Rate of Temperature Change in Short-Length Round Timbers

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This paper discusses factors affecting the rate of temperature change in wood and includes time-temperature curves for use in determining the temperature obtained at different points in short-length log sections when heated in steam or hot water. One group of curves shows the temperature distribution from the ends to the mid-length and at different points between the circumference and axis, of logs 12, 24, 36, 48, and 60 in. in diameter. Other charts show the time-temperature distribution at various points between the circumference and axis at the mid-length of log sections of any diameter between 12 and 60 in. Examples are given illustrating the procedure for using the charts in finding the temperature at a parcular point, under different heating conditions.

INTRODUCTION

GREEN logs of various species are often heated in steam or in hot water to condition them for veneer-cutting. These logs are relatively short, and the ratio of diameter to length is large in comparison with round timbers in the form of posts, poles, and piling. In timbers in which the ratio of diameter to length is small, the effect of longitudinal heating can extend only a limited distance, while the portion between the end regions is raised to any desired temperature. Because of the comparatively short length of veneer logs, however, it is necessary to consider the effect of heat transfer in the longitudinal as well as in the transverse direction. The relative amount of heating that takes place in these two directions depends upon a number of variables, such as cross-sectional dimensions, length, heating period, and temperature of the heating medium.

The purpose of this paper is to discuss methods of finding the rate of temperature change in timbers of this kind. No attempt is made to discuss the temperatures needed for satisfactory results in veneer-cutting, for some woods apparently require less heating than others. When the optimum temperature is known, however, the data presented provide a guide for selecting the required heating period for logs of different species and of different dimensions. The data also furnish a useful aid in studying the effect of different heating conditions on veneer-cutting operations.

A large amount of experimental work has been done at the Forest Products Laboratory to study the rate of temperature change in wood when heated in different mediums and to investigate the effect of variables, such as species, moisture content, density, or specific gravity, and variation in the rate of heating in the radial, tangential, and longitudinal directions. **A** number of papers have been published which discuss the results of these experiments (1 through 10).[2]

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society.

EFFECT OF VARIABLES ON RATE OF TEMPERATURE CHANGE

Heating Medium. Experiments with different heating mediums show that for the same heating period and the same temperature conditions, steam heats wood faster than water, and water heats faster than such liquids as preservative oils (6). In general, water heated wood about 5 to 10 per cent more slowly than steam. All of the liquids, however, heated the wood somewhat faster than hot plates (6, 8). The slowest rate of heating was obtained in air at a low humidity, but the rate was increased as the humidity of the air was increased (10).

Variation in the rate at which wood heats in different mediums is apparently caused by such variables as type of surface contact, rate of circulation when liquids and gases are used, specific heat of the heating substance, and heat of vaporization as in the case of steam.

Differences in Moisture Content. Wood seasoned well below 30 per cent moisture content, which is about the fiber saturation point, was found to heat more slowly than green wood. Results obtained in heating green wood nevertheless indicate that differences in moisture content above the fiber saturation point have no significant effect on the rate of heating in the range of temperatures commonly employed (4). For practical purposes,. therefore, it may be assumed that green timbers of any given species will heat at about the same rate at any moisture content above the fiber saturation point.

Direction of Grain. Studies made of the relative rate of heating in the radial and tangential directions (at right angles and parallel to the rings, respectively) showed there was no important difference in the rate of temperature change for these two directions, but the rate of heating in the longitudinal direction or along the fibers was found to be from $2^1/_4$ to $2^3/_4$ times faster than in the transverse direction (2, 6, 7). For practical purposes it should be sufficiently accurate to assume that on the average the rate of temperature change longitudinally is about $2'/_1$ times faster than it is across the grain or in a plane at right angles to the axis of the tree. This ratio has been used in the calculation of temperature data that will be discussed later.

Density or Specific Gravity and Species. Experiments on specimens of the same species and on those of different species having different specific gravities showed that with any given heating medium, the rate of heating varied inversely with the specific gravity (6). The rate of temperature change is apparently influenced by the density of the wood rather than by the species, which affects the results merely to the extent that different species may vary in density.

The specific gravity of wood is usually based upon the volume at current moisture content and weight when oven-dry.

Table 1 shows the average specific-gravity values for various species of hardwoods and softwoods. These values are based upon volume when green and weight when oven-dry and are convenient to use when employing the time-temperature curves

[2] Numbers in parentheses refer to the Bibliography at MACLEAN,1f the paper.

TABLE 1 AVERAGE SPECIFIC GRAVITY OF VARIOUS SPECIES OF SOFTWOODS AND HARDWOODS[a]

Species	Average specific gravity (based on volume when green, and weight when oven-dry)
Hardwoods:	
Ash, white (Fraxinus americthe)	0.55
Aspen, bigtooth (Populus grandidentata)	0.35
Basswood, American (Tilia americana)	0.32
Beech, American (Fagus grandifolia)	0.56
Birch, yellow (Betula lutea)	0.55
Chestnut, American (Castanea dentata)	0.40
Elm, American (Ulmus americana)	0.46
Elm, rock (Ulmus thomasii)	0.57
Hackberry (Celtis occidentalis)	0.49
Hickory, mockernut (Hicoria alba)	0.64
Maple, silver (Acer saccharinum)	0.44
Maple, sugar (Acer saccharum)	0.56
Oak, red (commercial) (Quercus sp.)	0.56
Oak, white (commercial) (Quercus sp.)	0.59
Pecan (Hicoria pecan)	0.60
Sweetgum (Liquidambar styracifiva)	0.44
Sycamore, American (Platanus occidentalis)	0.46
Tupelo, black (Nyasa sylvatica)	0.46
Tupelo, water (Nyasa aquatica)	0.46
Walnut, black (Juglans nigra)	0.51
Yellow poplar (Liriodendron tuhpifera)	0.38
Softwoods:	
Baldcypresa (Taxodium distichum)	0.42
Douglas fir, coast (Pseudotsuga taxifolia)	0.45
Douglas fir, Rocky Mountain (Pseudotsuga taxifolia)	0.40
Fir, white (Abies sp.)	0.37
Hemlock, Eastern (Tsuga canadensis)	0.38
Hemlock, Western (Tsuga heterophylla)	0.38
Larch, Western (Larix occidentalis)	0.48
Pine, Eastern white (Pinus strobus)	0.34
Pine, jack (Pinus banksiana)	0.39
Pine, loblolly (Pinus taeda)	0.47
Pine, longleaf (Pinus palustris)	0.54
Pine, ponderosa (Pinus ponderosa)	0.38
Pine, red (Pious resinosa)	0.44
Pine, shore (Pinus contorta)	0.38
Pine, shortleaf (Pinus echinata)	0.46
Pine, slash (Pinus caribaea)	0.56
Pine, sugar (Pinus lambertiana)	0.35
Pine, Western white (Pinus monticola)	0.36
Red cedar, Eastern (Juniperus virginiana)	0.44
Red cedar, Western (Thuja plicate)GREEN	0.31
Redwood (Sequoia sempervirens)	0.38
Spruce, Engelmann (Picea engelmanni)	0.31
Spruce, Sitka (Picea sitchensis)	0.37
Spruce, white (Picea glauca)	0.37
Tamarack (Larix laricina)	0.49
White-cedar, Northern (Thuja occidentalis)	0.29

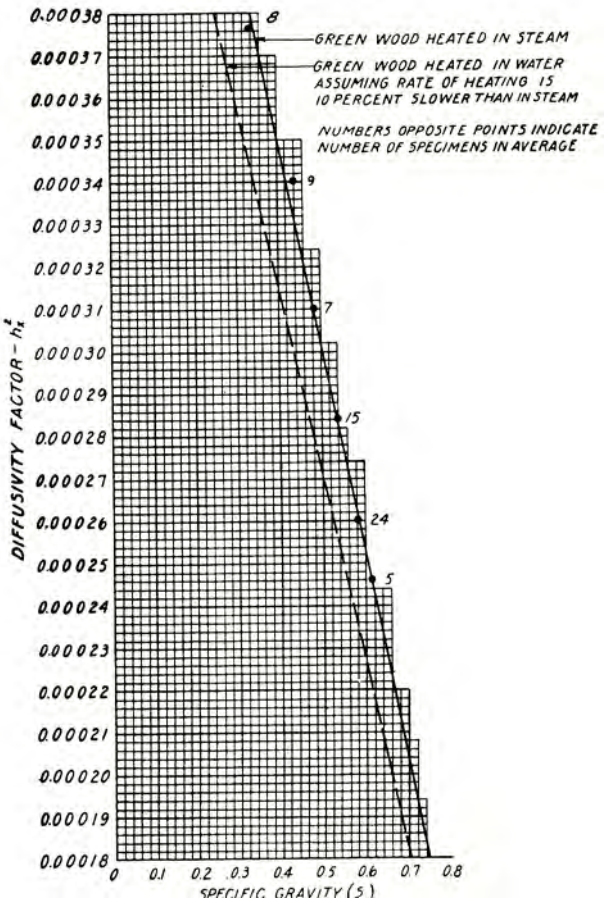
[a] Data for other species can be found in "Strength and Related Properties of Woods Grown in the United States," U. S. Dept. of Agriculture, technical Bulletin 479.

shown in the figures. The method of using th6 data on density, or specific gravity, is discussed in the following section.

COMPUTATION OF TEMPERATURE

The rate of temperature change in any solid depends upon the diffusivity which, like conductivity, may be considered a constant for normal ranges of temperature. This factor is a measure of the rate of temperature change and may be defined as the change in temperature produced in a unit volume of the substance by the quantity of heat that passes in unit time through unit area of a layer of unit thickness and having unit difference of temperature between the faces. If the diffusivity is represented by the symbol h^2, the specific heat by c, the density by p, and the conductivity by K, the relation of these variables is shown by the algebraic expression, $h^2 = \frac{K}{cp}$ Thus the diffusivity varies inversely as the product of specific heat and density, and directly as the conductivity.

Fig. 1 shows the relation of diffusivity and the specific gravity of wood determined from tests made on green material heated in steam. As previously mentioned, water was found to heat a little more slowly than steam, and for the purpose of calculation it should be satisfactory to assume that the rate of heating in water is about 90 per cent as fast as in steam. On this basis the dotted line in Fig. 1 represents the relation of diffusivity and specific gravity for green wood heated in water. If, for example, green wood having a specific gravity of about 0.5 were heated in steam, the diffusivity would be determined from Fig. 1 as about



Fm. 1 RELATION OF DIFFUSIVITY AND SPECIFIC GRAVITY

0.000302, while the corresponding diffusivity for heating in water would be about 0.000272. These diffusivity factors are in inch-second units.

Since the rate of temperature change depends to a certain extent upon the method of heating, the diffusivity values determined from experimental tests may be considered as values which apply for the heating medium used.

Wood is a substance that is naturally more or less variable in structure; hence temperature calculations cannot be made with the same precision that would be possible if it were a homogeneous material or a substance having the same physical properties in all directions. Nevertheless, the calculated temperature values, which are based upon average results from experimental tests, are sufficiently accurate for practical purposes, and whatever differences there may be between the computed temperatures and those actually obtained in a given timber will be small provided care is taken to maintain the heating conditions assumed.

In the early part of the heating period a variable amount of time is usually needed to bring the heating medium to the desired maximum temperature. A gradual change in the surface temperature is therefore obtained during this period, while the temperature calculations are necessarily based upon the assumption that the surfaces of the timbers are maintained at a constant temperature from the time the heating period is started.

If the temperature rise is fairly uniform, a convenient way to make an adjustment for the lower average temperature to which the wood is exposed during the early period of heating is to assume that the full temperature is applied for one half of this period. For example, if 2 hr were taken to bring the heating medium to the maximum temperature, the 2-hr period would be

counted as 1 hr at the maximum temperature. While this is an approximate method of adjusting for the period of variable temperature, it is generally satisfactory for moderate periods of changing surface temperature when the rate of temperature rise of the heating medium is reasonably uniform. A consideration of this lower average temperature at the beginning of the heating period naturally becomes of less importance as the total time of heating is increased. When the heating conditions are not controlled so that they are approximately uniform over most of the heating period, individual judgment must be exercised in determining the total time the timbers should be heated to compensate for the variable conditions.

FACTORS INVOLVED IN TEMPERATURE CALCULATIONS

Equation [4] in the Appendix was derived for computing the temperature data shown in Figs. 4 to 16, inclusive. Definitions of the symbols used and a brief outline of the development of this equation are also given in the Appendix.

An examination shows that the following factors are included in the fundamental equation:

1 Initial wood temperature = U_0
2 Temperature of heating medium = U_1
3 Diffusivity factor for radial heating = h^2
4 Diffusivity factor for longitudinal heating = q^2
5 Heating period = T (sec)
6 Radius of timber = a (in.)
7 Length of timber = L (in.)
8 Distance from nearest end surface, of point at which temperature is to be computed = Z (in.)
9 Distance from axis, of point at which temperature is to be computed = r ***(in)***
10 Temperature at the point (r, Z) under consideration = U

Fig. 2 shows the distance from the end surface and the distance from the circumference at which temperatures were computed for plotting the time-temperature curves. Fig. 3 is a chart for finding the corresponding wood temperatures when the heating conditions are different from those assumed in computing the temperature data plotted.

In the preparation of the time-temperature curves shown in Figs. 4 to 16, inclusive, the initial wood temperature U_0 was taken as 70 F, the heating-medium temperature U_1 as 212 F, the diffusivity factor for radial heating h^2 as 0.000270 (in-sec), and the diffusivity factor q^2 for longitudinal heating as about 2.5 times h^2, in round figures 0.000680, making the ratio slightly over 2.5, or about 2.52. It may be noted in Fig. 1 that the diffusivity factor of 0.000270 corresponds to about that for green wood having a specific gravity of 0.5, when heated in water.

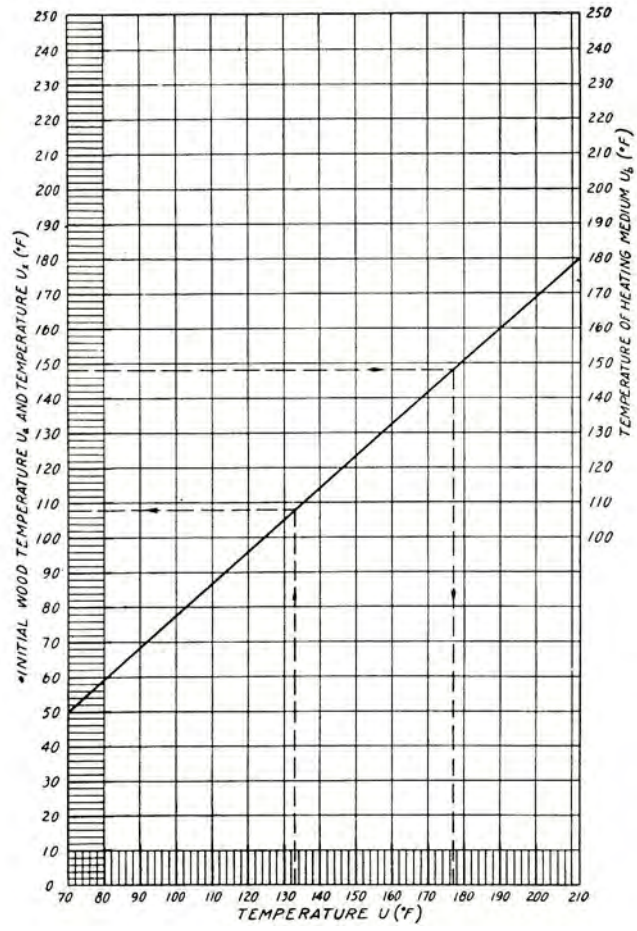
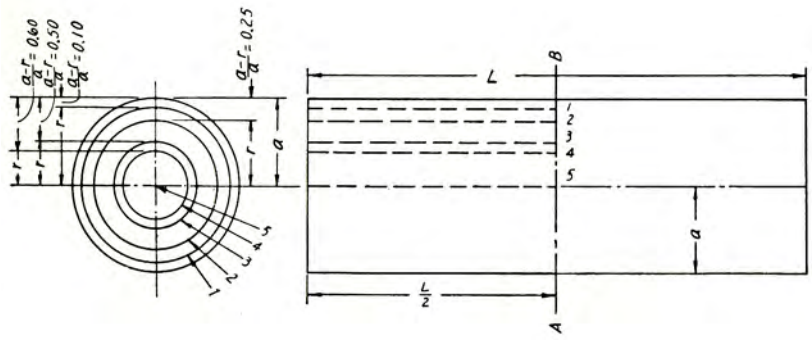


FIG. 3 CHART FOR FINDING CORRESPONDING TEMPERATURE U_x WITHIN TIMBER WHEN INITIAL WOOD TEMPERATURE Ua, HEATING-MEDIUM TEMPERATURE Ub, OR BOTH, ARE DIFFERENT FROM THOSE ASSUMED IN COMPUTING PLOTTED TEMPERATURES

(Dotted lines illustrate examples.)



TEMPERATURE CURVES IN GROUP I ARE TEMPERATURES FROM END SURFACE TO MID-SECTION A-B AT DIFFERENT INSTANCES FROM THE CIRCUMFERENCE, NUMBERED I. 2,3, 4, AND
TEMPERATURE CURVES IN GROUP 2 SHOW TEMPERATURE DISTRIBUTION FROM CIRCUMFERENCE TO CENTER OF MID-SECT/ON A-8, AND TEMPERATURE CURVES //V GROUPS SHOW TEMPERATURES IN MID-SECT/ON AT DISTANCES OF 0.25 dt.)r 0 SO a , 0.75a , AND a INCHES FROM THE CIRCUMFERENCE WHERE a=RADIUS IN INCHES

FIG. 2 SKETCH SHOWING DISTANCE FROM END SURFACE AND DISTANCE FROM CIRCUMFERENCE AT WHICH TEMPERATURES ARE COMPUTED

Since the diffusivity for the longitudinal direction q^2 is taken as $2.52h^2$, the exponential function

$$e^{-T\left(h^2\lambda_n^2 + \frac{\pi^2 q^2 m^2}{L^2}\right)}$$

in the general Equation [3] can be written in the form

$$e^{-h^2T\left(\lambda_n^2 + \frac{2.52\pi^2 m^2}{L^2}\right)}$$

and a similar substitution can be made for q^2 in the terms of Equation [4] (Appendix). A change in the diffusivity factor for radial heating will therefore represent a proportional change in the longitudinal diffusivity factor q^1.

Although the temperature curves given in the various charts were computed by assuming the particular diffusivities and temperature conditions cited, these curves may be used for any predetermined conditions of temperature and for any species having diffusivities different

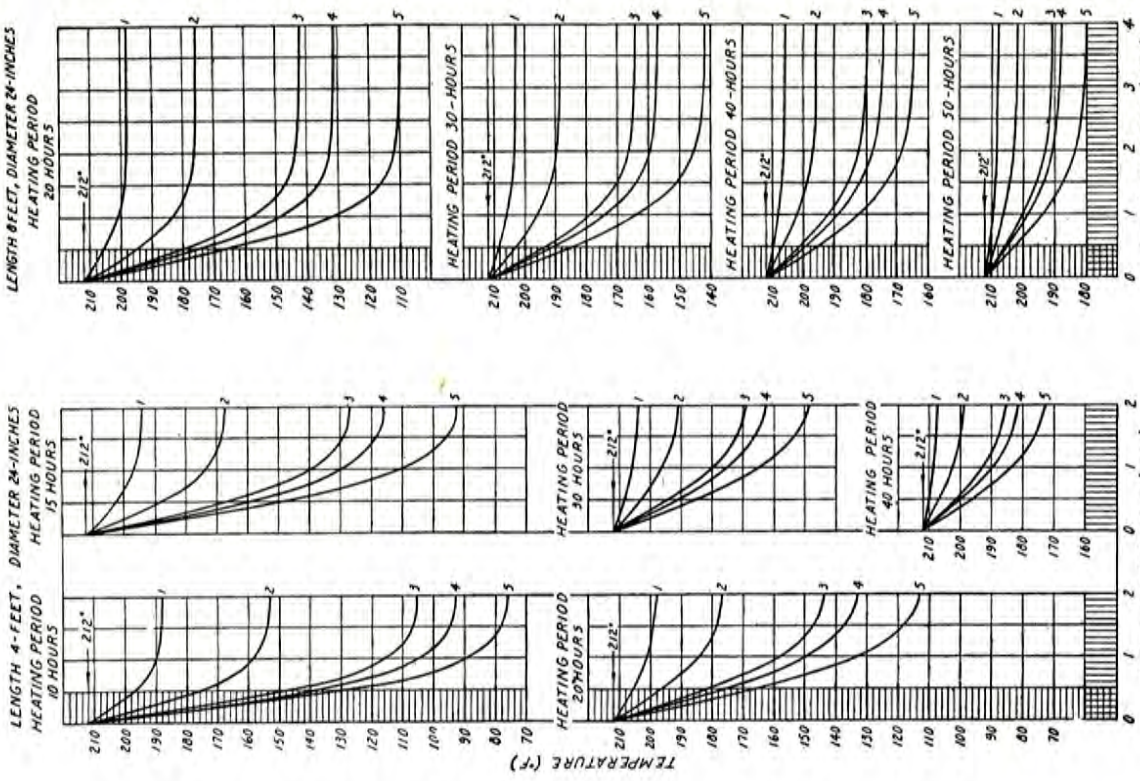


FIG. 5 TEMPERATURES OBTAINED AT DIFFERENT DISTANCES FROM END OF LOGS 4 AND 8 FT LONG AND 24 IN. DIAM

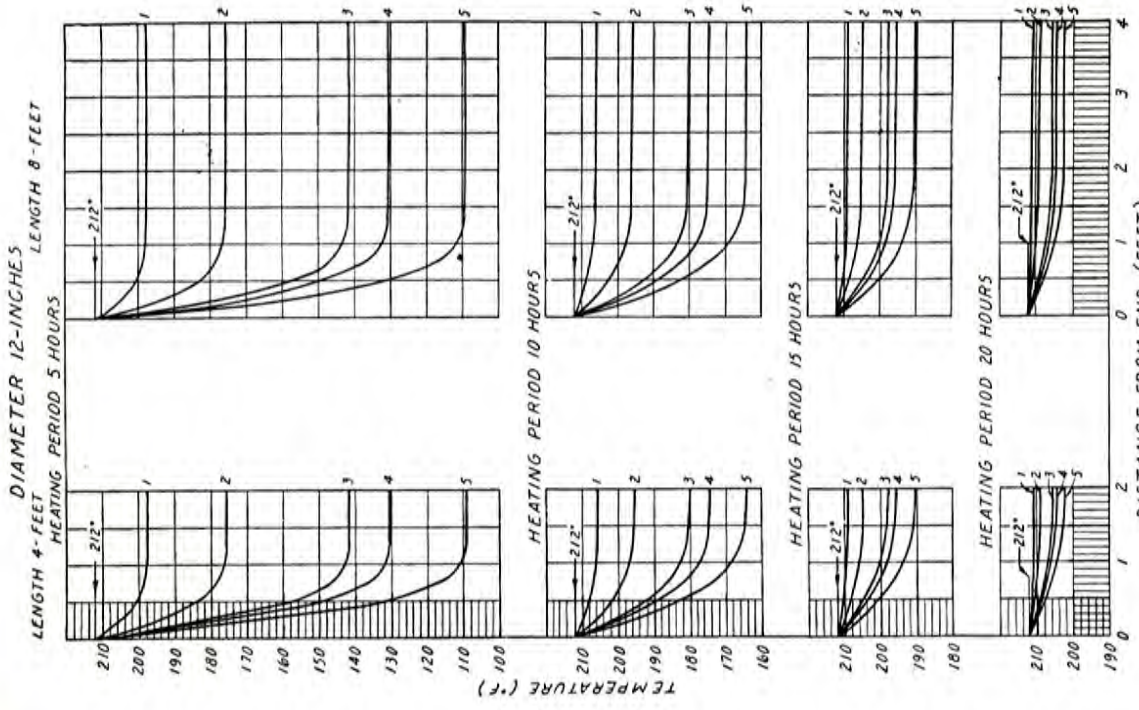


FIG. 4 TEMPERATURES OBTAINED AT DIFFERENT DISTANCES FROM END OF LOGS 4 A ND 8 FT LONG AND 12 IN. DIAM

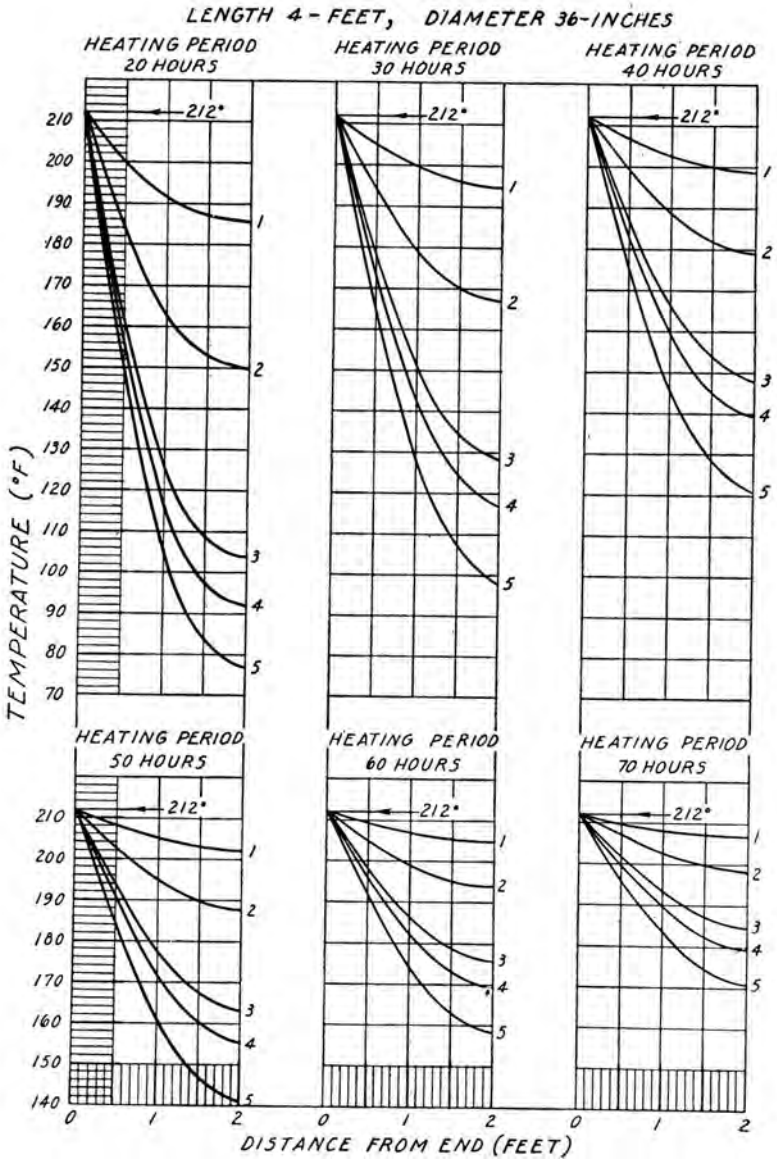


FIG. 6 TEMPERATURES OBTAINED AT DIFFERENT DISTANCES FROM END OF LOG 4 FT LONG AND 36 IN. DIAM

from those assumed. The method of doing this is explained in the Appendix under the heading, "Method of Using the Figures When the Diffusivity, Heating Conditions, or Both Are Different From Those Assumed in Computing the Time-Temperature Curves." Examples illustrating the use of the curves under different conditions will be given later.

DATA PLOTTED IN TEMPERATURE CURVES

The temperature curves comprise three groups in which the initial wood temperature was taken as 70 F and the heating-medium temperature as 212 F.

Group 1. Figs. 4 to 11, inclusive, show the computed temperature distribution from the end to the mid-length of logs 4 and 8 ft long and having dipleters of 1, 2, 3, 4, and 5 ft. Each set of curves is for a different heating period, in most cases varying in intervals of 5 to 10 hr, depending upon the diameter.

Five temperature curves showing the temperature at the different distances from the circumference shown in Fig. 2 are given for each heating period. The top curve in Figs. 4 to 11, inclusive, shows the temperature in the zone or isothermal region at a distance from the circumference equal to $^1/_{50}$ (a) where a is the radius. The second, third, fourth, and fifth curves from the top show the corresponding temperatures in the zones that are atdistances of $^1/_4$ (a), $^1/_2$ (a), $^3/_5$ (a), and a, respectively, from the circumference. In other words, if a is the radius of the log and r is the distance from the center or axis to the isothermal region at which the temperature is computed, then (a — r) is the distance from the surface to the point at which the temperature is given. The ratio $\left(\frac{a-r}{a}\right)$ is therefore equal to 0.10, 0.25, 0.50, 0.60, and 1.0 for the curves shown. In the last case, r = 0; hence the curve for this distance from the circumference shows the temperature variation along the axis from the end to the mid-length of the timber. The distance from the circumference to the point on the radius at which the temperature is computed is evidently equal to the numerical value of $\left(\frac{a-r}{a}\right)$ multiplied by the radius. For example, if the radius is 12 in. and $\left(\frac{a-r}{a}\right)$ =

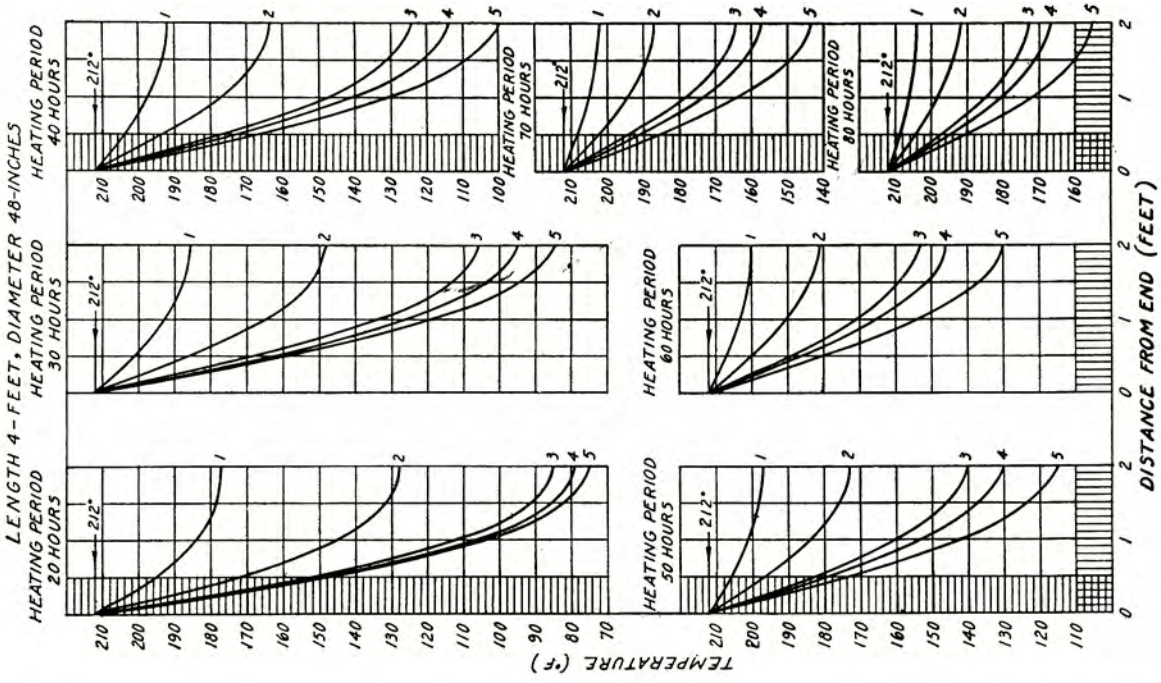


FIG. 8 TEMPERATURES OBTAINED AT DIFFERENT DISTANCES FROM END OF LOG 4 FT LONG AND 48 IN. DIAM

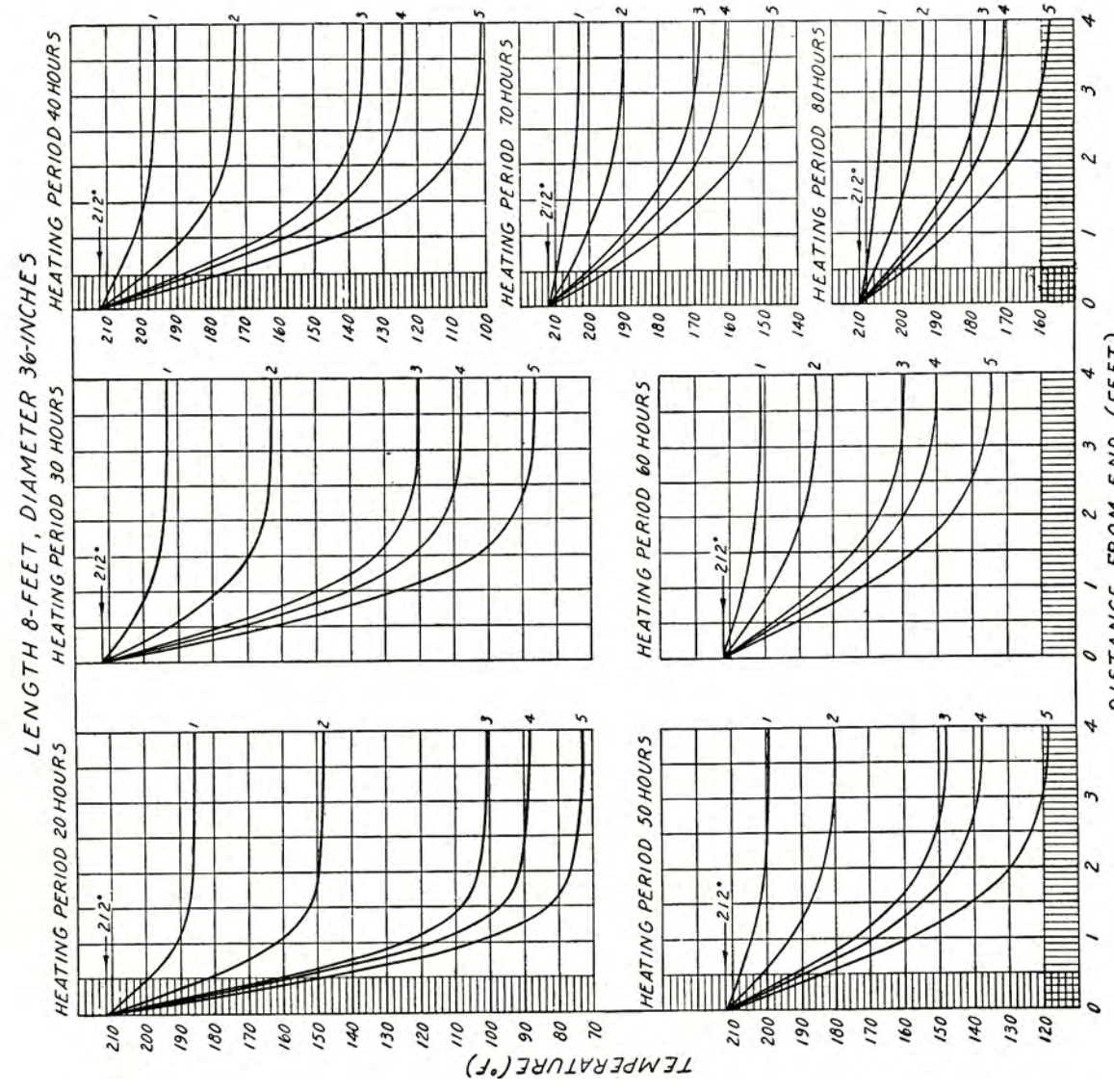


FIG. 7 TEMPERATURES OBTAINED AT DIFFERENT DISTANCES FROM END OF LOG 8 FT LONG AND 36 IN. DIAM

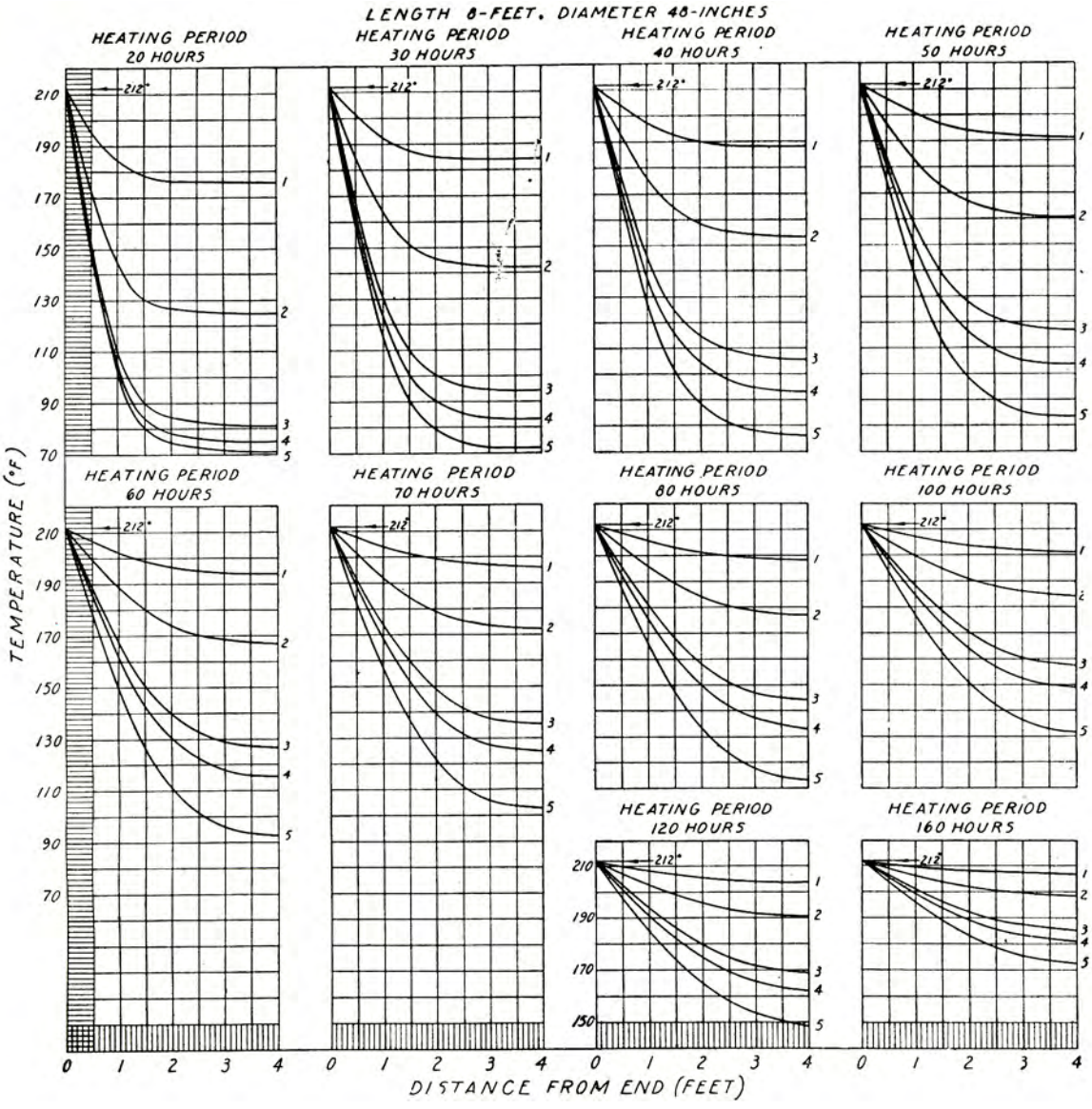


FIG. 9 TEMPERATURES **OBTAINED AT DIFFERENT DISTANCES FROM END OF LOG 8 FT LONG AND 48 IN. DIAM**

0.10, the distance from the **surface is 12(0.10) or 1.2 in. The** curves are shown for only **one** half **the length since the rate of** temperature change is assumed to be **the same from either end.**

Group 2. Figs. 12 to 14, inclusive, **show the computed temperature** distribution for various heating **periods at all distances** from the circumference to the center **at the transverse mid-section** of logs 4 and 8 ft long, for diameters of **1, 2, 3, 4, and 5 ft. Similar** curves could be prepared for a **transverse section at any distance from the end surface by plotting** the **temperature data from the** curves shown in group **1 as was** done **in preparing the curves in** group 2. The temperature at the transverse **mid-section, however,** is generally of most interest, since the lowest **temperatures** are obtained in this region. The curves given in **group 2 (Figs. 12** to 14, inclusive) show the temperatures obtained **at the end of 5-** to 10-hr heating periods, except for particularly **long heating** periods or when the rate of temperature change was very slow. Under conditions in which this rate was unusually slow, somewhat longer time differences were used.

The time required to obtain a given temperatuie (which is found between any two time-temperature curves) can be determined with sufficient accuracy for practical purposes from the proportional distance on the vertical line intersecting the two curves between which the temperature is found. This assumes that the change in temperature between any two time-temperature curves is directly proportional to the time heat is applied during this time interval. For example, assume the required temperature is 170 F at 8 in. from the surface of a log 36 in. diam and 8 ft long. In this case $\left(\frac{a-r}{a}\right) = \frac{8}{18}$ or approximately 0.45. The vertical line from 0.45, Fig. 13B, intersects the horizontal line from 170 F at a point between the temperature curves for heating periods of 60 and 70 hr. The temperature after heating for 60 hr is 164 F, and that after heating for 70 hr is about 173 F, or an increase of 9 deg in 10 hr. When the proportional change in temperature is assumed to be the same as the proportional change in time of heating the temperature ratio is $\frac{170-164}{173-164} = {}^{6}/_{9}$ or ${}^{1}/_{3}$. Since the time interval is 10 hr, the time is found from the relation, 2:3 = x:10 and x (the time that is required to in-

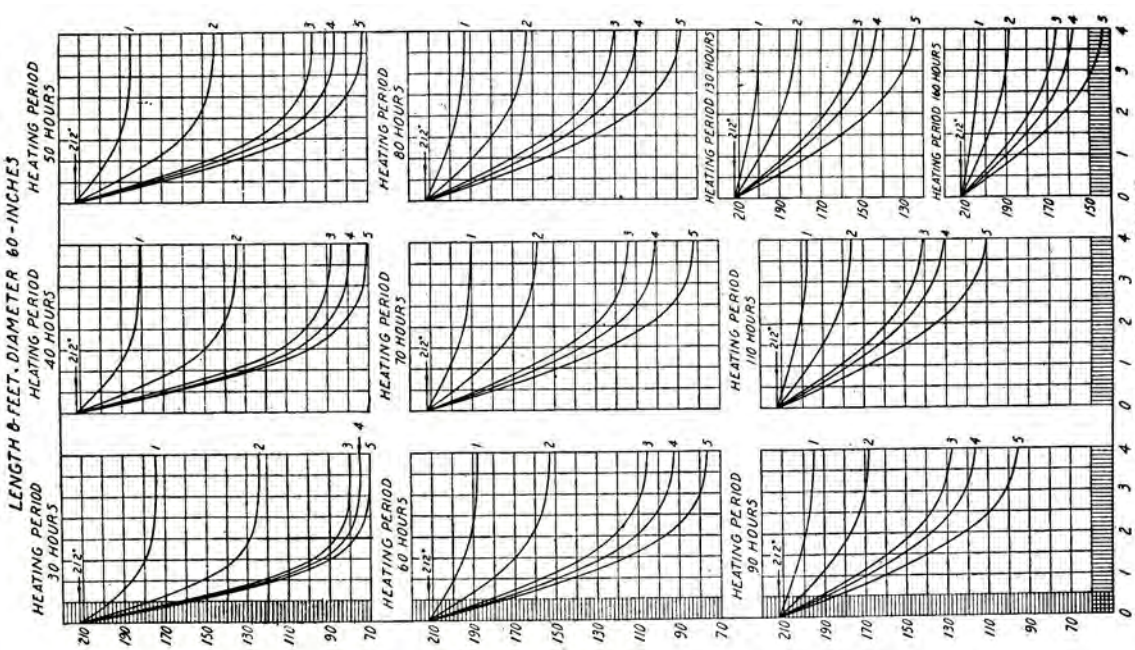


FIG. 11 TEMPERATURES OBTAINED AT DIFFERENT DISTANCES FROM END OF LOG 8 FT LONG AND 60 IN. DIAM

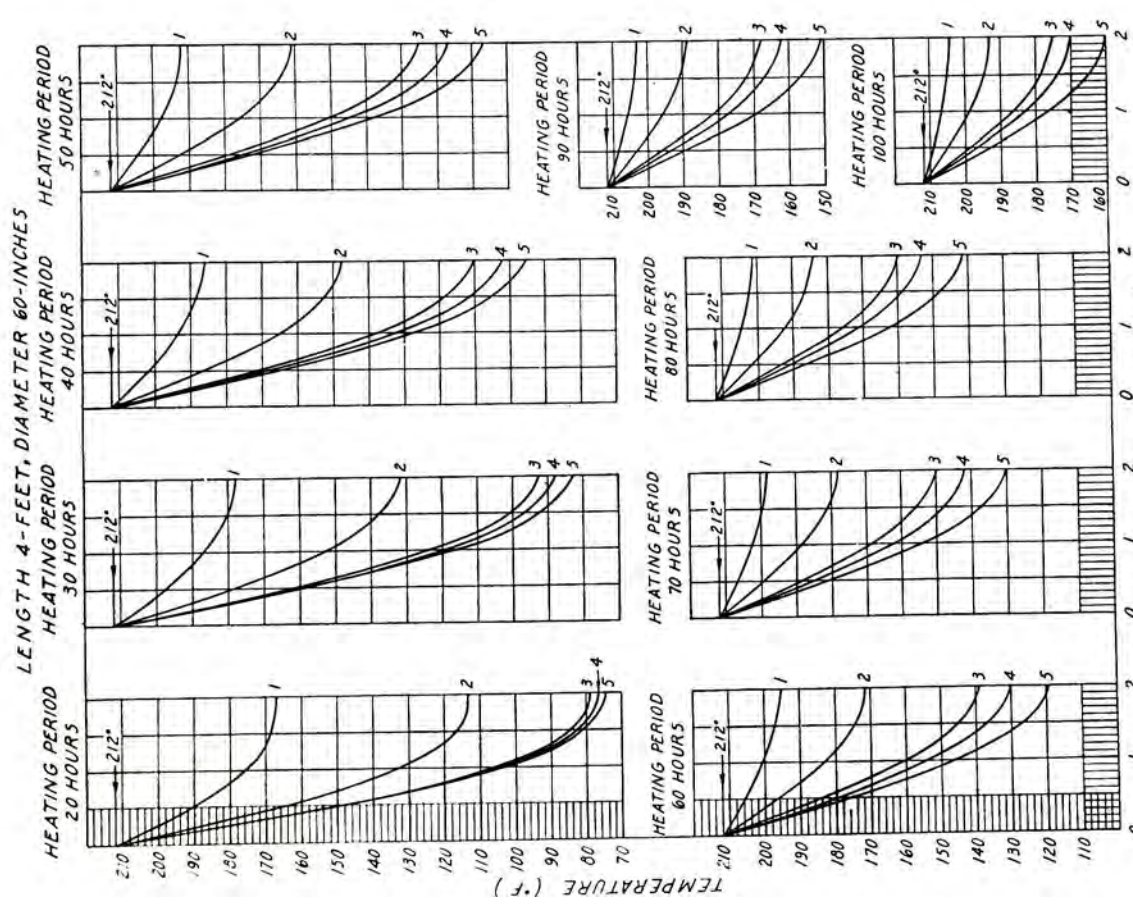


FIG. 10 TEMPERATURES OBTAINED AT DIFFERENT DISTANCES FROM END OF LOG 4 FT LONG AND 60 IN. DIAM

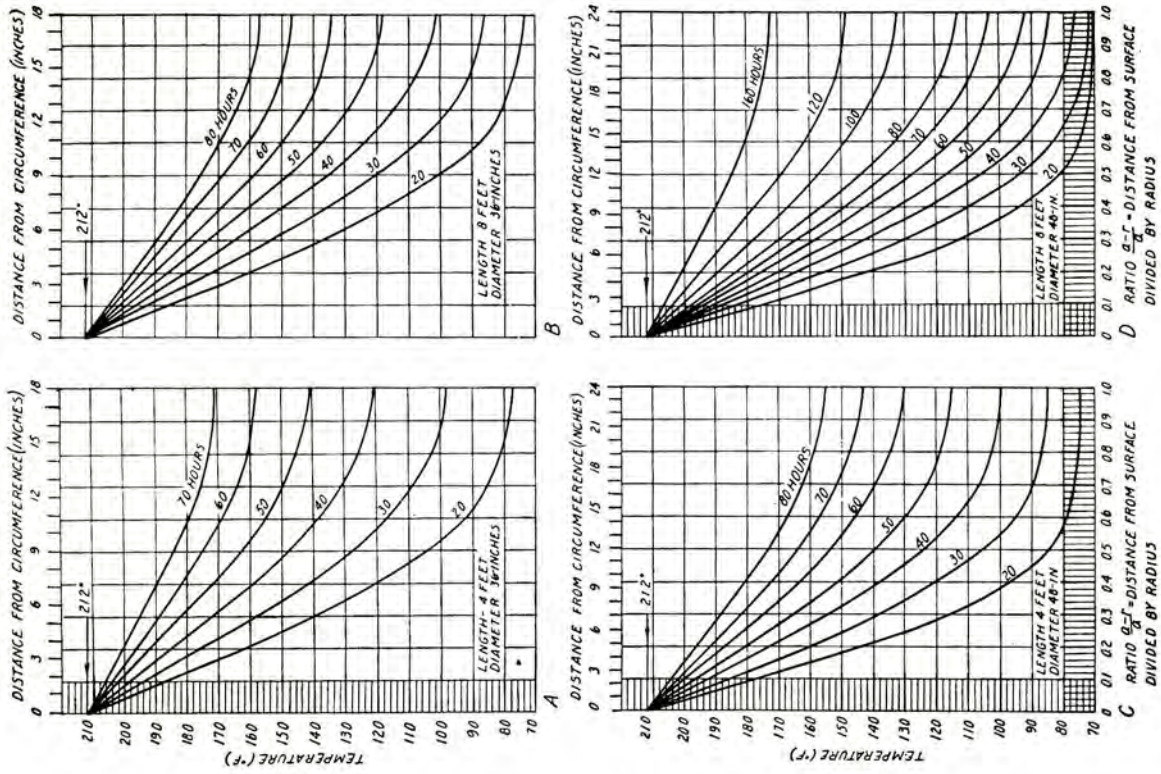


FIG. 12 TEMPERATURE DISTRIBUTION FROM CIRCUMFERENCE TO CENTER OF MID-SECTION; LOGS 4 AND 8 FT LONG AND 12 AND 24 IN. DIAM

FIG. 13 TEMPERATURE DISTRIBUTION FROM CIRCUMFERENCE TO CENTER OF MID-SECTION; LOGS 4 AND 8 FT LONG AND 36 AND 48 IN. DIAM

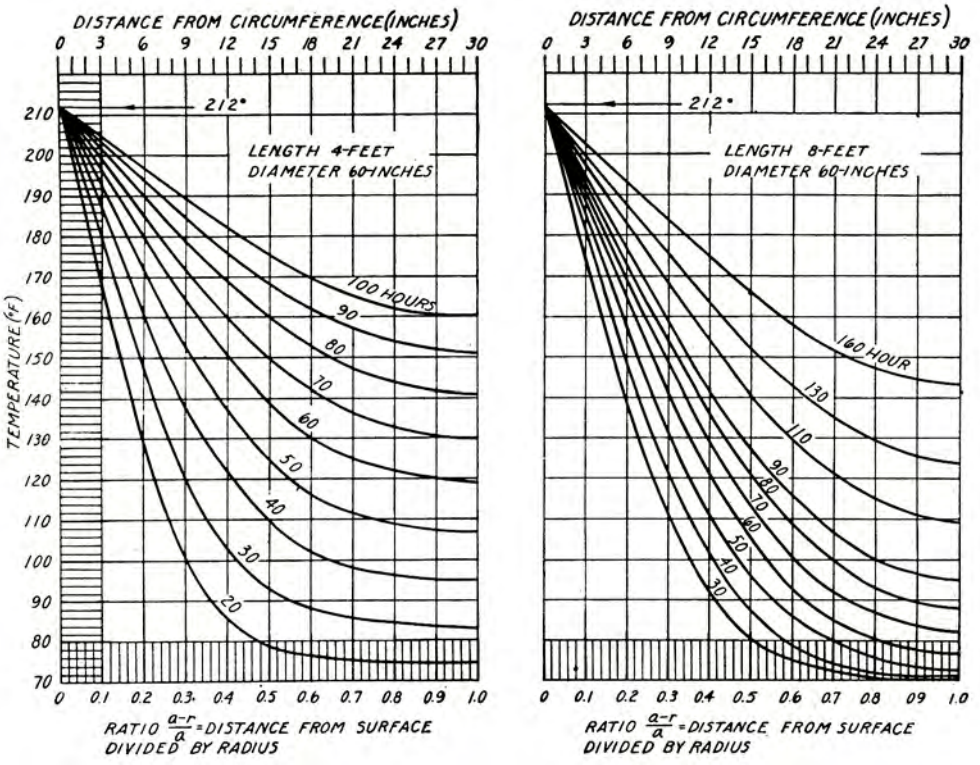


FIG. 14 TEMPERATURE DISTRIBUTION FROM CIRCUMFERENCE TO CENTER OF MID-SECTION; LOGS 4 AND 8 FT LONG AND 60 IN. DIAM

crease the temperature from 164 F to 170 F) is ($^2/_3$) 10 or 6 $^2/_3$ hr. The total time necessary to reach a temperature of 170 F is then 60 + 6.7 = 66.7 hr.

The same method would be used in finding the corresponding temperature when the heating period is known. If, for example, the heating period is 23.5 hr, and it is desired to find from Fig. 12D the temperature obtained at a point in the log half way between the axis and circumference, the figure shows that after 20 hr heating the temperature is 142 F, and after 30 hr heating it is 164 F, or an increase of 22 deg in 10 hr. The temperature increase above 142 F is then computed by the proportion (23.5 — 20) :10 = x:22, from which it is found that the temperature change $x = \left(\frac{3.5}{10}\right) 22 = 7.7$ deg F. The temperature obtained after heating for 23.5 hr is then obtained by adding 7.7 deg to 142 F and is approximately 150 F.

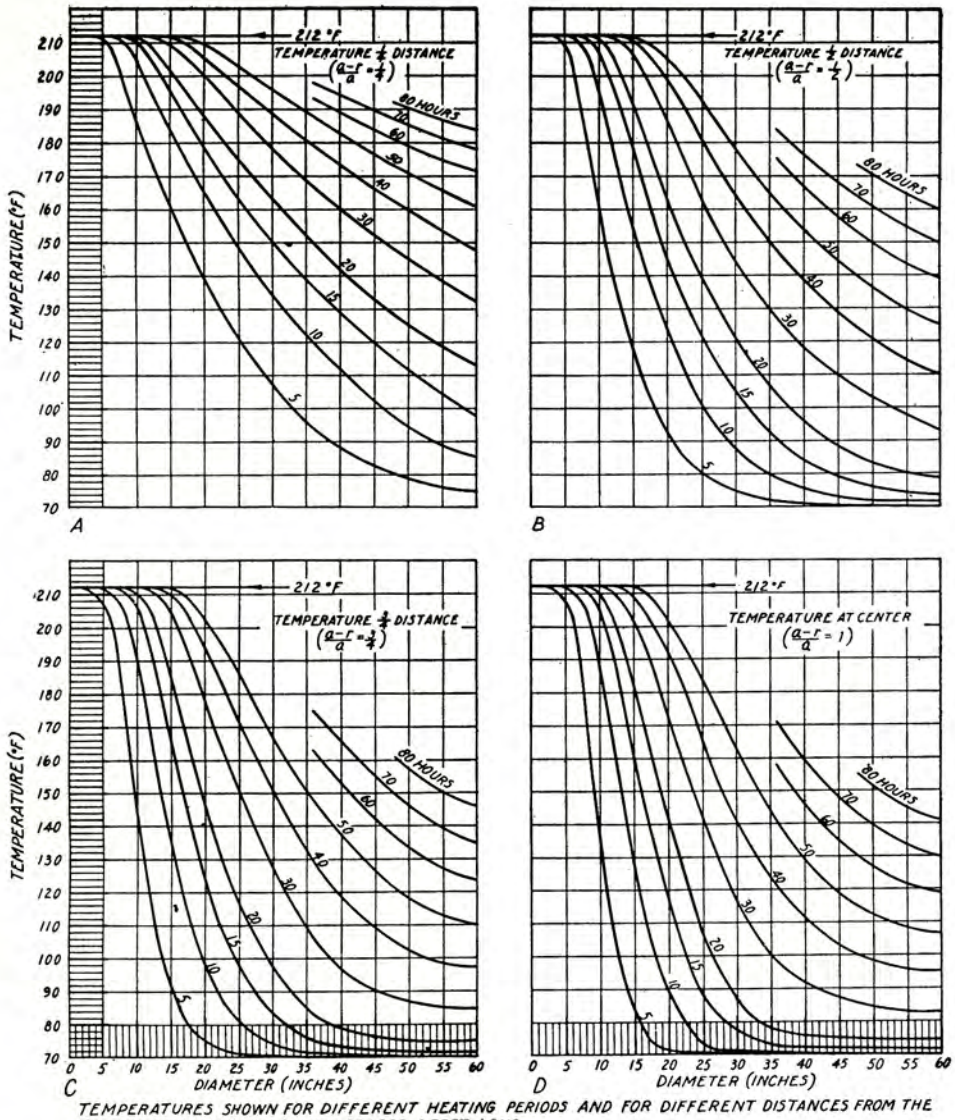
If desired, the temperatures can be plotted against heating periods for any particular point in the timber, as for example at the center. Such curves would furnish a convenient method for reading directly the temperature obtained in a given time, or the corresponding heating period required to reach a definite temperature.

Group 3. In order to obtain curves that would show the temperature at various distances from the circumference at the mid-length of logs of any diameter between 12 and 60 in., data used in plotting the curves in group 2 were also used in preparing the curves shown in Figs. 15 and 16. Each curve in Figs. 15 and 16 was prepared by plotting the temperatures for each heating period for the logs of different diameters. Curves were then drawn through these plotted points for the same heating period. For heating periods over 50 hr, data were plotted only for timbers 36 in. or more in diameter.

Temperatures plotted in group 3, Figs. 15 and 16, are for points at distances from the circumferential surface of $^1/_4$, $^1/_2$, and $^3/_4$ the radius, and for the center or where $\frac{a - r}{a} = 1$. When the required heating period or temperature is found between two time-temperature curves, the heating period or temperature can be found by using the proportional relation as explained for the curves in group 2.

The data show that it is not feasible to heat the large-diameter timbers at a considerable distance from the surface because of the extremely long heating periods required. For example, under the conditions assumed in making the temperature computations, Fig. 15B shows that if a temperature of 150 F is desired at a distance of 12 in. from the circumference of a log 4 ft diam, the heating period would be nearly 58 hr or 2.4 days. In such cases it would probably be better to heat to the minimum required temperature at a shorter distance from the circumference. After cutting to this depth the log could be reheated and with the reduced diameter the rate of heating would be much faster. This procedure would also avoid overheating the wood near the surface and at the ends.

The temperature curves shown in group 3 will be particularly useful for general purposes, since they show the relation of temperature and heating period for all diameters between 12 and 60 in. If it is desired to find the temperature at distances from the circumference that are intermediate between $\frac{a - r}{a} = 0.25$, 0.50, a 0.75, and 1.0, which are the distances shown in this group of curves, the data for any distance required may be found in the figures for group 2. These temperatures can then be plotted for the different diameter timbers as was done in preparing the curves for group 3. Temperatures for these four points can also be plotted against the proportional distance from the surface (as in Figs. 12 to 14, inclusive) for any diameter and any heating period. Smooth curves drawn through these plotted points will



TEMPERATURES SHOWN FOR DIFFERENT HEATING PERIODS AND FOR DIFFERENT DISTANCES FROM THE CIRCUMFERENCE TO THE CENTER OF LOGS 4 FEET LONG.
(TEMPERATURES FOR TRANSVERSE SECTION 2 FEET FROM END)

[FIG. 15 TEMPERATURES OBTAINED AT DISTANCES OF 1/4, 1/2, AND 3/4 RADIUS AND AT CENTER OF LOGS 4 FT LONG

then show the approximate temperature for all distances from the surface to the center or axis of the timber.

Method of Using Fig. 3. Fig. 3 is a chart that furnishes a convenient method for finding the corresponding wood temperature when the heating conditions are different from those assumed in computing the temperature curves shown in the different groups.

The initial wood temperature U_a and the temperature U_z obtained in the wood after any given heating period are read on the left vertical scale, while the heating-medium temperature U_b is read on the right vertical scale. Values of U, the corresponding computed wood temperatures found from the figures, are given on the bottom scale.

In using Fig. 3, it is merely necessary to place a straightedge connecting the initial wood temperature U_a on the left scale with the heating-medium temperature U_b on the right scale. This is illustrated by the line connecting the initial wood temperature U_a = 50 F on the left scale with the heating temperature U_b = 180 F on the right scale. Assume, for example, that with the foregoing temperature conditions it is desired to find the corresponding computed temperature U (see Appendix for definition of symbols) at a point in the timber when the required **temperature** U_z is 148 F. This is found as follows:

Pass horizontally from 148 F on the left scale to the point of intersection with the line joining 50 and 180 F. Directly below this point of intersection U is found on the horizontal scale to be 177 F. The procedure is shown by the dotted line from 148 F. Again, let it be assumed that data from one of the charts shows that a temperature U of 133 F is obtained after heating for a period of T hr, and it is desired to find the temperature U_a that would be obtained at the same point, in the same time, and in the same timber when the initial wood temperature U_a is 50 F and the heating-medium temperature U_b is 180 F, instead of 70 and 212 F, the temperatures assumed in computing the data used in plotting the curves.

The temperature U_z is found in this case by passing vertically from U = 133 F on the horizontal scale to the line connecting 50 and 180 F. Directly across on the left scale U_z is found to be 108 F. These steps are shown in Fig. 3 by a dotted line connecting the temperature U = 133 F, and U_z = 108 F. This, is simply reversing the procedure for finding U when U_z is known.

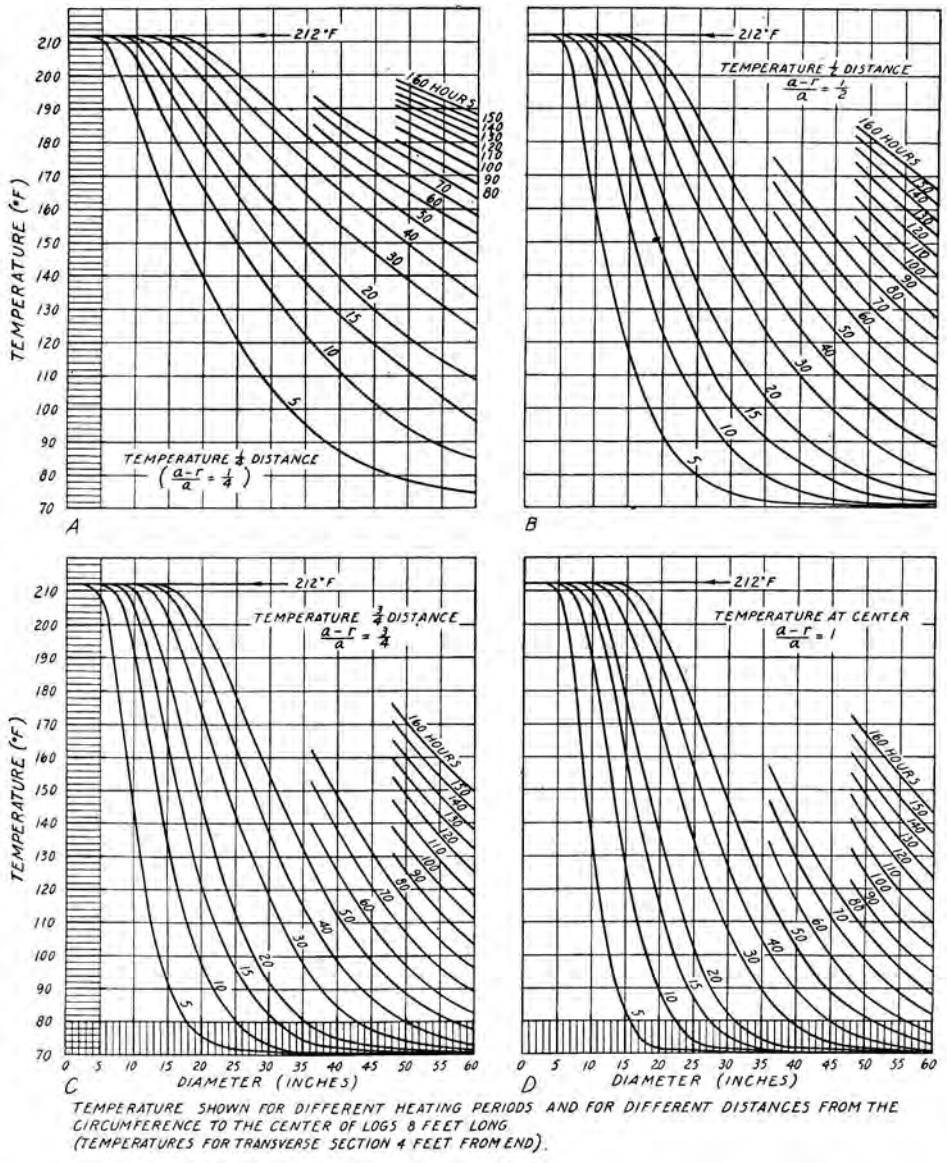


FIG. 16 TEMPERATURES OBTAINED AT DISTANCES OF 1/4, 1/2, AND 3/4 RADIUS AND AT CENTER OF LOGS 8 FT LONG

Fig. 3 is convenient to use in finding how the wood temperatures vary when different heating conditions are employed. For example, in the illustrations given, heating under the lower temperatures assumed for U_a and U_b the wood temperature will be (177 — 148) or 29 deg F lower at the point under consideration, than if the initial wood temperature had been 70 F, and the heating-medium temperature had been 212 F, which were the temperatures assumed in computing the curves. Similarly when any point is at a temperature U of 133 F (with the temperature conditions assumed in computing the curves), when heating under the lower temperature with U_a = 50 F and U_b = 180 F, the same point would be at a temperature of 108 F or 25 deg F lower. This chart can be used for finding either U_z or U instead of computing these temperatures from Equations [B_1] and [C_1] in the Appendix.

In general, problems relating to temperature changes will be either:

1 To find the time τ_b required to obtain a specified temperature U_z at some particular point in a timber; or

2 To determine the temperature U_z that will be obtained when the time τ_b and temperature U are known. In any problem the following variables are involved: Variables found by means of the temperature charts are U and T, and variables relating to the timber in question are U_a, U_b, U_z, and τ_b. The relation between T and T_b is shown by Equation [A] (Appendix), which is $T_b h_z^2 = T(0.00027)$

When U_a, U_b, and U_z are assumed, U is found from Equation [C_1] (Appendix) or from Fig. 3. The corresponding time T required to obtain the temperature U can then be found from the proper chart, and the time T_b can be determined from Equation [A].

When T_b, U_a, and U_b are given, T is found from Equation *[A]*, and the corresponding temperature U can be found from the proper chart. The temperature U_z can then be determined by means of Equation [B_1] (Appendix) or from Fig. 3.

EXAMPLES SHOWING How CHARTS ARE USED'

Example I. Data are as follows: Diameter of log = 22 in.;

Refer to Appendix for definitions of symbols used.

length of log = 8 ft; species, yellow birch; heating medium, water; water temperature during heating period = 180 F = Ub; and initial wood temperature = 50 F =

Required to find the heating period (T_b) needed to obtain a temperature of 140 F (U_z) halfway between the circumference and center at the mid-length. In this case $\frac{a-r}{a} = \frac{1}{2}$. Therefore, for a log of this diameter, the distance from the surface = V, (11) = 5.5 in.

From Table 1 the average specific gravity of yellow birch (based on weight when oven-dry and volume when green) is found to be 0.55, and the diffusivity hx^2 found in Fig. 1 for this specific gravity is about 0.000249 when the heating medium is water. In this example El_a, $(Ib$, and U_z are given, while U, T, and Tb are to be determined.

The first step is to find U, using Equation [C, (Appendix), or Fig. 3. From Equation [C1]

$$U = 212 - \left[\frac{(180-140)(142)}{(180-50)}\right] = 168 \text{ F}$$

This can be found readily from Fig. 3 by placing a straightedge connecting 50 F on the left scale with 180 F on the right scale, as shown by tile straight line on this figure. Passing horizontally from 140 F on the left scale to this line the temperature U is found directly below on the lower scale to be 168 F, which is the temperature found by using Equation [C,].

The second step is to find from Fig. 16(B) the time T required to reach a temperature of 168 F in a timber 22 in diam and 8 ft long. Passing from 168 F in Fig. 16(B) to a point directly over the diameter of 22 in. shown on the abscissa, it is found that this point is between the curves for heating periods of 20 and 30 hr. The temperature after heating for 20 hr is found to be 153 F, and after heating for 30 hr is 174 F, or a temperature difference of 21 deg F. The proportion of the vertical distance between the two curves for the temperature of 168 F is then computed as $\frac{168-153}{174-153} = \frac{15}{21}$; and since the time interval is 10 hr, the time T is computed as $\left[20 + \frac{15}{21}(10)\right]$ or about 27.1 hr.

This is the time that would be required if the diffusivity were the same as that assumed in computing the temperature curves; but since the diffusivity for yellow birch was found to be about 0.000249, the time T_b must be computed from the relation shown by Equation [A] (Appendix) that gives

$$T_b = \frac{T(0.00027)}{0.000249} = (27.1)(1.084)$$

or 29.4 hr, which is the time to be determined.

If steam at 212 F were used as the heating medium, the diffusivity as indicated in Fig. 1, would be about 0.000276. With U_a = 50 F, U_b = 212 F, and the required temperature U_x = 140 F, the corresponding temperature U is determined from Equation [C, (Appendix) or from Fig. 3 to be 149 F. From Fig. 16(B), the time T required to reach a temperature of 149 F in a log 22 in. diam is found to be about 19 hr, and the required heating period Tb is $\frac{19(0.00027)}{0.000276} = 18.6$ hr.

Example 2. With the same data as given for example 1 find the temperature that would be obtained at $^1/_4$ the distance from the circumference to the center when the heating period T_b is 25 hr. In this example, $\frac{a-r}{a} = {}^1/_4$, and the distance from the circumference = $^1/_4$ (11) = $2^3/_4$ in. instead of $5^1/_2$ in. as in example 1. In this example U_a, U_b, and T_b are given, and T, U, and U_x are to be found.

First, find T from Equation [A] (Appendix). In this case

$$T = \frac{T_b(h_x^2)}{h^2} = \frac{25(0.000249)}{0.00027} = 23 \text{ hr, approximately}$$

Next, find U. for this heating period. From Fig. 16(A) the temperature U is found to be about 185 F, after heating for 23 hr.

Then find the temperature U_x from Equation [B_1] (Appendix), or from Fig. 3.

Using Equation [B_1]

$$U_x = 180 - \left[\frac{(180-50)(212-185)}{142}\right] = 155 \text{ F}$$

which is the temperature to be determined. This temperature U_x can also be found from Fig. 3 by placing a straightedge on 50 F on the left scale and on 180 F, right scale. Reading up from 185 F on the bottom scale to the line connecting 50 F and 180 F, and directly across to the left scale, the temperature U_x is found to be about 155 F as computed from Equation [B_1] (Appendix).

If steam at 212 F were used as the heating medium the diffusivity would be 0.000276 (as found for example 1), and T would be $\frac{25\ (0.000276)}{0.00027} = 25.6$ hr. The temperature U for this heating period is found from Fig. 16(A) to be about 188 F. When U_a = 50 F, and U_b = 212 F, the corresponding temperature U_x is found from Equation [B_1] or from Fig. 3, to be about 185 F.

Example 3. Data are as follows: Diameter of log = 48 in.; length of log = 8 ft; species, Coast Douglas fir; heating medium, water; water temperature during heating period = 190 F = Ub; and initial wood temperature = 60 F = U_x.

Required to find the temperature U_z obtained at a distance of 15 in. (Z) from the end surface and at a distance from the circumference equal to $^1/_4$ the radius when the log section has been heated for 30 hr (T_b). In this example $\frac{a-r}{a} = {}^1/_4$; hence the distance from the circumference = $^1/_4$ of 24 or 6 in. In this example U_a, U_b, and Tb are given, and T, U, and U_x are to be determined.

From Table 1 the average specific gravity of green Coast Douglas fir is found to be 0.45, and from Fig. 1 the corresponding diffusivity for heating in water is found to be about 0.000293 = h_x^2.

The first step is to find the heating period T using Equation $[A]$ (Appendix)

$$T = \frac{T_b(h_x^2)}{0.00027} = \frac{30(0.000293)}{0.00027} = 32.6 \text{ hr}$$

Because of the higher diffusivity of the Douglas fir, in comparison with the diffusivity of 0.00027 used in computing the temperature curves, the Douglas fir heats $\frac{(0.000293-0.00027)}{0.00027}100 = 8.5$ per cent faster, and the temperature attained in 30 hr at the point under consideration would be the same (for the same heating conditions)*as that reached after 32.6 hr heating when the wood has a diffusivity of 0.00027;

The second step is to find the temperature U in Fig. 9. From this figure the temperature U is found to be 155 F $\left(\text{for } Z = 15 \text{ in. and } \frac{a-r}{a} = \frac{1}{4}\right)$ after a heating period of 30 hr, and to be 169 F when the heating period is 40 hr. After 30 hr heating, the temperature at the point given would then be computed as

$$\left[155 + \left(\frac{32.6 - 30}{10}\right)(169 - 155)\right] = 159 \text{ F approximately}$$

This is the temperature U, or the temperature that would be obtained if $U. = U_0 = 70$ F and $Ub = U1 = 212$ F; but since $U.$ is assumed as 60 F, and Ub as 190 F, U_z is determined by using Equation [B_1] (Appendix), or from Fig. 3, and is found to be about 141 F which is the temperature to be determined.

This is similar to example 2, except that the point under consideration is between the end surface and the transverse mid-section, and the data are obtained from temperature curves shown in group 1.

Example 4. With the data given in example 3 find the heating period Tb required to obtain a temperature of $U_x = 160$ F for the same point in the log. In this example U_a, U_b, and U_x are given, and U, T, and T_b are to be determined.

The first step is to find the corresponding temperature U when $U_a = 60$ F, $U_b = 190$ F, and $U_x = 160$ F. From Equation [C_1] (Appendix), or from Fig. 3, U is found to be about 179 F.

Fig. 9 shows that when the distance from the end = 15 in. and the distance from the circumference $(a - r) = 6$ in. or $^1/_4$ the radius, the temperature of 179 F is between the temperature curves for heating periods of 50 and 60 hr. After heating for 50 hr, the temperature is 177.5 F, and after heating for 60 hr, it is about 184 F. The heating period required to reach 179 F would then be computed as

$$\left[\left(\frac{179 - 177.5}{184 - 177.5}\right) 10 + 50\right] = 52.3 \text{ hr}$$

Since this is the time T required when the diffusivity is 0.00027 the time Tb is as follows

$$T_b = \frac{52.3(0.00027)}{0.000293} = 48.2 \text{ hr}$$

This is similar to example 1 except that the point at which the temperature is to be determined is between the end surface and transverse mid-section.

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Appendix

METHOD OF COMPUTING TEMPERATURES IN ROUND TIMBERS OF SHORT LENGTH WHERE END HEATING AS WELL AS HEATING IN THE RADIAL DIRECTION MUST BE TAKEN INTO CONSIDERATION

In order to derive an equation for computing the temperature U at any point r inches from the axis and Z inches from one of the end surfaces, it is necessary to solve the following partial differential equation

$$\frac{\delta U}{\delta T} = \left[h^2 \left\{\frac{\delta^2 U}{\delta r^2} + \frac{1}{r}\frac{\delta U}{\delta r}\right\} + q^2 \frac{\delta^2 U}{\delta Z^2}\right] \ldots\ldots\ldots [1]$$

In this equation h^2's the diffusivity factor for heating in the radial direction and q^2 is the diffusivity factor for heating in the longitudinal direction.

Equation [1] must be solved, subject to the following boundary conditions: Let $\theta = (U - U_1)$, where U_1 is the temperature of the heating medium and U *is* the temperature at the point under consideration at any time T.

Let U_0 = initial wood temperature; a = average radius of log, and L = length in inches.

If $\theta = (U - U_1)$, $U = U_0$ and $\theta = (U_0 - U_1)$ when $T = 0$

$$\left.\begin{aligned} \theta = 0 \text{ when } r &= a \\ Z &= 0 \\ Z &= L \end{aligned}\right\} \quad \text{and } T > 0$$

$\theta = f(r,Z)$ when $T = 0$

Substituting θ in Equation [1]

$$\frac{\delta \theta}{\delta T} = \left[h^2 \left\{\frac{\delta^2 \theta}{\delta r^2} + \frac{1}{r}\frac{\delta \theta}{\delta r}\right\} + q^2 \frac{\delta^2 \theta}{\delta Z^2}\right] \ldots\ldots\ldots [1a]$$

A solution of differential Equation [1a] to meet the boundary conditions gives

$$\theta = \sum_{m=1}^{m=\infty} \sum_{n=1}^{n=\infty} Am,ne^{-T\left(h^2\lambda_n^2 + \frac{q^2\pi^2m^2}{L^2}\right)} \sin\frac{m\pi Z}{L} J_0(\lambda_n r). \quad [2]$$

where

$$Am,n = \frac{4}{a^2L[J_1(\lambda_n a)]^2}\int_0^a dr \int_0^L rf(r,\mu)J_0(\lambda_n r)\sin\frac{m\pi\mu}{L}\,d\mu$$

When $f(r,\mu)$ is a constant and equal to $(U_0 - U_1)$, as in this case

$$Am,n = \frac{4(U_0 - U_1)}{a^2L[J_1(\lambda_n a)]^2}\int_0^a dr \int_0^L rJ_0(\lambda_n r)\sin\frac{m\pi\mu}{L}\,d\mu$$

$$= \left[\frac{4(U_0 - U_1)}{L\lambda_n a J_1(\lambda_n a)}\right]\left(\frac{L}{m\pi}\right)(1 - \cos m\pi)$$

$$= \frac{8(U_0 - U_1)}{m\pi\lambda_n a J_1(\lambda_n a)} \text{ when } m \text{ is odd, and } = 0 \text{ when } m \text{ is even}$$

Substituting the value

$$\frac{8(U_0 - U_1)}{m\pi\lambda_n a J_1(\lambda_n a)}$$

for $A_{m,n}$ and the temperature difference $(U - U_1)$ for θ, Equation [2] may be written

$$U - U_1 = 8(U_0 - U_1) \sum_{m=1}^{m=\infty} \sum_{n=1}^{n=\infty} \frac{1}{m\pi\lambda_n a J_1(\lambda_n a)} \sin \frac{m\pi Z}{L} J_0(\lambda_n r) e^{-T\left(h^2\lambda_n^2 + \frac{\pi^2 q^2 m^2}{L^2}\right)} \quad \ldots\ldots\ldots [3]$$

In the foregoing equations T is the time in seconds, To(X.r) and Ji(Xna) are Bessel functions of the zeroth and first order, respectively, and $(\lambda_n r) = x$ in $J_0(x)$ and $J_1(x)$.[4] Term λ_n is a root of $Jo(\lambda_n a) = 0$ and e is the base of the Napierian logarithms.

Writing the first few terms of Equation [3]

$$\begin{aligned} U = U_1 + (U_0 - U_1)\frac{8}{\pi}\Bigg[& \frac{1}{\lambda_1 a J_1(\lambda_1 a)} \sin \frac{\pi Z}{L} J_0(\lambda_1 r) e^{-T\left(h^2\lambda_1^2 + \frac{\pi^2 q^2}{L^2}\right)} \\ & + \frac{1}{\lambda_1 a J_1(\lambda_1 a)} \frac{1}{3} \sin \frac{3\pi Z}{L} J_0(\lambda_1 r) e^{-T\left(h^2\lambda_1^2 + \frac{9\pi^2 q^2}{L^2}\right)} \\ & + \frac{1}{\lambda_1 a J_1(\lambda_1 a)} \frac{1}{5} \sin \frac{5\pi Z}{L} J_0(\lambda_1 r) e^{-T\left(h^2\lambda_1^2 + \frac{25\pi^2 q^2}{L^2}\right)} + \ldots \\ & + \frac{1}{\lambda_2 a J_1(\lambda_2 a)} \sin \frac{\pi Z}{L} J_0(\lambda_2 r) e^{-T\left(h^2\lambda_2^2 + \frac{\pi^2 q^2}{L^2}\right)} \\ & + \frac{1}{\lambda_2 a J_1(\lambda_2 a)} \frac{1}{3} \sin \frac{3\pi Z}{L} J_0(\lambda_2 r) e^{-T\left(h^2\lambda_2^2 + \frac{9\pi^2 q^2}{L^2}\right)} + \ldots \\ & + \frac{1}{\lambda_3 a J_1(\lambda_3 a)} \sin \frac{\pi Z}{L} J_0(\lambda_3 r) e^{-T\left(h^2\lambda_3^2 + \frac{\pi^2 q^2}{L^2}\right)} \\ & + \frac{1}{\lambda_3 a J_1(\lambda_3 a)} \frac{1}{3} \sin \frac{3\pi Z}{L} J_0(\lambda_3 r) e^{-T\left(h^2\lambda_3^2 + \frac{9\pi^2 q^2}{L^2}\right)} + \ldots \\ & + \frac{1}{\lambda_4 a J_1(\lambda_4 a)} \sin \frac{\pi Z}{L} J_0(\lambda_4 r) e^{-T\left(h^2\lambda_4^2 + \frac{\pi^2 q^2}{L^2}\right)} \\ & + \frac{1}{\lambda_4 a J_1(\lambda_4 a)} \frac{1}{3} \sin \frac{3\pi Z}{L} J_0(\lambda_4 r) e^{-T\left(h^2\lambda_4^2 + \frac{9\pi^2 q^2}{L^2}\right)} + \ldots \Bigg]. \quad [4] \end{aligned}$$

The number of terms that must be used in making temperature calculations will depend to a considerable extent upon the value of the negative exponent of the Napierian log base e. Since this exponent involves the time T and the dimensions of the timber (as well as the diffusivity factors h^2 and q^2), the series converges more rapidly as the time T increases, and as the diameter and length of the timber decrease, since the values of X vary inversely with the radius, and the second term of the negative exponent varies inversely as the square of the length L.

[4] Tables of Bessel's functions for different values of x and roots of $J_n(x) = 0$ are given in "Smithsonian Physical Tables," Smithsonian Institute, Washington, D. C., and in "Fourier Series and Spherical Harmonics," by W. E. Byerly, Ginn and Co., 1893, New York, N. Y.

METHOD OF USING FIGURES WHEN DIFFUSIVITY, HEATING CONDITIONS, OR BOTH, ARE DIFFERENT FROM THOSE ASSUMED IN COMPUTING THE TIME-TEMPERATURE CURVES

Equation [A] which follows, shows the relation of time and diffusivity, and Equations [B] and [C] show the method of finding the corresponding wood temperatures when the heating conditions are different from those assumed in computing the data for the temperature curves shown in the figures.

Symbols applying to computed data in figures are as follows:

U_0 = initial wood temperature, assumed as 70 F
U_1 = heating-medium temperature, assumed as 212 F
h^2 = diffusivity in radial direction = 0.00027 sq in. per sec
q^2 = diffusivity in longitudinal direction = $2.52(h^2) = 0.00068$
U = temperature obtained in given time T at some particular distance from surface. Values of U are temperatures within wood at distance from surface shown by figure
T = time of heating (in hours) necessary to obtain temperature U. This period is found from figure for diameter and length under consideration

Symbols, applying to temperature computations when the diffusivity, heating conditions, or both, are different from those used in computing the data plotted in the figures, are as follows :

U_x = any initial wood temperature
U_b = any heating-medium temperature
h_x^2 = any value of diffusivity for transverse direction, sq in. per sec

Since the diffusivity in the longitudinal direction (q^2) is taken as approximately $2^1/_2$ times the diffusivity in the transverse direction for the calculated temperatures, only values of h_x^2 need to be considered.

U_x = temperature obtained at definite point in timber when initial wood temperature U_a, heating temperature U_b, or both, are different from U_0 = 70 F, and U_1 = 212 F, the initial wood and heating-medium temperatures assumed in computing curves shown in figure
T_b = time of heating (in hours) required to obtain temperature U_x when either diffusivity h_x^2, initial wood temperature U_a, the heating-medium temperature U_b, or all three variables are different from corresponding values used in computing data for curves

Relation of Heating Period and Diffusivity. The relation of the heating period T and the heating period Tb is shown by the equation

$$Tb(h_x^2) = T(0.00027) \quad \ldots\ldots\ldots [A]$$

In this equation Tb is the heating period assumed or to be determined when the wood has a diffusivity k^2, and T is the heating period found from the figure for the timber in question. Values of h_x^2 for green wood of any given specific gravity can be found in Fig. 1, when either water or steam is used as the heating medium. Specific-gravity values for different woods, based on weight when oven-dry and volume when green, are given in Table 1.

Relation of Initial Wood Temperature, Heating-Medium Temperature, and Temperatures Obtained in Wood. A simple proportional relation exists between the temperatures obtained under any given heating conditions. For example, the relation of the temperatures obtained when the initial wood temperature U_a, the heating-medium temperature U_b, or both, are different from those assumed in making temperature computations are expressed by the equation

$$\frac{(U_b - U_x)}{(U_b - U_a)} = \frac{(U_1 - U)}{(U_1 - U_0)}$$

When U_a, U_b, and U are known it is necessary to determine U_x, the wood temperature obtained when U_a, U_b, or both are different from U_0 and U_1. Solving for U_x in terms of U_b, U_a, U_1, U_0, and U gives

$$U_x = U_b - \left[\frac{(U_b - U_a)(U_1 - U)}{(U_1 - U_0)}\right] \ldots\ldots\ldots [B]$$

When U_x, U_a, and U_b are known it is necessary to solve for U to find the heating period T from the proper figure. Solving for U in terms of U_a, U_b, U_x, U_0, and U_1 gives

$$U = U_1 - \left[\frac{(U_b - U_x)(U_1 - U_0)}{(U_b - U_a)}\right] \ldots\ldots\ldots [C]$$

If the values of U_0 = 70 F and U_1 = 212 F (the initial wood and heating-medium temperatures assumed in making the computations) are substituted in Equations *[B]* and *[C]*

$$U_x = U_b - \left[\frac{(U_b - U_a)(212 - U)}{142}\right] \ldots\ldots\ldots [B_1]$$

$$U = 212 - \left[\frac{(U_b - U_x)(142)}{(U_b - U_a)}\right] \ldots\ldots\ldots\ldots [C_1]$$

Values of U_x and U can also be determined by means of Fig. 3, as explained in the text.

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