MECHANICAL NONLINEAR SHEAR WALL MODEL

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INTRODUCTION

Shear walls play a dominant role in the static and dynamic analysis and design of low-rise timber buildings. Shear wall behavior is often difficult to describe. For example, low-rise timber buildings have shear walls made of lumber framing and various sheathing materials. The load-deformation plot of a shear wall, when subjected to horizontal loads, is nonlinear. The analysis of a shear wall building thus requires either a full-scale test or nonlinear analysis. Full-scale tests do demonstrate true nonlinear behavior; however, they are often too time consuming and expensive for the number of replications necessary for a statistically valid study. Nonlinear analysis often involves mathematical complexity and instability such that limiting assumptions are necessary to simplify the problems.

This study describes a mechanical model which simulates the nonlinear behavior of a shear wall and aids in the construction of inexpensive models of entire shear wall buildings. Static and dynamic measurements obtained using these models can provide reliable results for shear wall buildings. Tests on these wall models will be useful to: (1) Observe nonlinear behavior of the "parent" structure without having to conduct full-scale tests; (2) check the accuracy of linear analysis; and (3) verify the results of nonlinear analysis.

DESCRIPTION OF MODEL

The mechanical model is a hinged square frame of rigid bars and stiffened by two linear diagonal springs (Fig. 1) of equal stiffness. Either a single- or multiple-frame model may be constructed, depending on the wall characteristics to be simulated.

The springs, which simulate the in-plane horizontal resistance of a diaphragm, are prestressed and, as a consequence, have a limited, predetermined capacity to carry compressive force. This compressive force capacity is actually a reduction in tensile force. Both diagonal springs resist low-magnitude lateral frame force, but as the compressive force in one of the springs exceeds its prestressing force, this spring buckles and

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Note. —Discussion open until May 1,1985. To extend the dosing date one month, a written request must be filed with the ASCE Manager of Technical and Professional Publications. The manuscript for this paper was submitted for review and possible publication on January 23, 1984. This paper is part of the *Journal of Engineering Mechanics*, Vol. 110, No. 12, December, 1984. ASCE, ISSN 0733-9399/84/0012-1773/$01.00. Paper No. 19314.

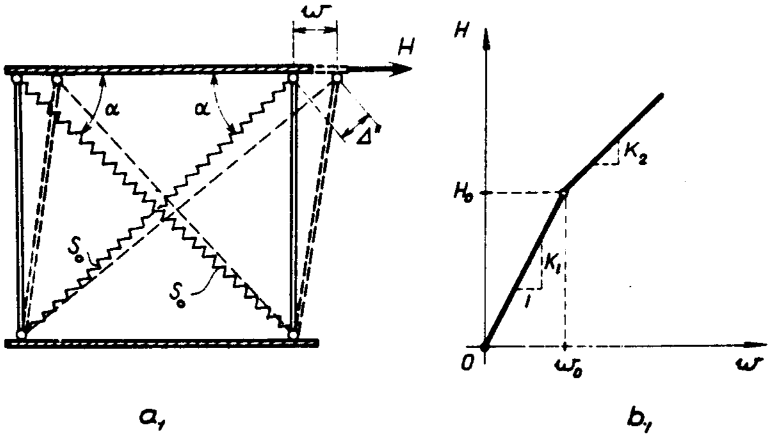

FIG. 1. —~~Sing~~le-Frame Mechanical Model with Springs of Stiffness *k* and Prestressing Force S_o

no longer resists lateral frame force. The horizontal stiffness of the frame diminishes to half its initial value and, consequently, the load-deformation plot has a bilinear form if changes of geometry are neglected. Inclusion of the changes of geometry causes the bilinear load-deformation curve to become continuously nonlinear.

Neglecting the effects of geometry changes and denoting, by S_o, the prestressing forces in the spring, the corresponding elongation of the springs, Δ_o and the horizontal displacement of the frame corresponding to spring buckling, w_o, can be obtained (Fig. 1):

$$\Delta_o = \frac{S_o}{k} \qquad (1a)$$

$$w_o = \frac{S_o}{k \cos \alpha} \qquad (1b)$$

in which *k* is the spring coefficient and a is the angle between the spring and horizontal. If the geometry changes between the deformed and undeformed configurations are neglected, a is constant. When the horizontal displacement of the frame, *w*, is less than w_o —~~the~~ frame deformation needed to relax one spring—both of the springs resist the lateral load, *H*. Consequently

$$H = K_1 w \qquad (2a)$$

$$K_1 = 2k \cos^2 \alpha \qquad (2b)$$

When $w > w_o$, one of the springs buckles and the horizontal stiffness of the frame diminishes to half its initial value (assuming both springs have equal stiffness). Thus

$$H = K_2 w \qquad (3a)$$

$$K_2 = k \cos^2 \alpha \qquad (3b)$$

The single-frame model may be extended in series to form a multiframe model which simulates a multilinear force-displacement shear wall

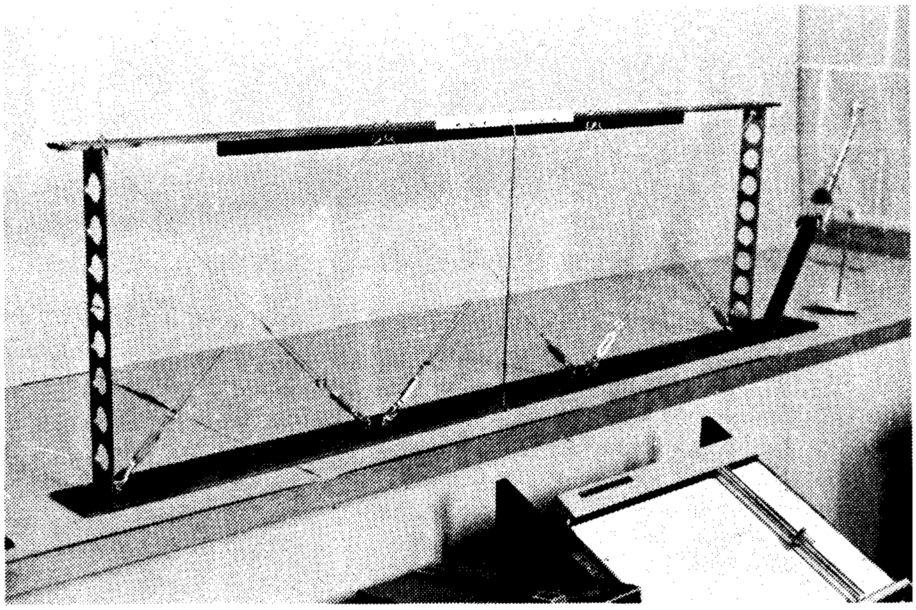

FIG. 2. —Multi-Frame Model Consisting of Three Single Frames

relationship. For example, consider a multi-frame model made of three single frames (Fig. 2). Assume the prestressing forces of the springs in frames 1, 2 and 3 are different although they fulfill the relationship $S_{01} < S_{02} < S_{03}$. Also assume the stiffness of each spring is equal. Eqs. 2 and 3 then become

$$H = 6kw\cos^2\alpha = K_1 w, \quad \text{if} \quad 0 \le w \le w_{01} \quad \text{(4a)}$$

$$H = 5kw\cos^2\alpha = K_2 w, \quad \text{if} \quad w_{01} \le w \le w_{02} \quad \text{(4b)}$$

$$H = 4kw\cos^2\alpha = K_3 w, \quad \text{if} \quad w_{02} \le w \le w_{03} \quad \text{(4c)}$$

$$H = 3kw\cos^2\alpha = K_4 w, \quad \text{if} \quad w_{03} \le w \quad \text{(4d)}$$

If geometry changes are considered, two additional effects are required for the H-w relationship. The first effect results from the angle

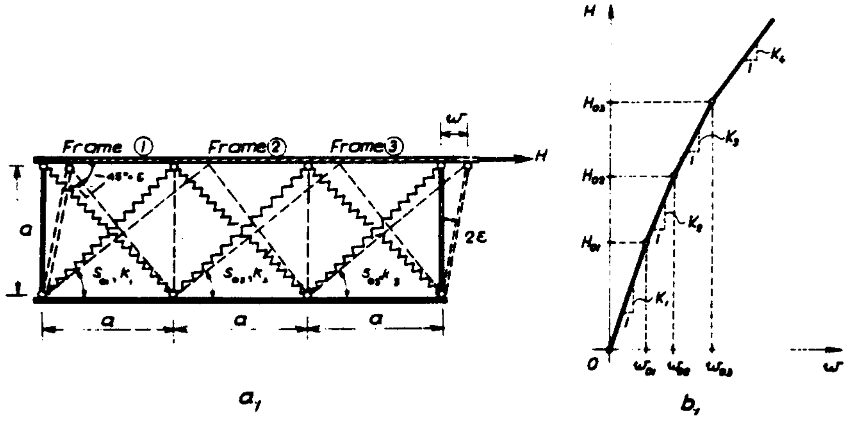

FIG. 3. — Deformed Three-frame Model Including Changes in Geometry

between the spring and the horizontal in the undeformed configuration, a, and in the deformed configuration, a + e, in which, for a single square frame with side dimension, a (Fig. 3)

$$\epsilon = \frac{1}{2}\sin^{-1}\left(\frac{w}{a}\right) \quad \dots\dots \quad (5)$$

The lengths of the two diagonals in the frame are also corrected to their deformed length.

The second effect is due to the weight of the top bar of the frame, W, acting through horizontal displacement, w, creating a secondary moment. The weights of the vertical frame members were neglected.

The relationship for an n-frame model considering geometry changes is derived in Ref. 1. By proper choice of the constants W_i, k_i, a_i, S_{oi}, and i, the stiffness diagram of the mechanical model can simulate the actual nonlinear load-displacement relationship of a shear wall.

EXPERIMENTAL STUDIES

A series of tests were conducted to investigate the static and dynamic responses of the models with different arrangements. A single-frame model ($n = 1$) and a three-frame model ($n = 3$) were used. The models had square dimensions ($a = 45°$). The spring stiffnesses, k_i; the prestressing forces of the springs, s,; and the vertical weight of the top bar of the frame, W_i ; were varied. The frames were subjected to static lateral loading to measure their stiffness characteristics and then were vibrated to determine their natural frequencies. A total of 59 different cases were investigated (1).

A typical load-displacement curve is given in Fig. 4 and a typical free vibration diagram in Fig. 5.

REVIEW

Load-displacement relationships analytically determined are compared to experimental results (Fig. 4). The agreement is generally good. The experimental results were always larger than those obtained by theory due to the friction at the frame connections. This effect was negligible at horizontal deformations less than w, .

Changing the frame parameters allows simulation of a great variety of shear walls and diaphragms. Increasing the prestressing force, S_o (and thus w_o), the linear part of the stiffness diagram is extended. Applying additional frame weight, W, flattens the stiffness diagram. Changing the spring stiffness, k, changes the initial stiffness of the frame and influences the nonlinear part of the diagram.

The single-frame model is generally adequate for simulating shear wall stiffness. The three-frame model is even more suitable because of the larger number of parameters that can be varied; however, it entails work of greater complexity.

The model also simulates the nonlinear dynamic response (Fig. 5) of shear walls and diaphragms with different stiffness characteristics. It was observed that, at amplitudes less than w_o, the model performed with nearly linear vibrations and the natural frequencies were constant. When

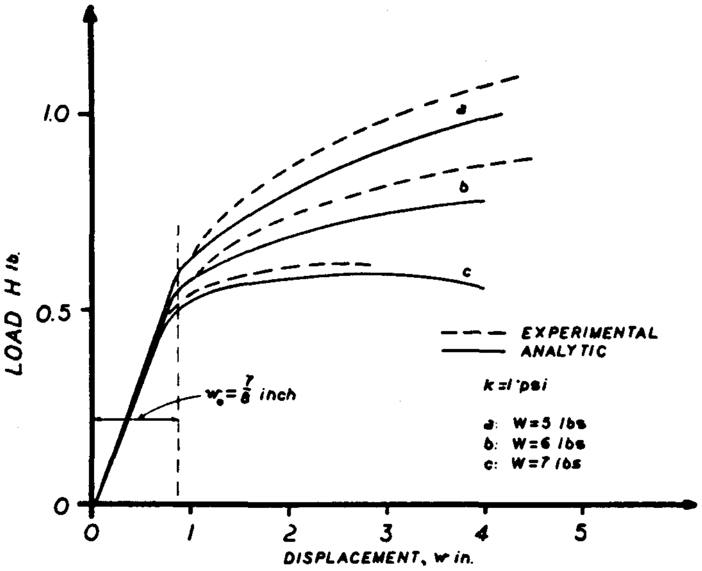

FIG. 4. —Typical Lateral Force-Displacement Relationship Experimentally Determined (1 lb = 0.45 kg; 1 in. = 2.5 cm)

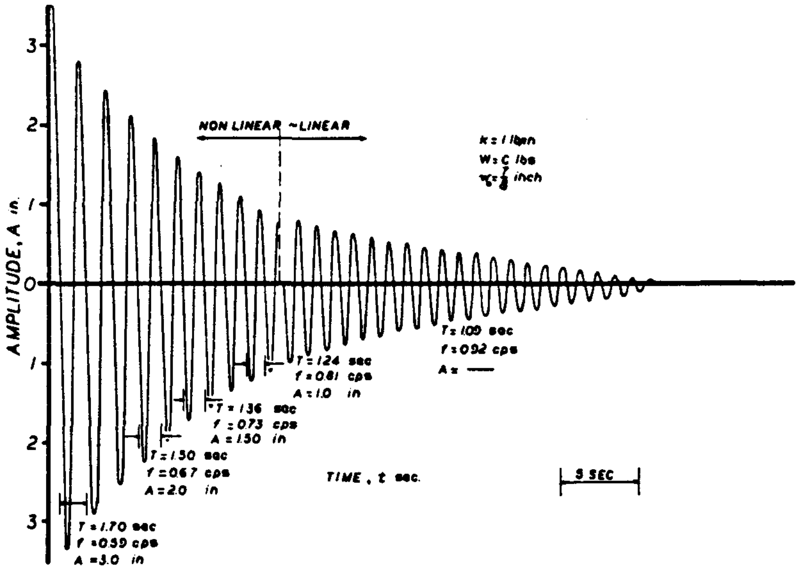

FIG. 5. —Typical Free Vibration Response

amplitudes exceeded w_o, the vibrations became nonlinear. As the amplitudes increased, the natural frequencies decreased due to the decreasing frame stiffness.

CONCLUSION

The individual single- and multi-frame models simulate very well the linear and nonlinear behavior of shear walls and diaphragms. Therefore they can form the elements of experimental models of entire shear wall structures. In these models, the floors can be replaced by rigid plates, while each vertical shear wall is simulated by a single or multi-frame. These models are suitable for the experimental investigation of the linear and nonlinear, static or dynamic response of shear wall structures.

APPENDIX.—REFERENCE

1. Naik, T. R., "A Model for Simulating the Nonlinear Behavior of Shear Walls," Research Report to the USDA Forest Prod. Lab. from the College of Engineering and Applied Science, Univ. of Wisconsin, Milwaukee, Wisc., Sept., 1982.