

## CRITERION FOR MIXED MODE FRACTURE IN WOOD

By S. Mall,<sup>1</sup> Joseph F. Murphy,<sup>2</sup> M. ASCE, and James E. Shottafer<sup>3</sup>

**ABSTRACT:** The mixed mode fracture failure of eastern red spruce (*Picea rubens*) was investigated by means of single edge notch and center crack specimens with various crack inclinations in the tangential-longitudinal plane with propagation in the longitudinal direction (i.e., TL system of crack propagation). These results demonstrate there is a definite interaction between failure stress intensity factors  $K_I$  and  $K_{II}$  during the mixed mode fracture of wood. These mixed mode data were supplemented by the fracture toughness ( $K_{IC}$ ) and pure mode II critical stress intensity factor ( $K_{IIC}$ ). Several mixed mode fracture failure criteria were compared. The criterion that could not be rejected is  $K_I/K_{IC} + (K_{II}^2/K_{IIC}^2) = 1$ .

### INTRODUCTION

The concepts of linear elastic fracture mechanics for investigating the fracture failure in wood have received a considerable amount of attention in the past several years (2,3,12,14). Recently, the first international meeting on "Wood Fracture" attracted 15 papers on this subject (4). The greatest amount of work reported in this context has been primarily concerned with the fracture of wood in the opening mode, Mode I. These studies have demonstrated that fracture toughness,  $K_{IC}$ , is a geometry-independent material property of wood. The observed scatter in the fracture toughness is expected and consistent with the other material properties of wood. Wood is a natural material and the tree is subjected to numerous and constantly changing influences, such as moisture, soil conditions, and growing space. The fracture failure of wood for sliding mode, mode II, has been investigated by Barrett and Foschi (5). They studied this phenomenon by employing small and large beams of western hemlock (*Tsuga heterophylla*) slit along the neutral plane and loaded in bending. Their results showed that a pure mode II critical stress intensity factor,  $K_{IIC}$ , is also a material property. Wood structural members are, however, often subjected to the complex loading condition that results in mixed mode fracture failure. Available information on wood fracture under the combined mode loading condition is limited, and it will be reviewed briefly in the following.

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<sup>1</sup>Asst. Prof., Mech. Engrg. Dept., Univ. of Maine, Orono, Me. 04469.

<sup>2</sup>Engr., Forest Production Lab., Forest Service, U.S. Dept. of Agr., Madison, Wisc. 53705.

<sup>3</sup>Prof., Forest Production Lab., Univ. of Maine, Orono, Me. 04469.

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The subject of the mixed mode fracture in wood was first studied by Wu (18). He conducted combined loading tests to produce mode I and II in center-notched plates of balsa wood (assumed *Ochroma pyranidale*) where the crack was collinear with the grain direction. Wu found the interaction between stress intensity factors  $K_I$  and  $K_{II}$  and proposed the criterion for the mixed mode fracture of the form

$$\frac{K_I}{K_{IC}} + \left( \frac{K_{II}}{K_{IIC}} \right)^2 = 1 \dots\dots\dots (1)$$

From experimental results of butt joints in pine (*Pinus radiata*), Leicester (13) proposed (as conservative) the following interaction between stress intensity factors  $K_I$  and  $K_{II}$  under the mixed mode stress conditions:

$$\frac{K_I}{K_{IC}} + \frac{K_{II}}{K_{IIC}} = 1 \dots\dots\dots (2)$$

Later, Williams and Birch (14) studied this phenomenon in Utile and Scots pine (*Entandrophragma utile* and *Pinus sylvestris*) utilizing straight and angled edge cracked specimens. They concluded, however, that the shear stress causing sliding mode has no effect on the failure in the mixed mode and proposed the following criterion for the failure:

$$\frac{K_I}{K_{IC}} = 1 \quad (K_{II} \geq 0) \dots\dots\dots (3)$$

Recently, Woo and Chow (17) investigated the mixed mode fracture in two species of wood, Kapur and Gagil (*Dryobalanops* spp. and *Hope segal*), with the help of single-edge-notch and center-crack specimens. Their results showed mixed mode fracture depends on both  $K_I$  and  $K_{II}$  stress intensity factors. They could not, however, determine any fracture criterion as expressed by Eqs. 1-3. This can be attributed to the lack of information regarding the mode II critical stress intensity factor in their experiments.

It seemed, therefore, a useful exercise to further explore the mixed mode fracture phenomenon of wood, particularly since many experimental results are necessary in order to develop a reliable fracture criterion. This will also be the first step towards the development of a design procedure for wood structural members, based on fracture failure, subjected to complex loading conditions. The purpose of this paper is to present our investigation of the mixed mode fracture of wood. The objective of our study was to investigate this phenomenon in a locally available wood to develop a mixed mode fracture criterion based on experimental results.

#### EXPERIMENTAL PROCEDURE

Wood is characterized as a cylindrically anisotropic material with three principal elastic directions: Longitudinal (L); radial (R); and tangential (T) directions. A rectangular piece, cut a distance from the center of the tree stem, is thus idealized as an orthotropic plate. The direction of crack propagation in an orthotropic plate is conventionally referred to in terms

of two principal elastic directions; the first indicating the direction normal to the crack surface and the second the direction of crack propagation. This leads to six possible principal systems of crack propagation (11), but in only four cases, TL, TR, RL, and RT, is coplanar crack propagation possible. The present study, however, was restricted to the TL system of crack propagation. Ten flat-sawn boards of eastern red spruce (*Picea rubens*) were purchased locally in the kiln-dried condition and were further conditioned in a humidity room at 22° C (68° F ± 5° F) and 65% relative humidity (ASTM D 143-78 (1)), resulting in a nominal equilibrium moisture content of 12% (±1%). These boards were straight-grained with the dominant system of propagation being TL.

Three types of specimens were selected to determine the critical stress intensity factors for pure opening mode I ( $K_{Ic}$ ), pure sliding mode II ( $K_{IIc}$ ), and mixed mode situation ( $K_I - K_{II}$ ). Figure 1 shows the configuration of the single-edge-notch specimen ( $2L = 203$  mm,  $2W = 64$  mm, thickness = 6 mm), employed in the present investigation. Experiments with six sets of this specimen having crack inclinations,  $\theta_r$ , of 0°, 30°, 45°, 70°, 80°, and 85° were conducted. For each set, a total of 10 specimens were machined from 10 boards (one from each board). The crack length,  $a$ , was 25.4 mm in all specimens, and the crack was collinear with the grain direction. The crack was carefully oriented along the grain direction, and this was further verified by checking the fracture angle which

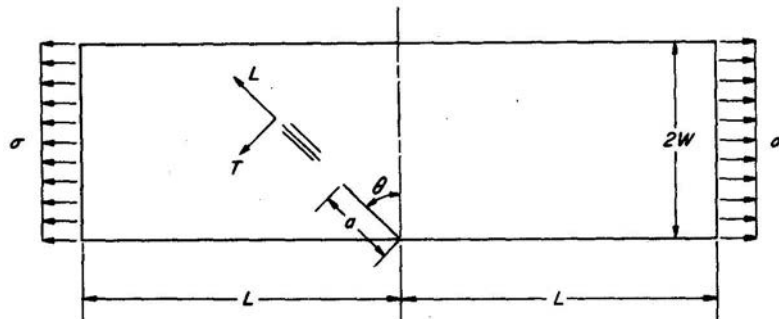


FIG. 1. —Single Edge Notch Specimen

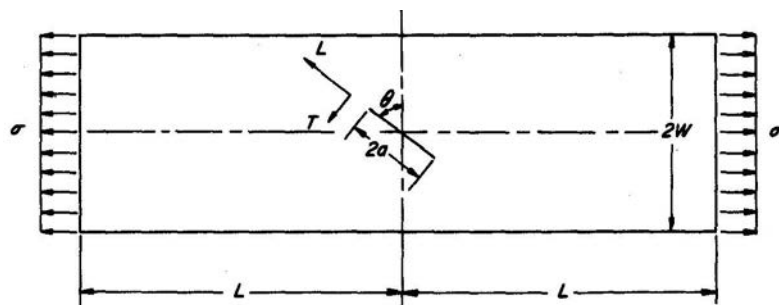


FIG. 2. —Center Crack Specimen

was always within  $\pm 1^\circ$  of the nominal crack inclination. The crack was cut with a very fine razor blade. The specimen with the crack perpendicular to the edge ( $\theta = 0^\circ$ ) provided the fracture toughness  $K_{IC}$ , and the inclined crack provided the failure stress intensity factors  $K_I$  and  $K_{II}$  in the presence of tensile and shear stresses.

A second series of tests utilized the center crack specimen as shown in Fig. 2. Overall dimensions of this specimen were similar to those of the single-edge-notch specimen ( $2L = 203$  mm,  $2W = 64$  mm, thickness = 6 mm). Five sets of this specimen having crack inclinations,  $\theta$ , of  $45^\circ$ ,  $60^\circ$ ,  $75^\circ$ ,  $80^\circ$ , and  $85^\circ$  were selected. These inclinations were chosen to overlap and extend the data obtained from the single-edge-notch specimen. The maximum ratios of  $K_{II}/K_I$  were 2.35 and 7.27 from single-edge-notch and center-crack specimens, respectively. The crack was again 25.4 mm long, and collinear with the grain direction. Fifty specimens were obtained from 10 boards, one from each board for each angle tested.

The critical mode II stress intensity factor,  $K_{IIC}$ , was determined from the compact shear specimen as proposed by Chisholm and Jones (10). The geometry of the compact shear specimen, employed in the present study, is shown in Fig. 3 where  $L = 140$  mm,  $W = 102$  mm,  $H = 25.4$  mm, and  $B = 6$  mm. The crack ( $a = 64$  mm) was machined with a jeweler's slotting cutter (thickness = 0.5 mm) on the milling machine, and the crack tip was finished with a razor-blade incision. The crack was collinear with the grain direction. Ten specimens, one from each board, were tested by employing a fixture similar to that of Chisholm and Jones (10).

All testing was done in the humidity room with an Instron Universal testing machine. All specimens were tested at a crosshead speed of 0.5 mm/s. The load-displacement relation, recorded in all experiments, was linear up to the critical fracture load for all specimens. The failing stress intensity factors for single-edge-notch and center-crack specimens were computed from

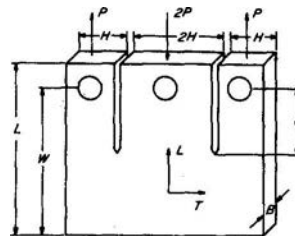
$$K_I = Y_1 \sigma \sqrt{\pi a} \cos^2 \theta \dots\dots\dots (4)$$

$$K_{II} = Y_2 \sigma \sqrt{\pi a} \cos \theta \sin \theta \dots\dots\dots (5)$$

and, in the case of the compact shear specimen

$$K_{II} = F \sigma \sqrt{\pi a} \dots\dots\dots (6)$$

in which  $\sigma$  is applied as gross stress.  $Y_1$ ,  $Y_2$ , and  $F$  are the geometric



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FIG. 3. —Compact Shear Specimen

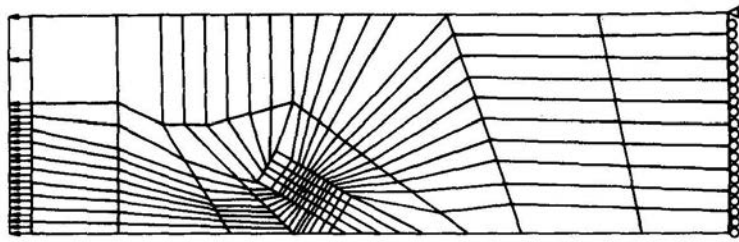


FIG. 4. — Typical Finite Element Breakdown for Single Edge Notch Specimen

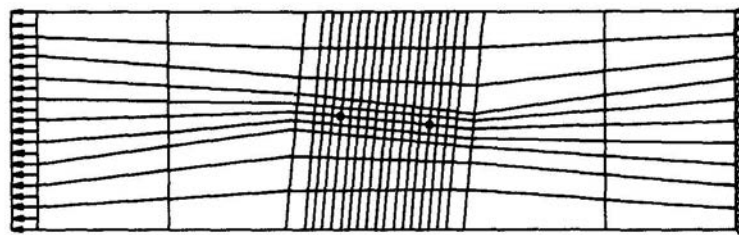


FIG. 5. — Typical Finite Element Breakdown for Center Crack Specimen

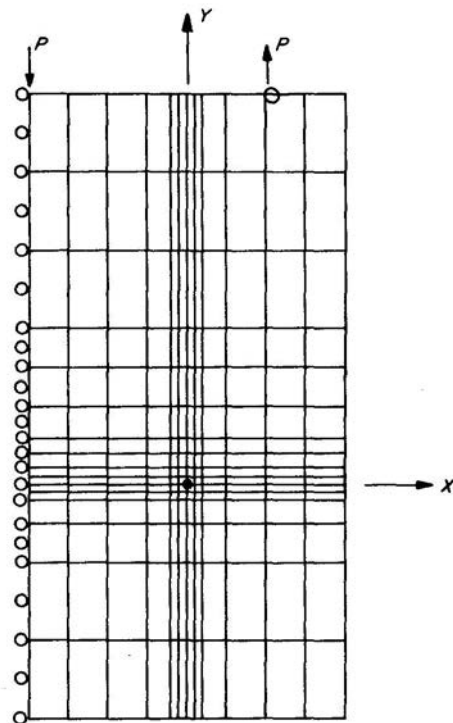


FIG. 6. — Finite Element Breakdown for Compact Shear Specimen (the Crack Tip was Pinned in the Y-Direction to Eliminate any Rigid-Body Movement)

correction factors that were computed from the finite element method as described in the next section.

### FINITE ELEMENT ANALYSIS

A finite element analysis was conducted to evaluate finite geometry correction factors for the previously mentioned three specimens. Typical finite element grids for single edge notch and center crack specimens are shown in Figs. 4 and 5, respectively. The finite element formulation utilized primarily eight-node quadrilateral elements along with six-node triangular elements, if required. The four-rectangular elements surrounding the crack tip were modified to incorporate the required inverse square root singularity characteristics of linear elastic fracture mechanics. This singularity was achieved by placing the mid-side node of the side connected to the crack tip at the quarter position adjacent to the crack tip (6). These finite element meshes consisted of a fine grid region near the crack tip where the size of the typical element was 1.3 mm x 1.3 mm. The symmetry about the center line of the double shear specimen allowed gridding of only half of the specimen, as shown in Fig. 6.

The stress intensity factors  $K_I$  and  $K_{II}$  were computed with the displacement method (9) from nodal point displacements on the crack surfaces generated from the finite element analysis using the classical crack tip displacement equations. The finite element analyses were first checked with the known solutions for the isotropic material. Analyses of single edge notch specimens for each crack inclination investigated were verified with the available solution by Bowie (8). Analyses of central crack specimens with crack inclinations of  $45^\circ$  and  $75^\circ$  were compared with Wilson's solution (16). The compact shear specimen's solution was compared with the boundary-collocation method (10). The present finite element analyses yielded stress intensity factors that were within  $\pm 5\%$  of their counterparts obtained by previous investigators. Then these finite element analyses were extended for the present orthotropic cases. The material properties of eastern red spruce employed in finite element analyses were: (1) Elastic modulus in longitudinal direction,  $E_L = 12,721$

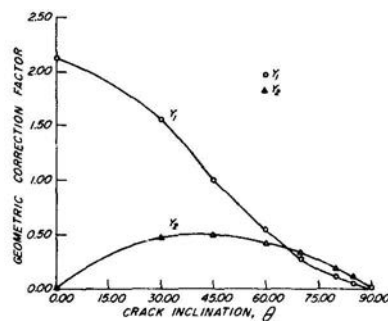


FIG. 7. — Geometric Correction Factors  $Y_1$  and  $Y_2$  (from Eqs. 4 and 5) for Single Edge Notch Specimen

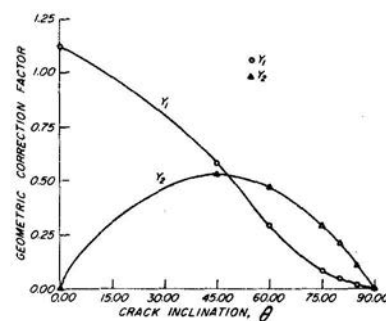


FIG. 8. — Geometric Correction Factors  $Y_1$  and  $Y_2$  (from Eqs. 4 and 5) for Center Crack Specimen

MPa; (2) elastic modulus in transverse direction,  $E_T = 629$  MPa; (3) shear modulus in the  $LT$  plane,  $G_{LT} = 743$  MPa; and (4) Poisson's ratio,  $\nu_{LT} = 0.42$ . These elastic parameters were computed from regression equations as proposed by Bodig and Goodman (7). The average specific gravity required for these regression equations was experimentally determined to be 0.42. Figures 7 and 8 show relations between geometric correction factors,  $Y_1$  and  $Y_2$ , as back calculated from Eqs. 4 and 5, and the crack inclination,  $\theta$ , for single edge notch and center crack specimens, respectively. The geometric correction factor for the compact shear specimen investigated here, and to be used in Eq. 6, is

$$F = 0.57 \dots\dots\dots (7)$$

**RESULTS AND ANALYSIS**

The failure stress intensity factors  $K_I$  and  $K_{II}$ , obtained from all specimens with various crack inclinations, are summarized in Table 1. Shown are the average values of critical stress intensity factors,  $K_I$  and  $K_{II}$ , coefficient of variation (COV) i.e., ratio of standard deviation to average value, and sample size. The observed variation in  $K_I$  and  $K_{II}$  is reasonable as compared to other mechanical properties of wood. The critical mode I and II stress intensity factors,  $K_{IC}$  and  $K_{IIC}$ , for eastern red spruce in the TL system of crack propagation were  $0.42\text{MPa}\sqrt{\text{m}}$  and  $2.18\text{MPa}\sqrt{\text{m}}$ , respectively. This species of wood has not been previously tested for these fracture mechanics parameters. However, present values of  $K_{IC}$  and  $K_{IIC}$  are comparable with their counterparts obtained for other species;  $K_{IC}$  for the TL mode of Douglas-fir (*Pseudotsuga menziesii*) is about  $0.38\text{MPa}\sqrt{\text{m}}$ (3), and  $K_{IIC}$  for western hemlock (*Tsuga heterophylla*) is about  $2.20\text{MPa}\sqrt{\text{m}}$ (5).

**TABLE 1. —Failure Stress Intensity Factors  $K_I$  and  $K_{II}$**

Specimen type (1)	Crack inclination angle (2)	Sample size (3)	Failure $K_{I,}$ in $\text{MPa}\sqrt{\text{m}}$		Failure $K_{II,}$ in $\text{MPa}\sqrt{\text{m}}$	
			Mean (4)	Coefficient of variation (5)	Mean (6)	Coefficient of variation (7)
Single edge notch	0°	10	0.42	0.12		
	30°	10	0.41	0.16	0.12	0.16
	45°	9	0.43	0.10	0.22	0.10
	70°	8	0.40	0.12	0.49	0.12
	80°	8	0.39	0.15	0.64	0.15
Central crack	85°	10	0.37	0.11	0.87	0.11
	45°	10	0.38	0.09	0.35	0.09
	60°	10	0.37	0.14	0.59	0.14
	75°	8	0.32	0.18	1.10	0.18
	80°	9	0.27	0.12	1.19	0.12
Compact shear		8	0.21	0.09	1.53	0.09
		10			2.18	0.09

TABLE 2. — Lack of Fit Values for Different Failure Criteria

Failure criterion (1)	p value (2)
$K_I/K_{IC} = 1$	<0.0001*
$K_I/K_{IC} + K_{II}/K_{IIC} = 1$	<0.0001
$K_I/K_{IC} + (K_{II}/K_{IIC})^2 = 1$	0.5629
$(K_I/K_{IC})^2 + K_{II}/K_{IIC} = 1$	0.0784
$(K_I/K_{IC})^2 + (K_{II}/K_{IIC})^2 = 1$	<0.0001

\*In a one-way analysis of variance for equality of  $K_I$  (where  $K_{II} = 0$ ), the p value is still less than 0.0001.

To show the interaction between failure  $K_I$  and failure  $K_{II}$ , all the experiments of both single-edge-notch and center-crack specimens are plotted in Fig. 9. This figure also shows all of the experimental data of  $K_{IC}$  and  $K_{IIC}$  on the axes. The data in Fig. 9 define the functional relationship between  $K_I$  and  $K_{II}$  for the unstable crack extension in the presence of tensile and in-plane shear stresses.

A lack of fit test (11) was performed on the data assuming five different failure models. Statistical lack of fit values are tabulated in Table 2. Since both  $K_I$  and  $K_{II}$  are functions of the load, error was measured along a ray from the origin. To assume  $K_{II}$  an independent variable and  $K_I$  dependent, or vice-versa, is incorrect. As can be seen from Table 2 the one failure that cannot be rejected due to lack of fit is the model proposed by Wu (18). That is

$$\frac{K_I}{K_{IC}} + \left(\frac{K_{II}}{K_{IIC}}\right)^2 = 1 \dots\dots\dots (8)$$

This relation is replotted in Fig. 10 using the average values for  $K_{IC}$  and  $K_{IIC}$ . The conservative linear relation of Leicester (13) and the criterion proposed by Williams and Birch (15) are also plotted in Fig. 10. The present experiments on eastern red spruce corroborate the findings of Wu (18) on balsa. This may be a fortuitous coincidence for these two species of wood, or possibly a trend for a wide range of species. The obvious question regarding the existence of a universal criterion for mixed mode fracture, either for a wide range of species or for all species of wood, cannot be answered until data for various species are measured. Leicester's relation is a conservative failure equation. This study's data do not support the conclusions of Williams and Birch (15) who tested Utile and Scots pine. They could not measure  $K_{IIC}$  directly but calculated  $K_{IIC}$ .

There are two notable things regarding the mixed mode fracture in wood: (1) Wood is a highly anisotropic material [ $(E_l/E_r) > 15$ ]; (2) fracture in wood always occurs parallel to the grain direction. Thus, it may appear that failure in the mixed mode closely corresponds to the failure phenomenon where the presence of shear stresses has no effect, as elaborated by Williams and Birch (15). However, the present investigation, as well as the previous works of Wu (18) and Woo and Chow (17), have clearly demonstrated a definite interaction between  $K_I$  and  $K_{II}$  during the mixed mode fracture of wood.

**SUMMARY**

Ten edge notch specimens were tested in tension to measure fracture toughness on critical mode I stress intensity factor,  $K_{Ic}$ , of eastern red

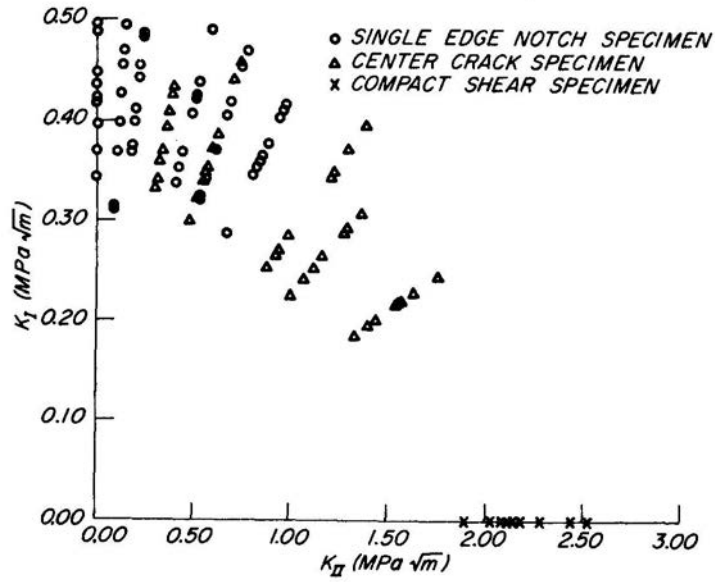


FIG. 9.—Interaction Between  $K_I$  and  $K_{II}$

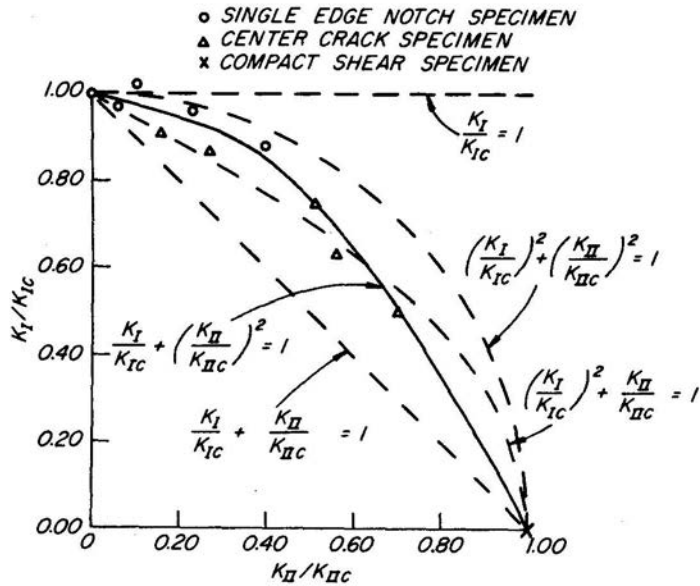


FIG. 10.—Relation Between  $K_I/K_{Ic}$  and  $K_{II}/K_{IIc}$  (Mean Values of Data)

spruce. Ten compact shear specimens were tested to measure the critical mode II stress intensity factor,  $K_{IIc}$ . Ninety edge notch and central crack specimens with crack angles from  $30^\circ$ - $85^\circ$  were tested in tension to evaluate five mixed mode fracture failure criteria. In all tests, crack propagation was in the longitudinal direction in the longitudinal-radial plane.

**CONCLUSIONS**

1. The fracture toughness,  $K_{Ic}$ , for the opening mode in case of the TL system of crack propagation for eastern red spruce was found to be  $0.42 \text{ MPa}\sqrt{\text{m}}$ .

2. The pure mode II critical stress intensity factor,  $K_{IIc}$ , for the TL system of crack propagation for eastern red spruce was found to be  $2.18 \text{ MPa}\sqrt{\text{m}}$ .

3. There is a definite interaction between  $K_I$  and  $K_{II}$  in the mixed mode fracture of eastern red spruce. The criterion for the mixed mode fracture failure of this species of wood can be expressed in the following form:

$$\frac{K_I}{K_{Ic}} + \left(\frac{K_{II}}{K_{IIc}}\right)^2 = 1 \dots\dots\dots (9)$$

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