bi_m	ϵ	Pn	Ko	Pd'_q	Pd'_m	W_q	W_m	t_{c0}	θ_{p0}
1	0.008	0.4	1.4	0.3	2.4	8	0	0	1.1	W_m	$11t_0$	θ_{p0}
2a	0.008	0.4	1.4	0.3	0.24	80	>0	0	2.0	W_m	$2t_0$	θ_{p0}
2b	0.008	0.4	1.4	0.3	0.24	80	>0	0	2.0	0.11	$2t_0$	$0.1\theta_0$
3	0.008	0.4	1.4	0.3	0.43	44.44	>0	>0	2.0	1.0	$2t_0$	$0.5\theta_0$

to be negligible until Fo > 110, when $\bar{Q}$ starts to branch out with increasing Fo, reaches a maximum. and then decreases. The higher the value of Pd'_q the sooner the maximum $\bar{Q}$ is reached. When the maximum $\bar{Q}$ is approached, the inside moisture potential tends to increase and the surfaces dry out fast while the temperature is still increasing, resulting in surface degrade. Therefore, by increasing DBT only, the drying time cannot be reduced and material degrade may occur (Fig. 2). To avoid material degrade, Pd'_q must be kept low.

According to Equation (9), with t replaced by its average value $\bar{t}$ and T by $\bar{T}$, and cases 1 and 2a of Table 1, we obtain

$$\frac{\bar{t}}{t_0} = 1 + 10\bar{T} \qquad (Pd'_q = 0) \tag{56}$$

$$\frac{\bar{t}}{t_0} = 1 + \bar{T} \qquad (Pd'_q > 0) \tag{57}$$

The data for $\bar{T}$ as a function of Fo for the Pd'_q values in Figure 2 are transformed with $\bar{T}$ being replaced by the average temperature ratio $\bar{t}/t_0$ according to Equations (56) and (57) (Fig. 3). Figure 3 shows that for the same curves of $\bar{Q}$ as a function of Fo for the several values of Pd'_q in Figure 2, the curve of the average temperature ratio $\bar{t}/t_0$ as a function of Fo for $Pd'_q = 0$ is much higher than the curves for $Pd'_q > 0$. This indicates that for $Pd'_q > 0$, the heat absorption of lumber is less than that for $Pd'_q = 0$ based on the assumed relations (56) and (57). Obviously, for Equation (57) to be valid and to achieve the economical process objectives, Pd'_q cannot be made arbitrarily small. The smallest allowable value for Pd'_q can only be determined experimentally.

Figure 4 presents the variations of average dimensionless temperature $\bar{T}$ and mass transfer potential $\bar{Q}$ as functions of Fo for several values of Pd'_m with $Pd'_q = 0.005$. The curves of $\bar{T}$ and $\bar{Q}$ as functions of Fo for $Pd'_m = 0$ (case 2b in Table 1) are the same as the corresponding curves for $Pd'_q = 0.005$ in Figure 2. As Pd'_m increases from 0 to 1, $\bar{T}$ varies only slightly while $\bar{Q}$ increases noticeably but nonuniformly with increasing Fo. As Pd'_m approaches 1, the increase of $\bar{Q}$ becomes increasingly small for Fo > 50.

The variations of average temperature ratio $\bar{t}/t_0$ as a function of Fo for the values of Pd'_m in Figure 5 are obtained from Equation (57). The average mass transfer potential ratio $\bar{q}/q_0$, is obtained from Equation (10) with q replaced by its average value $\bar{q}$, Q by $\bar{Q}$, and cases 2b and 3 of Table 1:

$$\frac{\bar{\theta}}{\theta_0} = 1 - 0.9\bar{\Theta} \qquad (Pd'_m = 0) \tag{58}$$

$$\frac{\bar{\theta}}{\theta_0} = 1 - 0.5\bar{\Theta} \qquad (Pd'_m > 0) \tag{59}$$

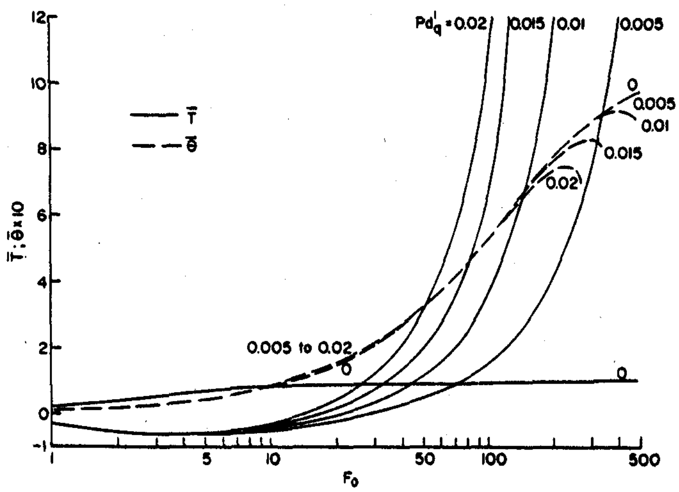

Figure 2—Variations of average dimensionless temperature $\bar{T}$ and mass transfer potential $\bar{\Theta}$ as functions of time Fo for several values of Pd'_q with $Pd'_m = 0$. (Lu = 0.008, Bi_q = 0.4, Bi_m = 1.4, ϵ = 0.3; for Pd'_q = 0, Pn = 2.4, Ko = 8, W_q = 1.1; for Pd'_q > 0, Pn = 0.24, Ko = 80, W_q = 2.) (ML90 5496)

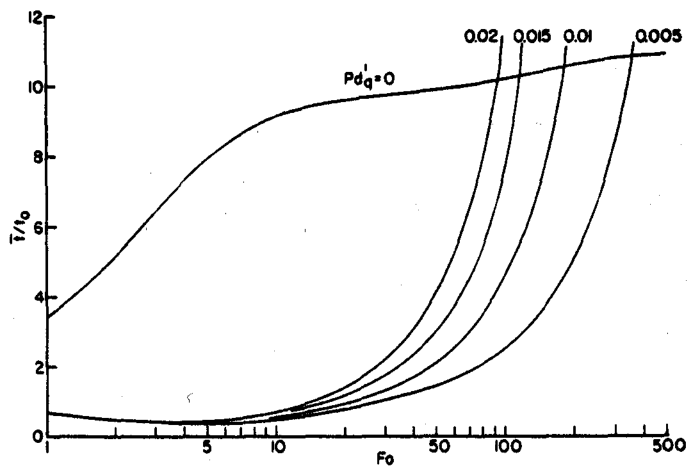

Figure 3—Variations of average temperature ratio $\bar{t}/t_0$ as a function of time Fo for several values of Pd'_q with $Pd'_m = 0$. (Lu = 0.008, Bi_q = 0.4, Bi_m = 1.4, ϵ = 0.3; for Pd'_q = 0, Pn = 2.4, Ko = 8, W_q = 1.1; for Pd'_q > 0, Pn = 0.24, Ko = 80, W_q = 2.) (ML90 5497)

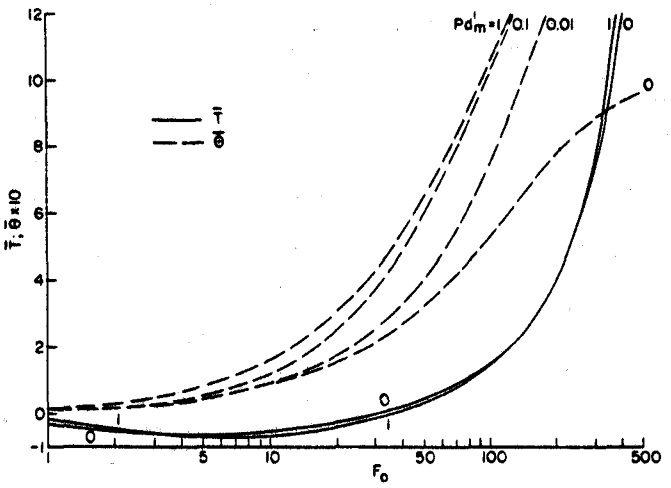

Figure 4—Variations of average dimensionless temperature $\bar{T}$ and mass transfer potential $\bar{\Theta}$ as functions of time Fo for several values of Pd'_m with $Pd'_q = 0.005$. (Lu = 0.008, $Bi_q = 0.4$, $Bi_m = 1.4$, $\epsilon = 0.3$, $W_q = 2$; for $Pd'_m = 0$, $Pn = 0.24$, $Ko = 80$, $W_m = 0.11$; for $Pd'_m > 0$, $Pn = 0.43$, $Ko = 44.44$, $W_m = 1$.) (ML90 5498)

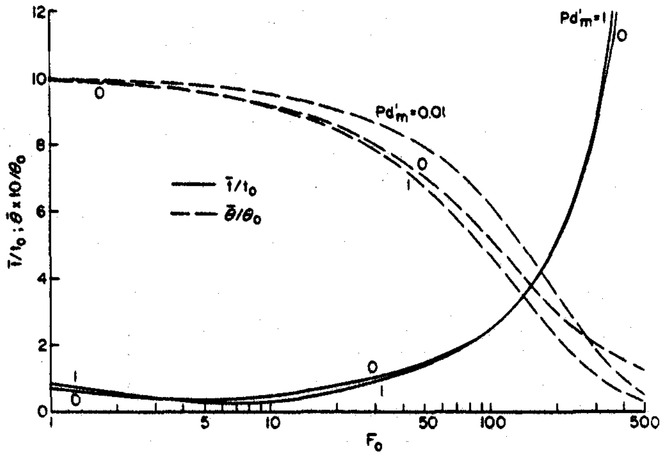

Figure 5—Variations of average temperature ratio $\bar{t}/t_0$ and mass transfer potential ratio $\bar{\theta}/\theta_0$ as functions of time Fo for several values of Pd'_m with $Pd'_q = 0.005$. (Lu = 0.008, $Bi_q = 0.4$, $Bi_m = 1.4$, $\epsilon = 0.3$, $W_q = 2$; for $Pd'_m = 0$, $Pn = 0.24$, $Ko = 80$, $W_m = 0.11$; for $Pd'_m > 0$, $Pn = 0.43$, $Ko = 44.44$ $W_m = 1$.) (ML90 5499)

The data for $\bar{Q}$ as a function of Fo for the values of Pd'_m in Figure 4 are transformed with $\bar{Q}$ being replaced by the average mass transfer potential ratio $\bar{q}/q_0$ according to Equations (58) and (59) (Fig. 5). In Figure 5. for Pd'_m = 0 and 1, the two corresponding curves of $\bar{t}/t_0$ are closely entwined with each other; for Pd'_m to increase from 0 to 0.01, the drying time actually increases as the $\bar{q}/q_0$ curve for Pd'_m = 0.01 stays above that for Pd'_m = 0 until Fo = 280 at $\bar{q}/q_0$ = 0.22, where the two curves intersect with each other. For Pd'_m = 1, the drying time is somewhat reduced, reaching Fo = 215 as compared to Fo = 305 for Pd'_m = 0 at $\bar{q}/q_0$ = 0.2. The curve for Pd'_m = 0.1 is slightly above that for Pd'_m = 1 and is not plotted. The relationship between the drying time and Pd'_m is thus seen to be nonuniform, but a value of Pd'_m approaching 1 has a definite advantage especially for Fo > 10.

Note that if a larger value for Pd'_q as shown in Figure 2 was selected for Figure 5, for a given value of Pd'_m the ratio q/q_0 for the surfaces (X = ± 1) will approach 0 when $\bar{q}/q_0$ is still positive as time Fo increases. When or shortly before that happens, the drying process must be suspended and the environment altered to avoid material degrade. In application, the drying process can be readily divided into several zones, in each of which different boundary conditions can be prescribed to obtain optimum results of reduced lumber heat absorption and drying time. Such a process is, of course, beyond the scope of the present study.

CONCLUSIONS

This paper presents an analytical method for solving the Luikov system of heat and mass transfer equations with exponentially time dependent boundary conditions. The solutions are composed of a homogeneous solution and a particular solution. When the boundary conditions change, the homogeneous solution remains the same; only the particular solution needs to be changed. Also, the particular solution may contain several parts, each corresponding to a part of the boundary conditions. Thus, the method can reduce considerably the mathematical derivations when the boundary conditions are complex. In the literature, the integral transform techniques have been recommended for solving the Luikov system of equations. Such techniques are very difficult to apply for the type of problem considered in the study reported here.

From the numerical example of lumber drying, the solutions suggest that a drying process be divided into several zones with different boundary conditions in each zone to achieve the goals of reducing lumber heat absorption and drying time. The method presented in this study can handle different boundary conditions with ease and is therefore very attractive for such applications.

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