Thomas J. Urbanik[1]

Force Plate for Corrugated Container Vibration Tests

REFERENCE: Urbanik, T. J., **"Force Plate for Corrugated Container Vibration Tests,"** ***Journal of Testing and Evaluation,*** JTEVA, Vol. 18, No. 5, Sept. 1990. pp. 359–362.

ABSTRACT: A stack of corrugated containers subjected to over-the-road vibrations behaves like a distributed spring-mass system. Severe stack vibrations can crush the bottom container, fatigue the product cushioning. and damage the product. Laboratory tests were conducted to characterize the spring-mass system response to vibration by using a force plate combined with a single accelerometer. By using the force plate, only two data acquisition channels are needed to measure the driving acceleration and response force beneath a container stack. The acceleration and force data become input to a model that reduces the stack to an equivalent single-degree-of-freedom system and identifies its properties. The merits of alternate stacking patterns and interior packaging can then be quantified in terms of their generalized mass, stiffness, and damping and the effect of thew properties on the stack response. Cushioning vibration characteristics are determined for flexible product-container interactions. Container designs can be improved based on understanding the physics of transportation vibrations.

KEY WORDS: shipping environment, shock and vibration, preshipment container testing, unit load transfer function

Preshipment vibration testing is one way to reduce costs of product damage and overpackaging. Advances in this technology with respect to corrugated fiberboard containers depend largely on how well we understand the physics of stacked container vibrations [*1*]. Current standardized procedures for vibration tests of shipping containers attempt to replicate the distribution environment (ASTM D 4728, D 999, and D 4169) and then subjectively interpret the results. Just as cushioning curves are used to select an optimum thickness material to protect against shock, a better characterization of container slacks can provide a method for reducing the effects of vibration.

A stack of containers is a multiple-degree-of-freedom (MDOF) system. Measuring the behavior of a stack as a dynamic structure translates into a problem of determining values of the constants in the equations of motion. Distributed mass, distributed flexibility, and imperfect rectilinear motion of a stack complicate data acquisition and theoretical characterization. Attaching accelerometers to points of concentrated mass is not practical because of the number of data channels needed to capture the degrees of freedom. In addition, according to theory, monitoring only one convenient mass concentration is not an accurate way to characterize stiffness or damping of the system. A better approach is to first simplify the stack to an equivalent single-degree-of-freedom (SDOF) system with generalized mass, stiffness, and damping, according to the form of an assumed response function.

The design and application of a force plate for stacked cantainer vibration tests are described in this report. The force plate can be used to determine the generalized SDOF properties of an MDOF system. Our primary objective in building the plate was to experimentally confirm the conclusions set forth by an earlier theory [*2*]. We needed the instrumentation to measure the dynamic reaction beneath a stack of vibrating containers. According to theory, the dynamic compressive force on the bottom container in a stack can be minimized by optimizing the interior packaging. Our experiments did indeed agree with theory. Because the force plate played such an integral role in acquiring the kind of data we needed, this report will highlight the contribution of the force plate to the experiment. Using the force plate, we could successfully characterize whole container stacks and their individual container and cushion components,

Materials and Methods

Force Plate Design

A force plate consists of a rigid platen supported by load cells. The electronic signals from the cells combine to yield a single signal. When the force plate is placed on top of a vibration table, the direction of the measured force exactly aligns with that of the driving acceleration. Unlike uniaxial accelerometers attached to critical points in a stack, a force plate can Sense the average mass equivalent of mass moments of inertia. The stiffness of the plate needs to be such that the plate does not provide a significant source of resonance over the range of driving frequencies. A signal taken from the driving actuator or vibration table usually provides the source for controlling electrohydraulic systems [*3*]. By controlling the table vibration with an accelerometer attached beneath the force plate, the plate is eliminated as a degree of freedom in analyzing the stack response.

The force plate in our study was made of solid maple 76.2 mm thick (Fig. 1). The load cells were positioned 483 mm apart. A plate with a mass of 28 kg was found to produce natural frequencies of 63 and 46 Hz when centrally loaded with 119 and 210 kg, respectively. These calibration results, obtained with lead-shot bags and iron weights, only approximate the effect of rigid and concentrated loads, yet adequately define the upper operating frequency of the plate.

Data characterizing the test specimen shown in Fig. 1 can be summarized in terms of the generalized mass, stiffness, and damping of an equivalent SDOF system. Variations and com-

Manuscript received 10/22/89; accepted for publication 2/12/90.

---

[1] Research General Engineer, USDA Forest Service, Forest Products Laboratory, Madison, WI 53705-2398.

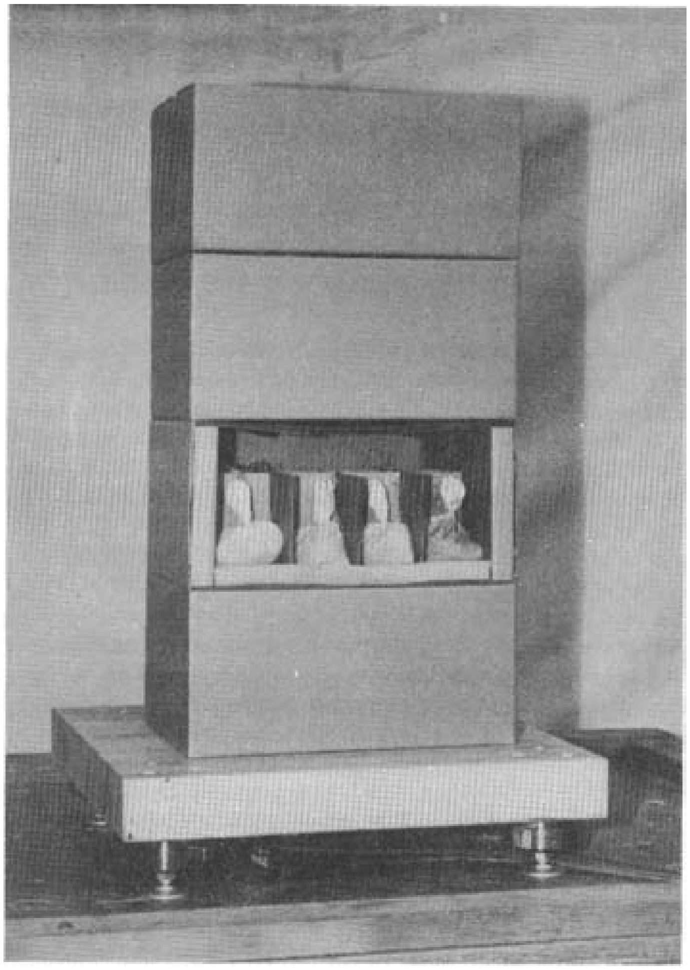

FIG. 1—*Force plate with load cells for determining the reaction force beneath a stock of vibrating containers. Cutaway view shows specimen components. An accelerometer (not shown), attached to the plate, controls the forcing acceleration.*

binations of end pad height, cushion stiffness, contained mass, and stack height can be compared in terms of their effect on generalized stack properties. Figure 2 shows the specimen configuration for measuring cushion properties. The bottom box is rigid, compared to the cushion, and is left empty. The product weight in the upper box rests on a cushion defined by an interior pad and the diaphragm action of the container flaps.

Force Plate Mechanics

The schematic and free body diagram of a plate and stack combination appear in Fig. 3. In addition to the stack reaction force F_c of interest, the magnitude of the measured force plate output F_p includes the plate static weight W_p and the force W_p^* due to inertial resistance of the distributed mass and stiffness of the plate. Equating the forces acting on the plate and rearranging terms yields

$$F_c = F_p - W_p - W_p^* A_p/g \tag{1}$$

where A_p is the acceleration of the plate's center of dynamic mass and g is the gravitational constant.

The plate itself, if too flexible, can act like a distributed spring-mass system. The value of W_p^*, representing the inertial resistance force of an equivalent concentrated mass, needs to be determined by a calibration test. After considering experimental variation, we found that when input was below 20 Hz, the generalized dynamic weight (that is, the inertial force when $A_p = g$) of the force plate equaled the static weight of the plate. The plate flexural stiffness K_p and damping D_p could likewise be determined. However, knowing these values is not critical if the accelerometer attached to the plate to yield A_p is also used to generate the electrohydraulic feedback signal.

If the container stack is considered as an equivalent concentrated mass, stack acceleration A_c is determined by rearranging the terms equating the forces acting at the center of dynamic mass:

$$A_c = (F_c - W_c)g/W_c^* \tag{2}$$

Here W_c is the static weight of the containers and W_c^* becomes the generalized weight of an equivalent SDOF system.

The theory needed to analyze a SDOF system can be found in an elementary vibration handbook [*4*]. The steady-state response of the stack to a forced periodic plate vibration is characterized by the stack transmissibility *Tr* as a function of forcing frequency *f*. Transmissibility is the ratio of the response amplitude to the forcing amplitude:

$$Tr(f) = |A_c|/|A_p| \tag{3}$$

By determining the acceleration response amplitude from the periodic force according to Eq 2, then substituting into Eq 3, we derive

$$Tr(f) = [(|F_c| - W_c)g/W_c^*]/|A_p| \tag{4}$$

Then by equating Eq 4 to the form of the transmissibility curve [*2*] and multiplying by W_c^* we obtain

$$(|F_c| - W_c)g/|A_p| = W_c^* \sqrt{\frac{1 + (2pf/f_n)^2}{(1 - [f/f_n]^2)^2 + (2pf/f_n)^2}} \tag{5}$$

where f_n is the natural frequency of the generalized SDOF system and p the fraction of critical damping. Equation 5 is limited to systems with linear stiffness and linear viscous damping. Other forms of the transmissibility curve could be substituted to investigate nonlinear systems.

Application

Determining the properties of a stack consists of acquiring periodic signals A_p and F_p over a range of frequencies and recording them in a digital format for later analysis. For our applications, we used a digital oscillosope combined with a microcomputer and associated software (Fig. 4). From the digitized time-varying magnitudes of A_p and F_p, the summation F_c is determined from Eq 1 point by point and recorded. Then, the wave amplitudes $|A_p|$ and $|F_c|$ are calculated at each frequency. We found

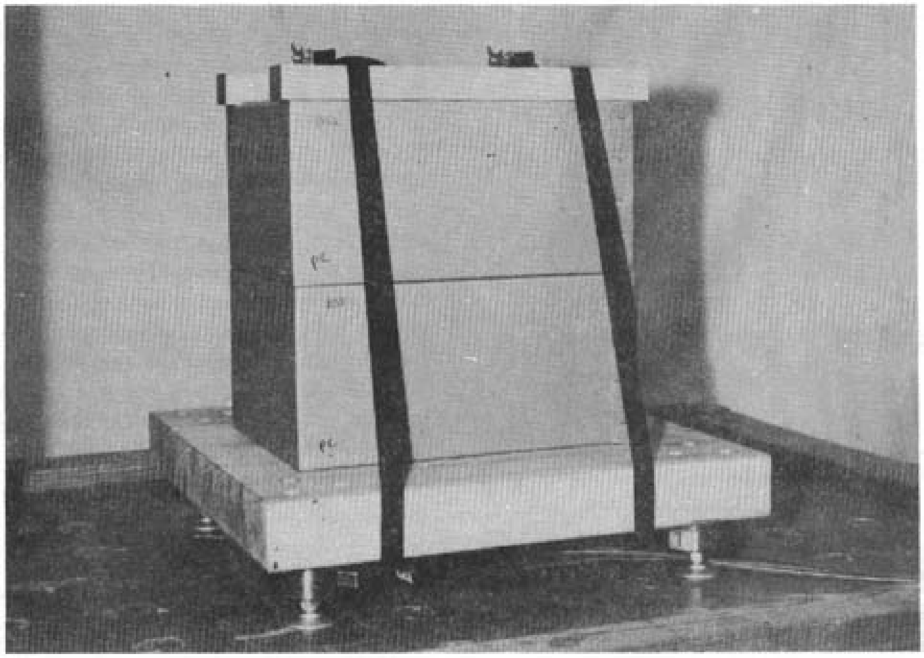

FIG. 2—*Application of force plate to determine the stiffness and damping characteristics of cushion and product combination in the upper box. The bottom box is empty and effectively rigid.*

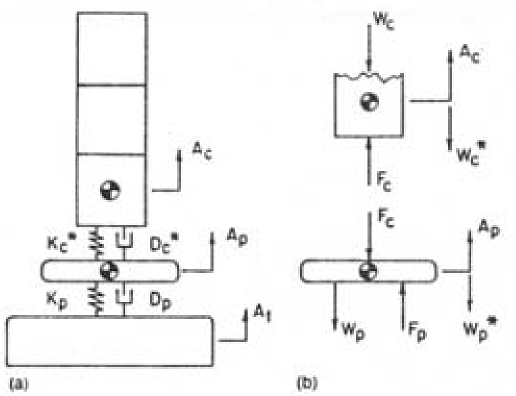

FIG. 3—*Schematic and free body diagrams of plate and stack combination:* (a) *Container stack and force plact on vibration table.* (b) *Free body diagrams of equivalent SDOF systems. All force vectors are assumed to pass through the center of dynamic mass.*

that calculating the RMS amplitudes was an objective approach to analyzing noisy and nonsinusoidal signals.

The response characteristics f_n and ρ and property W_c^* of the SDOF system, appearing on the right-hand side of Eq 5, can be considered as constants in a regression equation having f as its independent variable. The dependent variable Y would be given by the expression on the left. By incorporating Eq 5 into a curve-fitting program, the constants can thus be evaluated by fitting data defining Y as a function off. General nonlinear regression programs for doing this can be found among public domain software [5]. The values of generalized stiffness K_c^* and generalized damping D_c^* characterizing the stack are calculated from

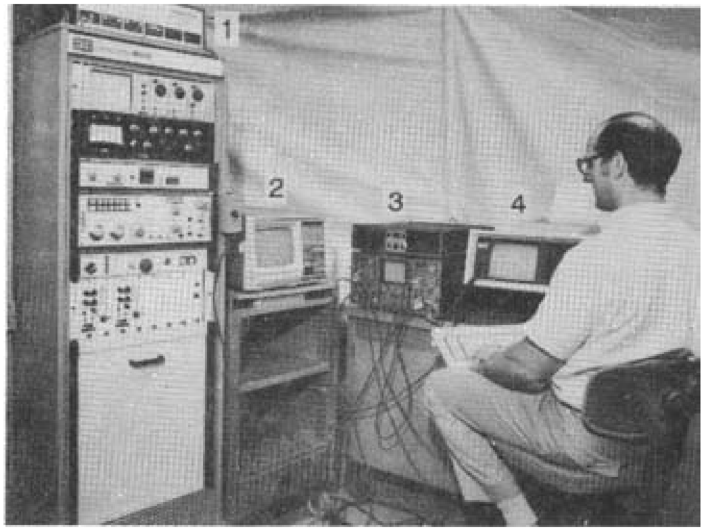

FIG. 4—*Instruments for controlling force plate vibration, acquiring data, and determining transmissibility response of specimen.* (1) *Analog load cell summing amplifier.* (2) *Frequency analyzer.* (3) *Multichannel digital storage oscilloscope for recording time domain data.* (4) *High speed microcomputer with digital-analogue conversion ability for vibration control and analysis of storage oscilloscope data.* (5) *Signal conditioners.*

$$K_c^* = 4(\pi f_n)^2\, W_c^*/g \tag{6}$$

$$D_c^* = 2\rho\sqrt{K_c^* W_c^*/g} \tag{7}$$

where f_n has units of hertz.

The response observed for a seven-tier stack and the dimen-

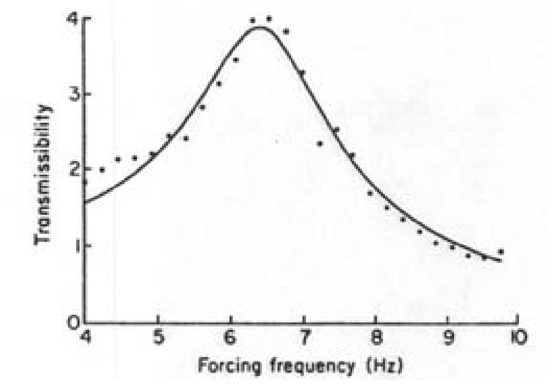

FIG. 5—*Form of output from container slack analysis. The points represent data. The curve is a plot of Eq 4 obtained after fitting Eq 5 to the data.* W_c *= 1147 N. Linear theory predicts* W_c^* *= 642 N,* f_n *= 6.51 Hz,* ρ *= 0.134,* K_c^* *= 109 kN/m, and* D_c^* *= 715 N s/m.*

sionless transmissibility curve defined by Eq 4 are compared in Fig. 5. This example represents a case where a stack was observed to behave according to linear vibration theory.

Conclusions

This paper presents a practical and theoretically consistent approach for characterizing the vibration response of stacked containers with the generalized properties of an equivalent single-degree-of-frreedom system. Acceleration and force signals acquired from a force plate are analyed by microcomputer software to determine the reaction force beneath thestack. Specimen stacks can be tested rapidly without the need to bury accelerometers among the products. Stacks can be compared in terms of generalized mass, stiffness, and damping as functions of the packaging design. An electrohydraulic vibration table is controlled via the signal obtained from an accelerometer attached to the force plate underside, thus minimizing interference caused by plate flexing. A plate stiff enough to test the effects of typical transportation frequencies on typical stack weights can be fabricated from inexpensive materials. The plate can be used to measure the average stiffness and damping of cushions under the actual loading condition imposed by the interior package and product. Our results can be extended to single load cell systems for testing small specimens.

References

[*1*] "Future Trends: Cushioning—The More You Know, The More You'll Save," *Packaging,* Aug. 1988. pp. 80–81.

[*2*] Urbanik, T. J., "Vibrational Loading Mechanism of Unitized Corrugated Containers with Cushions and Non-Load-Bearing Contents," *Shock and Vibration Bulletin,* Vol. 54 (Part 3), 1984, pp. 111-121.

[*3*] Brandenburg, R. K. and Lee. J. J., *Fundamentals of Packaging Dynamics,* MTS Systems Corporation, Minneapolis, Minn., 1985, Chapter 5.

[*4*] Harris, C. M. and Crede, C. E., *Shock and Vibration Handbook,* 3rd ed., McGraw-Hill, New York, 1988.

[*5*] Madison Academic Computing Center, NREG77-Fortran 77 Nonlinear Regression Routine, University of Wisconsin, Madison, 1985.