SHEAR STRENGTH IN PRINCIPAL PLANE OF WOOD

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ABSTRACT: **In this study, the writers used Tsai and Wu's tensor polynomial theory to rederive a formula originally derived by Cowin for shear strength variation in a plane of material symmetry of orthotropic materials. Experimental work was then performed (based on a method by Arcan, Hashin, and Voloshin) on Sitka spruce specimens to verify the formula. The theory and experiment agreed closely. When the angle between shear force and grain on the longitudinal-tangential plane increased from 0-30° shear strength decreased more than 43%. Engineers should take this relationship into account when designing wood structural members.**

INTRODUCTION

In this study, the writers used Tsai and Wu's (8) tensor polynomial theory to rederive a formula originally derived by Cowin (3) for shear strength variation in a plane of material symmetry of orthotropic materials. We then performed experimental work on Sitka spruce specimens to verify the formula.

Background. —Wood may be described as an orthotropic material with unique and independent mechanical properties in the directions of three mutually perpendicular axes—longitudinal, radial, and tangential. These axes, called the principal axes, are shown in Fig. 1. The longitudinal axis L is parallel to the fiber grain; the radial axis R is perpendicular to the grain and normal to the growth rings; and the tangential axis T is perpendicular to the grain and tangent to the growth rings. The planes formed by these axes are planes of material symmetry or principal planes.

A structural member made of wood, such as a rectangular beam or column, also has three geometrical axes parallel with the edges of the member. Stresses acting in the member are usually described in terms of the geometrical axes, whereas mechanical properties of wood are always described in terms of its principal axes. The two sets of axes are not always parallel, producing intersections between the grain (longitudinal axis) and the sides of the member—a condition called cross grain. (Cross grain occurs because the grain often forms a somewhat helical pattern or because of tree crook and sweep.) Therefore, for engineering applications, it is necessary to evaluate the mechanical properties with respect to the geometrical axes of the member.

Literature. — Studies of the strength properties of wood as a function of grain angle are sparse and rare in the literature. Compressive strength of wood at angles to the grain was first investigated by Hankinson (5). The Hankinson formula, while being strictly empirical, has been found by Norris (6), Goodman and Bodig (4), and Cowin (3) to fit their ex-

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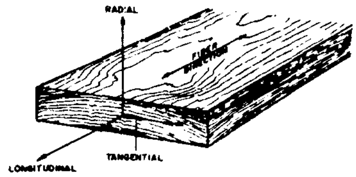

FIG. 1.—Principal Axes of Wood

perimental data reasonably well. In his effort to derive the Hankinson formula from the Tsai-Wu theory (8), Cowin (3) obtained a formula for shear strength in a plane of symmetry of an orthotropic material. At that time, the only experimental data on wood in the literature to verify his formula were the data on pine by Ashkenazi (2). Cowin found that his formula could represent the general trend of Ashkenazi's data. When Ashkenazi performed his shear strength tests on pine, no shear test method was known that could cause wood specimens to fail in pure shear. This fact coupled with the inhomogeneity of wood properties can be used to explain the relatively poor comparison between theory and experiment in Ref. 3.

Cowin followed the conventional approach of decomposing the applied stresses with respect to the material axes to obtain his shear strength formula. However, according to Tsai and Wu (8), their theory also permits transformation of the material axes with respect to the applied stresses to obtain a solution-an approach used in the present study to achieve a more compact derivision of Cowin's result. Experimental work performed in this study to verify the formula derived from the Tsai-Wu theory to predict shear strength variation in a plane of material symmetry of wood was based on a method by Arcan et al. (1).

METHODOLOGY

In this study, we derived a formula for predicting shear strength of wood as a function of grain angle and conducted tests to verify the formula.

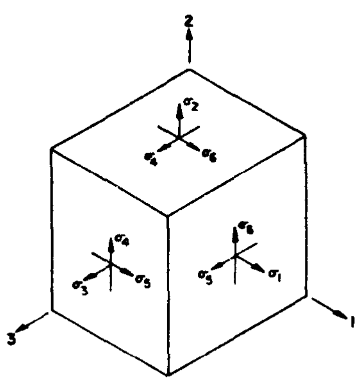

FIG. 2.—Positive Normal and Shear Stresses

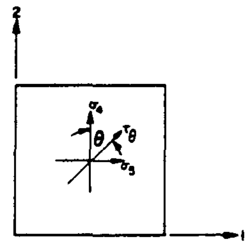

FIG. 3.—Shear Stresses on 1-2 Plane

Formula for Shear Strength in Plane of Material Symmetry. — In deriving the formula for shear strength, we used the coordinate system and notation used by Tsai and Wu (8). The positive stresses on the planes of material symmetry are shown in Fig. 2. Consider the shear stress τ_θ acting on the 1-2 plane in Fig. 3. Because this plane is a plane of material symmetry, the shear strength must remain the same if the sense of τ_θ is reversed. Therefore, the quadratic strength theory in Ref. 8 cannot contain the linear term of τ_θ. It can be put in the following form

$$F'_{44}\bar{\tau}_\theta^2 = 1 \quad \text{(1)}$$

in which $\bar{\tau}_\theta$ = shear strength on the θ plane; and F'_{44} is a component of a strength tensor of the fourth rank. As shown in Tsai and Wu (8), F'_{44} can be transformed in terms of θ as follows

$$F'_{44} = \frac{F_{44}(1 + \cos 2\theta)}{2} + \frac{F_{55}(1 - \cos 2\theta)}{2} = F_{44}\cos^2\theta + F_{55}\sin^2\theta \quad \text{(2)}$$

in which F_{44} and F_{55} are components of a strength tensor associated with the square of σ_4 and σ_5, respectively, in the theory.

Substituting Eq. 2 into Eq. 1 yields

$$\bar{\tau}_\theta^2 = \frac{1}{F_{44}\cos^2\theta + F_{55}\sin^2\theta} \quad \text{(3)}$$

Let $\bar{\sigma}_4$ and $\bar{\sigma}_5$ be the shear strengths corresponding to the shear stresses σ_4 and σ_5, respectively. At q = 0, $\bar{\tau}_\theta = \bar{\sigma}_4$, and $F_{44} = 1/\bar{\sigma}_4^2$; at $\theta = \pi/2$, $\bar{\tau}_\theta = \bar{\sigma}_5$, and $F_{55} = 1/\bar{\sigma}_5^2$. Thus, Eq. 3 reduces to

$$\bar{\tau}_\theta^2 = \frac{\bar{\sigma}_4^2\bar{\sigma}_5^2}{\bar{\sigma}_5^2\cos^2\theta + \bar{\sigma}_4^2\sin^2\theta} \quad \text{(4)}$$

which agrees with the formula obtained by Cowin (3). The square of the shear strength follows a Hankinson-type strength criterion.

Eq. 4 should hold for shear stresses acting on any plane of material symmetry. In the experimental work described in the following section, the 2-axis will be identified with the longitudinal axis and the l-axis will be identified with the tangential axis in Fig. 1. τ_θ will then act on the longitudinal-tangential plane of the wood specimens.

In this study, experimental data for $\tau = 0°$ ($\bar{\sigma}_4$) and $\theta = 90°$ ($\bar{\sigma}_5$) were used in Eq. 4 to plot the expected shear strength for all angles between $\theta = 0°$ and $\theta = 90°$ (Fig. 6).

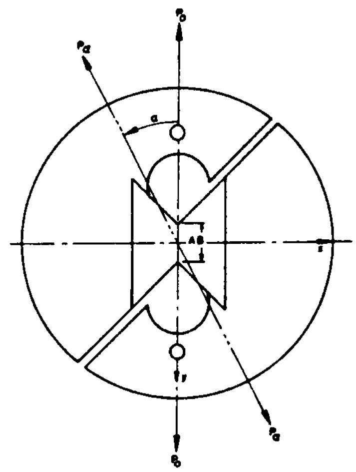

FIG. 4.—Test Fixture and Specimen

Experimental Procedures.—We prepared the test fixture and the specimen following Arcan, Hashin, and Voloshin (1), as shown in Fig. 4.

Fixture. —The test fixture is composed of two identical pieces of aluminum which together form a circular disk with antisymmetric cutouts. The straight sides of each portion of the disk are either perpendicular or oriented at 45° to the coordinate axes in Fig. 4. The fixture is 10 in. in diam, 1/2 in. thick, and weighs 2.6 lb.

Specimen. —The specimens were bow-tie-shaped pieces from a Sitka spruce board 3 in. by 10 in. by 10 ft in size with the 10-in. dimension being in the radial direction. The bow tie shape is created by two 90° notches. The complete history of the board is not known, but it had been in outdoor storage under roof for several years at the Forest Products Laboratory. It was free of visible drying defects and was at approximately 12% moisture content. The critical section of the specimen is the narrowest section at the center AB in Fig. 4. The normal and shear stresses in the cross section at AB are

$$\sigma_x = \sigma_y = \frac{P_\alpha \sin \alpha}{A} \quad (5)$$

$$\tau = \frac{P_\alpha \cos \alpha}{A} \quad (6)$$

in which P_α is the applied load at an angle α from AB and the cross-sectional area at AB is *A*. When α is zero, the normal stresses vanish and a state of pure shear exists at AB.

The section at AB of the specimen has a length of 1 in. and a thickness of 1/2 in.; the width of the specimen perpendicular to AB is 2-1/2 in. The cross section was on the longitudinal-tangential plane at a specified angle θ to the grain. Four angles were selected, viz, 0° (parallel to grain shear), 30°, 60°, and 90° (rolling shear), to verify Eq. 4. After the specimens had been fabricated, they were stored in an environmental room with constant 74° F temperature and 56% relative humidity.

Assembly and Loading. —*The* specimens removed from the environmental room were bonded to the fixture with a commercially available epoxy adhesive and secured in place for at least 12 hr for the adhesive to be cured in the test room.

The tensile loads were applied to failure with an Instron test machine with $\alpha = 0$ in Fig. 4. After each test, the failed specimen was removed and the aluminum fixture cleaned to be used again.

Results and Conclusions

Experimental Results. —Fig. *5(a)* shows a failed specimen with AB at 0° to the grain. The typical failure mode is shown in Fig. *5(b)* as the angle between AB and the grain increased to more than 30°. In the specimen shown in Fig. *5(b),* failure started at the two ends of section AB because of shear and then followed more or less the radial direction toward the two long sides of the specimen. The direction of failure propagation could be attributed to distortion of cell cross sections, stress concentrations at ray tissue, early-latewood boundaries, effect of increasing growth ring curvature, and other anatomical features which led to the separation of the primary bonds in the radial direction. However, the shear strength of section AB is dependent only on the initiation of failure at the section and not on the direction of fracture propagation after failure. The data obtained can, therefore, be used to calculate the shear strength along the critical section of the specimen according to Eq. 6 with α equal to zero.

The experimental results are presented in Table 1 and Fig. 6. The coefficient of variation for each sample is about 15–20%, which is common in the evaluation of mechanical properties of wood (7).

The shear strength decreased drastically with the angle to grain for small values of the angle. When the angle changes from 0°-30°, it dropped from 906 psi (6.25 MPa) to 513 psi (3.54 MPa), a decrease of more than 43%.

Comparison of Theoretical and Experimental Values. —The mean values of the test data at θ = 30° and 60° agree closely with the predictions

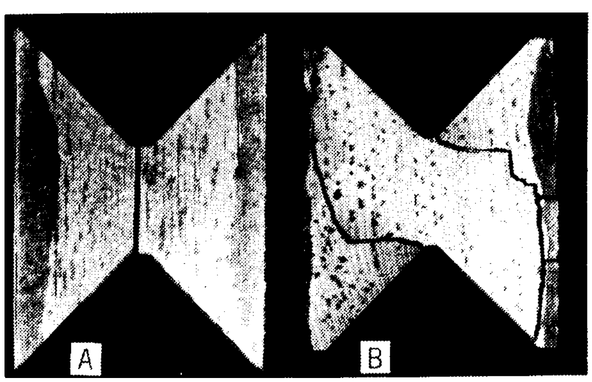

FIG. 5.–Failed Sitka Spruce Specimens with Long Edge of Critical Section: *(a)* Parallel to Grain; *(b)* Perpendicular to Grain, in Longitudinal-tangential Plane

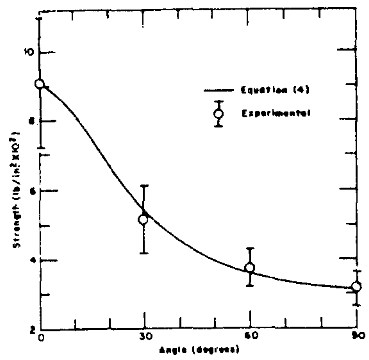

FIG. 6.—Mean and Standard Deviation Of Shear Strength versus Angle to Grain in Longitudinal-Tangential Plane of Sitka Spruce Specimens

TABLE 1.—Shear Strength in Longitudinal-Tangential Plane of Sitka Spruce Specimens

Angle between shear force and grain, θ, in degrees (1)	Number of tests (2)	Average shear strength, in pounds per square inch (3)	Range, in pounds per square inch (4)	Coefficient of variation, as a percentage (5)
0	25	906	602–1,288	20.6
30	10	513	341–689	19.1
60	10	372	290–456	14.9
90	10	315	238–407	16.0

Note: 1 psi = 6.8947 kPa. Average moisture content from 10 separate specimens from same board was 11.8%; average specific gravity from 10 separate specimens from same board was 0.38.

of Eq. 4 (Fig. 6). (In this comparison, the theoretical predictions at θ = 30° and 60° are, of course, based on the mean values at θ = 0° and 90° from the test data.) This close agreement goes far beyond Cowin's (3) finding that the Tsai-Wu theory predicts the general trend of the data. In fact the data of this study are closely predicted by the formula at both θ = 30° and θ = 60°.

Conclusions

The close correspondence between the experimental and theoretical results in this study demonstrates that this formula has potential for use in predicting shear strength of wood in a variety of cross-grain situations. The magnitude of the decrease in shear strength with increasing grain angle clearly demonstrates the importance of the effect of grain angle on the shear strength of wood, a factor that has so far received little attention from the practicing wood engineer. Because cross grain cannot be avoided in structural wood members, the information protided in this report should prove to be valuable to the practicing wood engineer as well as the wood grading agency.

Appendix I.—References

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Appendix II.—Notation

The following symbols are used in this paper:

A = cross-sectional area;
F_{ii} = strength tensors of fourth rank;
P_α = applied load along angle α;
α = angle from grain in L-R plane;
θ = angle from grain in L-T plane;
σ_i = stress components;
$\bar{\sigma}_i$ = strength components; and
$\bar{\tau}_\theta$ = shear strength along angle θ.